CHAPTER 2

Single- and Three-Phase Power

Objective

-Understand the difference between time and the phasor domains.

-Explain the magnitude and angular relation between the phase and line voltages in a WYE and DELTA systems.

-Calculate real, reactive, and apparent power in single-phase and three-phase circuits.

-Define power factor.

-Solve power factor correction problems

Review

The total energy delivered to a load divided by the time required to deliver it yields the average power delivered

$$P_{avg} = \frac{W_{in}}{\triangle t}$$

Efficiency of the delivered power

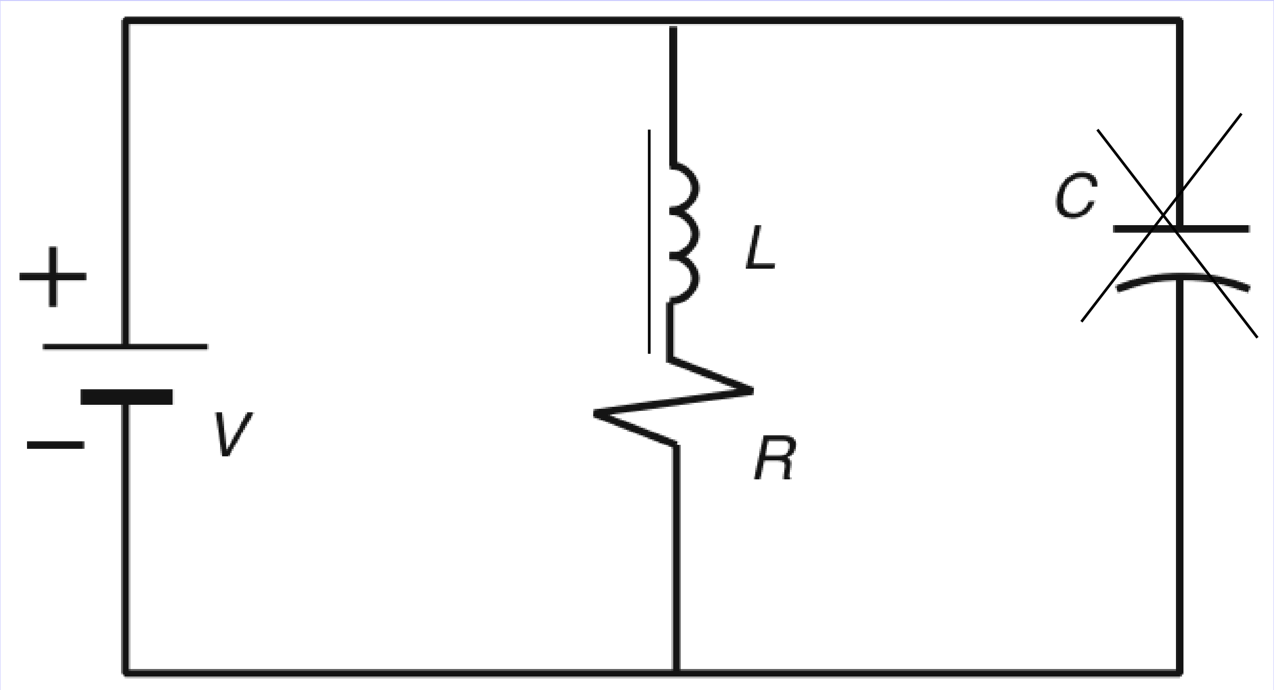
$$W_{in} = \frac{W_{out}}{\eta}$$

Efficiency of the system

Units: horsepower, watts

1 Watt = 1 Joule delivered in a 1 second pulse

DC power



DC system:

- Power delivered to the load does not fluctuate.

- If the transmission line is long power is lost in the line.

$$E = V$$

$$I = \frac{E}{R_{load}}$$

$$P = VI = EI = \frac{E^2}{R_{load}}$$

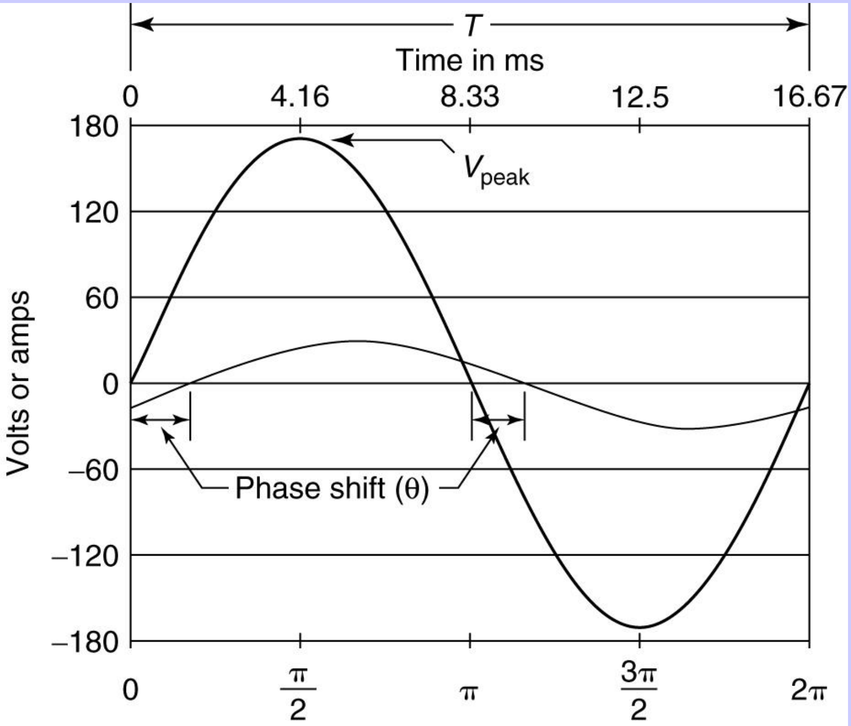
Important:

-Inductors become short circuits to DC.

-Capacitors become open circuits to DC.

Single-Phase AC Circuits

Alternating current derives its name from the fact that the voltage and current are sinusoidal functions.



$$V(t) = 170 sin(wt) volts = 120\sqrt{2} sin(wt) volts$$

$$I(t) = 30 sin(wt - \frac{\pi}{6}) amps = 21.21\sqrt{2} sin(wt - \frac{\pi}{6}) amps$$

$$f = \frac{1}{T} = \frac{1}{16.67ms} = 60Hz$$

$$\omega = 2\pi f \frac{rad}{s}$$

Related to the peak value is the the RMS of the waveform.

RMS is found by taking the square root of the mean of the waveform squared:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^t [v(t)]^2 dt}$$

The RMS is effective value of the AC voltage or current : the RMS current or voltage is the value that would cause the same amount of heat in a resistor as a DC current or voltage with the same value.

For sinusoidal functions:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = 0.707(V_{peak})$$

Phasors

Convenient way to represent voltages and currents using phasor form (work in phasor domain)

In its simplest form, a phasor can be thought as a complex number that has magnitude and phase of a sinusoidal function and can be represented partially as a vector in the complex plane.

IMPORTANT: The magnitude of a phasor will always be RMS value of the sinusoid it is representing.

