Chapter 5

TRANSFORMERS

Objective

- Understand the transformer nameplate
- Describe the basic construction features of a transformer.
- Explain the relationship between voltage, current, impedance, and power in a transformer.
- Define transformer exciting current.
- Develop transformer equivalent circuits from open-circuit and short-circuit test data.
- Analyze transformer operation.
- Calculate transformer voltage regulation and efficiency.
- Use K-factor-rated transformer to solve nonlinear load problems.
- Explain the four standard three-phase transformer configurations

Introduction

A **transformer** is an electrical device that transfers energy from one circuit to another purely by magnetic coupling.

Relative motion of the parts of the transformer is not required for transfer of energy.

Transformers are often used to convert between high and low voltages and to change impedance.

Transformers alone **cannot** do the following:
- Convert DC to AC or vice versa
- Change the voltage or current of DC
- Change the AC supply frequency.

However, transformers are **components** of the systems that perform all these functions.

Transformer Nameplate Data

Transformer nameplates contain information about the size of the transformer in terms of how much apparent power (rated in kVA) it is designated to deliver to the load on a continuous basis as well primary and secondary voltages and currents.

Example: 75 kVA, 720-240*120V

U-W   primary winding is rated U volts and secondary winding is rated V volts

U/W   indicates that two voltages are from the same winding and that both voltages are available

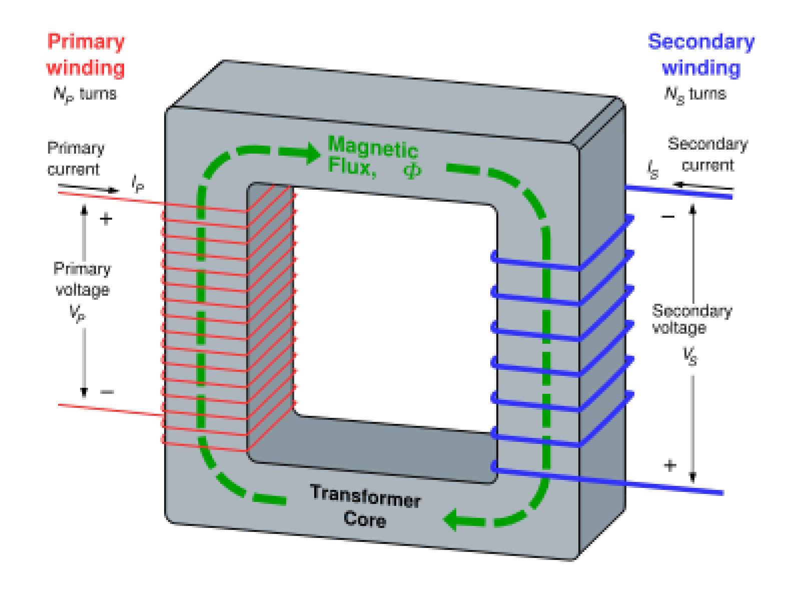
U*V   two part winding that can be connected in series or parallel to give higher voltage but only one voltage is available at a time.

U Y/W the Y indicates a 3-phase winding connected in a WYE configuration.

Example

Basic principles

An idealized step-down transformer showing resultant flux in the core



The **transformer** may be considered as a simple two-wheel 'gearbox' for electrical voltage and current.

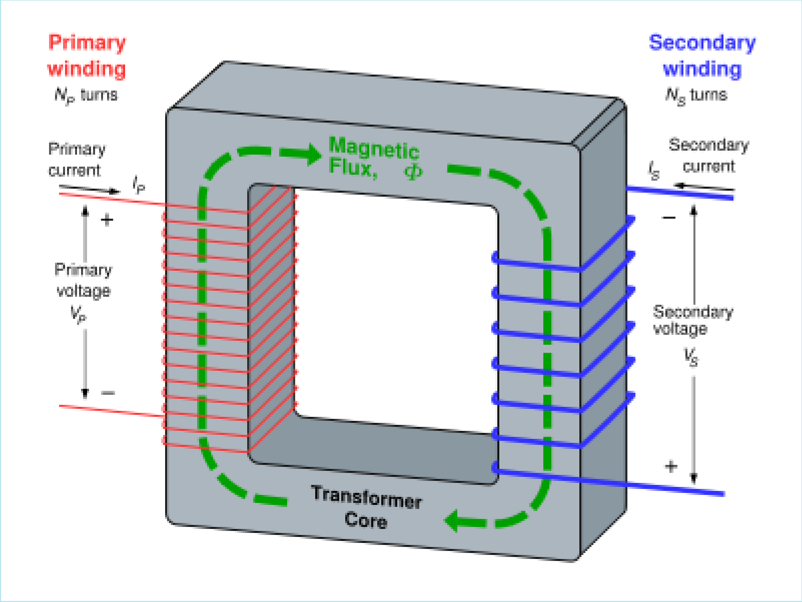
The primary winding is analogous to the input shaft

The secondary winding is analogous to the output shaft.

In this comparison, *current is equivalent to shaft speed and voltage to shaft torque*.

In a gearbox, mechanical power (speed multiplied by torque) is constant (neglecting losses) and is equivalent to electrical power (voltage multiplied by current) which is also constant.

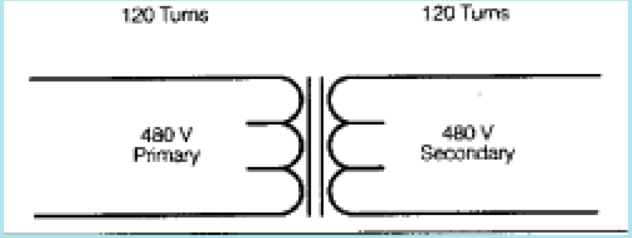
An idealized step-down transformer showing resultant flux in the core



The gear ratio is equivalent to the transformer step-up or step-down ratio.

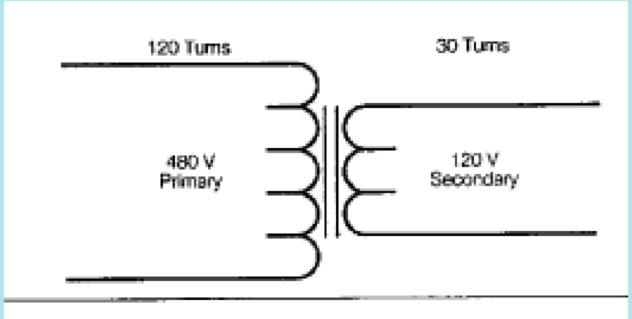
A **step-up transformer** acts analogously to a reduction gear (in which mechanical power is transferred from a small, rapidly rotating gear to a large, slowly rotating gear): it trades current (speed) for voltage (torque), by transferring power from a primary coil to a secondary coil having more turns.

A **step-down transformer** acts analogously to a multiplier gear (in which mechanical power is transferred from a large gear to a small gear): it trades voltage (torque) for current (speed), by transferring power from a primary coil to a secondary coil having fewer turns.



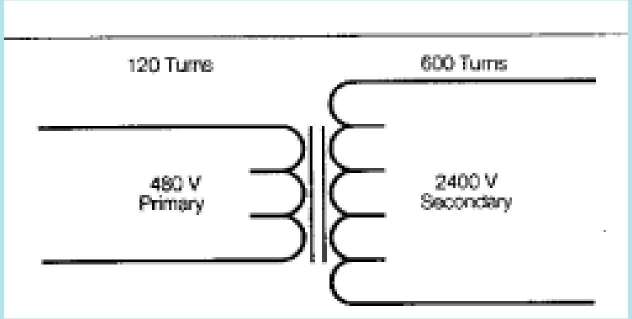
1/1 Transformer

When the primary winding and the secondary winding have the same amount of turns there is no change voltage, the ratio is 1/1 unity.



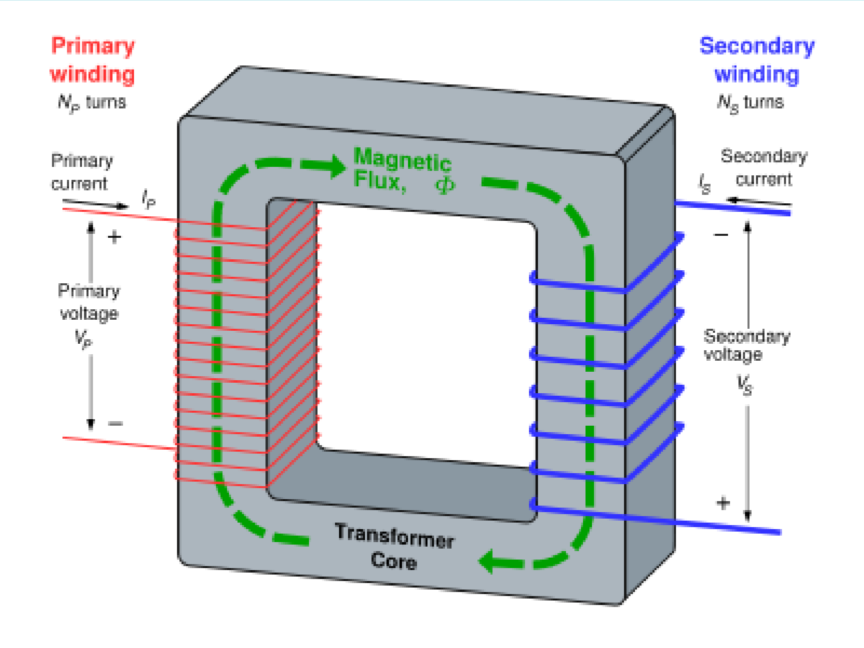
Step-Down Transformer

If there are fewer turns in the secondary winding than in the primary winding, the secondary voltage will be lower than the primary.



Step Up Transformers

If there are fewer turns in the primary winding than in the secondary winding, the secondary voltage will be higher than the secondary circuit.



A simple transformer consists of two electrical conductors called the **primary winding** and the **secondary winding**. If a time-varying voltage v_P is applied to the primary winding of N_P turns, a current will flow in it producing a magnetomotive force (MMF).

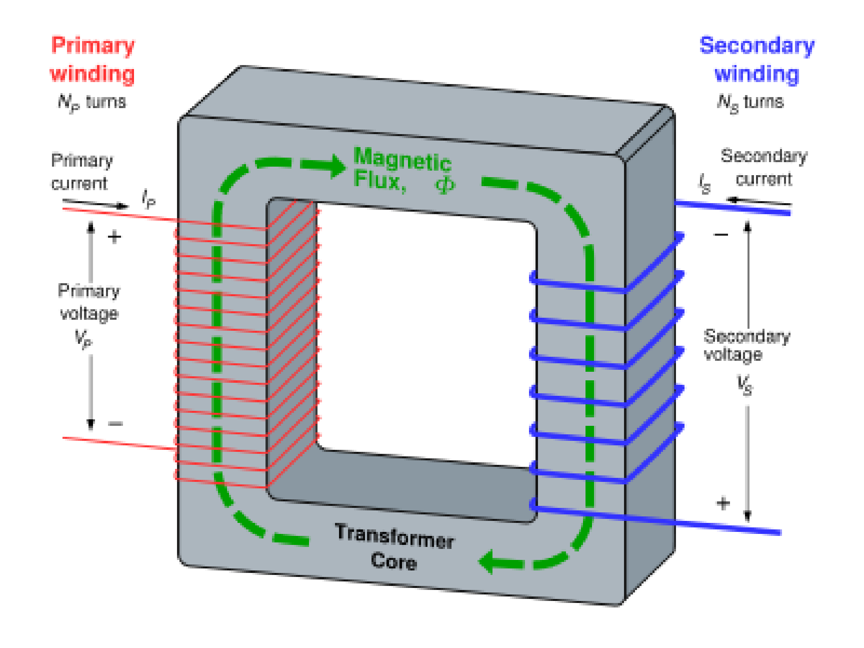
The primary MMF produces a varying magnetic flux Φ_P in the core.

