

CHAPTER 4

Distribution Transformers

Introduction

A **transformer** is an electrical device that transfers energy from one circuit to another purely by magnetic coupling.

Relative motion of the parts of the transformer is not required for transfer of energy.

Transformers are often used to convert between high and low voltages and to change impedance.

Transformers alone **cannot** do the following:

- Convert DC to AC or vice versa
- Change the voltage or current of DC
- Change the AC supply frequency.

However, transformers are **components** of the systems that perform all these functions.

Transformer Nameplate Data



Transformer nameplates contain information about the size of the transformer in terms of how much apparent power (rated in kVA) it is designated to deliver to the load on a continuous basis as well primary and secondary voltages and currents.

Example: 75 kVA, 720-240*120V

U-W primary winding is rated U volts and secondary winding is rated V volts

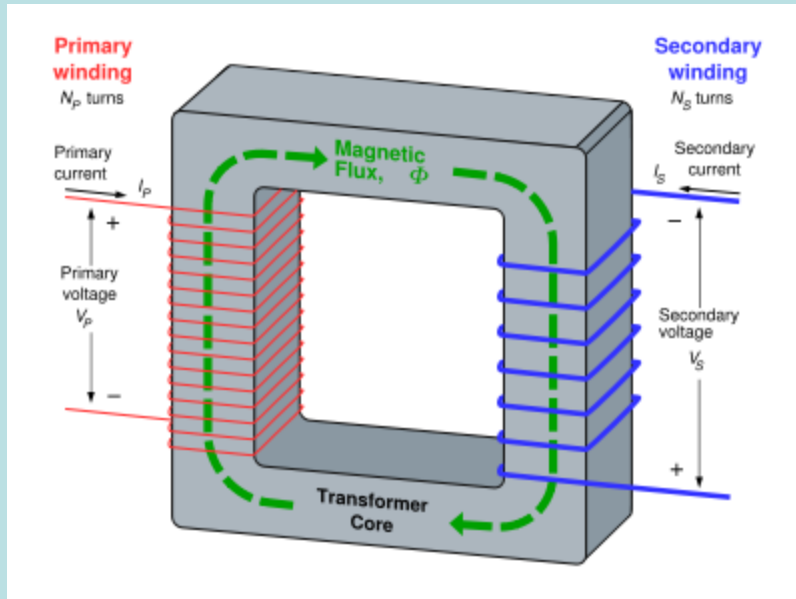
U/W indicates that two voltages are from the same winding and that both voltages are available

U*V two part winding that can be connected in series or parallel to give higher voltage but only one voltage is available at a time.

U Y/W the Y indicates a 3-phase winding connected in a WYE configuration.

Basic principles

An idealized step-down transformer showing resultant flux in the core



The **transformer** may be considered as a simple two-wheel 'gearbox' for electrical voltage and current.

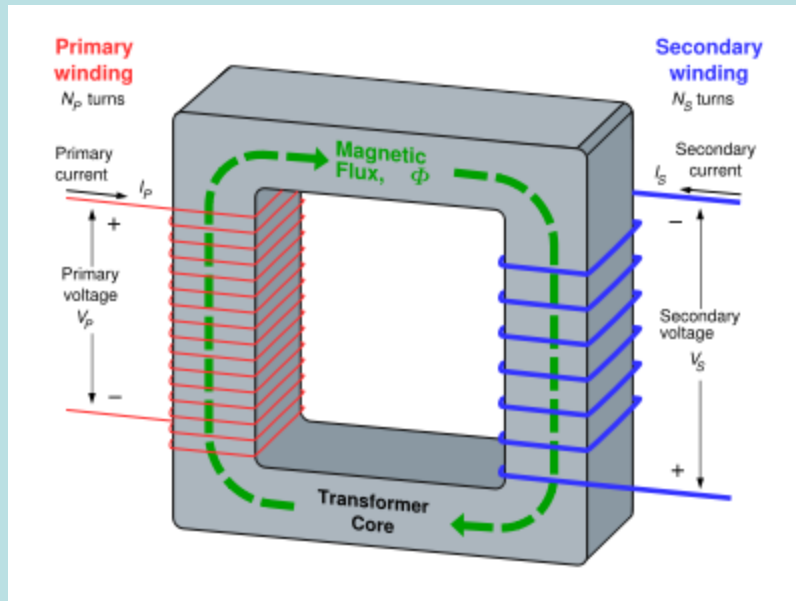
The primary winding is analogous to the input shaft

The secondary winding is analogous to the output shaft.

