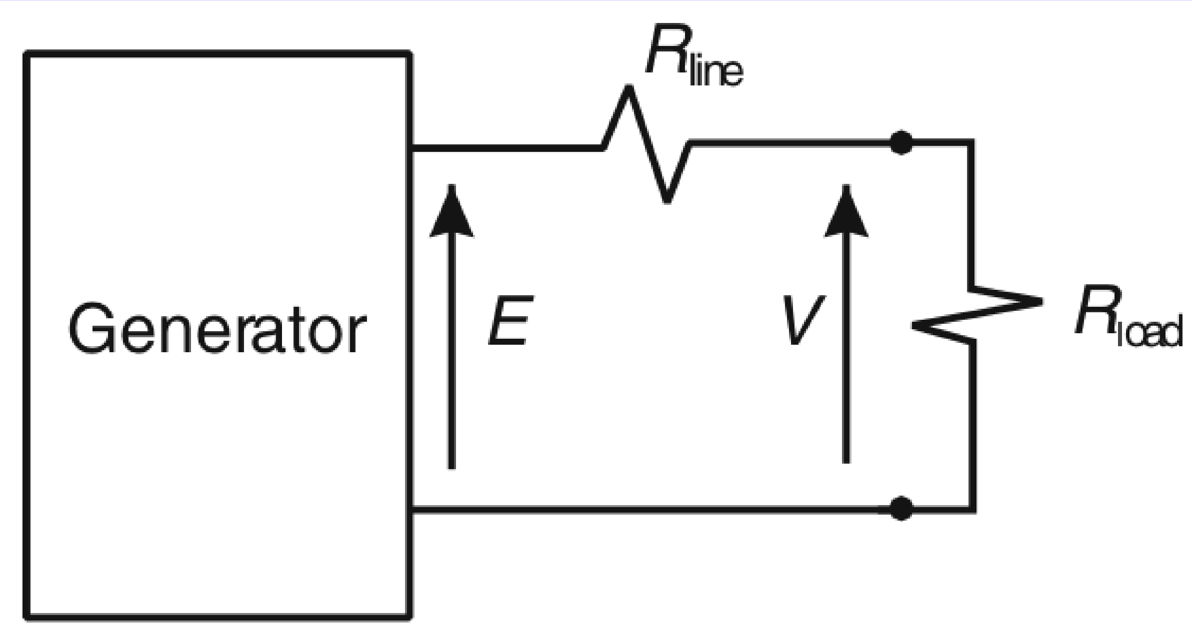
CHAPTER 2

Basic Concepts, Three-Phase Review, and Per Unit

AC power versus DC power



DC system:

- Power delivered to the load does not fluctuate.
- If the transmission line is long power is lost in the line.

Case 1: $R_{line} \cong 0$

$$E = V$$

$$I = \frac{E}{R_{load}}$$

$$P = VI = EI = \frac{E^2}{R_{load}}$$

Case 2: $R_{line} \neq 0$

$$P_{loss} = I^2 R_{line}$$

$$E = V - I R_{line}$$

AC system:

Power is the product of voltage and current, increasing the voltage would decrease the current for a given amount of power. The line loss and voltage drop would both decrease as well. Difficult to change the voltage in a DC system.

AC systems: ***alternate the current and voltage***

$$V(t) = V_{max} sin(wt)$$

$$I(t) = I_{max} sin(wt)$$

$$P(t) = V_{max} sin(wt) I_{max} sin(wt) = \frac{V_{max} I_{max}}{2}[1 - cos(2wt)]$$

Power has an average value but pulsates around the average value at twice the system frequency. We can use transformer to step up the voltage for the transmission. DC system delivers constant power but to change the voltage is difficult therefore line losses are more significant.

Advantages of AC systems:

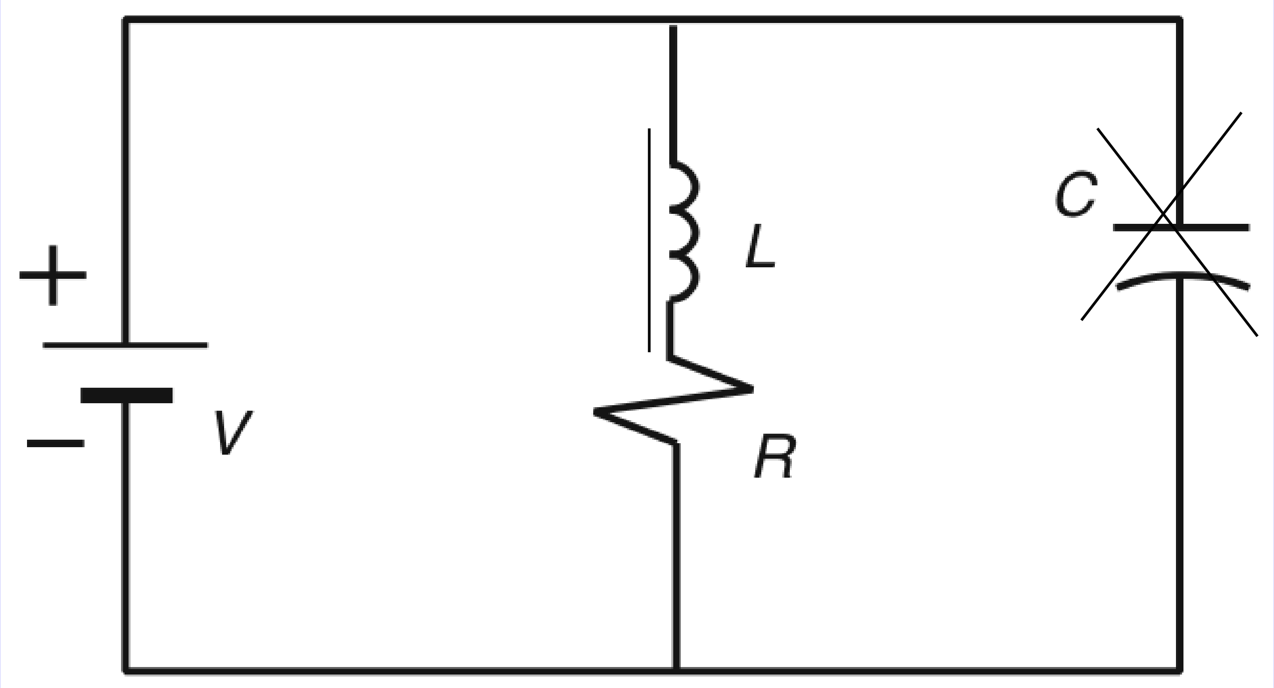
Voltage can be changed using transformers.

AC generators are cheap to build and maintain than DC generators

The ability to transmit over longer distances with lower losses by using transformers

Polyphase AC system can deliver constant power (discussed later)

DC power



DC system:

- Power delivered to the load does not fluctuate.

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$$E = V$$

$$I = \frac{E}{R_{load}}$$

$$P = VI = EI = \frac{E^2}{R_{load}}$$

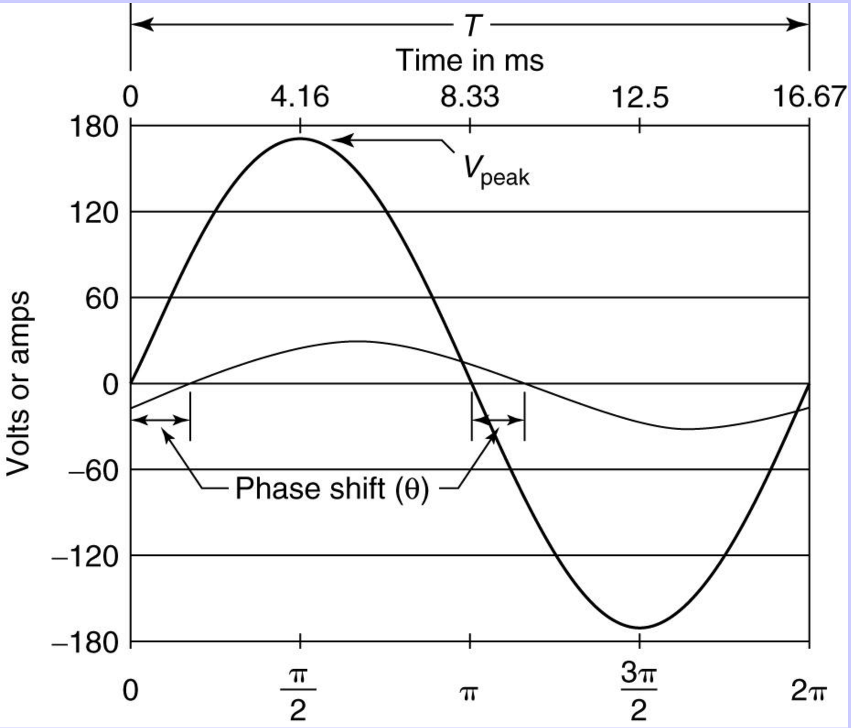
Important:

-Inductors become short circuits to DC.

-Capacitors become open circuits to DC.

Single-Phase AC Circuits

Alternating current derives its name from the fact that the voltage and current are sinusoidal functions.



$$V(t) = 170sin(wt)volts = 120\sqrt{2}sin(wt)volts$$

$$I(t) = 30sin(wt - \frac{\pi}{6})amps = 21.21\sqrt{2}sin(wt - \frac{\pi}{6})amps$$

$$f = \frac{1}{T} = \frac{1}{16.67ms} = 60Hz$$

$$\omega = 2\pi f \frac{rad}{s}$$

Related to the peak value is the the RMS of the waveform.

RMS is found by taking the square root of the mean of the waveform squared:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^t [v(t)]^2 dt}$$

The RMS is effective value of the AC voltage or current : the RMS current or voltage is the value that would cause the same amount of heat in a resistor as a DC current or voltage with the same value.
For sinusoidal functions:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = 0.707(V_{peak})$$

Phasors

Convenient way to represent voltages and currents using phasor form (work in phasor domain)

In its simplest form, a phasor can be thought as a complex number that has magnitude and phase of a sinusoidal function and can be represented partially as a vector in the complex plane.

IMPORTANT: The magnitude of a phasor will always be RMS value of the sinusoid it is representing.