In accordance with Faraday's Law, the voltage induced across the primary winding is proportional to the rate of change of flux :

$$v_P = N_P \frac{d\Phi_P}{dt}$$

Similarly, the voltage induced across the secondary winding is:

$$v_S = N_S \frac{d\Phi_S}{dt}$$



With perfect flux coupling, the flux in the secondary winding will be equal to that in the primary winding, and so we can equate Φ_P and Φ_S.

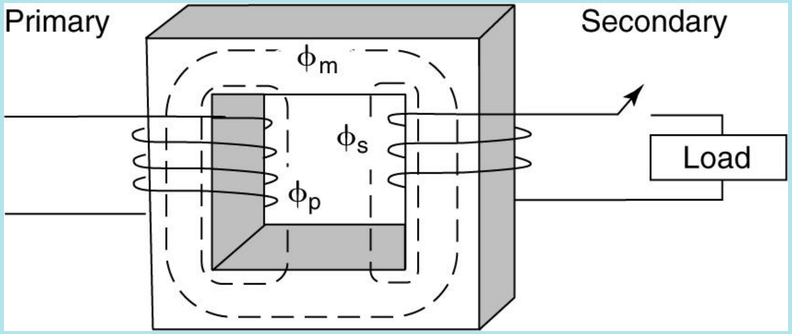
$$v_P = N_P \frac{d\Phi_P}{dt} \bigg/ v_S = N_S \frac{d\Phi_S}{dt} = \frac{v_P}{v_S} = \frac{N_P}{N_S}.$$

Hence, in an ideal transformer, the ratio of the primary and secondary voltages is equal to the ratio of the number of turns in their windings, or alternatively, the voltage per turn is the same for both windings.

This leads to the most common use of the transformer: to convert electrical energy at one voltage to energy at a different voltage by means of windings with different numbers of turns.

Example

The Universal EMF equation



Faraday's law tells: $e = N\dfrac{d\phi}{dt}$

If we apply sinusoidal voltage to the transformer:

$$e(t) = \sqrt{2}E_{RMS}sin(wt)$$

Flux is given by: $\phi(t) = \dfrac{1}{N}\displaystyle\int_0^t \sqrt{2}E_{RMS}sin(wt)dt$

$$\phi(t) = \frac{-\sqrt{2}E_{RMS}}{wN}cos(wt) = \frac{-\sqrt{2}E_{RMS}}{2\pi fN}cos(wt) = -\phi_{max}cos(wt)$$

$$E_{RMS} = \frac{2\pi fN\phi_{max}}{\sqrt{2}}$$

$$E_{RMS} = 4.44fN\phi_{max} = 4.44fNB_{max}A_c$$

This equation demonstrates a definite relation between the voltage in a coil, the flux density, and the size of the core. The designer must make trade-offs among the variables when design a transformer.

Example

Voltage and Current

For the ideal transformer, all the flux is confined to the iron core and thus links the primary and secondary.

$$E_{RMS} = 4.44 f N \phi_{max} = 4.44 f N B_{max} A_c$$

$$\Downarrow$$

$$E_p = 4.44 f N_p \phi_{max}$$

$$E_s = 4.44 f N_s \phi_{max}$$

$$\Downarrow$$

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a$$

Turns ratio

For step-down transformer, the primary side has more turns than secondary, therefore a >1;

For step-up transformer, the primary side has fewer turns than secondary, therefore a <1;

Because the losses are zero in the ideal transformer, the apparent power in and out of the transformer must be the same:

$$P_{in} = P_{out} = V_p I_p = V_s I_s$$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$$

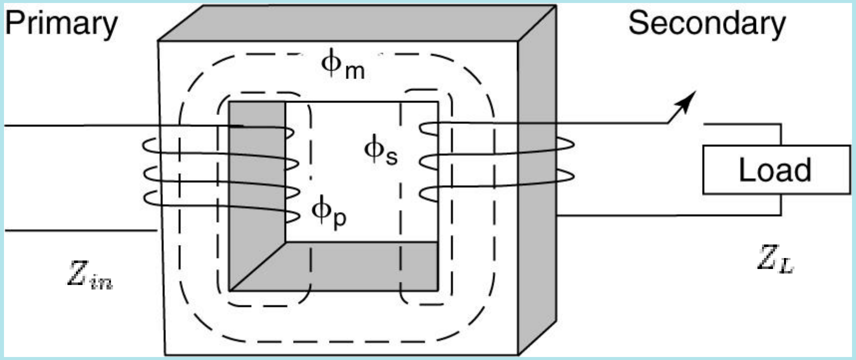
Ratio of the currents is inverse of the voltage ratio or the inverse of the turns ratio.
It makes sense: if we raise the voltage level to a load with a step-up transformer , then the secondary current drawn by the load would have to be less than the primary current, science the apparent power is constant

Example

Example

Impedance

Due to the fact that the transformer changes the voltage and current levels in opposite directions, it also changes the apparent impedance as seen from the two sides of the transformer.



Ohm's law applied at the load: $Z_L = \frac{V_s}{I_s}$

Recollect: $\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$

$$Z_L = \frac{\frac{V_p}{a}}{aI_p} = \frac{V_p}{a^2 I_p} = \frac{Z_{in}}{a^2}$$

$$Z_{in} = a^2 Z_L$$

The Reflected (referred) impedance (the impedance looking into the primary side of the transformer)

When we move an impedance from the secondary to the primary side of the transformer we multiply by the turns ratio squared.
When moving the impedance from the primary to the secondary, we divide it by the turns ratio squared.

This process is called referring the impedance to the side we move it, and allows us to use transformers to match impedances between a source and a load

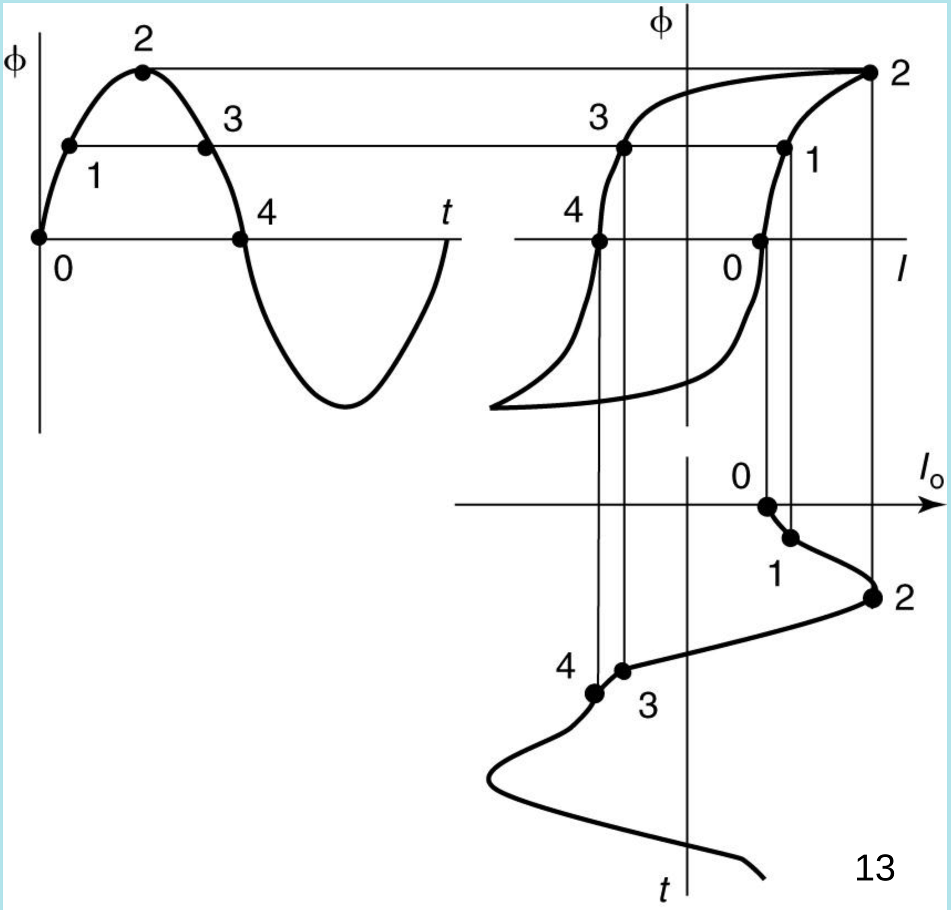
Example

Exciting Current

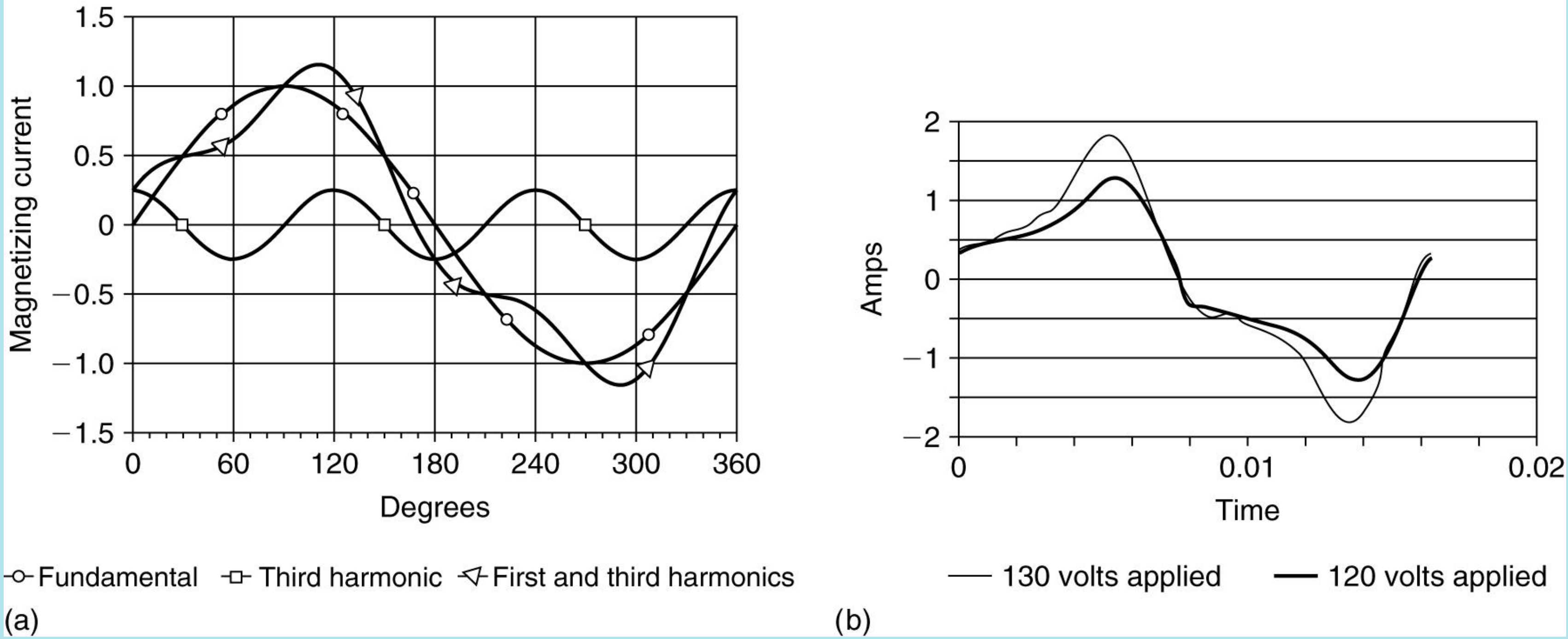
In real life we deal with real transformers which require current in the primary winding to establish the flux in the core. The current that establishes the flux is called the exiting current. Magnitude of the exciting current is usually about 1%-5% of the rated current of the primary.