In this comparison, *current is equivalent to shaft speed and voltage to shaft torque.*

In a gearbox, mechanical power (speed multiplied by torque) is constant (neglecting losses) and is equivalent to electrical power (voltage multiplied by current) which is also constant.

An idealized step-down transformer showing resultant flux in the core



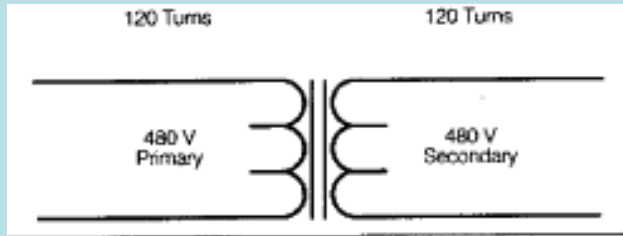
The gear ratio is equivalent to the transformer step-up or step-down ratio.

A **step-up transformer** acts analogously to a reduction gear (in which mechanical power is transferred from a small, rapidly rotating gear to a large, slowly rotating gear): it trades current (speed) for voltage (torque), by transferring power from a primary coil to a secondary coil having more turns.

A **step-down transformer** acts analogously to a multiplier gear (in which mechanical power is transferred from a large gear to a small gear): it trades voltage (torque) for current (speed), by transferring power from a primary coil to a secondary coil having fewer turns.

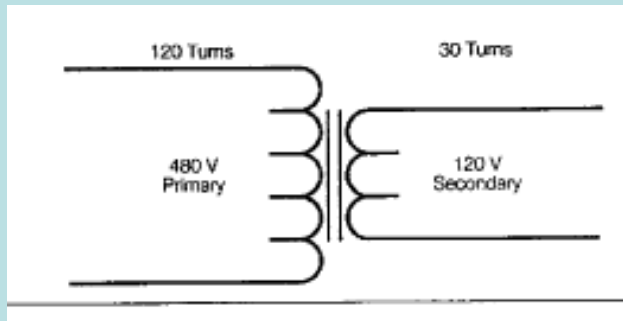
1/1 Transformer

When the primary winding and the secondary winding have the same amount of turns there is no change voltage, the ratio is 1/1 unity.



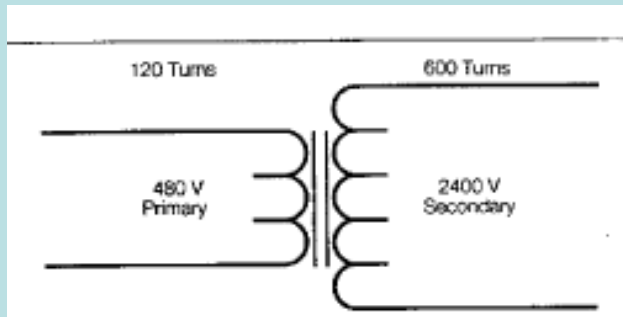
Step-Down Transformer

If there are fewer turns in the secondary winding than in the primary winding, the secondary voltage will be lower than the primary.