Voltage phasor magnitude

$$V(t) = 170sin(wt)volts = 120\sqrt{2}sin(wt)volts$$

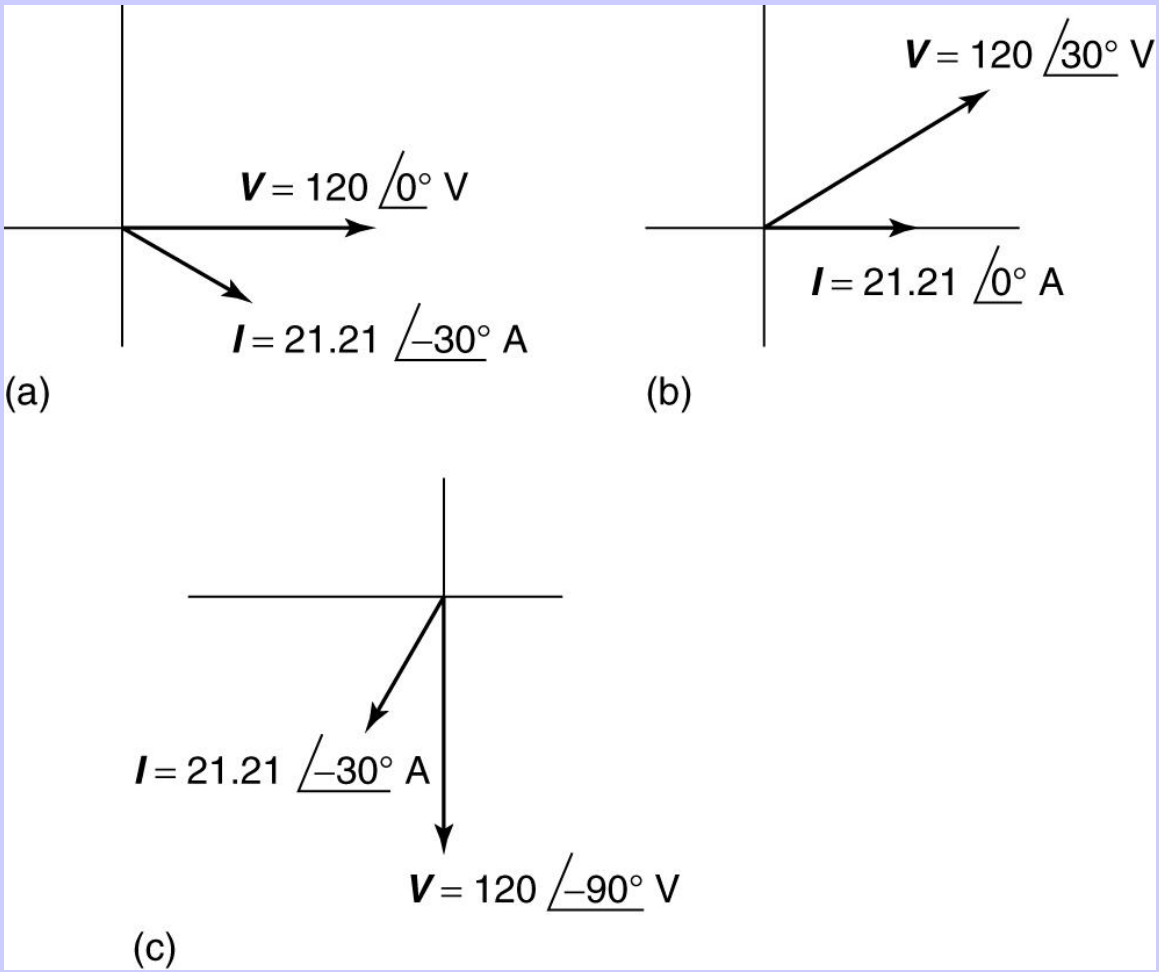
$$I(t) = 30sin(wt - \frac{\pi}{6})amps = 21.21\sqrt{2}sin(wt - \frac{\pi}{6})amps$$

Current phasor magnitude; current is lagging the voltage by 30 degrees

Example:

$$V(t) = 170sin(wt)volts = 120\sqrt{2}sin(wt)volts$$

$$I(t) = 30sin(wt - \frac{\pi}{6})amps = 21.21\sqrt{2}sin(wt - \frac{\pi}{6})amps$$



a. Choose the voltage as the reference phasor:

current lags the voltage by 30 degrees, so current phasor will be 30 degrees behind the voltage

$$V = 120\angle 0^o$$
$$I = 21.21\angle -30^o$$

b. Choose the current as the reference phasor:

current lags the voltage by 30 degrees, so the voltage phasor will be +30 degrees

$$V = 120\angle 30^o$$
$$I = 21.21\angle 0^o$$

c. Choose the cosine function as the reference phasor:

The sine function lags the cosine by 90 degrees, thus the voltage phasor should be at -90 degrees, current must be 30 degrees behind the voltage, so it is at -120 degrees.

$$V = 120\angle -90^o$$
$$I = 21.21\angle -120^o$$

Symbol notation for several complex quantities

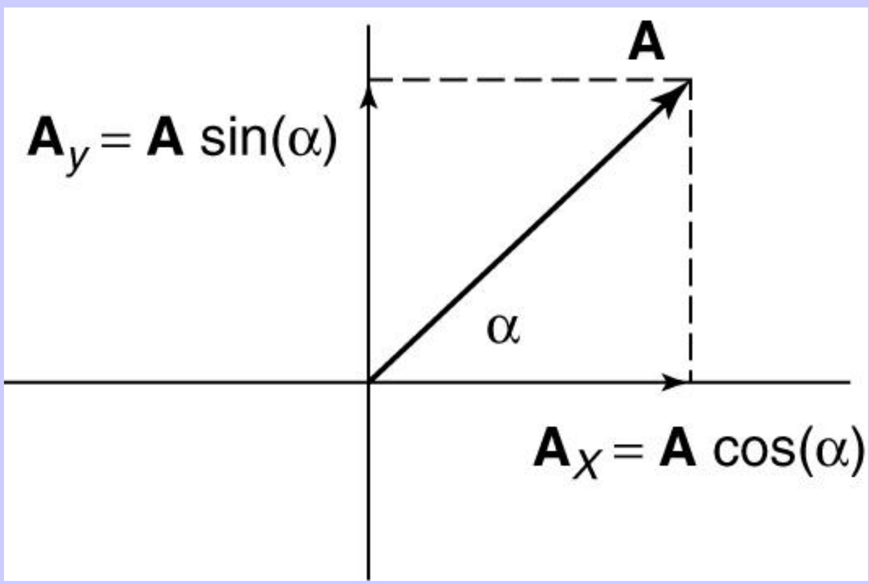
TABLE 2–2
Symbol Notation for Several Complex Quantities

Quantity	Phasor or complex number symbol	Magnitude symbol	Polar form (examples)	Rectangular form
Voltage	$\mathbf{V}$	$\lvert\mathbf{V}\rvert$ or V	$V\angle\alpha$ $\lvert\mathbf{V}\rvert\angle\alpha$	$V_x + \mathrm{j}V_y$
Current	$\mathbf{I}$	$\lvert\mathbf{I}\rvert$ or I	$I\angle\beta$ $\lvert\mathbf{I}\rvert\angle\beta$	$I_x + \mathrm{j}I_y$
Complex power	$\mathbf{S}$	$\lvert\mathbf{S}\rvert$ or S	$S\angle\theta$ $\lvert\mathbf{S}\rvert\angle\theta$	$P + \mathrm{j}Q$
Impedance	$\mathbf{Z}$	$\lvert\mathbf{Z}\rvert$ or Z	$Z\angle\theta$ $\lvert\mathbf{Z}\rvert\angle\theta$	$R + \mathrm{j}X_L$ $R - \mathrm{j}X_C$