Voltage phasor magnitude

$$V(t) = 170sin(wt)volts = 120\sqrt{2}sin(wt)volts$$

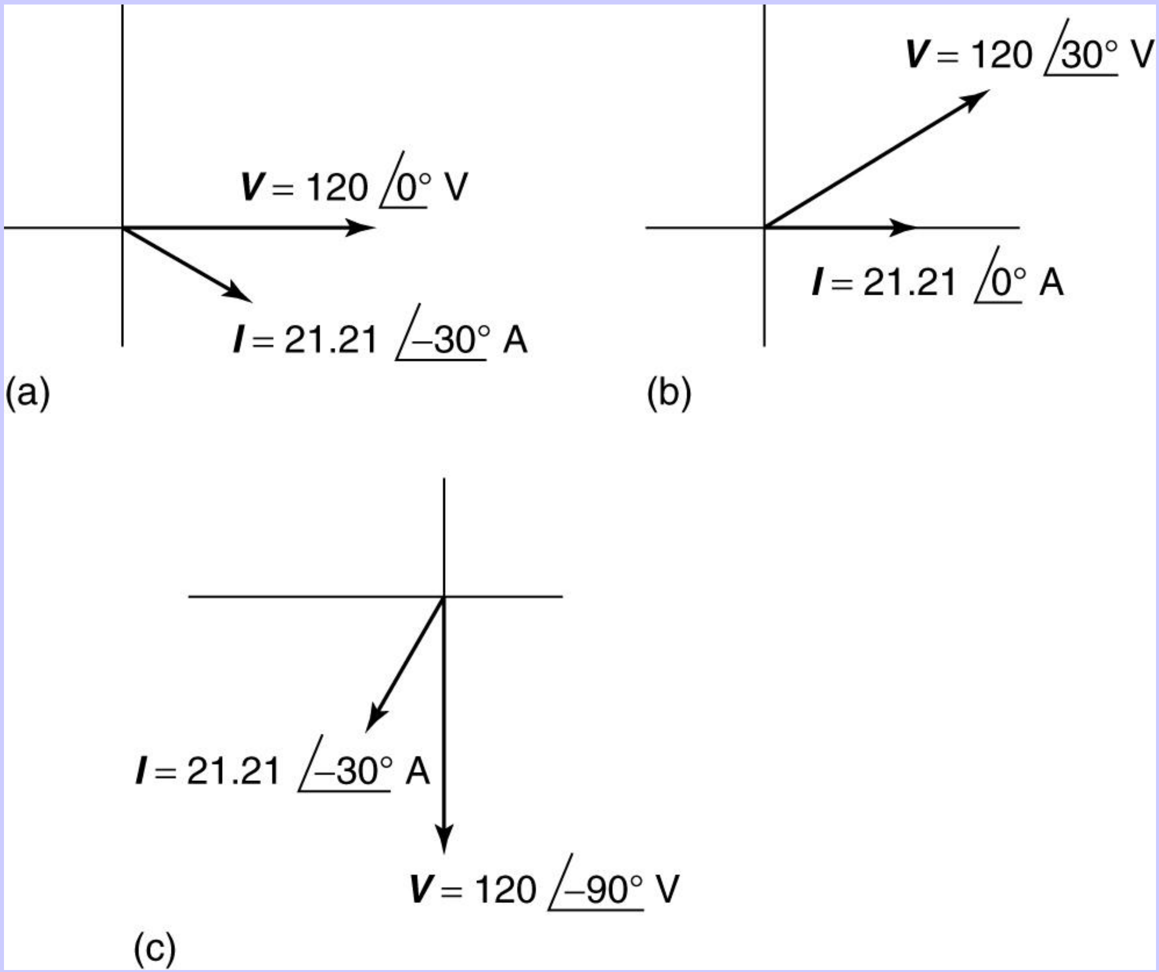
$$I(t) = 30sin(wt - \frac{\pi}{6})amps = 21.21\sqrt{2}sin(wt - \frac{\pi}{6})amps$$

Current phasor magnitude; current is lagging the voltage by 30 degrees

Example:

$$V(t) = 170sin(wt)volts = 120\sqrt{2}sin(wt)volts$$

$$I(t) = 30sin(wt - \frac{\pi}{6})amps = 21.21\sqrt{2}sin(wt - \frac{\pi}{6})amps$$



a. Choose the voltage as the reference phasor:

current lags the voltage by 30 degrees, so current phasor will be 30 degrees behind the voltage

$$V = 120\angle 0^o$$
$$I = 21.21\angle -30^o$$

b. Choose the current as the reference phasor:

current lags the voltage by 30 degrees, so the voltage phasor will be +30 degrees

$$V = 120\angle 30^o$$
$$I = 21.21\angle 0^o$$

c. Choose the cosine function as the reference phasor:

The sine function lags the cosine by 90 degrees, thus the voltage phasor should be at -90 degrees, current must be 30 degrees behind the voltage, so it is at -120 degrees.

$$V = 120\angle -90^o$$
$$I = 21.21\angle -120^o$$

Symbol notation for several complex quantities

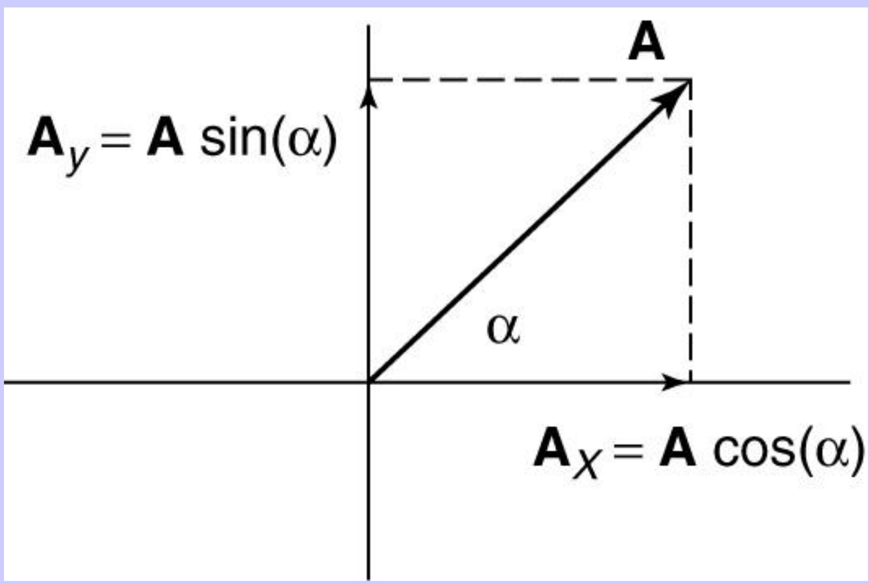
TABLE 2–2
Symbol Notation for Several Complex Quantities

Quantity	Phasor or complex number symbol	Magnitude symbol	Polar form (examples)	Rectangular form
Voltage	$\mathbf{V}$	$\lvert\mathbf{V}\rvert$ or V	$V\angle\alpha$ $\lvert\mathbf{V}\rvert\angle\alpha$	$V_x + jV_y$
Current	$\mathbf{I}$	$\lvert\mathbf{I}\rvert$ or I	$I\angle\beta$ $\lvert\mathbf{I}\rvert\angle\beta$	$I_x + jI_y$
Complex power	$\mathbf{S}$	$\lvert\mathbf{S}\rvert$ or S	$S\angle\theta$ $\lvert\mathbf{S}\rvert\angle\theta$	$P + jQ$
Impedance	$\mathbf{Z}$	$\lvert\mathbf{Z}\rvert$ or Z	$Z\angle\theta$ $\lvert\mathbf{Z}\rvert\angle\theta$	$R + jX_L$ $R - jX_C$

Mathematical operations with phasors

Conversion between rectangular and polar coordinates

Phasor with magnitude “A” and angle alpha



$$A_r = A\cos\alpha$$

$$A_j = A\sin\alpha$$

$$A = A_r^2 + A_j^2$$

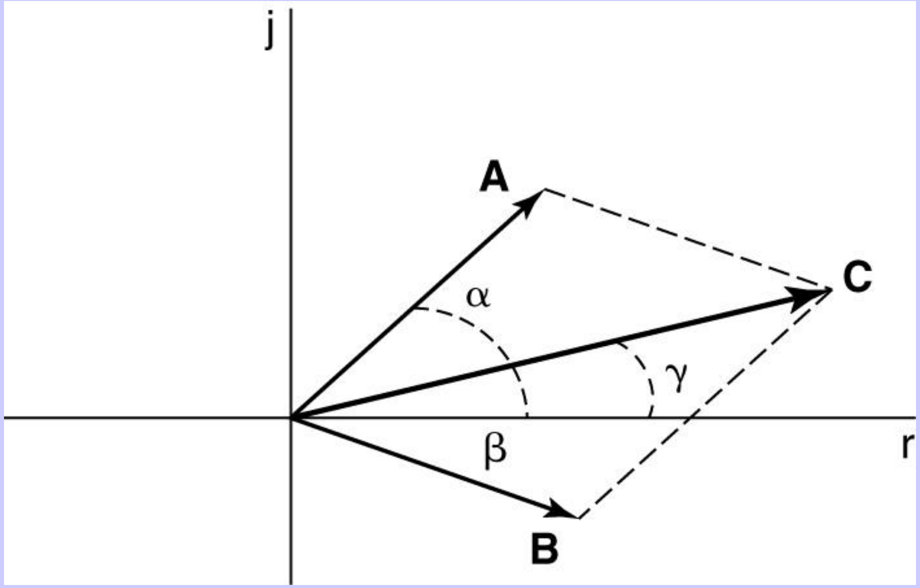
$$\alpha = \tan^{-1}\frac{A_j}{Ar}$$

Addition of complex numbers:

$$\mathbf{A} = A_r + jA_j$$

$$\mathbf{B} = B_r + jB_j$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = (A_r + B_r) + j(A_j + B_j) = C_r + jC_j = C\angle\gamma$$