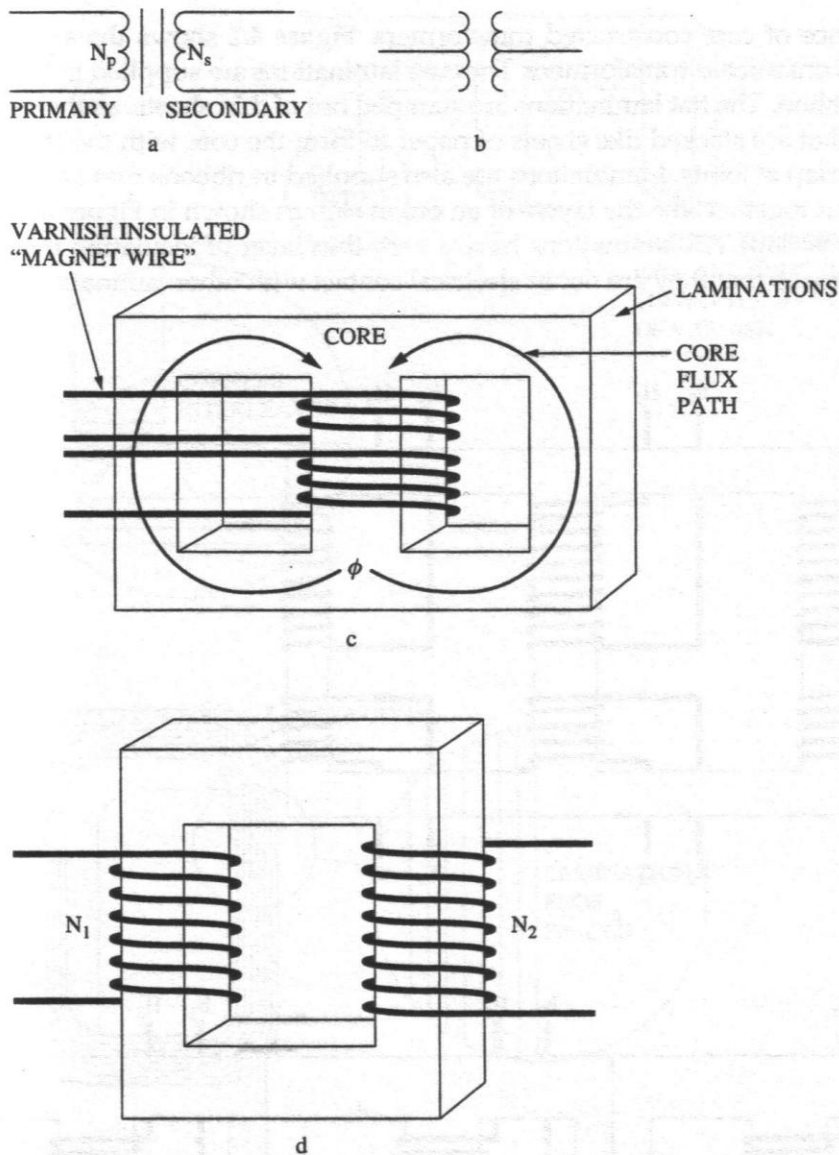


Step Up Transformers

If there are fewer turns in the primary winding than in the secondary winding, the secondary voltage will be higher than the secondary circuit.



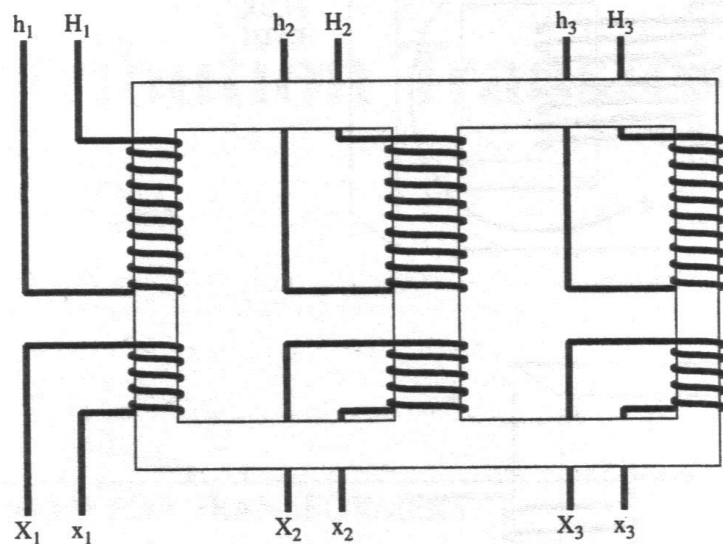
Shell and core constructed single-phase transformers