According to Faraday's law if we apply a sinusoidal voltage to the transformer, then the flux will also be sinusoidal.

Due to the non-linearity of B-H curve, the current will not be sinusoidal even if the flux is sinusoidal and the current will be out of phase with flux.



The current is not sinusoidal but it is periodic, thus can be represented by a Fourier series.



a. Harmonic content of exciting current.

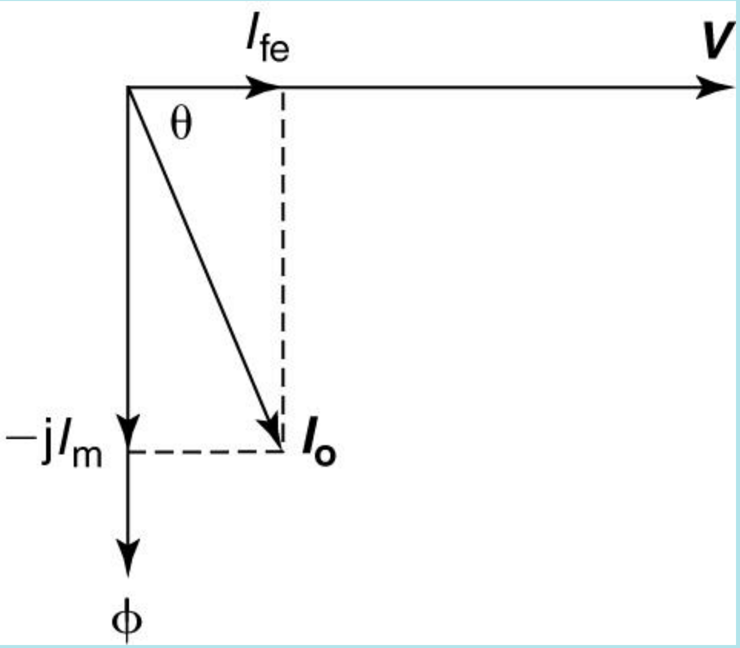
b. Measured exciting current.
A slightly higher applied voltage causes the transformer to draw a much higher exiting current which would also increase the core losses.

RMS value of the exiting current is calculated :

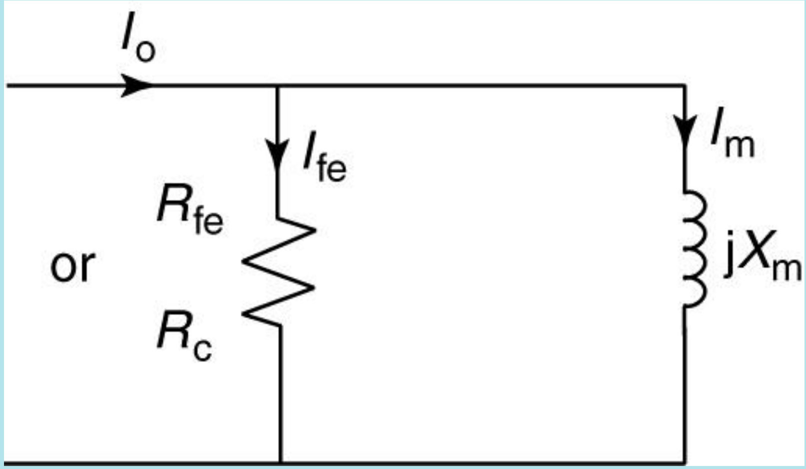
$$I_{0,RMS} = \sqrt{\frac{\int_0^{2\pi}[i_0(wt)]^2 d(wt)}{2\pi}}$$

The exciting current is not in phase with the flux.
The voltage is 90 degrees ahead of the flux, science voltage is the derivative of the flux.

The exciting current phasor lies between the voltage phasor and the flux phasor, therefore the current can be separated into two components:
- One in phase with the voltage, Ife. Represents real power being consumed and is called: **Core-loss current**
- One in phase with the flux, Im. Represents reactive power and is called: **Magnetizing current**

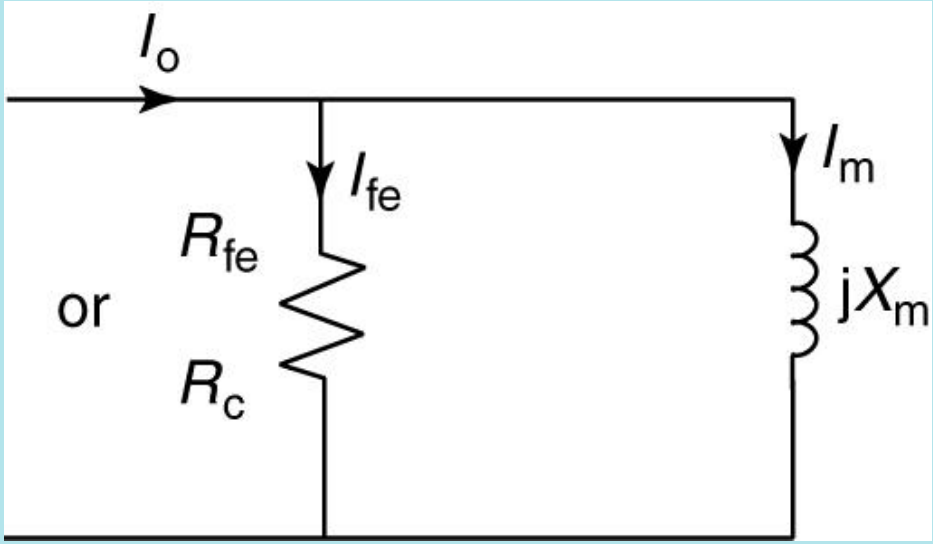


Phasor diagram of exciting current



Equivalent circuit of transformer core

The resistance Rc consumes real power corresponding to the core loss of the transformer.
The inductive reactance Xm draws the current to create the magnetic field in the transformer core.



We can calculate R_{fe} and X_m :

1. With no load on the transformer measure:
   the RMS current into the transformer
   the input voltage
   the real power into transformer

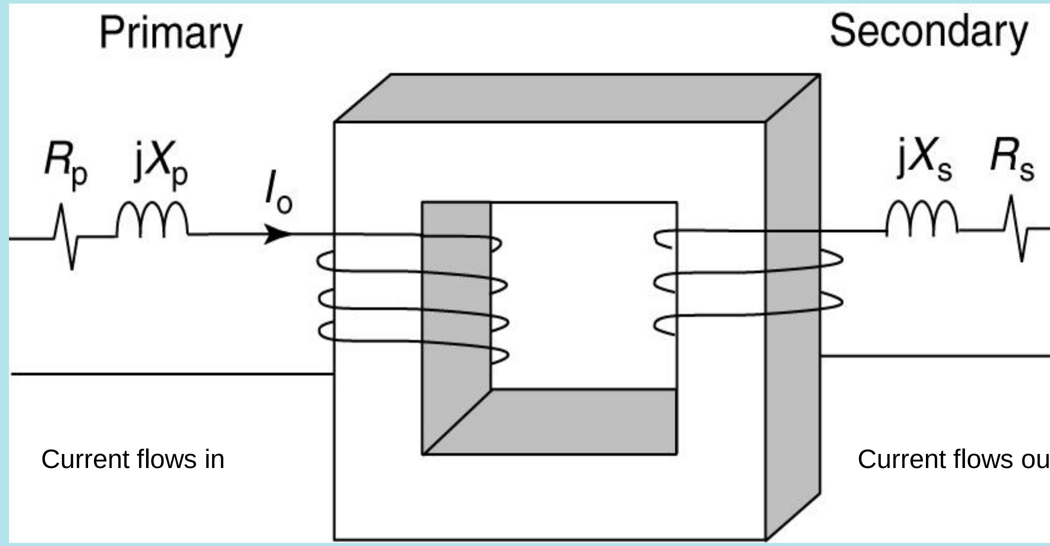
2. The product of the voltage and the current gives the apparent power, S.

3. Science the real power is known, we can find the power factor and the impedance angle.
   The elements then can be calculated.

Transformer Equivalent Circuits

All non-ideal transformers have:

- Winding resistance
- A core with finite permeability
- Leakage flux
- Hysteresis
- Eddy current losses



$$V_p = R_p I_p + N_p \frac{d\phi}{dt}$$

$$V_p = \underbrace{R_p I_p + N_p \frac{d\phi_p}{dt}} + N_p \frac{d\phi_m}{dt}$$

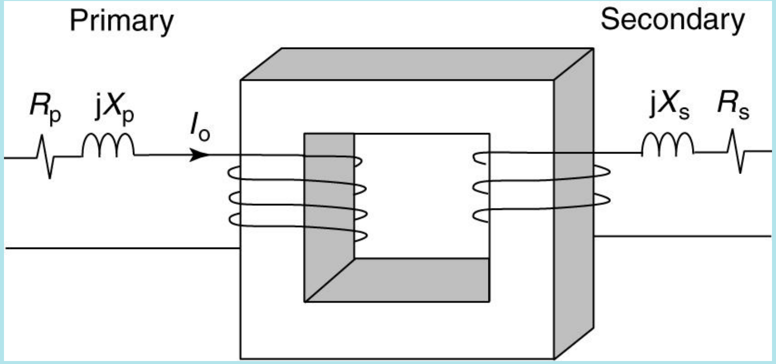
$$V_s = -R_s I_s + N_s \frac{d\phi}{dt}$$