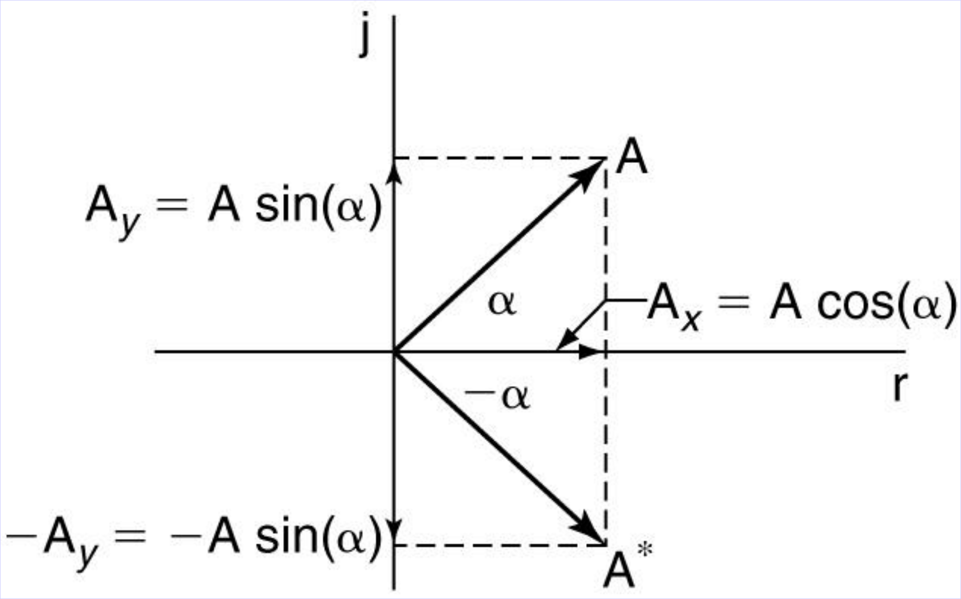
Multiplication of phasors:

$$\mathbf{D} = \mathbf{A} \times \mathbf{B} = A\angle\alpha \times B\angle\beta = AB\angle(\alpha+\beta) = D\angle\delta$$

Division of phasors:

$$\mathbf{E} = \frac{\mathbf{A}}{\mathbf{B}} = \frac{A\angle\alpha}{B\angle\beta} = \frac{A}{B}\angle(\alpha-\beta) = \mathbf{E}\angle\epsilon$$

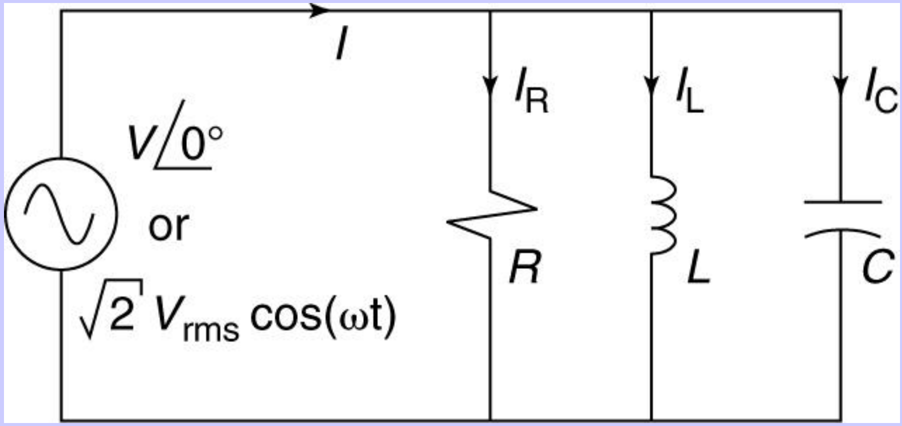
Conjugate:



$$\mathbf{A} = A\angle\alpha = A_r + jA_j$$
$$\mathbf{A}^* = A\angle(-\alpha) = A_r - jA_j$$

Example

Impedance in AC Circuits



Impedance of the inductor:

$$e(t) = L\frac{di(t)}{dt}$$

$$v(t) = V_{peak}\cos(wt)$$

$$i(t) = \frac{1}{L}\int v(t)dt = \frac{1}{L}\int \sqrt{2}V_{rms}\cos(wt)dt$$

$$i(t) = \frac{V_{rms}}{wL}\sqrt{2}\sin(wt)dt$$

Current in the inductor:

$$I_L = \frac{V}{X_L}\angle -90^o = -j\frac{V}{X_L} = \frac{V}{jX_L} = \frac{V}{jwL}$$

NOTE:

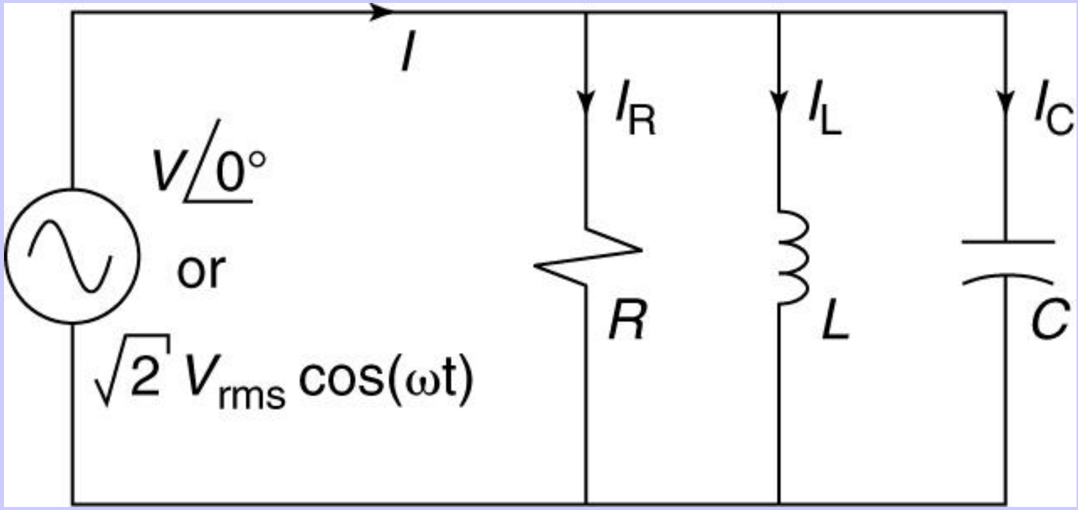
1. There is a phase shift between the voltage and the current. The current is lagging the voltage by 90 degrees for the inductor.
2. The RMS of the current is the RMS of the voltage divided by (wL)

Impedance of the inductor:

$$Z_L = jwL$$

Use Ohm's low in the phasor domain for the inductor by defining the **inductive reactance**

$$X_L = wL$$



Impedance of the capacitor:

$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = C\frac{d}{dt}\sqrt{2}V_{rms}\cos(wt)$$

$$i(t) = -wCV_{rms}\sqrt{2}\sin(wt)$$

NOTE:
1. The current is a negative sine wave, which leads the voltage (cosine wave) by 90 degrees.
2. The RMS of the current is related to the RMS of the voltage by (wC).

Capacitance reactance: $$X_C = \frac{1}{wC}$$

Impedance: $$Z_C = -j\frac{1}{wC} = \frac{1}{jwC}$$

Current in the capacitor: $$I_C = \frac{V}{X_C}\angle 90^o = j\frac{V}{X_C} = jwCV$$

The current in the capacitor will lead the voltage by 90 degrees.

example

Skin Effect

The **skin effect** is the tendency of an alternating electric current (AC) to distribute itself within a conductor so that the current density near the surface of the conductor is greater than that at its core. That is, the electric current tends to flow at the "skin" of the conductor.

The skin effect causes the effective resistance of the conductor to increase with the frequency of the current.

Skin depth is defined as the depth below the surface of the conductor at which the current density decays to 1/e (about 0.37) of the current density at the surface.

The skin depth in copper at various frequencies:

Frequency	**Skin Depth**
60 Hz	8.57 mm
10 kHz	0.66 mm
100 kHz	0.21 mm
1 MHz	66 µm
10 MHz	21 µm

Mitigation

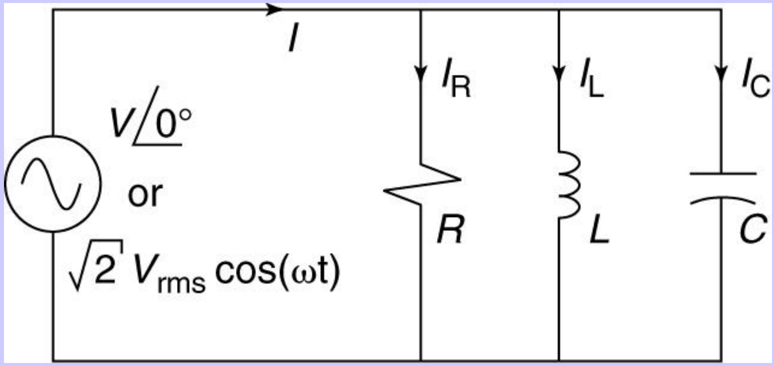
A type of cable called litz wire (from the German *Litzendraht*, braided wire) is used to mitigate the skin effect for frequencies of a few kilohertz to about one megahertz.

It consists of a number of insulated wire strands woven together in a carefully designed pattern, so that the overall magnetic field acts equally on all the wires and causes the total current to be distributed equally among them.

Litz wire is often used in the windings of high-frequency transformers, to increase their efficiency.

Large power transformers will be wound with conductors of similar construction to Litz wire, but of larger cross-section.