Mathematical operations with phasors

Conversion between rectangular and polar coordinates

Phasor with magnitude “A” and angle alpha



$$A_r = A\cos\alpha$$

$$A_j = A\sin\alpha$$

$$A = A_r^2 + A_j^2$$

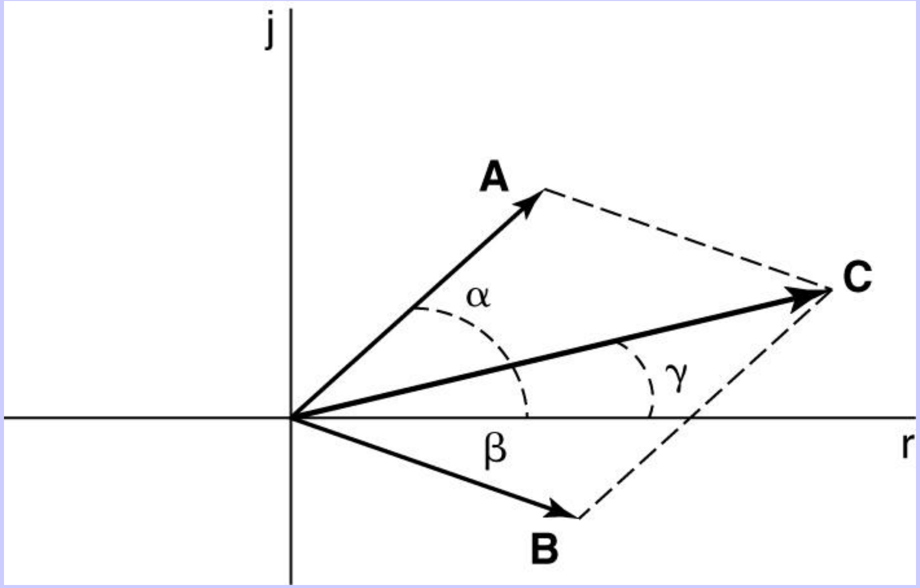
$$\alpha = \tan^{-1}\frac{A_j}{A_r}$$

Addition of complex numbers:

$$\mathbf{A} = A_r + jA_j$$

$$\mathbf{B} = B_r + jB_j$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = (A_r + B_r) + j(A_j + B_j) = C_r + jC_j = C\angle\gamma$$



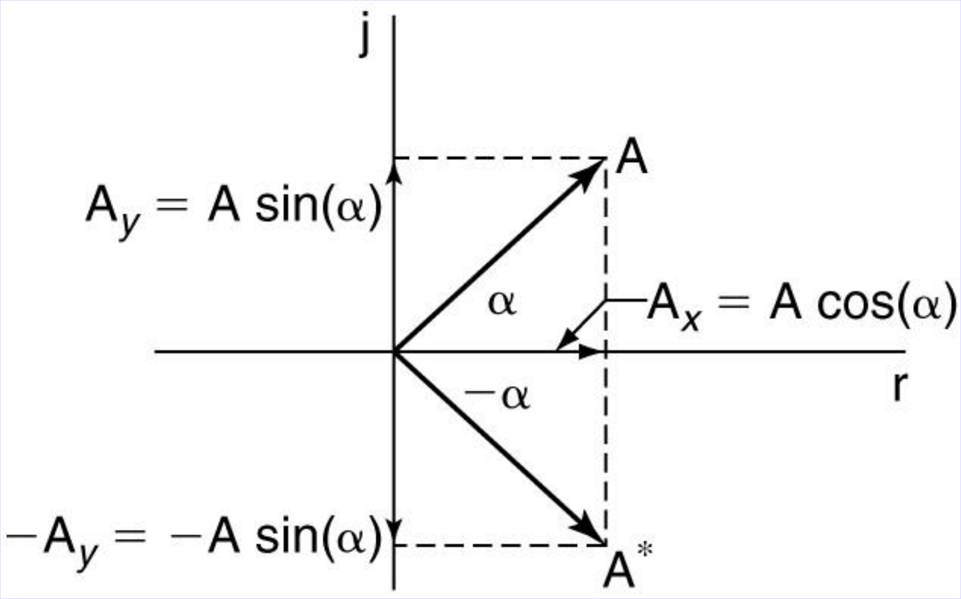
Multiplication of phasors:

$$\mathbf{D} = \mathbf{A} \times \mathbf{B} = A\angle\alpha \times B\angle\beta = AB\angle(\alpha+\beta) = D\angle\delta$$

Division of phasors:

$$\mathbf{E} = \frac{\mathbf{A}}{\mathbf{B}} = \frac{A\angle\alpha}{B\angle\beta} = \frac{A}{B}\angle(\alpha-\beta) = \mathbf{E}\angle\epsilon$$

Conjugate:

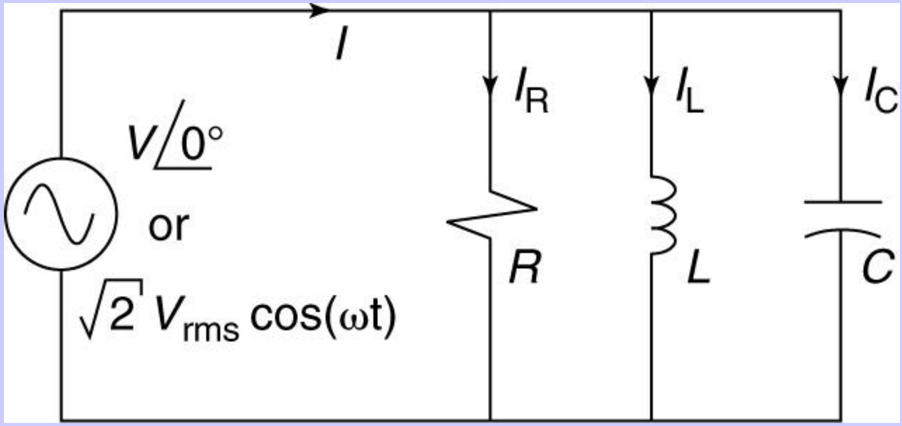


$$\mathbf{A} = A\angle\alpha = A_r + jA_j$$

$$\mathbf{A}^* = A\angle(-\alpha) = A_r - jA_j$$

Example

Impedance in AC Circuits



Impedance of the inductor:

$$e(t) = L\frac{di(t)}{dt}$$

$$v(t) = V_{peak}\cos(wt)$$

$$i(t) = \frac{1}{L}\int v(t)dt = \frac{1}{L}\int \sqrt{2}V_{rms}\cos(wt)dt$$

$$i(t) = \frac{V_{rms}}{wL}\sqrt{2}\sin(wt)dt$$

Current in the inductor:

$$I_L = \frac{V}{X_L}\angle -90^o = -j\frac{V}{X_L} = \frac{V}{jX_L} = \frac{V}{jwL}$$

NOTE:

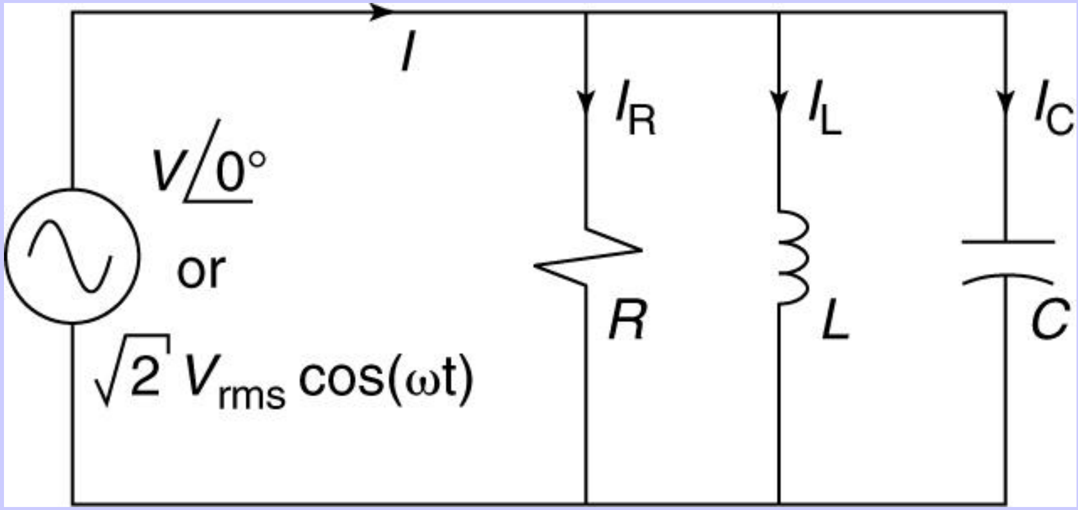
1. There is a phase shift between the voltage and the current. **The current is lagging the voltage by 90 degrees for the inductor.**
2. The RMS of the current is the RMS of the voltage divided by (wL)

Impedance of the inductor:

$$Z_L = jwL$$

Use Ohm's low in the phasor domain for the inductor by defining the **inductive reactance**

$$X_L = wL$$



Impedance of the capacitor:

$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = C\frac{d}{dt}\sqrt{2}V_{rms}\cos(wt)$$

$$i(t) = -wCV_{rms}\sqrt{2}\sin(wt)$$

NOTE:

1. **<u>The current is a negative sine wave, which leads the voltage (cosine wave) by 90 degrees.</u>**
2. The RMS of the current is related to the RMS of the voltage by (wC).

<u>Capacitance reactance:</u> $X_C = \frac{1}{wC}$

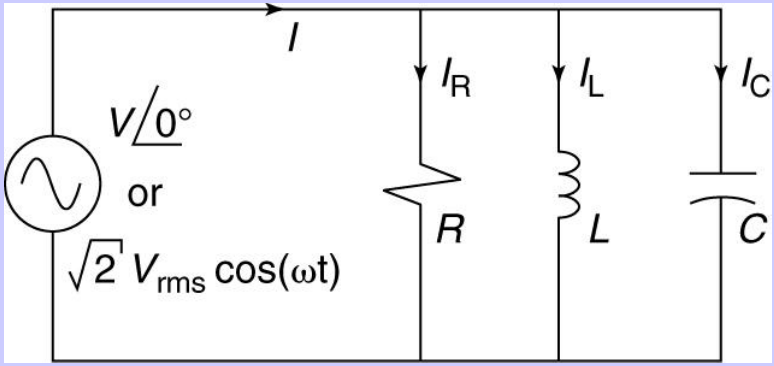
<u>Impedance:</u> $Z_C = -j\frac{1}{wC} = \frac{1}{jwC}$

<u>Current in the capacitor:</u> $I_C = \frac{V}{X_C}\angle 90^o = j\frac{V}{X_C} = jwCV$

The current in the capacitor will lead the voltage by 90 degrees.

example

Power in Single-Phase AC Circuits



Real Power (Active Power) energy consumed by the load that is converted to heat and is not be returned to its initial stage.
Real power used by the resistor:

$$P_R = V_R \times I_R = \frac{V_R^2}{R} = I_R^2 R$$

Reactive power (Inductor)
Instantaneous current through the inductor varies sinusoidally.
Inductor stores energy as a function of the current, thus magnetic field varies with time.
As current increases the inductor stores more energy in its magnetic field and the power flows from the source to the inductor.
As current decreases the inductor releases energy back to the source.
The inductor does not use any power on average, there is energy moving back and forth between the source and the Inductor. **This is called REACTIVE POWER.**

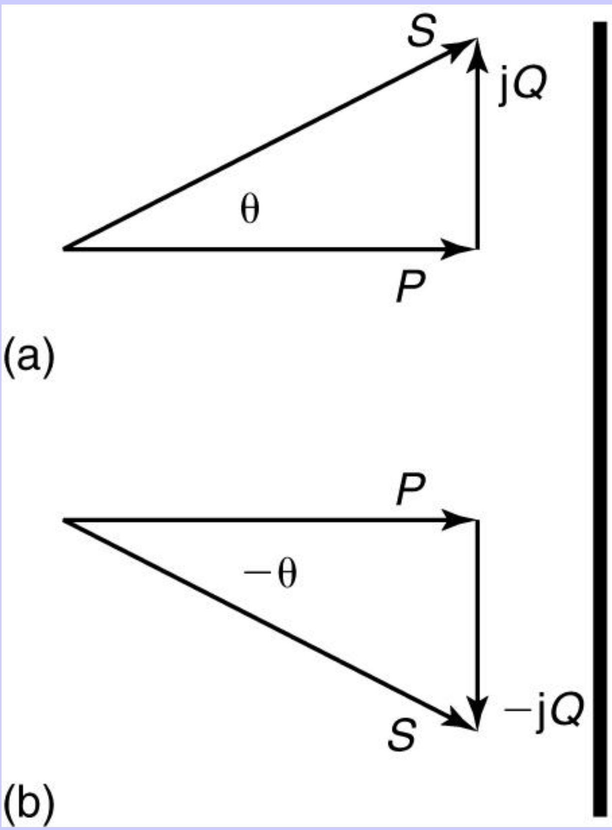
$$Q_L = V_L \times I_L = \frac{V_L^2}{X_L} = I_L^2 X_L$$

Reactive power (Capacitor)
Voltage across the capacitor varies sinusoidally.
Capacitor stores energy as a function of the voltage, thus capacitor's electric field varies with time.
Capacitor draws energy from the source as it charges, and returns energy as it discharges.
The voltage across the capacitor and the current through the inductor are 90 degrees out of phase, thus *when inductor is charging the capacitor discharges.*

$$Q_C = -V_C \times I_C = -\frac{V_C^2}{X_C} = -I_C^2 X_C$$

example

Complex Power



Real power

Reactive power

$$\mathbf{S} = P + jQ$$

$$S = |\mathbf{S}| = \sqrt{P^2 + Q^2}$$

Magnitude of the complex (apparent) power

$$P = S\cos(\theta)$$

Relationship between the real and apparent power.

$$Q = S\sin(\theta)$$

Relationship between the reactive and apparent power.