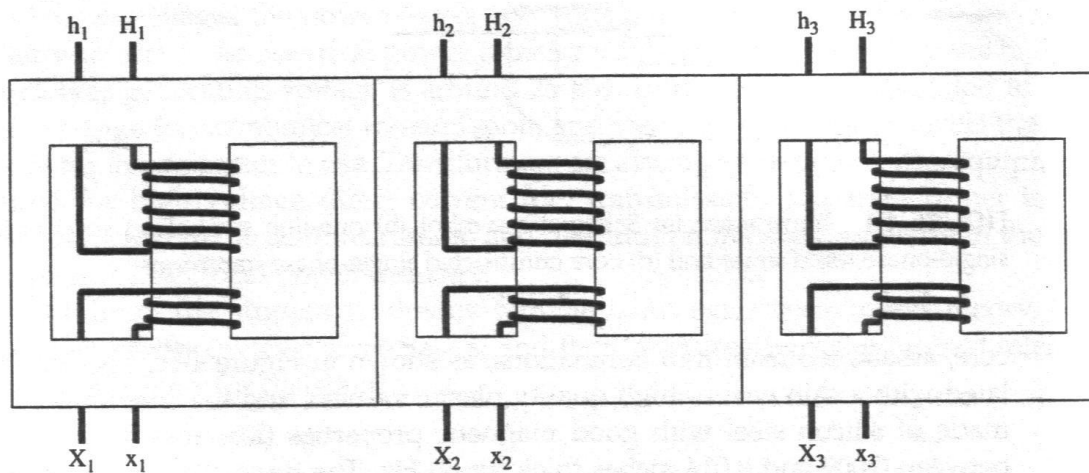
The shell type has less magnetic flux leakage than the core type but requires more core material.

Shell constructed three-phase transformers are less frequently used because of their added cost, and the good performance of core constructed transformers.