Power in Single-Phase AC Circuits



<u>Real Power (Active Power)</u> energy consumed by the load that is converted to heat and is not be returned to its initial stage.
Real power used by the resistor:

$$P_R = V_R \times I_R = \frac{V_R^2}{R} = I_R^2 R$$

<u>Reactive power (Inductor)</u>
Instantaneous current through the inductor varies sinusoidally.
<u>Inductor stores energy as a function of the current</u>, thus magnetic field varies with time.
As current increases the inductor stores more energy in its magnetic field and the power flows from the source to the inductor.
As current decreases the inductor releases energy back To the source.
The inductor does not use any power on average, there is Energy moving back and forth between the source and the Inductor. **This is called REACTIVE POWER.**

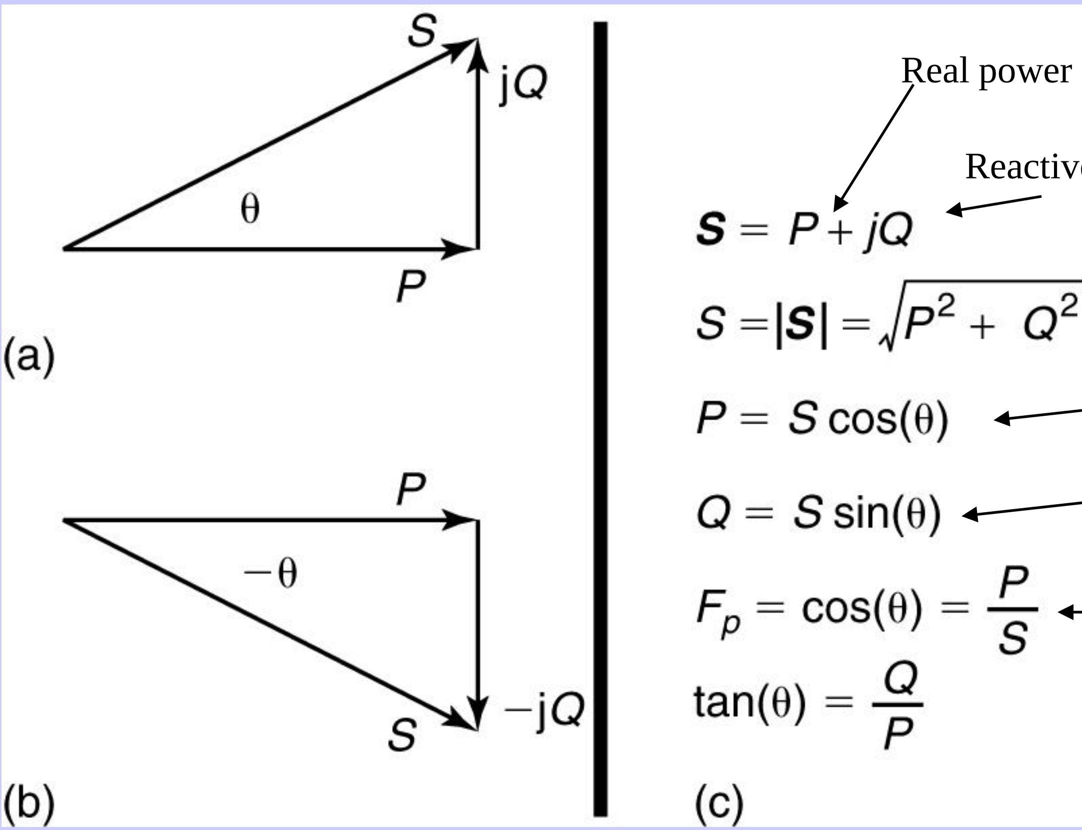
$$Q_L = V_L \times I_L = \frac{V_L^2}{X_L} = I_L^2 X_L$$

<u>Reactive power (Capacitor)</u>
Voltage across the capacitor varies sinusoidally.
<u>Capacitor stores energy as a function of the voltage</u>, thus capacitor's electric field varies with time.
Capacitor draws energy from the source as it charges, and returns energy as it discharges,
The voltage across the capacitor and the current through the inductor are 90 degrees out of phase, thus when inductor is charging the capacitor discharges.

$$Q_C = -V_C \times I_C = -\frac{V_C^2}{X_C} = -I_C^2 X_C$$

example

Complex Power



Real power

Reactive power

Magnitude of the complex (apparent) power

Relationship between the real and apparent power.

Relationship between the reactive and apparent power.

Power factor

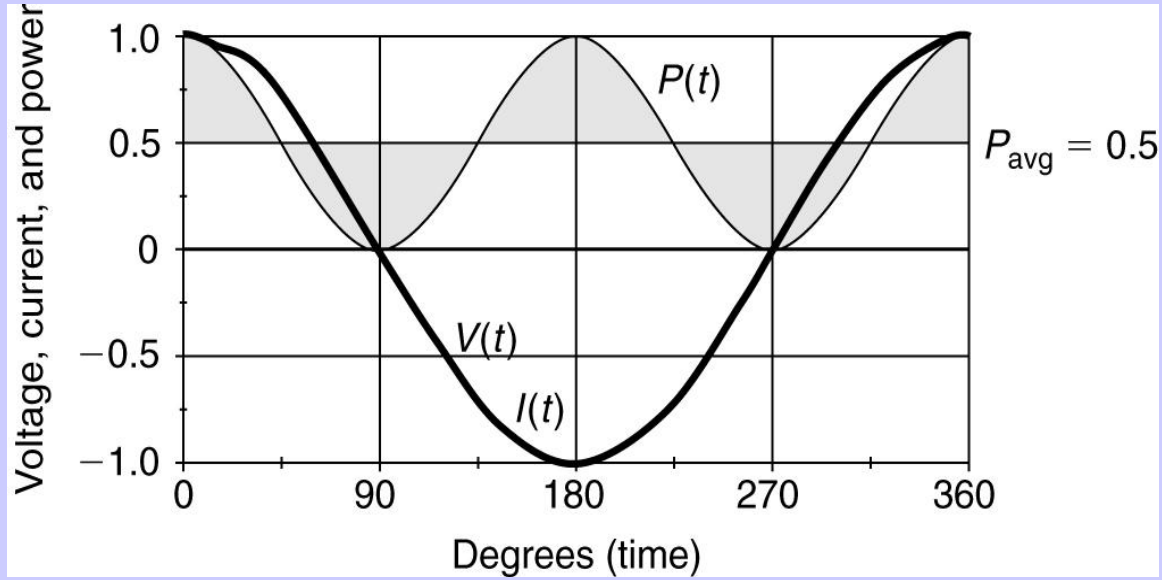
Angle theta is the angle of the current with respect to the voltage.
If the current lags the voltage (inductive load) then theta is negative.
If the current leads the voltage (capacitive load) then theta is positive.
Power factor is always positive and less than 1.

example

Instantaneous AC Power

Resistive load.

Shows instantaneous voltage, current and power for a resistive load



Voltage and current both have a peak value of 1.0 and RMS of 0.707.

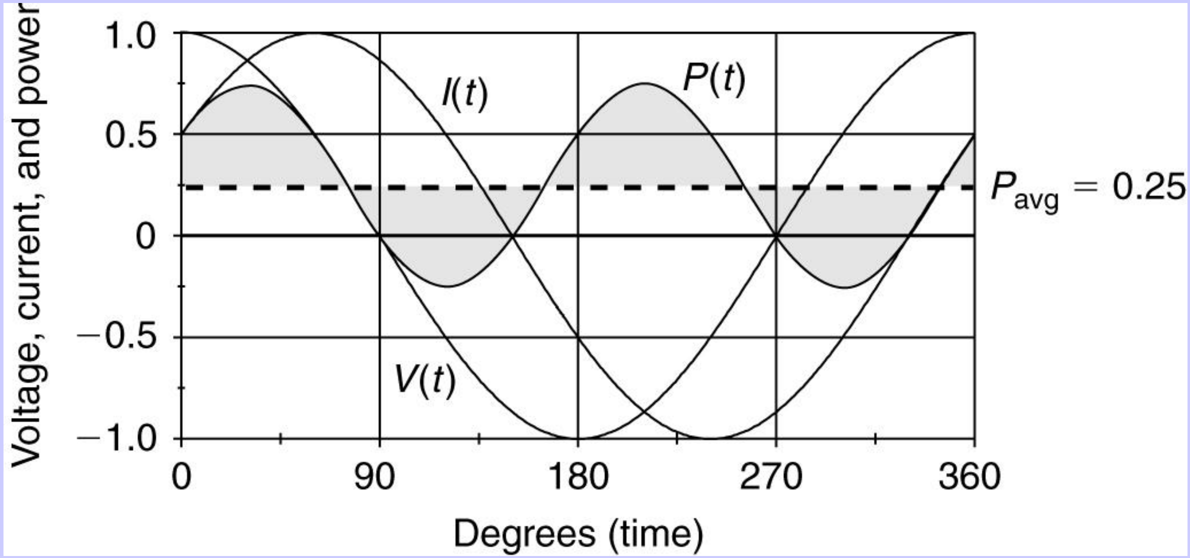
The apparent power $P_{app} = V_{rms} \times I_{rms}$

Since the voltage and current are always in phase, their product is always positive and, so power always flows from the source to the resistive load.

In this case the angle between the voltage and the current is zero so the power factor (= cos (angle)) is unity.

Resistive-inductive (*R*-*L*) load

Shows instantaneous voltage, current and power for a resistive-inductive load



Current lags the voltage by 60 degrees.

Since the voltage and current are out of phase their product may be negative and as a result there are times when the instantaneous power is negative. This represents a return of energy from the load to the source. That energy comes from the magnetic field of the system inductance.

Apparent power is 0.5 VA

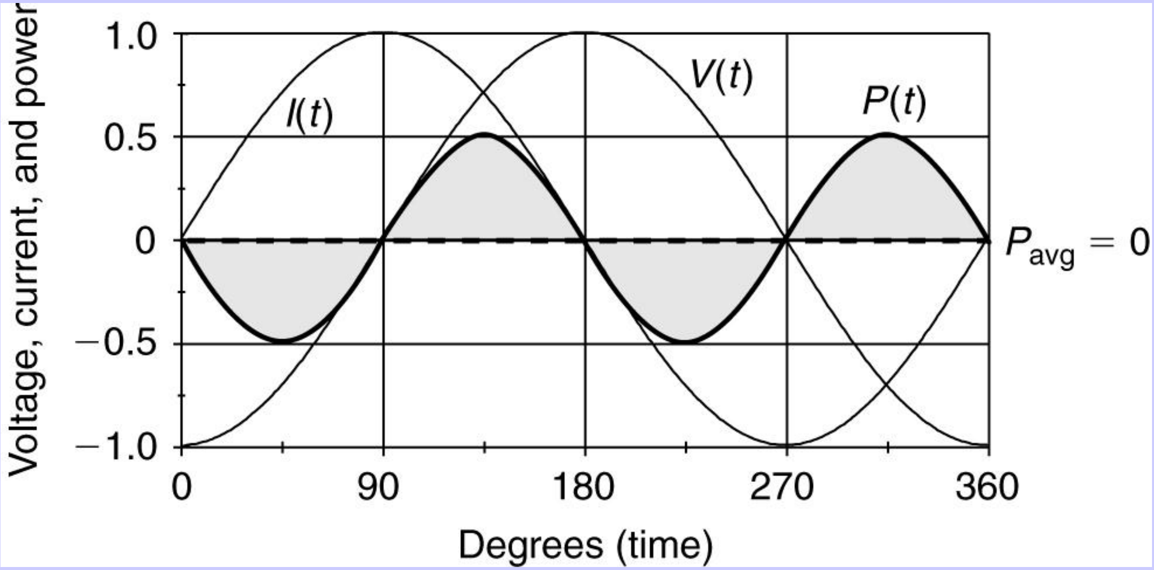
$$P_{app} = V_{rms} \times I_{rms}$$

Average power is 0.25 W=0.5cos (60)

Power factor

Purely inductive load.