$$F_p = \cos(\theta) = \frac{P}{S}$$

Power factor

$$\tan(\theta) = \frac{Q}{P}$$

(c)

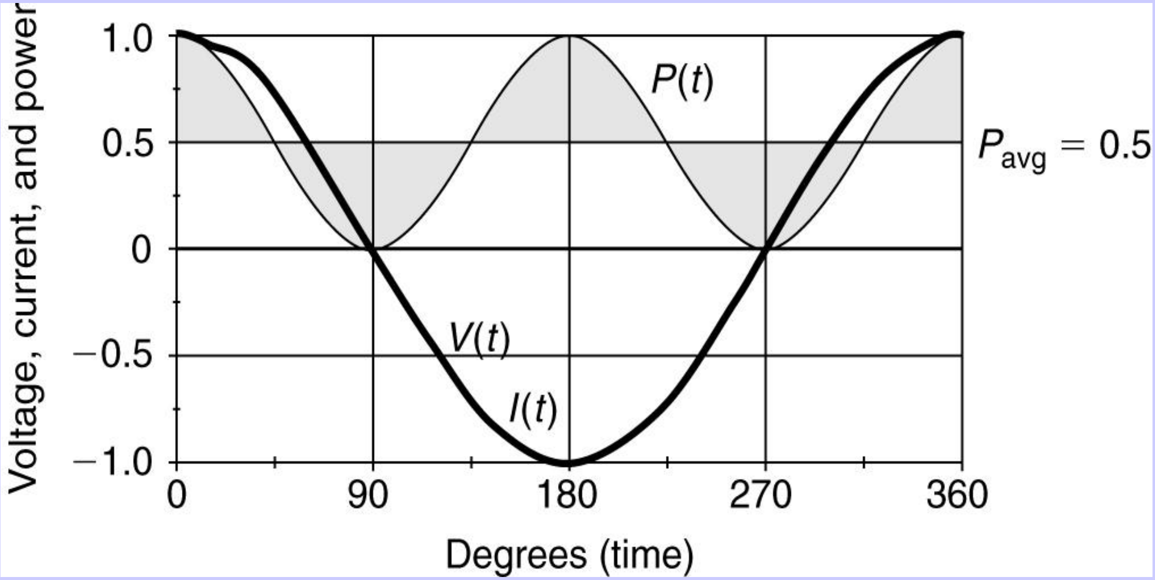
Angle theta is the angle of the current with respect to the voltage.
If the current lags the voltage (inductive load) then theta is negative.
If the current leads the voltage (capacitive load) then theta is positive.
Power factor is always positive and less than 1.

example

Instantaneous AC Power

Resistive load.

Shows instantaneous voltage, current and power for a resistive load



Voltage and current both have a peak value of 1.0 and RMS of 0.707.

The apparent power $P_{app} = V_{rms} \times I_{rms}$

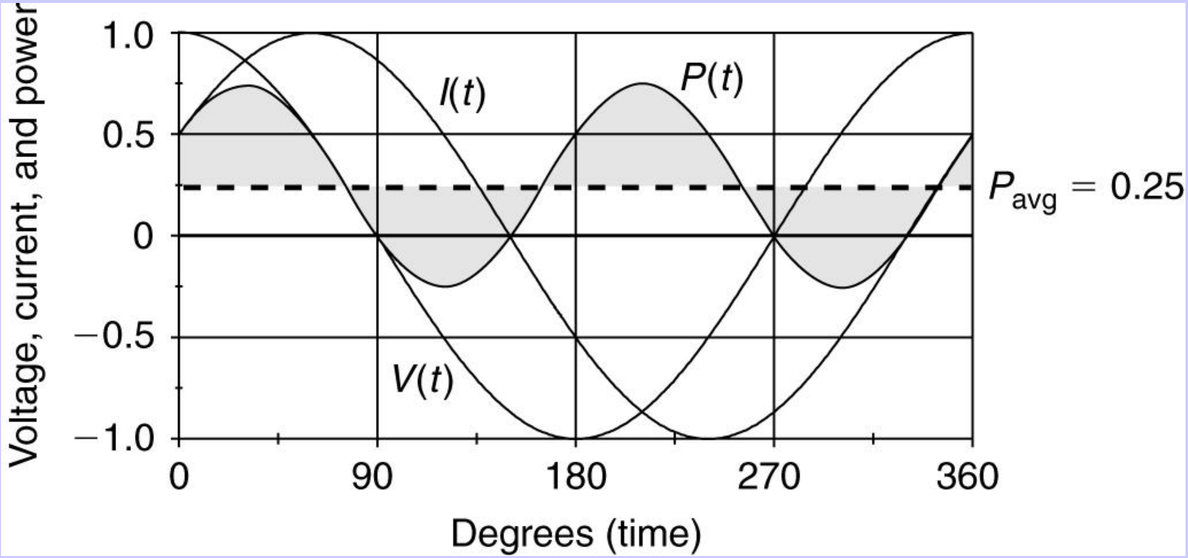
Since the voltage and current are always in phase, their product is always positive and, so power always flows from the source to the resistive load.

In this case the angle between the voltage and the current is zero so the power factor (= cos (angle)) is unity.

example

Resistive-inductive (*R*-*L*) load

Shows instantaneous voltage, current and power for a resistive-inductive load



Current lags the voltage by 60 degrees.

Since the voltage and current are out of phase their product may be negative and as a result there are times when the instantaneous power is negative. This represents a return of energy from the load to the source. That energy comes from the magnetic field of the system inductance.

Apparent power is 0.5 VA

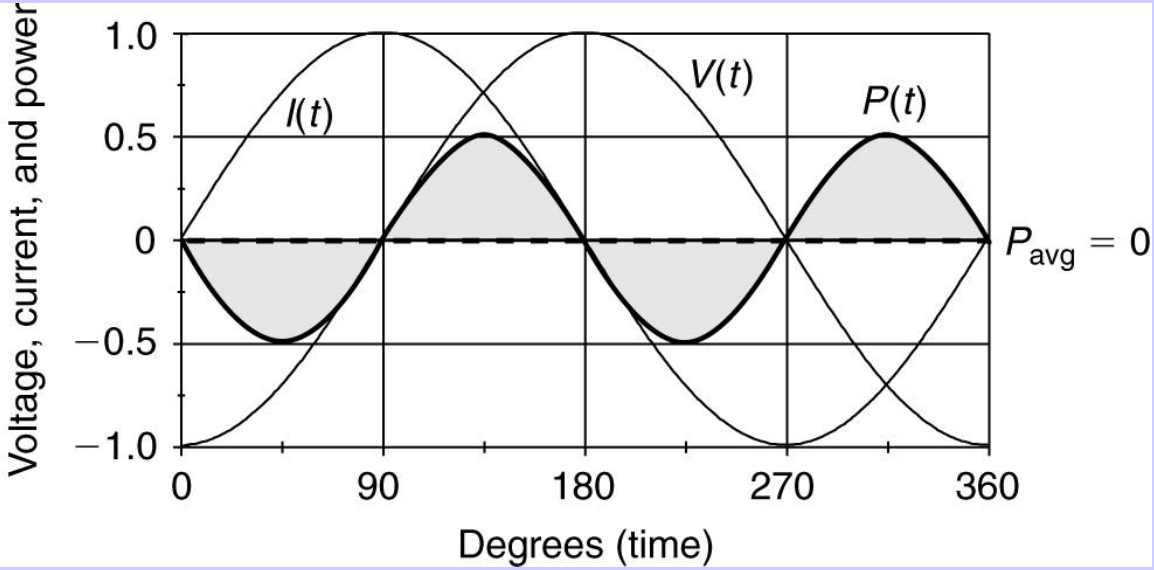
$$P_{app} = V_{rms} \times I_{rms}$$

Average power is 0.25 W=0.5cos (60)

Power factor

Purely inductive load.