Shell and core constructed 3-phase transformers

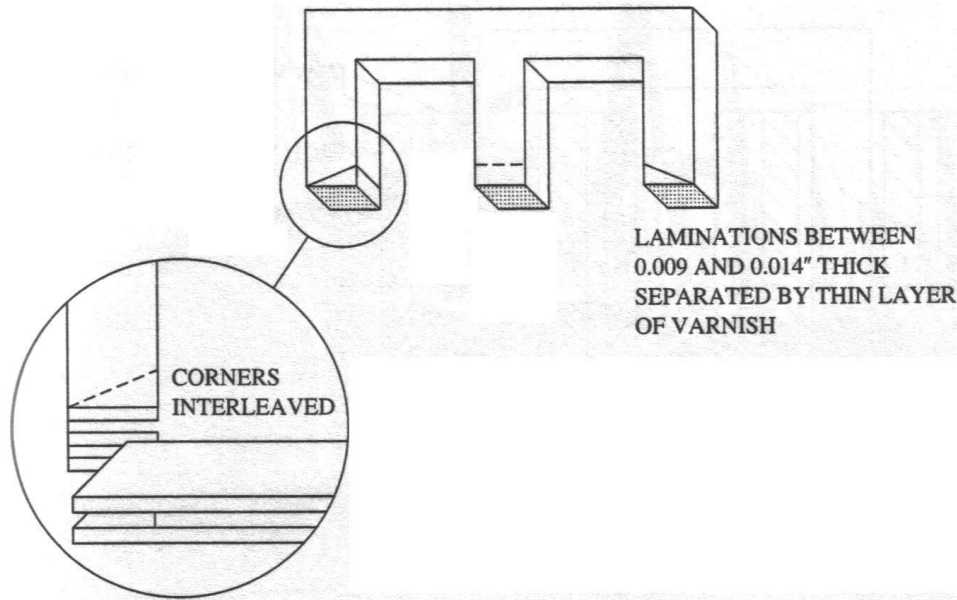


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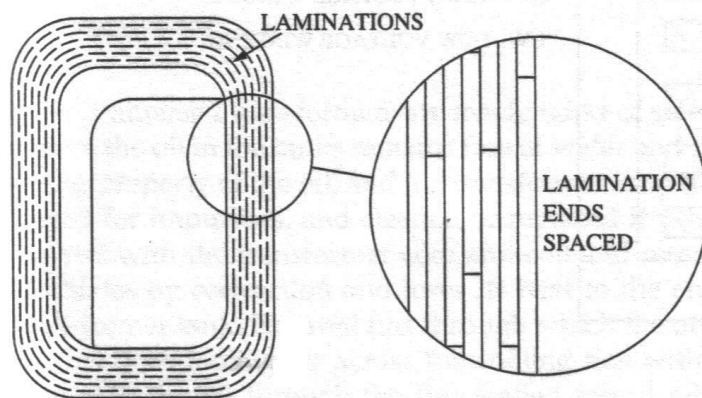


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Core structure



a



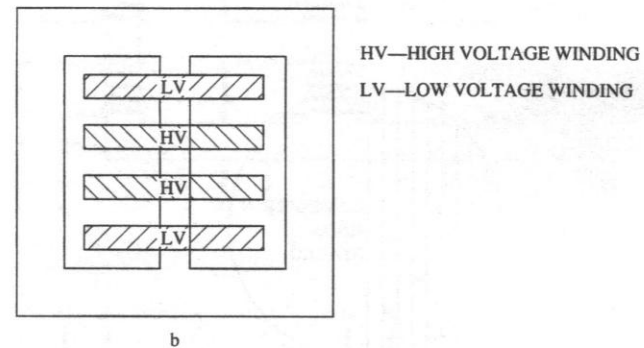
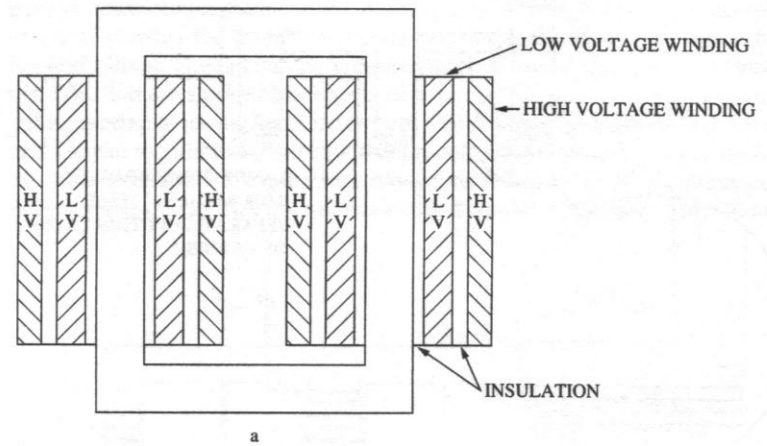
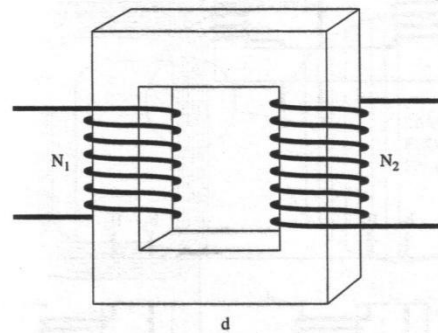
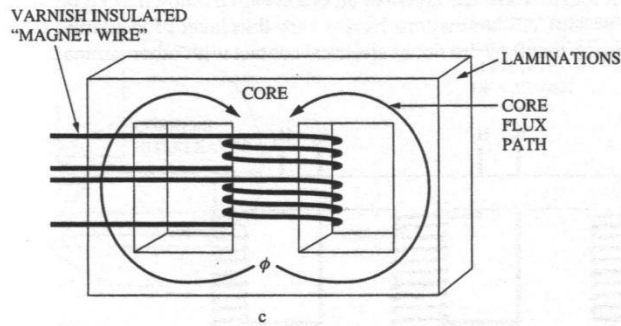
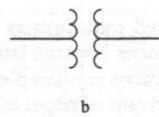
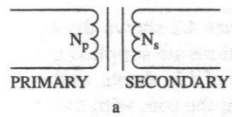
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The core laminations are supplied in two forms: flat and ribbon.