Shows instantaneous voltage, current and power for a purely inductive load



Current lags the voltage by 90 degrees.

Instantaneous power has an average of zero. Energy flows from source to the load half of the time and from the load back to the source the other half of the time.

Apparent power is 0.5 VA.

Real power is zero.

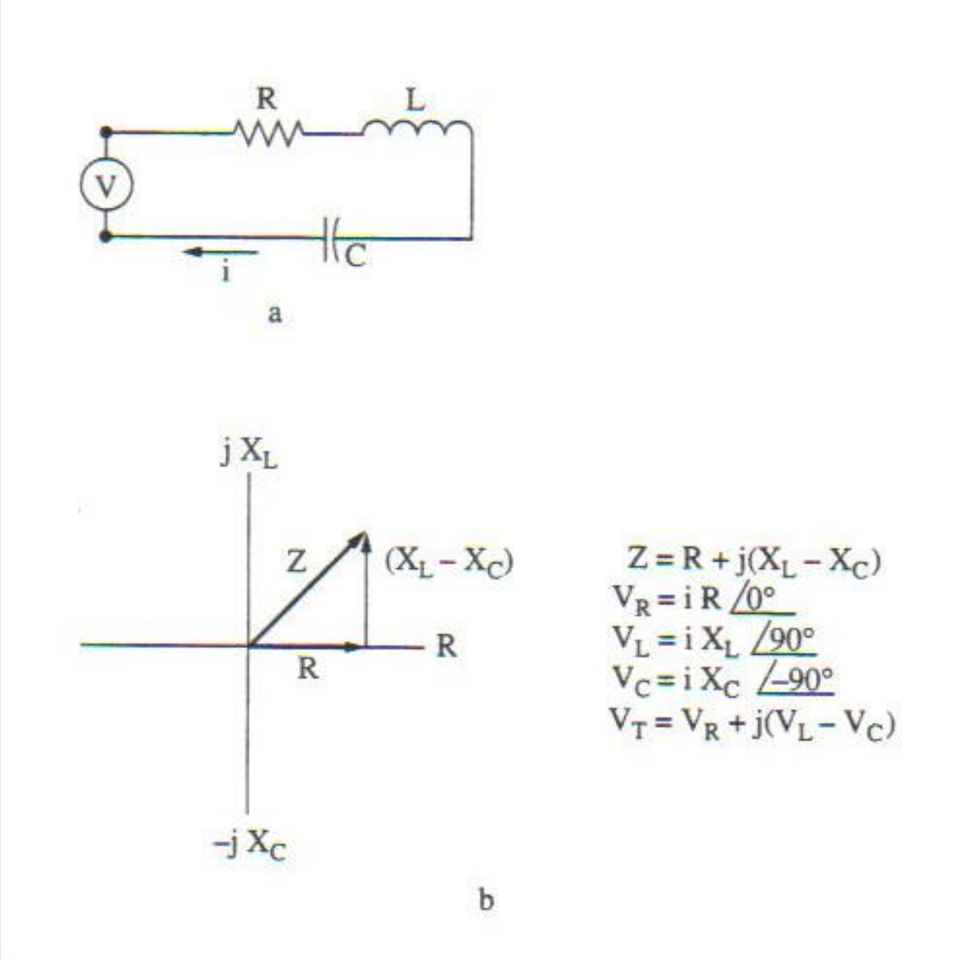
Reactive power is 0.5 Volt-Amperes-Reactive=0.5sin(90)

Power factor

LOAD

The load is the apparatus or devices to which the electrical system provides power. The electrical system loads may do work, such as electric motors or computers, provide comfort, as air conditioning and electric lighting, or entertainment and information, like television.

Impedance diagram for a series RLC circuit:

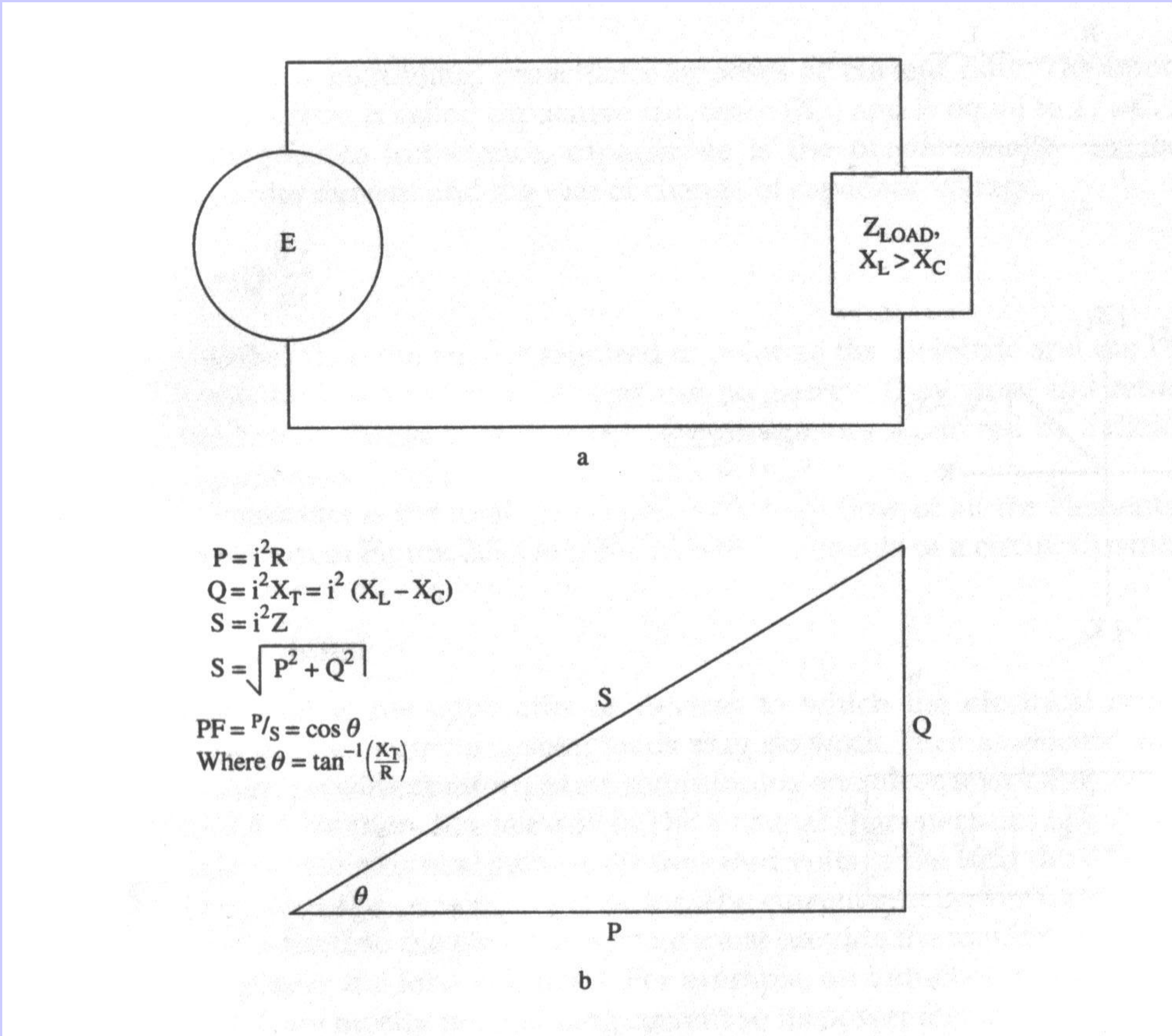


Z – total impedance

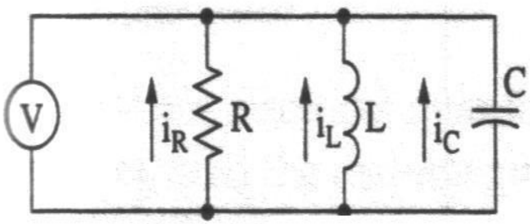
X_L- inductive reactance

X_C- capacitance reactance

The relation between apparent, reactive, and real power:

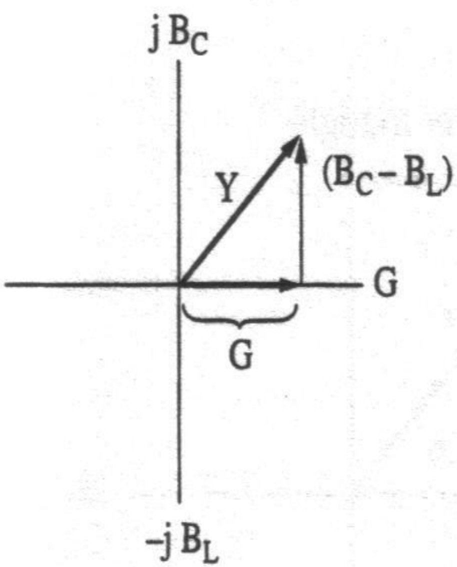


Admittance diagram for parallel RLC circuit



c

Z – total impedance
X_L- inductive reactance
X_C- capacitance reactance



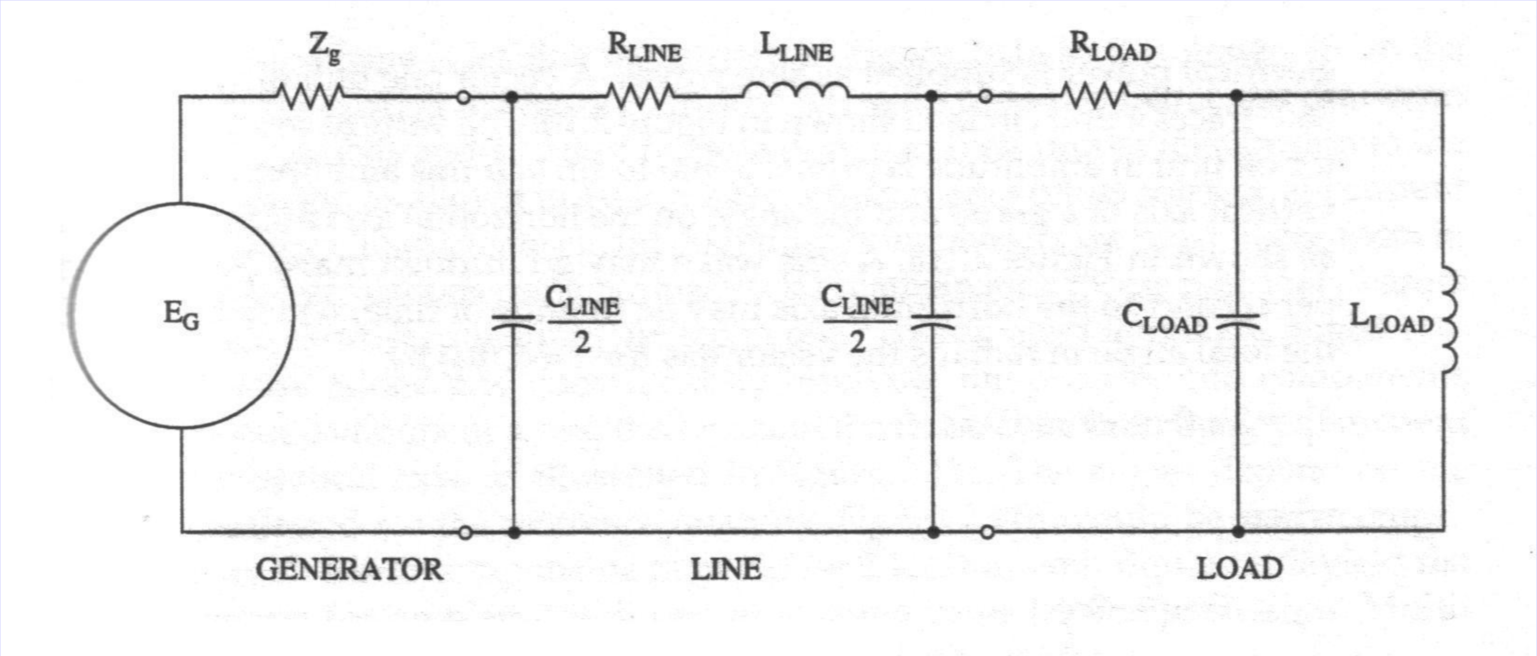
d

$$G = 1/R$$
$$B_C = 1/X_C \angle 90^\circ$$
$$B_L = 1/X_L \angle -90^\circ$$
$$Y = 1/Z = G + j(B_C - B_L)$$

The Line

The source of electricity and the load must be connected. The transmission and distribution lines provide the connection.
The line has all of the basic circuit elements: capacitance, inductance, and resistance. The line resistance is increased by the skin effect. The length and load of the line determine the relative impact of each of the basic elements on transfer of power through the line.

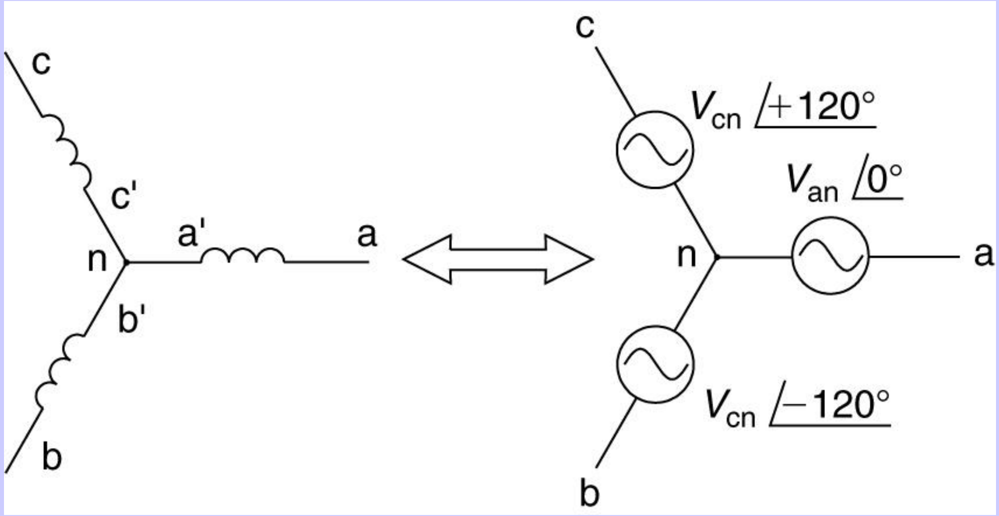
Simple power system equivalent circuit (for medium length line):



Three-Phase AC Circuits

Most industrial and commercial electrical power systems employ a 3-phase configuration.

3-phase systems consist of 3 equal in magnitude voltages and 120 degrees out of phase relative to each other.

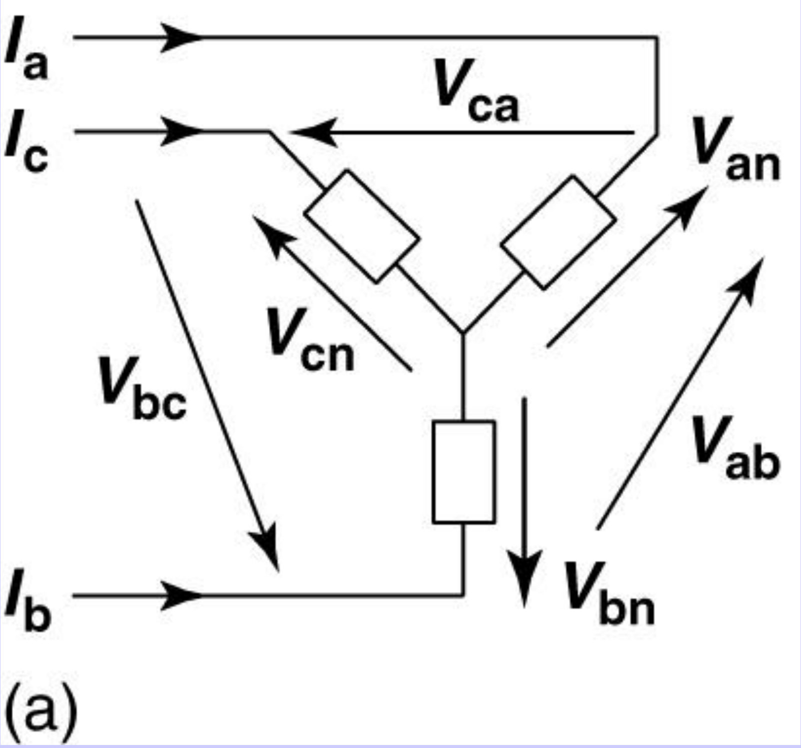


Example: 3-phase generator contains coils that are physically spaced 120 degrees apart and when the rotor turns, the voltages induced in the coils are electrically 120 degrees apart.

The power flowing in 3-phase system has a constant value

example

Three-Phase WYE Configuration



Wye:

Consists of three load components connected with a common point called **neutral.**

Line-to-neutral, phase-to-neutral, branch voltage or simply phase voltage V_P :

the voltage between each phase (line or hot) conductor and the neutral, $V_{cn} V_{an} V_{bn}$

Lin-to-line or simply line voltage V_L :

the voltage between any two conductors, $V_{bc} V_{ca} V_{ab}$

$|V_L| = \sqrt{3}|V_P|$ $\quad$ $I_L = I_P$

Advantage of 3-phase Wye configuration: availability of two voltage levels.

Common system voltages: 277/480 and 120/208V.

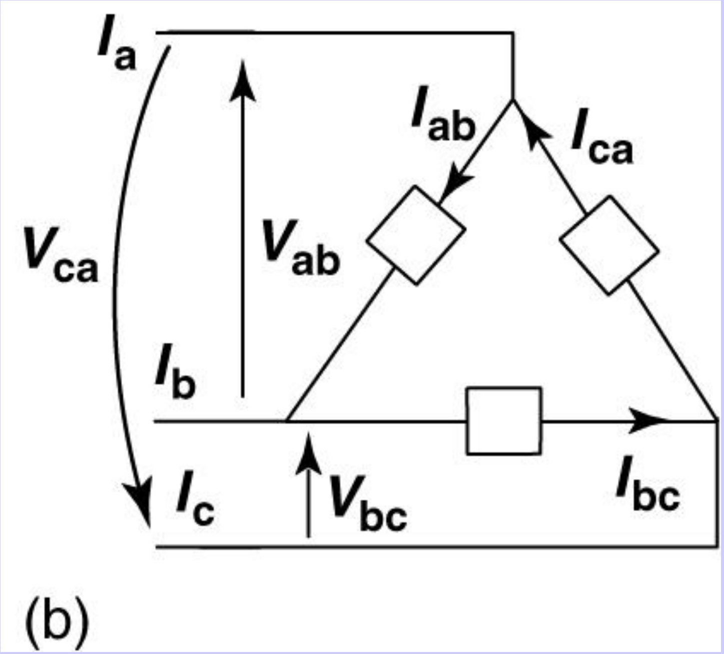
Three-Phase DELTA Configuration

Delta:

Consists of three load components connected end-to-end way and has no neutral point. Phases are connected in a triangle.