Shows instantaneous voltage, current and power for a purely inductive load



Current lags the voltage by 90 degrees.

Instantaneous power has an average of zero. Energy flows from source to the load half of the time and from the load back to the source the other half of the time.

Apparent power is 0.5 VA.

Real power is zero.

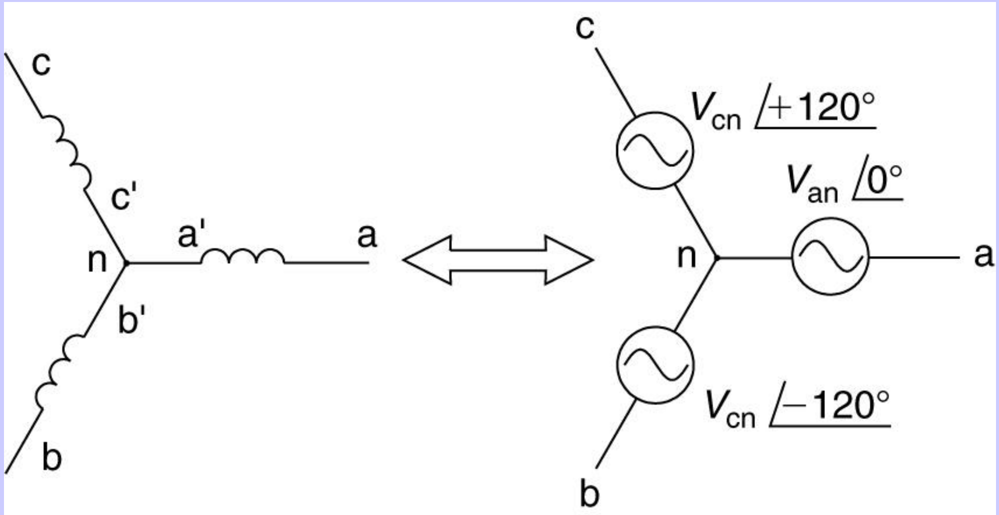
Reactive power is 0.5 Volt-Amperes-Reactive=0.5sin(90)

Power factor

Three-Phase AC Circuits

Most industrial and commercial electrical power systems employ a 3-phase configuration.

3-phase systems consist of 3 equal in magnitude voltages and 120 degrees out of phase relative to each other.

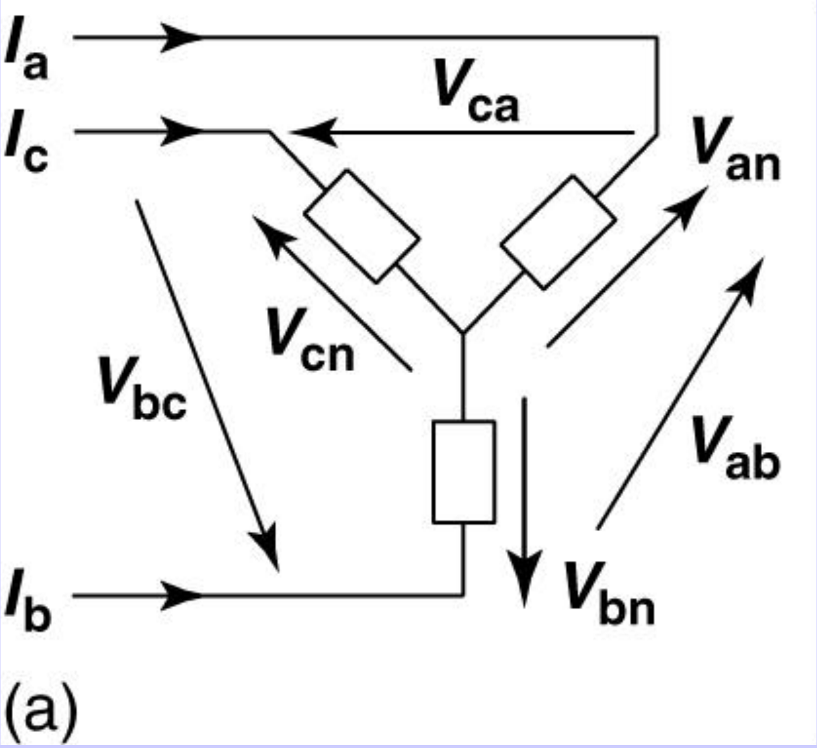


Example: 3-phase generator contains coils that are physically spaced 120 degrees apart and when the rotor turns, the voltages induced in the coils are electrically 120 degrees apart.

The power flowing in 3-phase system has a constant value

example

Three-Phase WYE Configuration



Wye:

Consists of three load components connected with a common point called **neutral.**

Line-to-neutral, phase-to-neutral, branch voltage or simply phase voltage V_P :

the voltage between each phase (line or hot) conductor and the neutral, $V_{cn} V_{an} V_{bn}$

Lin-to-line or simply line voltage V_L :

the voltage between any two conductors, $V_{bc} V_{ca} V_{ab}$

$|V_L| = \sqrt{3}|V_P|$ $\quad$ $I_L = I_P$

Advantage of 3-phase Wye configuration: availability of two voltage levels.

Common system voltages: 277/480 and 120/208V.

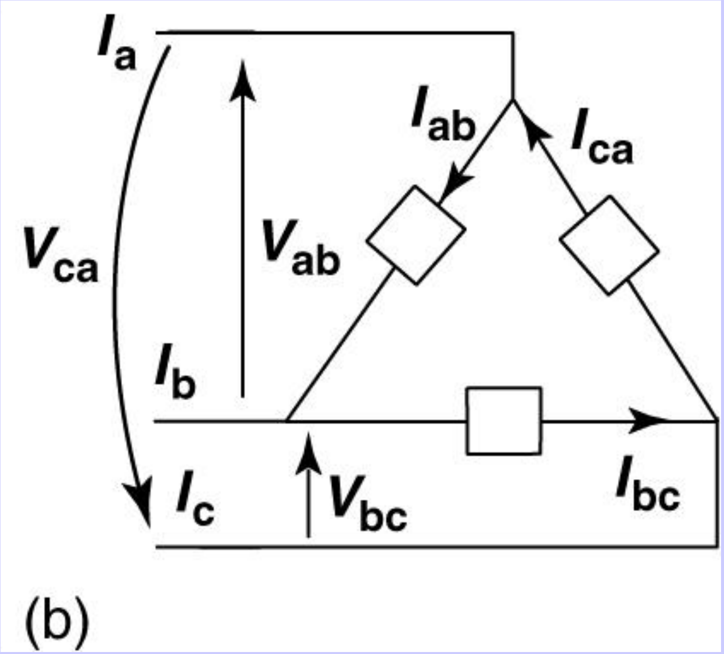
Three-Phase DELTA Configuration

Delta:

Consists of three load components connected end-to-end way and has no neutral point. Phases are connected in a triangle.