The flat laminations are stamped out of thin sheets, that are stacked like sheets of paper to form the core with the laminations cut to overlap at joints.

Ribbons laminations are rolled out and put together like the layers of an onion skin.

All laminations have a very thin layer of insulating material on their surface so that they are not in electrical contact with other laminations.



Higher voltage transformers are wound with either layer or disk type windings for greater separation, with disk favored for very high voltages. The entire core and winding is placed in a support structure and the winding leads brought out to terminations.

Transformer classification by cooling:

1. *Dry transformers.*
2. *Oil immersed transformers*

Dry types are air cooled, primarily by convection.

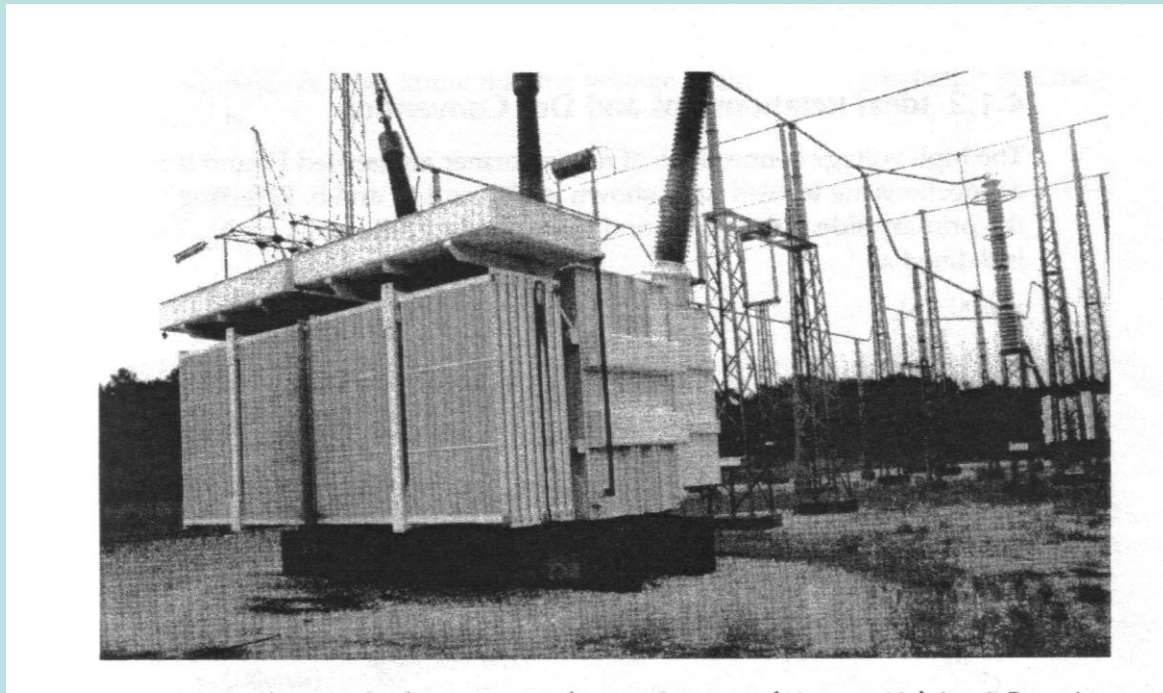
Oil immersed transformers in which the windings and core are immersed in oil, are both cooled and helped in insulation by the oil.

Almost all utility power and distribution transformers are oil.

Dry types are used primarily where minimum cost is a factor and the transformer is supplied by the customer such as in apartment house and a building distribution systems.

Because the cooling of dry transformers is by convection they are very intolerant of overloads.

Dry types must be in an enclosure for safety.