The vertices of the triangle are connected to the power system and the current that flows on those conductors is called the line current I_L : $I_a I_b I_c$

The current flowing in the branches, which are connected between the lines, is called the phase current I_P : $I_{bc} I_{ca} I_{ab}$

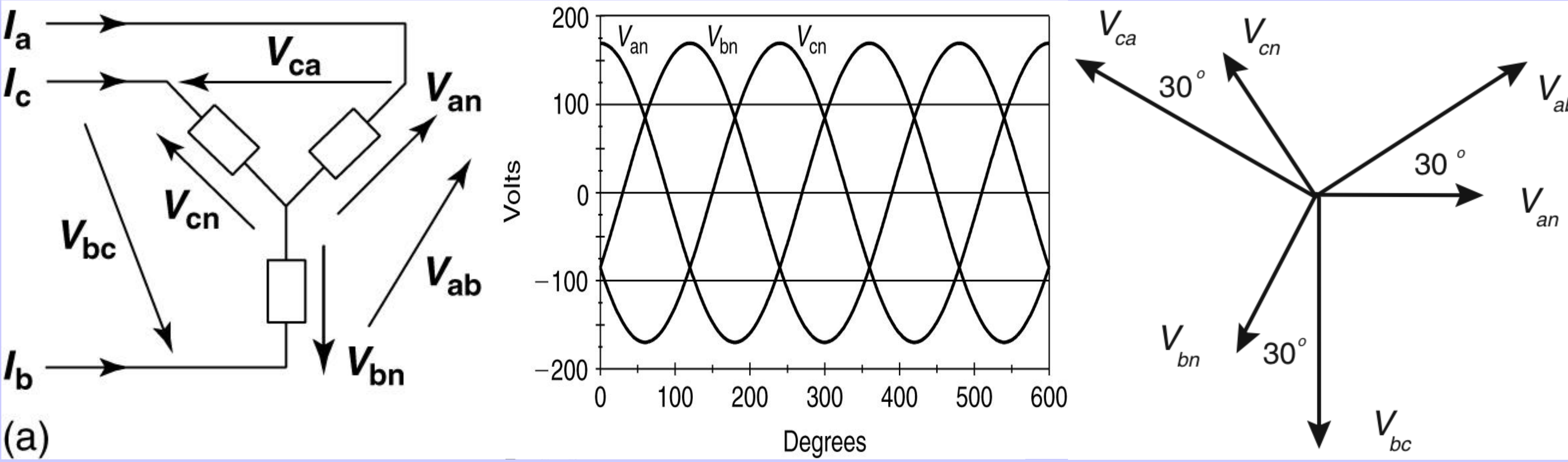


$V_L = V_P$

$|I_L| = \sqrt{3}|I_P|$

Common system voltages: 240V and 480V.

Angular relationship between V_L and V_P



Positive ABC phase sequence (phasors are assumed to rotate in the counterclockwise direction)

Assume the reference voltage $V_{an}\angle 0^o$ and with positive phase sequence $V_{bn}\angle -120^o$ $V_{cn}\angle 120^o$

Using Wye circuit we write equations around one of the loops according to the Kirchoff's law:

$$V_{ab} - V_{an} + V_{bn} = 0 \qquad V_{ab} = V_{an} - V_{bn}$$

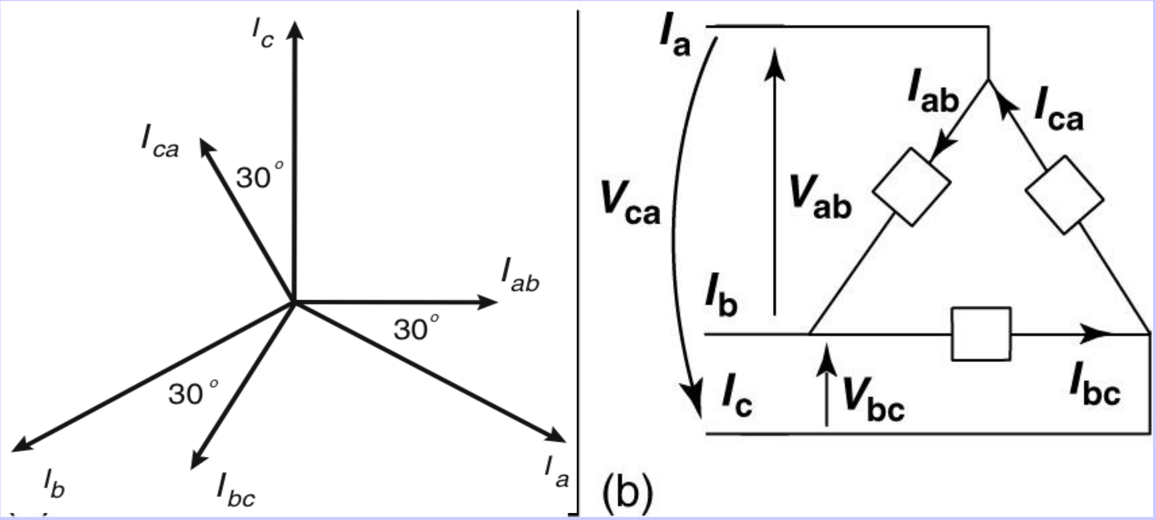
Since V_{an} and V_{bn} have the same magnitude V_P and separated by 120 degrees we can write:

$$V_{ab} = V_P\angle 0^o - V_P\angle -120^o = V_P\angle 0^o + V_P\angle 60^o$$

Adding two phasors we get: $V_{ab} = \sqrt{3}V_P\angle 30^o$

The line voltage leads the phase voltage by 30 degrees and the line voltage is the √3 larger in magnitude than the phase voltage.

Angular relationship between I_L and I_P



The angular relationship between the line and phase current in a delta system and the relationship of their magnitudes:

$$I_a = \sqrt{3} I_P \angle -30^\circ$$

The line current lags the phase current by 30 degrees, assuming a standard ABC phase rotation, and the line current is √3 larger in magnitude than the phase current

Balanced and unbalanced 3-Phase systems

A ***balanced system*** is one in which the line and phase currents and voltages in all three phases are equal in magnitude and separated by 120 degrees, and the impedances in all three phases are identical.
An ***unbalanced system*** is one in which any of the foregoing requirements are not met.

Power calculation in balanced 3-phase systems:

WYE system | Delta system

For a single-phase system, the apparent power is the product of the phase voltage and phase current

$$|S_P| = |V_P||I_P|$$

For a balanced 3-phase system , the total 3-phase apparent power is three times the power consumed by one phase

$$|S_{3P}| = 3|S_P| = 3|V_P||I_P|$$

WYE system:

$$|V_L| = \sqrt{3}|V_P|$$

$$|I_L| = |I_P|$$

$$|S_{3P}| = 3|V_P||I_P| = 3|\frac{V_L}{\sqrt{3}}||I_L| = \sqrt{3}|V_P||I_P|$$

Delta system:

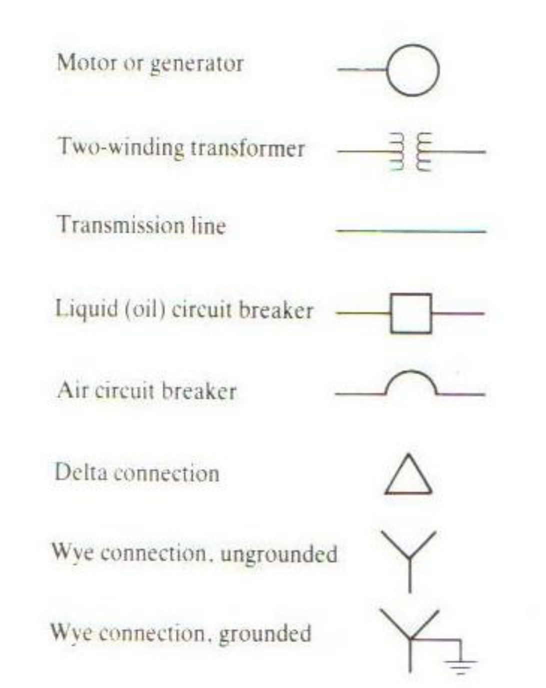
$$|V_L| = |V_P|$$

$$|I_L| = \sqrt{3}|I_P|$$

$$|S_{3P}| = 3|V_P||I_P| = 3|\frac{I_L}{\sqrt{3}}||V_L| = \sqrt{3}|V_P||I_P|$$

Power System Representation

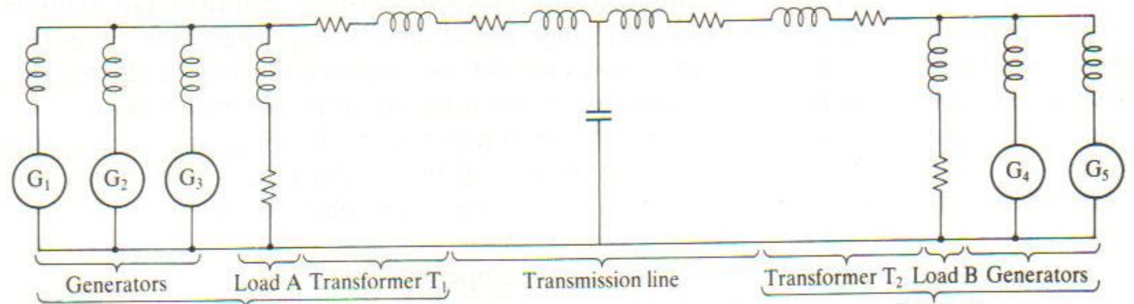
The **basic components** of a power system are generators, transformers, transmission lines, and loads.
The interconnections among these components in the power system is usually represented using **one-line diagram**.
For analysis, the equivalent circuits of the components are shown in a **reactance diagram or an impedance diagram**.