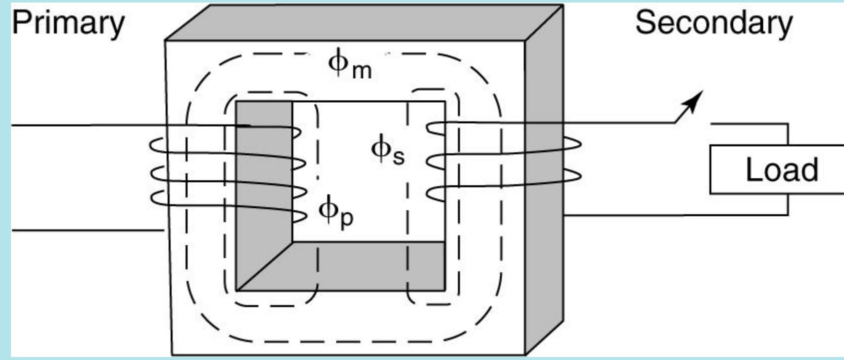
$$V_s = -R_s I_s - N_s \frac{d\phi_s}{dt} + N_s \frac{d\phi_m}{dt}$$

Mutual flux that
links both coils

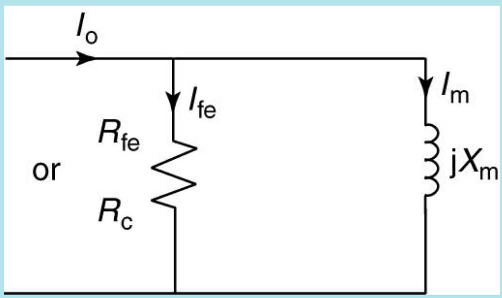
Primary leakage
flux

Can be represented as a resistor
and inductor in series with transformer

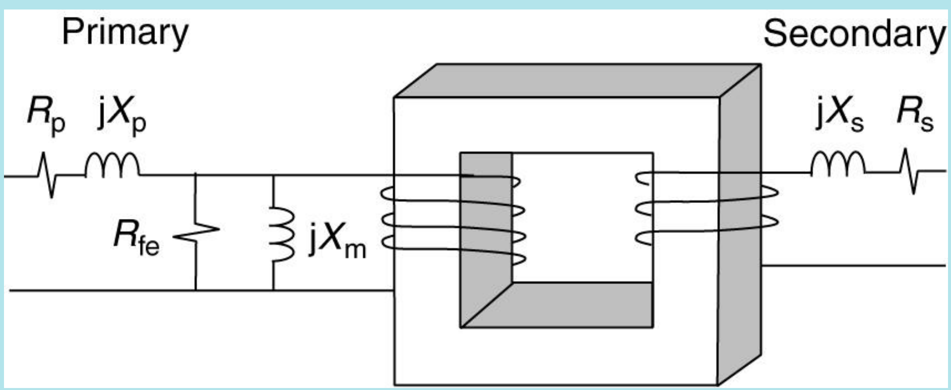




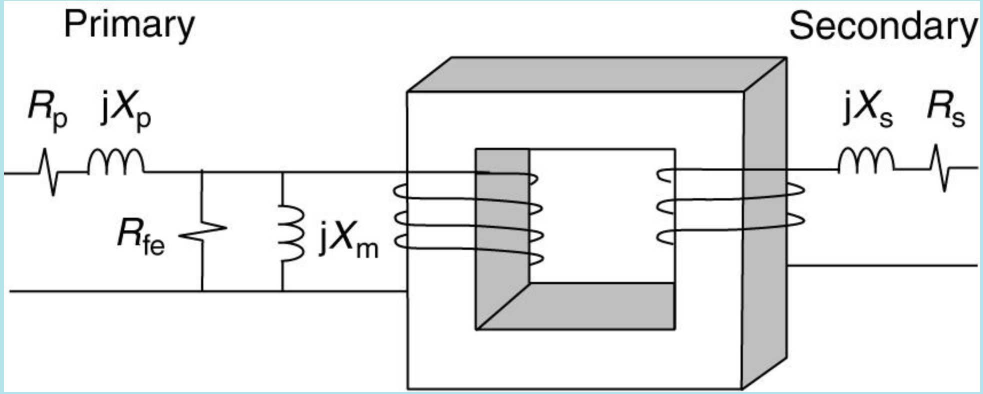
Non-ideal (real) transformer

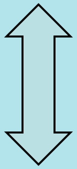


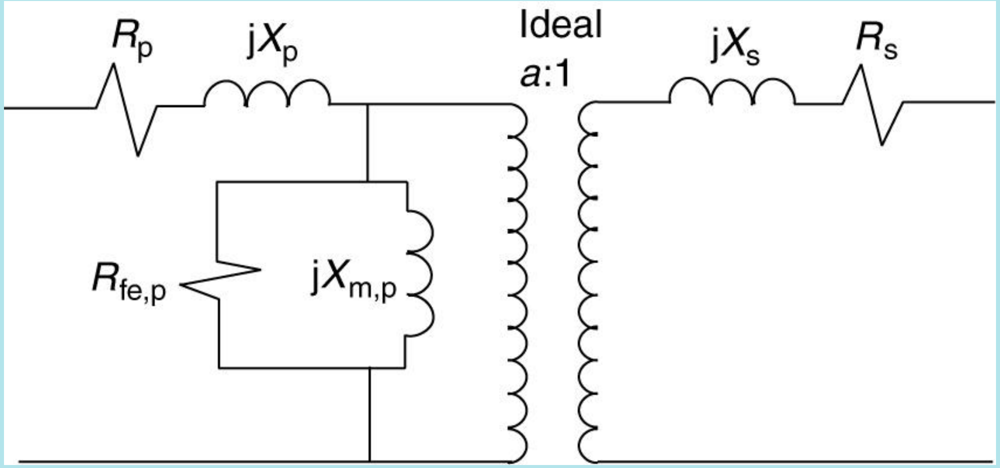
Magnetizing branch



The transformer is now **ideal** and the circuit elements account for the losses and voltage drops in the real transformer.



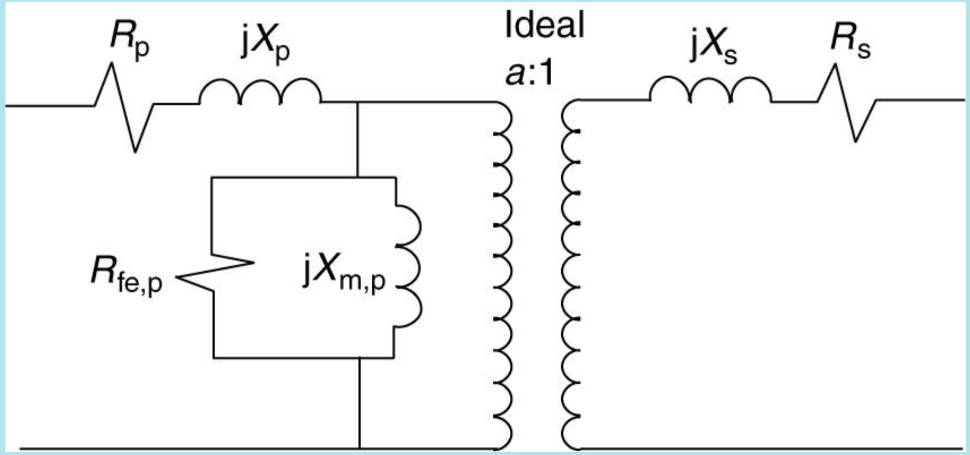


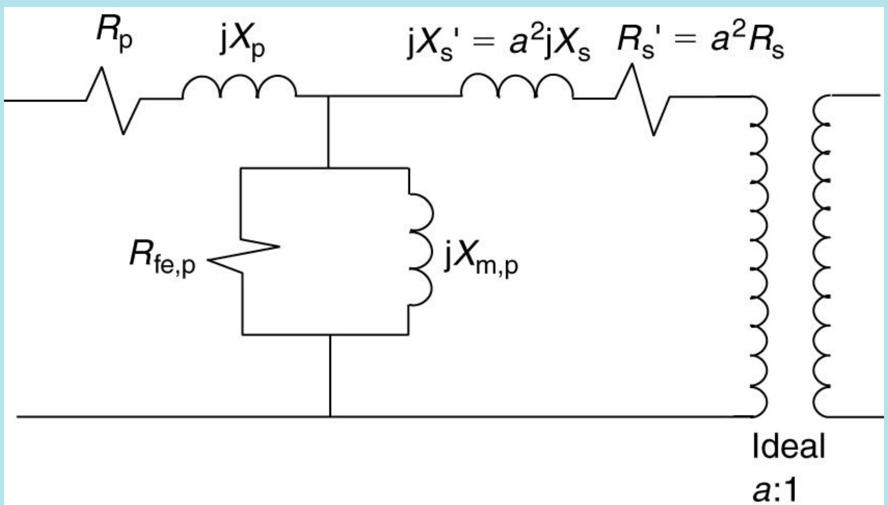


<u>Equivalent T-circuit</u>

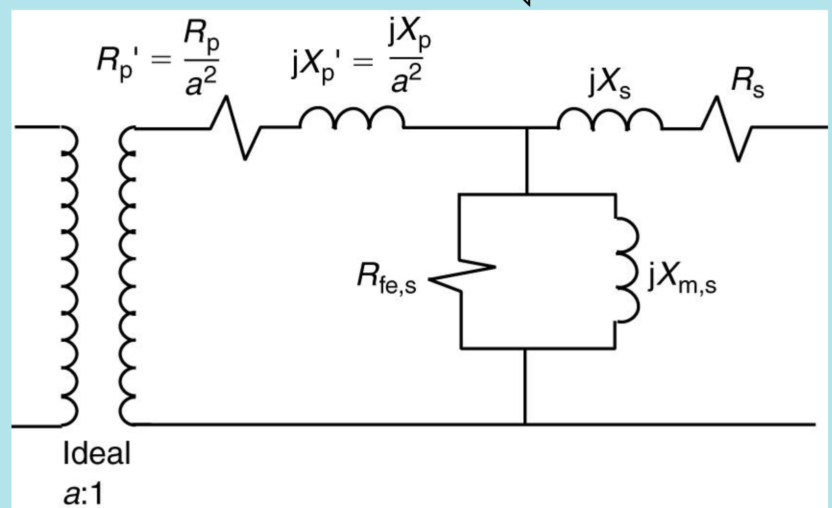
Third order circuit. It takes third order differential equation to solve it.

We can refer the impedances of the secondary to the primary side (or vise versa) yielding the equivalent circuits





All resistances and reactances have been referred to the primary side



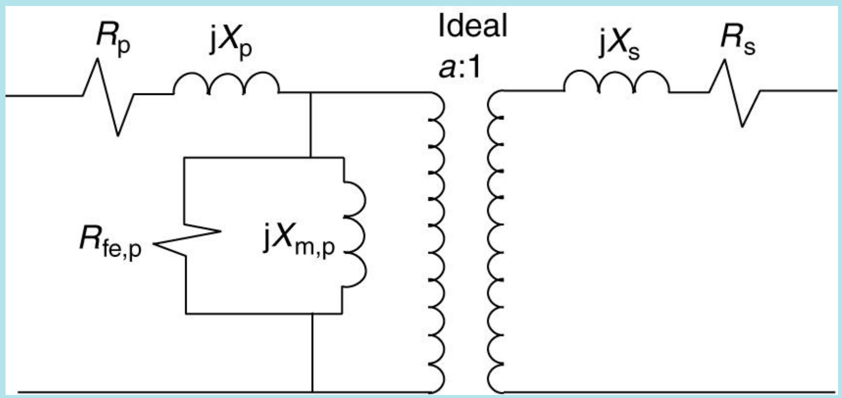
All resistances and reactances have been referred to the secondary side

$$Z_{in} = a^2 Z_L$$

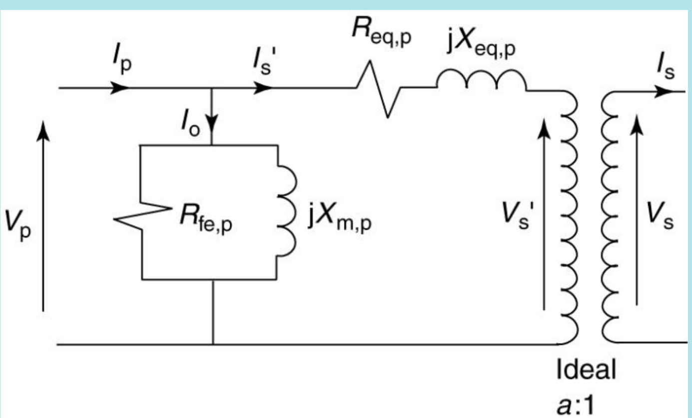
Cantilever Equivalent Circuit.