The vertices of the triangle are connected to the power system and the current that flows on those conductors is called the line current I_L : $I_a I_b I_c$

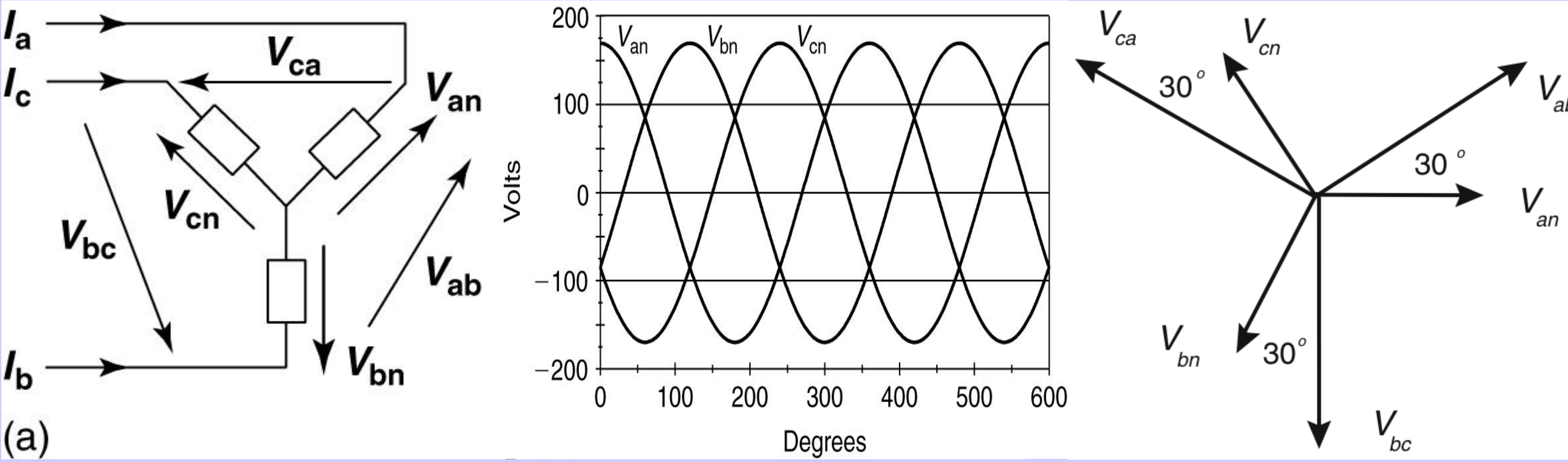
The current flowing in the branches, which are connected between the lines, is called the phase current I_P : $I_{bc} I_{ca} I_{ab}$



$V_L = V_P$ $\quad$ $|I_L| = \sqrt{3}|I_P|$

Common system voltages: 240V and 480V.

Angular relationship between V_L and V_P



Positive ABC phase sequence (phasors are assumed to rotate in the counterclockwise direction)

Assume the reference voltage $V_{an}\angle 0^o$ and with positive phase sequence $V_{bn}\angle -120^o$ $V_{cn}\angle 120^o$

Using Wye circuit we write equations around one of the loops according to the Kirchoff's law:

$$V_{ab} - V_{an} + V_{bn} = 0 \qquad V_{ab} = V_{an} - V_{bn}$$

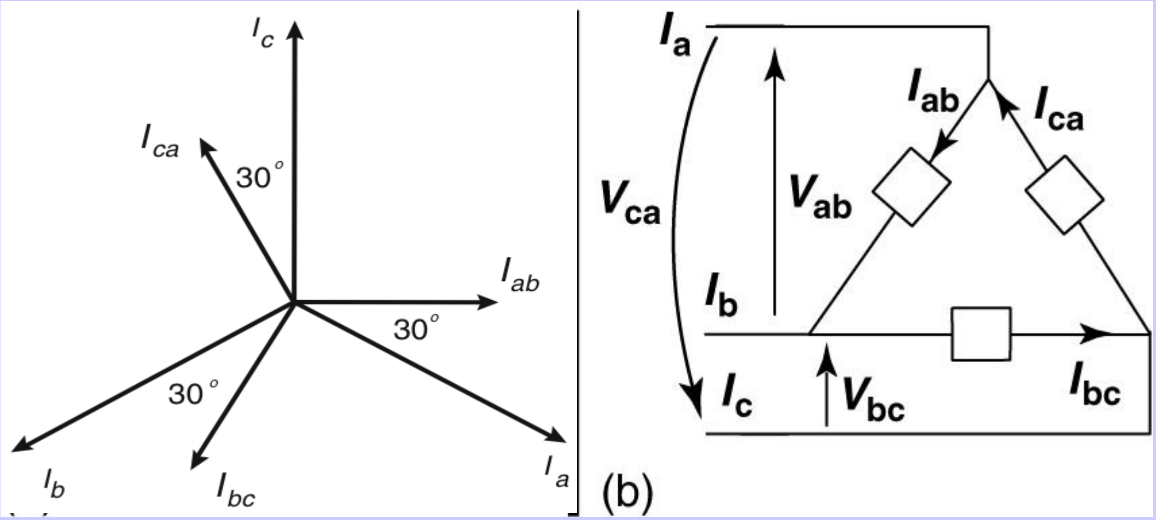
Since V_{an} and V_{bn} have the same magnitude V_P and separated by 120 degrees we can write:

$$V_{ab} = V_P\angle 0^o - V_P\angle -120^o = V_P\angle 0^o + V_P\angle 60^o$$

Adding two phasors we get: $V_{ab} = \sqrt{3}V_P\angle 30^o$

The line voltage leads the phase voltage by 30 degrees and the line voltage is the √3 larger in magnitude than the phase voltage.

Angular relationship between I_L and I_P



The angular relationship between the line and phase current in a delta system and the relationship of their magnitudes:

$$I_a = \sqrt{3} I_P \angle -30^\circ$$

The line current lags the phase current by 30 degrees, assuming a standard ABC phase rotation, and the line current is √3 larger in magnitude than the phase current

Balanced and unbalanced 3-Phase systems

A ***balanced system*** is one in which the line and phase currents and voltages in all three phases are equal in magnitude and separated by 120 degrees, and the impedances in all three phases are identical.
An ***unbalanced system*** is one in which any of the foregoing requirements are not met.

Power calculation in balanced 3-phase systems:

WYE system | Delta system

For a single-phase system, the apparent power is the product of the phase voltage and phase current

$$|S_P| = |V_P||I_P|$$

For a balanced 3-phase system , the total 3-phase apparent power is three times the power consumed by one phase

$$|S_{3P}| = 3|S_P| = 3|V_P||I_P|$$

WYE system:

$$|V_L| = \sqrt{3}|V_P| \qquad |I_L| = |I_P|$$

$$|S_{3P}| = 3|V_P||I_P| = 3|\frac{V_L}{\sqrt{3}}||I_L| = \sqrt{3}|V_L||I_L|$$

Delta system:

$$|V_L| = |V_P| \qquad |I_L| = \sqrt{3}|I_P|$$

$$|S_{3P}| = 3|V_P||I_P| = 3|\frac{V_L}{\sqrt{3}}||I_L| = \sqrt{3}|V_L||I_L|$$

examples

Power Factor Correction

Power factor is the ratio of real and to apparent power.