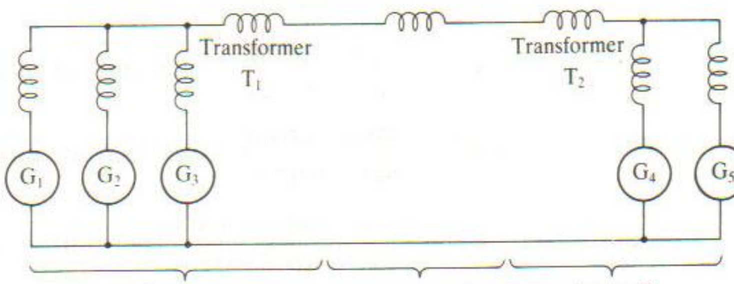
A one-line diagram for a power system consisting of two generating stations connected by a transmission line.
The advantage of such a one-line representation is its simplicity: one phase represents all three phases of the balanced system; the equivalent circuits of the components are replaced by their standard symbols.

The symbols used to represent the typical components of a power system.

IMPEDANCE AND REACTANCE DIAGRAMS



(a)



(b)

The one-line diagram serves as the basis for a circuit representation that includes the equivalent circuits of the components of the power system.

Such a representation is called an **impedance diagram, or a reactance diagram if resistances are neglected.**

Assumptions:

- A generator can be represented by a voltage source in series with an inductive reactance.
- The internal resistance of the generator is negligible compared to the reactance.
- The loads are inductive.
- The transformer core is ideal, and the transformer may be represented by a reactance.
- The transmission line is a medium-length line and can be denoted by “T” circuit. An alternative representation, such as “Π” circuit, is equally applicable.

The reactance diagram is drawn by neglecting all resistances, the static loads, and the capacitance of the transmission line.

The Per Unit Concept

A convenient method of calculating electrical power quantities.
A shorthand method of solving power system problems has been developed to eliminate many of the manipulations required in systems with more than one voltage.

$$Z_{ohms} = Z_{pu} Z_{base}$$

$$I_{amps} = I_{pu} I_{base}$$

$$V_{kV} = V_{pu} V_{base}$$

$$VA_{voltamperes} = VA_{pu} VA_{base}$$

examples

The Per Unit Concept (cont.)

$$base\ I = \frac{base\ kVA}{base\ kV}$$

$$base\ Z = \frac{base\ V}{base\ I}$$

$$base\ Z = \frac{base\ V^2}{base\ VA} = \frac{(base\ kV)^2 \times 1000}{base\ kVA} = \frac{(base\ kV)^2}{base\ MVA}$$

$$pu\ VA = \frac{VA}{base\ VA} = \frac{kVA}{base\ kVA}$$

$$pu\ V = \frac{V}{base\ V}$$

$$pu\ I = \frac{I}{base\ I}$$

$$pu\ Z = \frac{base\ V}{base\ I} = \frac{Z}{base\ Z} = \frac{Z \times (base\ VA)}{(base\ V)^2} = \frac{Z \times (base\ kVA)}{(base\ kV)^2 \times 1000}$$

$$actual\ value = (pu\ value) \times (base\ value)$$

The Per Unit Concept: Change of Base

The impedance of transformers, generators, and motors are often given in per unit or percent where the **base** used **is the machine nominal voltage and volt-amperes**.

The machine base is seldom the base of the system under analysis so a convenient method of changing the base of PU impedance is needed.

Consider a system in which base 2 is to be used for analysis, but base 1 is the base of a machine's PU impedance.

The actual machine impedance is the same in both system bases:

$$Z = pu\ Z_1 \frac{V_{b1}^2}{VA_{b1}}$$

$$Z = pu\ Z_2 \frac{V_{b2}^2}{VA_{b2}}$$

Since Z is the same in both systems:

$$pu\ Z_1 \frac{V_{b1}^2}{VA_{b1}} = pu\ Z_2 \frac{V_{b2}^2}{VA_{b2}}$$

Solving for PU Z_1

$$Z_{1pu} = Z_{2pu} \frac{V_{b2}^2}{V_{b1}^2} \frac{VA_{b1}}{VA_{b2}}$$

In terms of old base and new base:

$$new\ base\ Z_{pu} = old\ base\ Z_{pu} \frac{(old\ base\ V)^2 (new\ base\ VA)}{(new\ base\ V)^2 (old\ base\ VA)}$$

Per Unit with Balanced 3-Phase System

Method I:

Use the three-phase line to line voltages and volt-ampere and to calculate the line current:

$$VA = \sqrt{3} I_L \, V_{LL}$$

subscript LL is line to line. The PU voltage is the same whether line to line or phase to neutral values of voltage are used since $V_{LL} = \sqrt{3} V_{LN}$

where subscript LN is line to neutral.

The same is true for PU VA since $VA = 3 \times single - phase \; VA$

Calculated correctly the base impedance will be the same. The appropriate equations are:

$$base \; I = \frac{base \; three - phase \; kVA}{\sqrt{3} \; base \; kV_{LL}} = \frac{base \; phase \; kVA}{base \; kV_{LL}}$$

$$base \; Z = \frac{(base \; kV_{LL}/\sqrt{3})^2 \times 1000}{base \; three - phase \; kVA/3} = \frac{(base \; kV_{LL}/)^2 \times 1000}{base \; three - phase \; kVA} = \frac{(base \; kV_{LL})^2}{three - phase \; base \; MVA}$$

Method II:
In this method we convert all circuit parameters to phase-to-neutral values and convert back to line-to-line values at the end of the problem.
Delta-connected loads must be converted to their WYE equivalent impedance to obtain their PU impedance.
Recall: WYE-Delta conversion, with ABC = WYE values and XYZ = delta values:

$$Z_A = \frac{Z_Y Z_Z}{Z_X + Z_Y + Z_Z}$$

, in a balanced Delta system $Z_X = Z_Y = Z_Z = Z_D$

$$\Rightarrow \quad Z_A = \frac{Z_D^2}{3Z_D} = \frac{delta\ phase\ Z}{3}$$

$$Z_X = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$

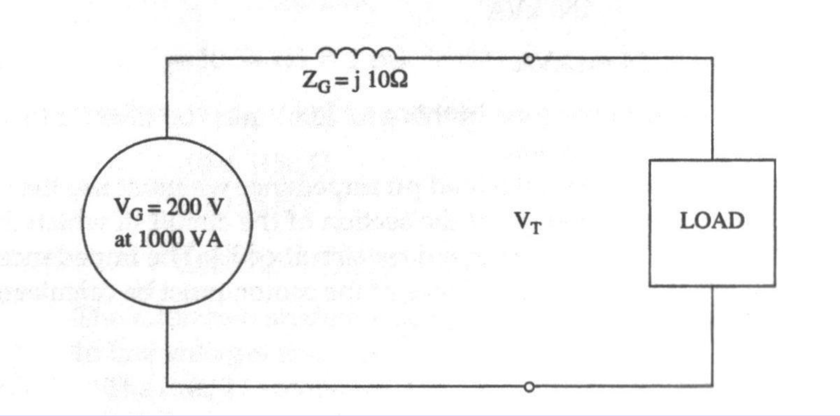
$$Z_A = Z_B = Z_C = Z_{WYE}$$

$$\Rightarrow \quad Z_D = \frac{3Z_{WYE}^2}{Z_{WYE}} = 3 \times (WYE\ impedance)$$

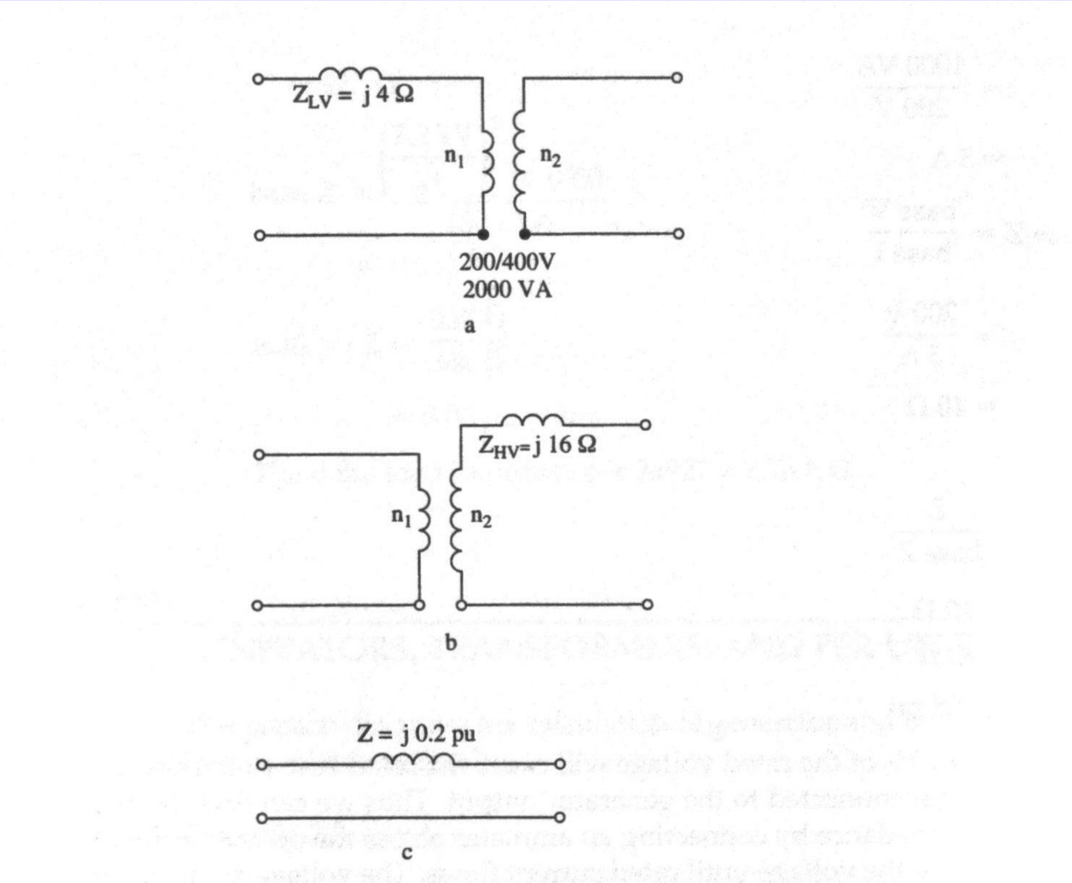
Example pages 55-58
58-60(ex 2.5,2.6),
61-68(ex 2.7,2.8)

Scan figures of the examples

Example 2.5



Example 2.6



Example 2.7

Example 2.8