The combination of the winding resistances is called ***equivalent resistance***.
The combination of the leakage reactances is called the ***equivalent reactance***.

The Cantilever Equivalent Circuit Model neglects the voltage drop of the exciting current in the primary coil but makes calculation the exciting current much easier because the primary voltage is applied directly to the magnetizing reactance and the core-loss resistance.



$$Z_{in} = a^2 Z_L$$

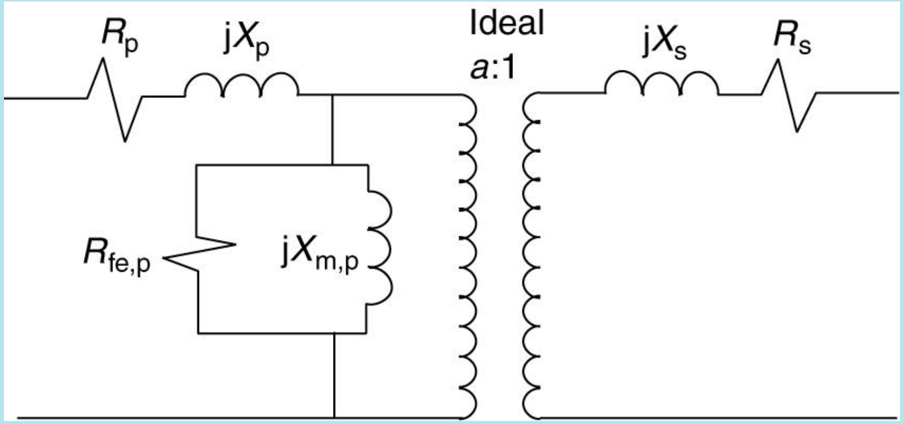


$$R_{eq.primary} = R_p + a^2 R_s = R_p + R_s'$$

$$X_{eq.primary} = X_p + a^2 X_s = X_p + X_s'$$

$$Z_{eq.primary} = R_{eq.primary} + X_{eq.primary}$$

Cantilever circuit referred to the primary.

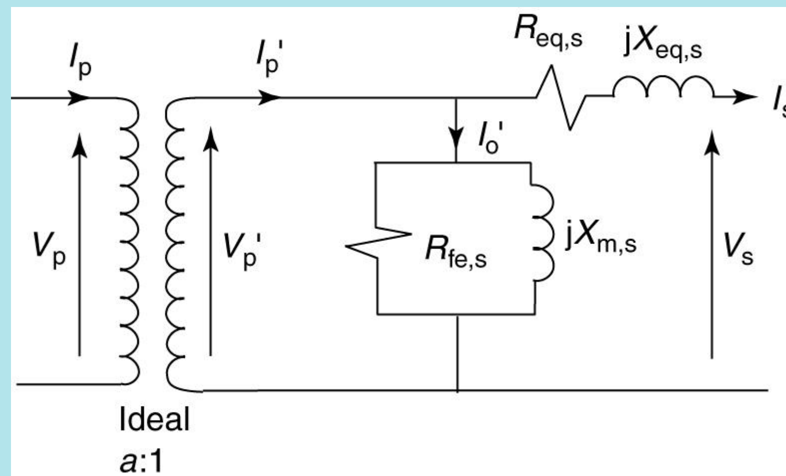


$$Z_{in} = a^2 Z_L$$

$$R_{eq.secondary} = \frac{R_p}{a^2} + R_s = R_p + R_s'$$

$$X_{eq.secondary} = \frac{X_p}{a^2} + X_s = X_p + X_s'$$

$$Z_{eq.secondary} = R_{eq.secondary} + X_{eq.secondary}$$



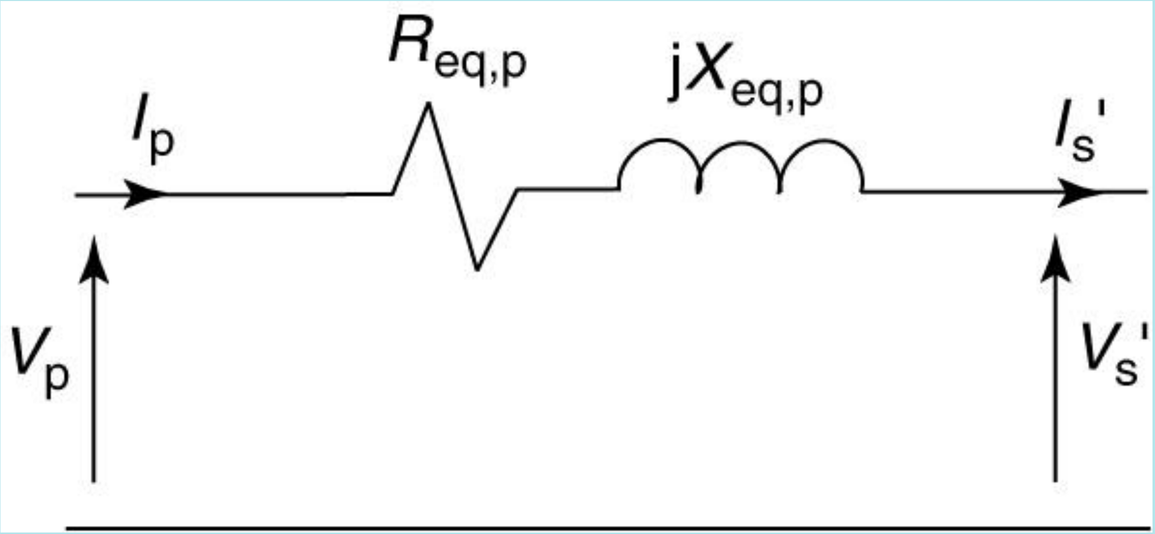
Cantilever circuit referred to the secondary.

Example

Series Equivalent Circuit

Note: In large-scale system studies, even cantilever model becomes too complex, so one final simplification is made.

We completely neglect the magnetizing branch of the transformer model. Only combined winding resistance and leakage reactance are included, resulting in the first order model (takes first order differential equation to solve it).



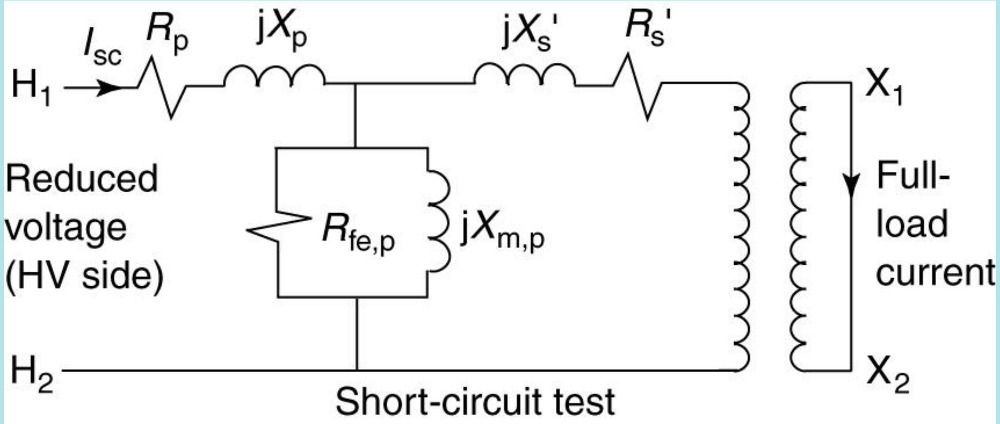
Series equivalent circuit.

Science there are no shunt elements, the primary and secondary currents are equal to each other.

Determining Circuit Parameters

For developed model to be useful, there must be a way to determine the values of the model parameters. We use two test to determine this parameters:

Short-circuit test One side of the transformer is shorted, and voltage is applied on the other side until rated current flows in the winding. The applied voltage, winding current, and input power are measured.



Technique: the low-voltage side of the transformer is shorted and voltage is applied to the high-voltage side, because it only takes about 4%-7% of rated voltage to cause rated current to flow in the winding.
The measurements are used to calculate value of Req and jXeq.

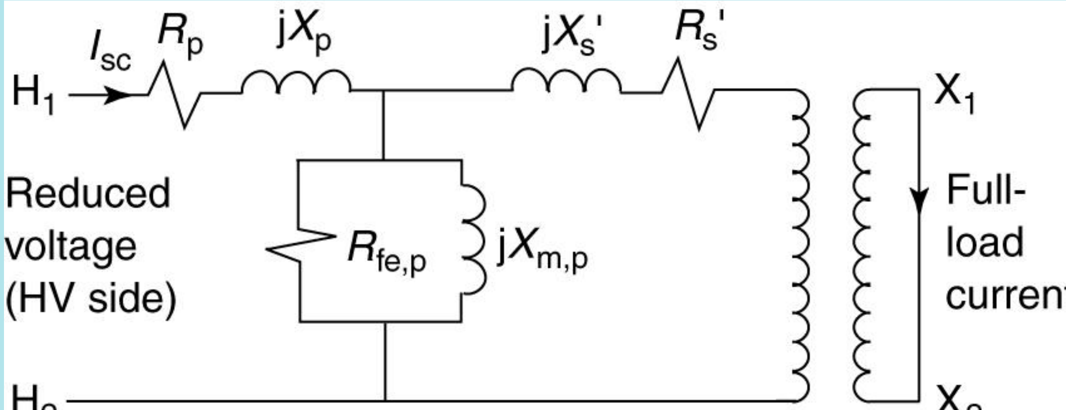
Input impedance to the transformer is the primary winding in series with the parallel combination of the secondary winding and the exciting branch:

$$Z_{in} = R_p + jX_p + [(R_s^{'} + jX_s^{'})||R_{fe,primary}||jX_{m,primary}]$$

The core branch elements are much larger that the winding impedance:

$$R_{fe,primary} >> R_s^{'} \qquad X_{m,primary} >> X_s^{'}$$

$$Z_{in} = R_p + jX_p + (R_s^{'} + jX_s^{'})$$
$$Z_{in} = R_{eq.primary} + X_{eq.primary}$$