It is a measure of how effectively the current is being converted into useful work output.

Most AC electrical power systems supply loads that are both *resistive* and *reactive.*

Power source delivers <u>real power</u> to the resistive component of the load and this real power is converted into the useful work.

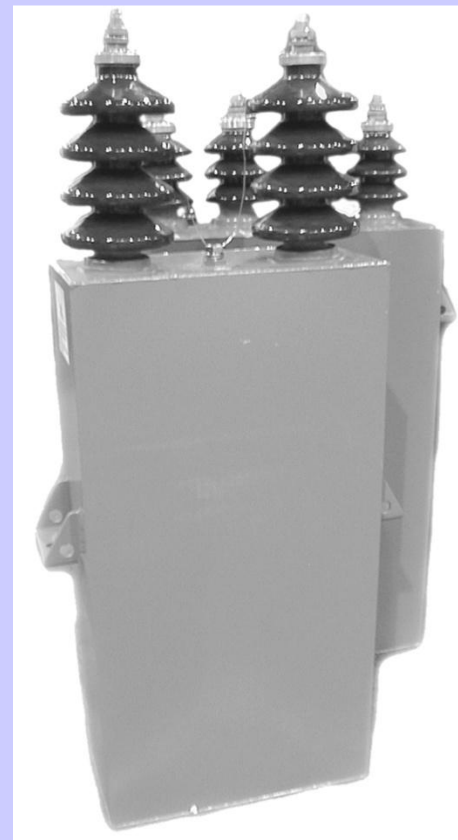
The power source must also deliver <u>reactive power</u> to the reactive components of the load. Reactive power does no useful work and is alternatively stored in the reactive elements of the load.

Although reactive power does no useful work, the source must have sufficient capacity to provide the reactive power, and the distribution system must be large enough to distribute reactive power.

Reactive power is, therefore, excess baggage. ***Power factor correction*** is a technique to reduce or eliminate this excess baggage.

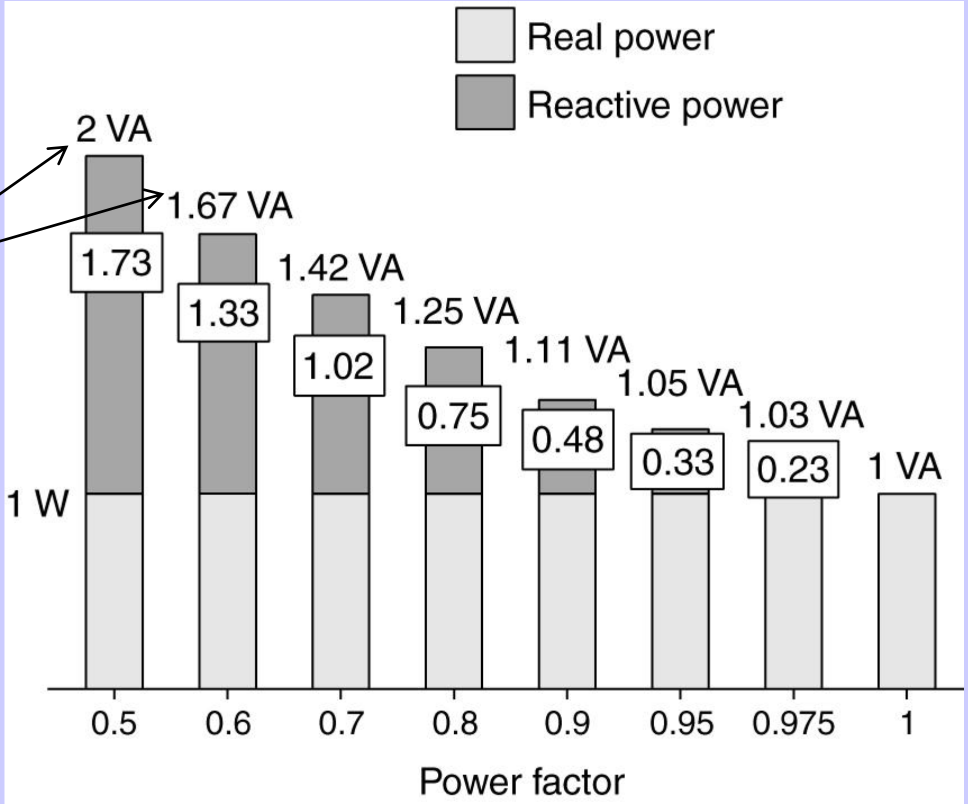
Capacitors are routinely used to correct low power factor in electrical distribution systems and can be connected either in Wye or Delta system.

Capacitor bank.



400 kVAR power factor correction capacitors.

Apparent power



As the power factor drops, the reactive power quickly becomes larger than the real power.

F.E. at a power factor 0.5 the apperent power becomes twice the real power, therefore twice as much current must be provided as we would at unity power factor do the same amount of useful work.

$$V \times I = \frac{joules}{coulomb} \times \frac{coulomb}{\text{s}} = \frac{joules}{\text{s}}$$

$$= watts = P_{elec} \qquad \textbf{(2–1)}$$

$$P_{\text{elec}} = V \times I = \frac{V^2}{R} = I^2R \qquad \textbf{(2–2)}$$

$$f = \frac{1}{T} \qquad \textbf{(2–4)}$$

$$X_L = \omega L \qquad \textbf{(2–11)}$$

$$\mathbf{Z} = R + \text{j}X \qquad \textbf{(2–13)}$$

$$X_{\text{C}} = \frac{1}{\omega C}$$

$$\mathbf{Z}_{\text{C}} = \frac{1}{\text{j}\omega C} = -\text{j}\frac{1}{\omega C} \qquad \textbf{(2–16)}$$

$$P_{\text{R}} = V_{\text{R}} \times I_{\text{R}} = \frac{V_{\text{R}}^2}{R} = I_{\text{R}}^2R \qquad \textbf{(2–18)}$$

$$Q_{\text{L}} = V_{\text{L}} \times I_{\text{L}} = \frac{V_{\text{L}}^2}{X_{\text{L}}} = I_{\text{L}}^2X_{\text{L}}$$

$$Q_{\text{C}} = -V_{\text{C}} \times I_{\text{C}} = -\frac{V_{\text{C}}^2}{X_{\text{C}}} = -I_{\text{C}}^2X_{\text{C}}$$

$$\boldsymbol{S} = P + \text{j}Q$$

$$\boldsymbol{S} = \boldsymbol{VI^*}$$

$$S = VI = \sqrt{P^2 + Q^2}$$

$$P = S[\cos(\theta)]$$

$$F_p = \cos(\theta)$$

$$Q = S[\sin(\theta)]$$

$$\boldsymbol{V}_{\mathbf{ab}} = \sqrt{3}V_{\text{P}}\angle 30°$$

$$\boldsymbol{I}_{\mathbf{a}} = \sqrt{3}\,I_{\text{P}}\angle -30°$$

$$|\boldsymbol{S}_{\mathbf{3P}}| = \sqrt{3}\,|\boldsymbol{V}_{\mathbf{L}}|\,|\boldsymbol{I}_{\mathbf{L}}|$$