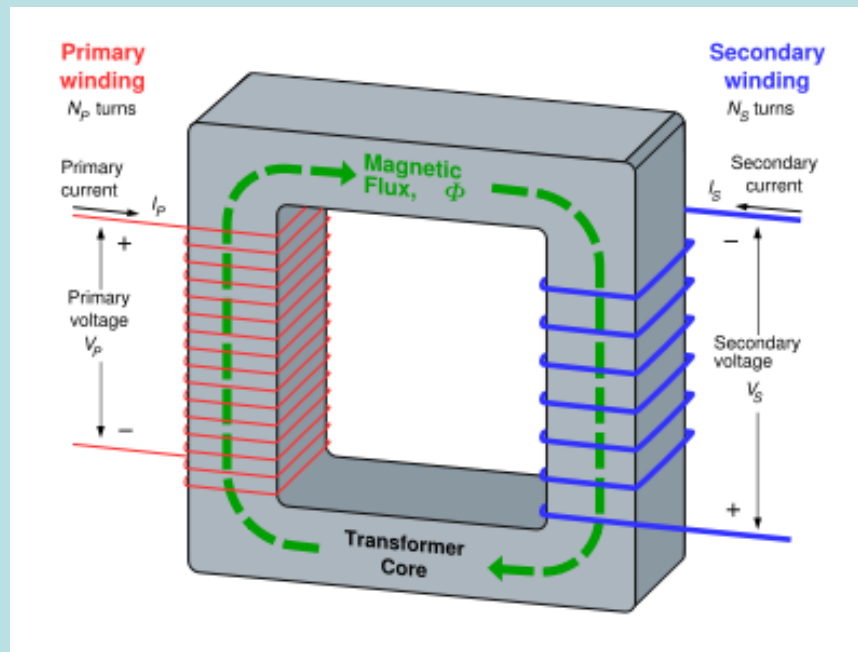
Oil immersed transformers are inside tanks of steel.

Oil in the tanks should remain free of water and impurities to ensure the insulating property of the oil. The oil circulates by convection and loses its heat to the environment through the steel transformer tank. External fins through which the oil can flow improves the cooling.

Each cooling improvement increases the capacity of the transformer.

Power transformer ratings often follow the following pattern: MVA-25/33/42, which means the capacity is 25 MVA with convection circulated oil (OA), 33 MVA with forced air added (FA), and 42 MVA with both forced oil and air (FOA).

Very high power transformers may have heat exchangers in which heat from the oil is transferred to water and the water is then cooled. Oil transformers are much more tolerant of overloads than dry types. Overloads that result in excessive heating of either type shortens the transformer life by degrading the insulation (and oil in oil types).



A simple transformer consists of two electrical conductors called the **primary winding** and the **secondary winding**. If a time-varying voltage V_P is applied to the primary winding of N_P turns, a current will flow in it producing a [magnetomotive force](#) (MMF).

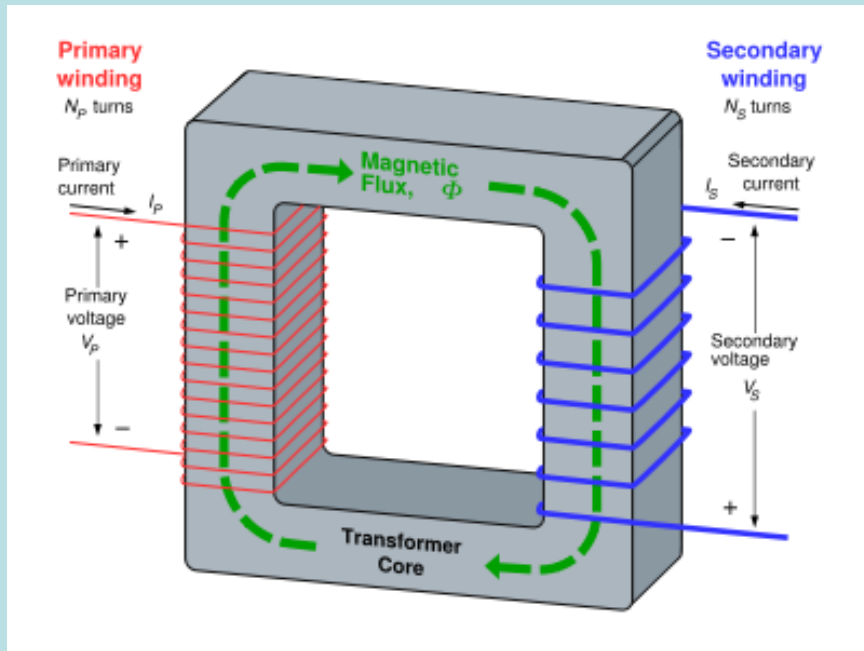
The primary MMF produces a varying [magnetic flux](#) Φ_P in the core.