Short-circuit test

To conduct the short-circuit test:
- Measure the voltage applied to the transformer high side Vsc
- Measure the short-circuit current in the high-side winding Isc
- Measure the power into the transformer Psc.
- Having these parameters we calculate the magnitude of input impedance:

$$|Z_{eq.primary}| \approx |Z_{sc}| = \frac{V_{sc}}{I_{sc}}$$

$$R_{eq,primary} \approx \frac{P_{sc}}{I_{sc}^2}$$

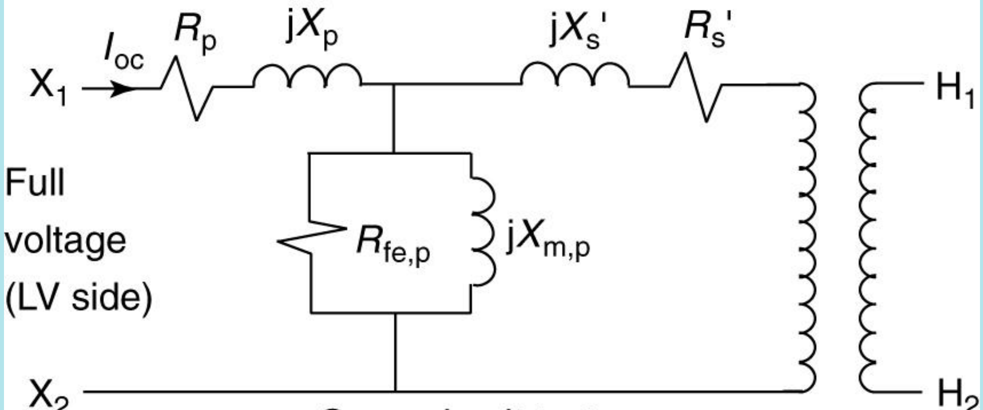
$$X_{eq,primary} = \sqrt{|Z_{eq.primary}|^2 - R_{eq,primary}^2}$$

Once we have values for the equivalent winding resistance and reactance we can apportion them to the two sides by assuming the windings have equal resistance and reactance when referred to the same side:

$$R_{primary} = R_s' = \frac{1}{2} R_{eq,primary}$$

$$X_{primary} = X_s' = \frac{1}{2} X_{eq,primary}$$

Open-circuit test



Open-circuit test

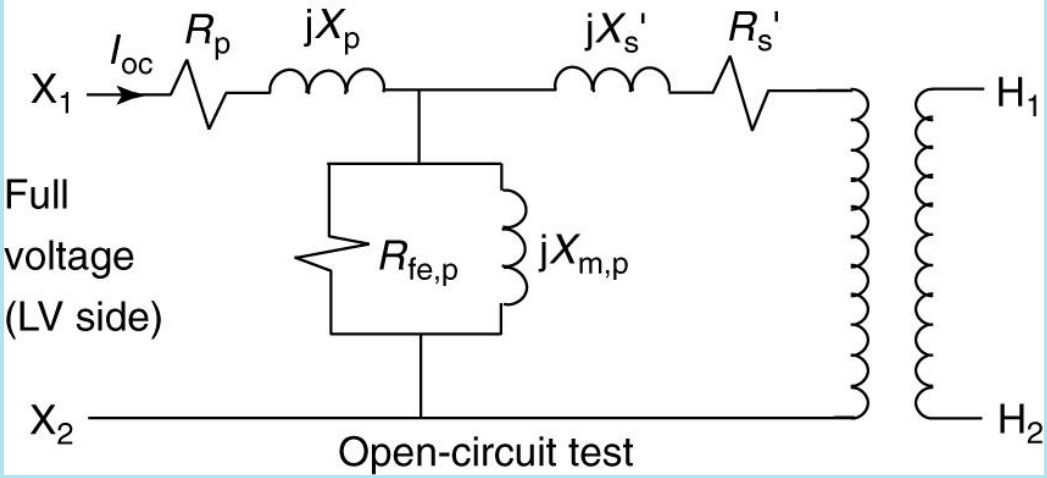
High-voltage side is opened and rated voltage is applied to the low-voltage side (primary side).

The low-voltage side is used to avoid high-voltage measurements.

With no load current, only exciting current Io flows and because the impedance of the primary side is small, the voltage across the magnetizing branch is approximately equal to the applied voltage.

Procedure: - Measure the voltage applied to the transformer low side Voc, the open-circuit current in the low-side winding Ioc and the power into the transformer during open circuit test Poc.

- Calculate resistance and reactance.



The input impedance during open-circuit test is the primary winding in series with the exciting branch:

$$Z_{in} = R_p + jX_p + (R_{fe,primary} || jX_{m,primary})$$

$$R_{fe,primary} >> R_{primary}$$

$$X_{m,primary} >> jX_{primary}$$

$$Z_{in} = R_{fe,primary} || jX_{m,primary}$$

Now we have the voltage applied to $R_{fe,primary}$ and the power dissipated, so we can calculate:

$$R_{fe,primary} = \frac{V_{oc}^2}{P_{oc}}$$

Because the magnetizing reactance is in parallel with $R_{fe,primary}$ we first need to find the reactive power to find reactance:

$$|S_{oc}| = V_{oc} \times I_{oc}$$

$$Q_{oc} = \sqrt{|S_{oc}|^2 - P_{oc}^2}$$

$$X_{m,primary} = \frac{V_{oc}^2}{Q_{oc}}$$

Transformer Efficiency

Efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}}$$

Input is the output plus losses:

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$$

LOSSES

Copper losses (the energy dissipated in the resistance of the windings)

Core losses (hysteresis and eddy current losses in ferromagnetic core of the transformer)

K-Factor-Rated Transformers

The overheating effect of nonlinear loads on wiring systems, transformers, motors, and generators can be severe.

K-Factor-Rated transformers are specifically designed to handle nonlinear loads.

They are typically constructed of thinner steel laminations, lower-loss steel, and larger conductors. This allows them to provide harmonic currents without overheating.

The K-factor rating of the transformer is an indication of the transformer's ability to deliver power to a load without exceeding the transformer' specified operating temperature limits.

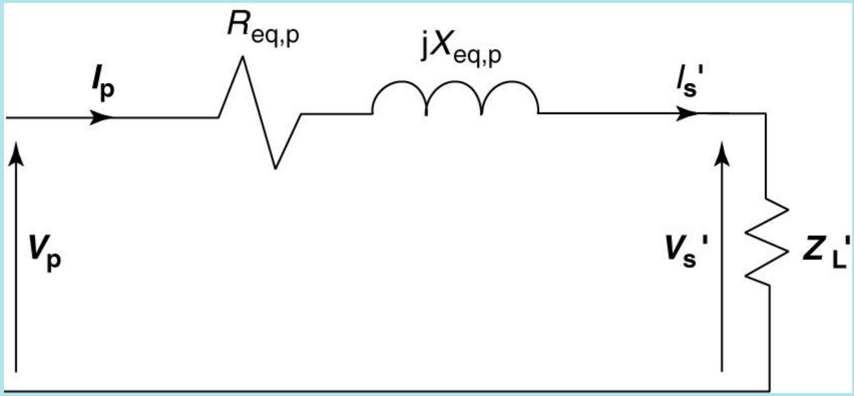
K-factor is defined as:

$$K = \sum h^2 \left(\frac{I_h}{I_{rms}}\right)^2$$

where h is the harmonic number, Ih is the RMS value of the harmonic current, and I RMS is the total RMS current.

Example: K-4 rated transformer means that this transformer can handle a load that produces harmonic currents that would cause four times the normal eddy current loss in a standard transformer

Voltage Regulation



The transformer windings have impedance so there will be a voltage drop across them that changes with current.
The secondary voltage will vary as the load changes.

Voltage regulation is a measure of the change in secondary voltage from no-load to full-load and is usually expressed as a percentage of the full-load voltage.

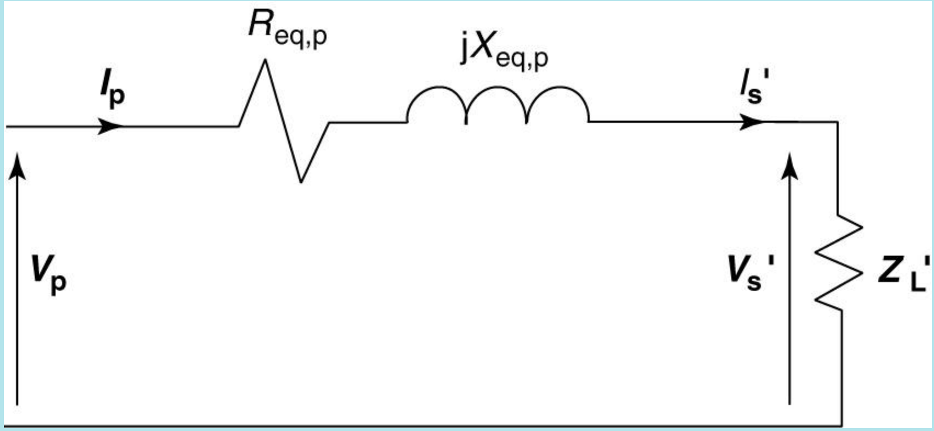
If there were **no load** on the transformer, the **current** would be **zero** and the referred secondary voltage would be equal to the primary voltage.

The **no-load voltage** (referred to the primary) of the transformer is the **primary voltage.**

As the load increases to **full load**, current flows in the windings of the transformer and there is a voltage drop across the transformer, and the referred value of the secondary voltage is no longer equal to the primary voltage.

Voltage regulation:

$$VR = \frac{|V_{nl}| - |V_{fl}|}{|Vfl|} \times 100$$



If we know the primary voltage and the current, we can calculate the secondary voltage:

$$V'_{nl} = V_p$$

$$V'_s = V'_{fl} = V_p - I'_{fl} Z_{eq,primary}$$

If we know the secondary voltage and current, then we can calculate the primary voltage, which is referred to the secondary and primary, respectively.

$$V_{fl} = V_s$$

$$V_{nl} = V'_p = V_s + I_{fl} Z_{eq,secondary}$$

Referred to the secondary

$$V'_{fl} = V'_s$$

$$V'_{nl} = V_p = V'_s + I'_{fl} Z_{eq,primary}$$

Referred to the primary

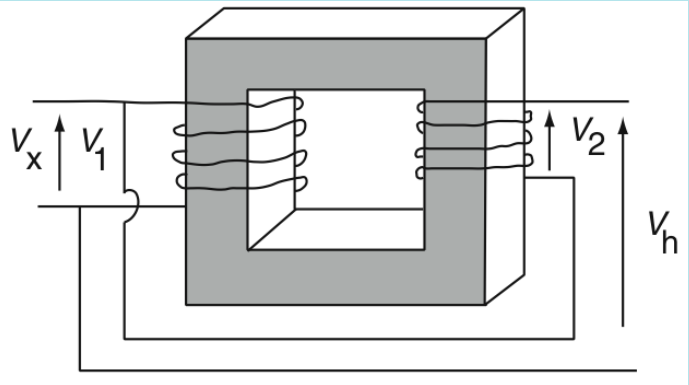
example

There are some applications, however, where poor regulation is desired.

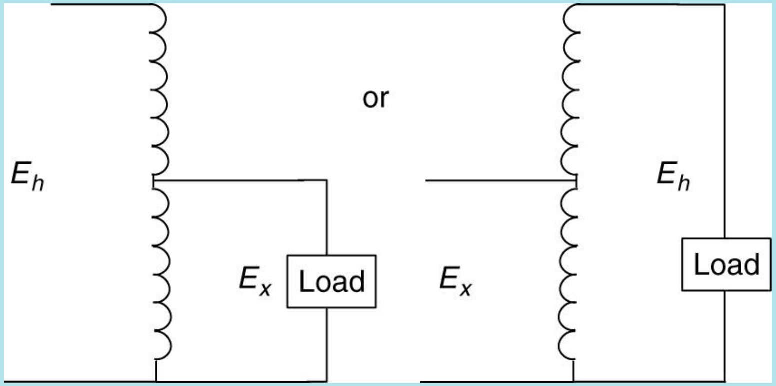
1. ***Discharge lighting***, where a step-up transformer is required to initially generate a high voltage (necessary to "ignite" the lamps), then the voltage is expected to drop off once the lamp begins to draw current. This is because discharge lamps' voltage requirements tend to be much lower after a current has been established through the arc path.
   In this case, a step-up transformer with poor voltage regulation suffices nicely for the task of conditioning power to the lamp.

2. ***Current control for AC arc welders***, which are nothing more than step-down transformers supplying low-voltage, high-current power for the welding process.
   A high voltage is desired to assist in "striking" the arc (getting it started), but like the discharge lamp, an arc doesn't require as much voltage to sustain itself once the air has been heated to the point of ionization.

Autotransformers



Elementary autotransformer.



Autotransformer schematic.

The low-voltage coil; is essentially placed on the top of the high-voltage coil and called the **series coil.** This connection is called **autotransformer** and can be used as a step-up or a step-down transformer.

Advantages of an autotransformer:

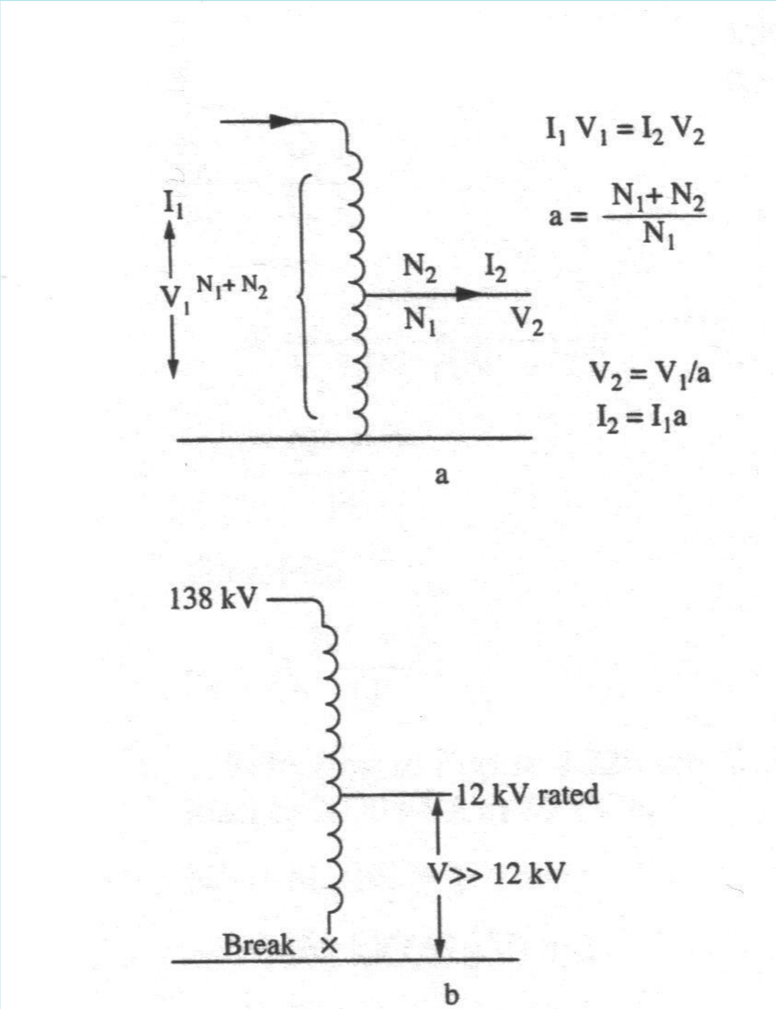
1. Higher power rating
2. Cheaper
3. More efficient
4. Low exciting current
5. Better voltage regulation

Disadvantages of an autotransformer:

1. Larger short-circuit currents available
2. No isolation between the primary and secondary
3. Most useful for relatively small voltage changes

As an autotransformer, the kVA rating is:

$$S_{auto} = S_{rated}\left(\frac{V_1 + V_2}{V_2}\right) = S_{rated}(a+1)$$



Autotransformers are one winding transformers that are often used in transmission and sub transmission substations.

Note that the primary and secondary sides are not isolated from each other. So if the secondary neutral, which is common to the primary neutral, is opened for any reason the full primary voltage could appear on the secondary side with disastrous results.

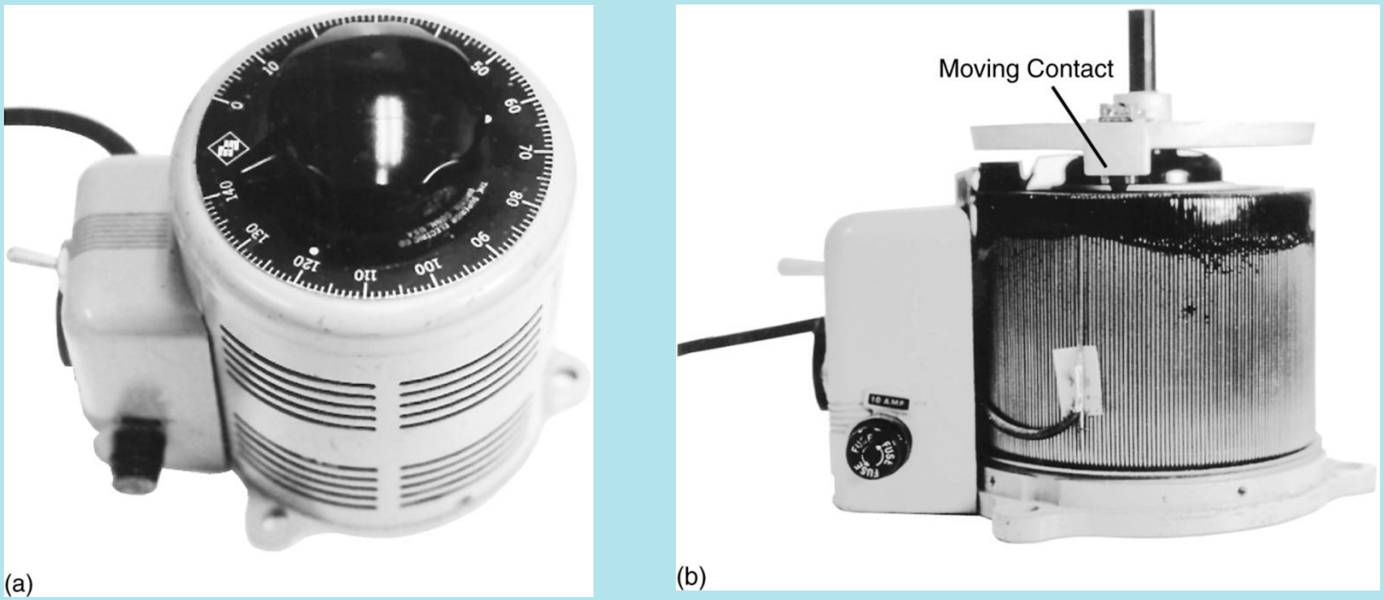
Unsafe conditions would result for people as well as damage to the low voltage side protective devices and equipment.

For this reason autotransformers are never used as the final transformer in distribution substations.

The ideal transformer relationships are approximately true for autotransformers as well as isolation (conventional) transformers.

Practical applications of autotransformers:

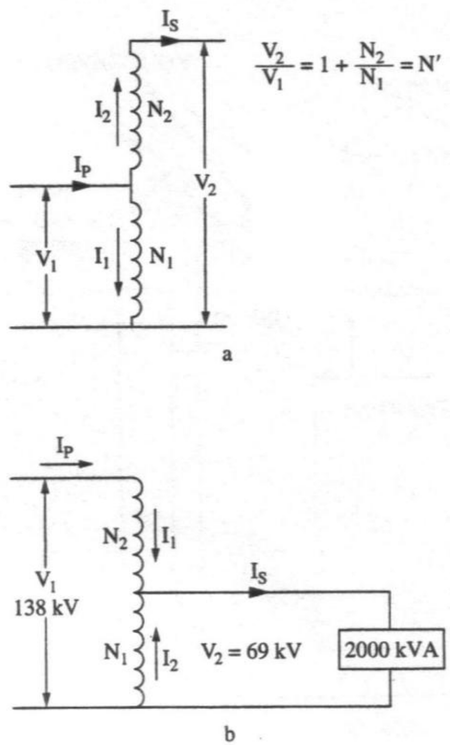
1. Connecting transmission lines of slightly different voltages (F.E 115kV and 138kV)
2. Compensating for voltage drop on long feeder circuits
3. Providing variable voltage control in the laboratory
4. Changing 208V to 240V or vice versa
5. Adjusting the output voltage of a transformer to keep the system voltage constant as the load varies.



Photograph of an autotransformer for variable-voltage control in the lab.

a. Laboratory autotransformer.
b. Cover removed to show coil and sliding contact.

The Autotransformer Rating Advantage



Autotransformer diagram for discussion of (a) ratio and (b) rating

$$\frac{S_A}{S_I} = \frac{I_1 V_1}{I_P V_1} = \frac{N' - 1}{N'}$$

$$S_A = S_I \frac{N' - 1}{N'}$$

$S_I = VA\ rating\ as\ an\ isolation\ trasformer = I_P V_1$

$S_A = VA\ rating\ as\ an\ autotrasformer = I_1 V_1$

Three-phase transformer connections

There are four major three-phase transformer connections :

1. WYE-WYE
2. Delta-Delta
3. WYE-Delta
4. Delta-WYE

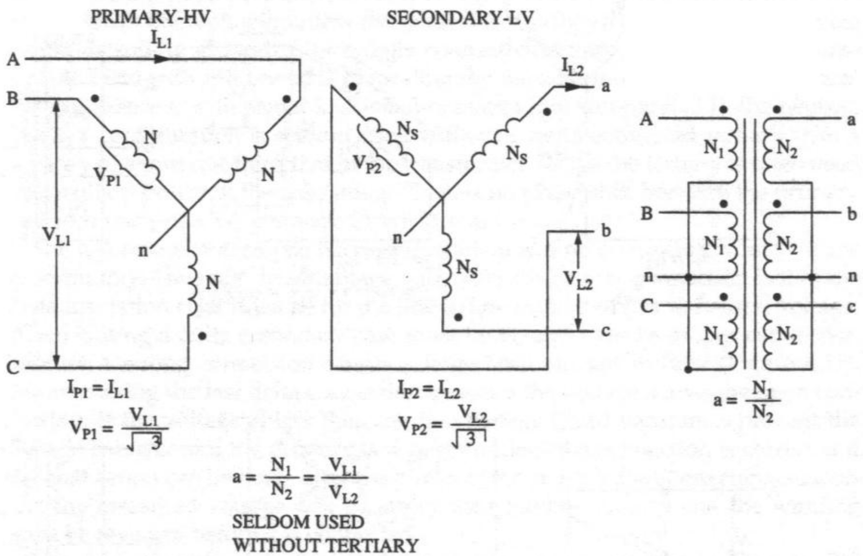
Three-phase transformers are less expensive than 3-single-phase transformers because less total core material is needed for the three-phase transformer and the packaging cost is reduced.

Additionally they take up less space, are lighter, require less on site external wiring for installation, and more efficient than three single-phase transformers.

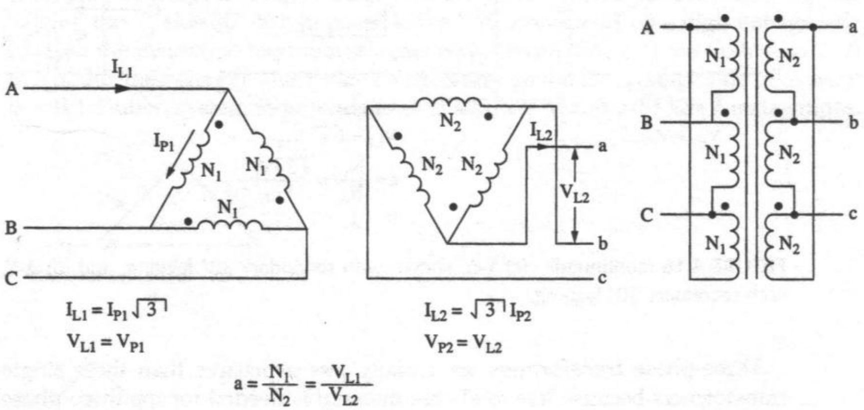
Single phase transformer has one voltage ratio which agrees with the turns ratio.

For 3-phase transformer:

1. Bank Ratio = BR = the ratio of line-to-line voltages.
2. Phase Ratio = PR = the ratio of the voltages in the coils of the transformer and thus agrees with the turns ratio.



a

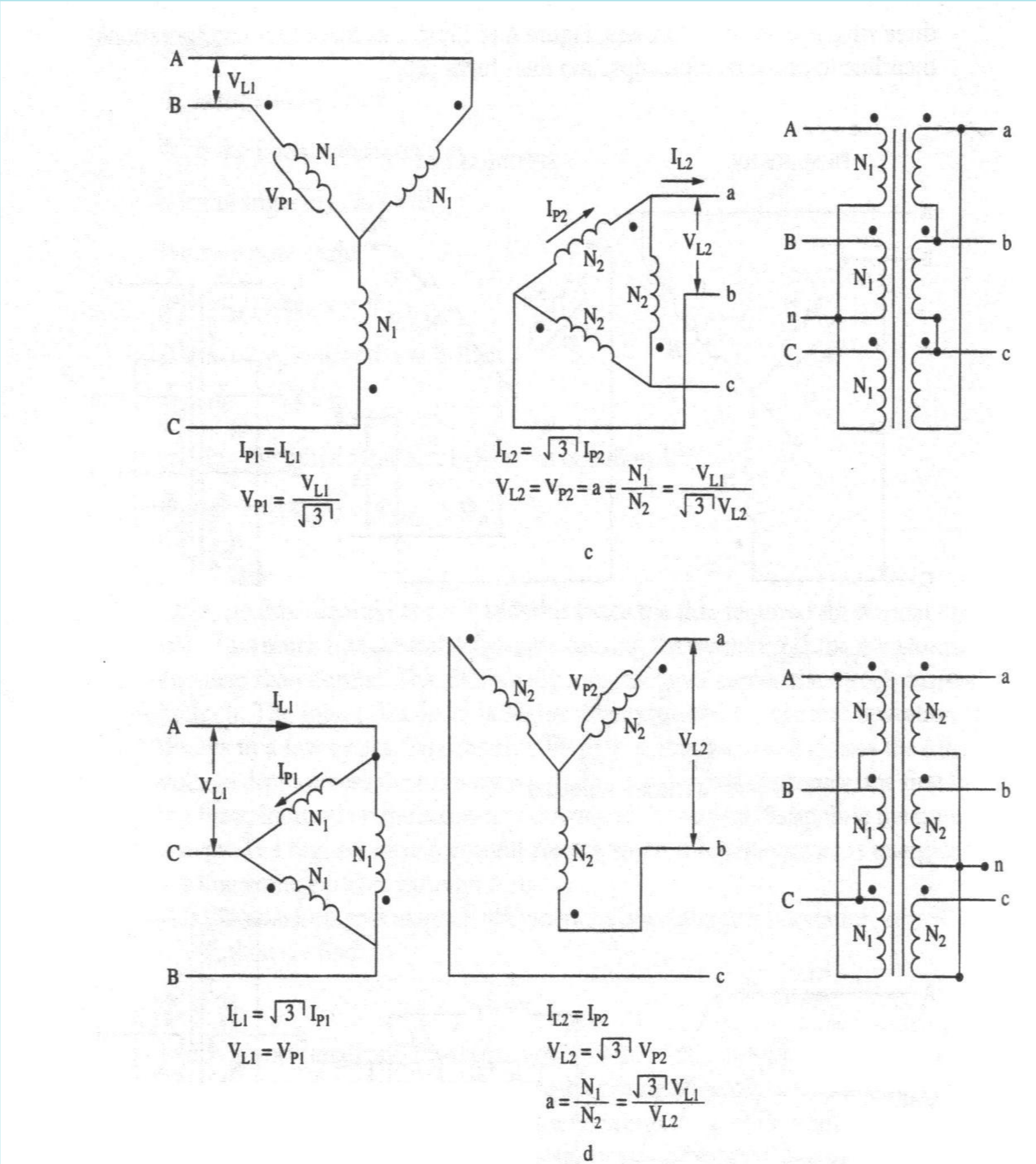


b

The WYE-WYE connection has significant third harmonic content on the secondary lines (unless the neutral point is grounded)

There is no phase shift between the primary and secondary of a Y- Y connected transformer.

The Delta-Delta connection has no harmonic problem and no phase shift from primary to secondary.
The only disadvantage with respect to a WYE connection is that the delta insulation class must be for the line to line instead of line to neutral voltage.



There is a 30 degrees phase shift in both connections. United States industry convention is to connect the secondary so it lags the high voltage primary by 30 degrees.

When possible the Y is connected to the high voltage side because the insulation requirements are lower (recall that Y phase voltage is 1/sqrt(3) that of the line voltage).

The Y may be necessary on the low voltage side because of the distribution system requirements as in 480/277 V and 208/120 V installations.

IMPORTANT NUMBERED EQUATIONS FROM THIS CHAPTER

$$E_{rms} = 4.44 f N \phi_{max} = 4.44 f N B_{max} A_c \qquad (5\text{–}6)$$

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a \qquad (5\text{–}7)$$

$$\frac{|\mathbf{I_p}|}{|\mathbf{I_s}|} = \frac{|\mathbf{V_s}|}{|\mathbf{V_p}|} = \frac{N_s}{N_p} = \frac{1}{a} \qquad (5\text{–}8)$$

$$R_{eq,s} = \frac{R_p}{a^2} + R_s = R_p' + R_s$$
$$X_{eq,s} = \frac{X_p}{a^2} + X_s = X_p' + X_s \qquad (5\text{–}20)$$

$$\mathbf{Z_{eq}} = R_{eq} + jX_{eq} \qquad (5\text{–}21)$$

$$|\mathbf{Z_{eq,p}}| \approx |\mathbf{Z_{sc}}| = \frac{V_{sc}}{I_{sc}} \qquad (5\text{–}22)$$

$$R_{eq,p} \approx \frac{P_{sc}}{I_{sc}^2} \qquad (5\text{–}23)$$

$$X_{eq,p} = \sqrt{|\mathbf{Z_{eq,p}}|^2 - R_{eq,p}^2} \qquad (5\text{–}24)$$

$$X_{eq,p} \approx \frac{Q_{sc}}{I_{sc}^2} \qquad (5\text{–}25)$$

$$R_p = R_s' = \frac{1}{2} R_{eq,p}$$
$$X_p = X_s' = \frac{1}{2} X_{eq,p} \qquad (5\text{–}26)$$

$$R_{fe,p} = \frac{V_{oc}^2}{P_{oc}} \qquad (5\text{–}27)$$

$$|\mathbf{S_{oc}}| = \mathbf{V_{oc}} \times I_{oc} \quad \text{and}$$
$$Q_{oc} = \sqrt{|\mathbf{S_{oc}}|^2 - P_{oc}^2} \qquad (5\text{–}28)$$

$$X_{m,p} = \frac{V_{oc}^2}{Q_{oc}} \qquad (5\text{–}29)$$

$$\eta = \frac{P_{out}}{P_{in}} \qquad (5\text{–}30)$$

$$\mathbf{Z_L} = \frac{\mathbf{V_s}}{\mathbf{I_s}}$$

$$\mathbf{Z_{in}} = a^2 \mathbf{Z_L}$$

$$|S_o| = 4.44 f N A_c B_{max} \ell_c H_{rms}$$

$$R_{eq,p} = R_p + a^2 R_s = R_p + R_s'$$
$$X_{eq,p} = X_p + a^2 X_s = X_p + X_s'$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$$
$$= 1 - \frac{P_{loss}}{P_{out} + P_{loss}}$$

$$\text{Volt Reg} = \frac{|\mathbf{V_{nl}}| - |\mathbf{V_{fl}}|}{|\mathbf{V_{fl}}|} \times 100\%$$

$$\mathbf{V_{nl}'} = \mathbf{V_p}$$
$$\mathbf{V_s'} = \mathbf{V_{fl}'} = \mathbf{V_p} - \mathbf{I_{fl}'}\mathbf{Z_{eq,p}}$$
$$\mathbf{V_{fl}} = \mathbf{V_s}$$
$$\mathbf{V_{nl}} = \mathbf{V_p'} = \mathbf{V_s} + \mathbf{I_{fl}}\mathbf{Z_{eq,s}}$$
$$\mathbf{V_{fl}'} = \mathbf{V_s'}$$
$$\mathbf{V_{nl}'} = \mathbf{V_p} = \mathbf{V_s'} + \mathbf{I_{fl}'}\mathbf{Z_{eq,p}}$$

$$K = \sum h^2 \left(\frac{I_h}{I_{rms}}\right)^2$$

$$S_{auto} = V_h I_2 \left(\frac{V_2}{V_2}\right) = V_2 I_2 \left(\frac{V_h}{V_2}\right)$$

$$S_{auto} = S_{rated}\left(\frac{V_1 + V_2}{V_2}\right)$$
$$= S_{rated}(a + 1)$$

$$I_{S/C} = \frac{(I_S)}{Z_{PU}}$$