In accordance with [Faraday's Law](#), the voltage induced across the primary winding is proportional to the rate of change of flux :

$$v_P = N_P \frac{d\Phi_P}{dt}$$

Similarly, the voltage induced across the secondary winding is:

$$v_S = N_S \frac{d\Phi_S}{dt}$$



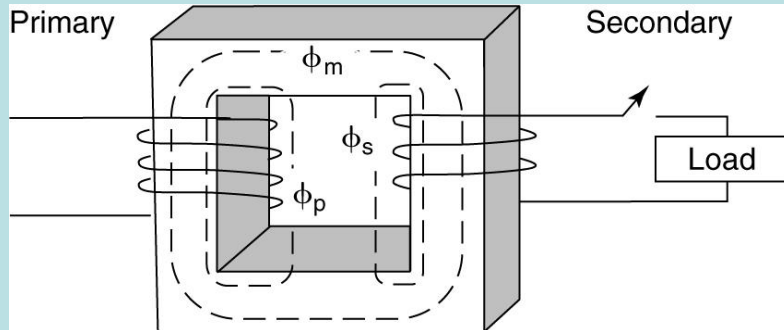
With perfect flux coupling, the flux in the secondary winding will be equal to that in the primary winding, and so we can equate Φ_P and Φ_S .

$$v_P = N_P \frac{d\Phi_P}{dt} \quad \Bigg/ \quad v_S = N_S \frac{d\Phi_S}{dt} \quad \Longrightarrow \quad \frac{v_P}{v_S} = \frac{N_P}{N_S}.$$

Hence, in an ideal transformer, the ratio of the primary and secondary voltages is equal to the ratio of the number of turns in their windings, or alternatively, the voltage per turn is the same for both windings.

This leads to the most common use of the transformer: to convert electrical energy at one voltage to energy at a different voltage by means of windings with different numbers of turns.

The Universal EMF equation



Faraday's law tells:

$$e = N \frac{d\phi}{dt}$$

If we apply sinusoidal voltage to the transformer:

$$e(t) = \sqrt{2}E_{RMS}\sin(\omega t)$$

Flux is given by:

$$\phi(t) = \frac{1}{N} \int_0^t \sqrt{2}E_{RMS}\sin(\omega t)dt$$

$$\begin{aligned}\phi(t) &= \frac{-\sqrt{2}E_{RMS}}{\omega N} \cos(\omega t) = \frac{-\sqrt{2}E_{RMS}}{2\pi f N} \cos(\omega t) = -\phi_{max} \cos(\omega t) \\ E_{RMS} &= \frac{2\pi f N \phi_{max}}{\sqrt{2}} \\ E_{RMS} &= 4.44 f N \phi_{max} = 4.44 f N B_{max} A_c\end{aligned}$$

This equation demonstrates a definite relation between the voltage in a coil, the flux density, and the size of the core. The designer must make trade-offs among the variables when design a transformer.

Voltage and Current

For the ideal transformer, all the flux is confined to the iron core and thus links the primary and secondary.

$$E_{RMS} = 4.44fN\phi_{max} = 4.44fNB_{max}A_c$$



$$E_p = 4.44fN_p\phi_{max}$$

$$E_s = 4.44fN_s\phi_{max}$$



$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a$$

Turns ratio

For step-down transformer, the primary side has more turns than secondary, therefore $a > 1$;

For step-up transformer, the primary side has fewer turns than secondary, therefore $a < 1$;

Because the losses are zero in the ideal transformer, the apparent power in and out of the transformer must be the same:

$$P_{in} = P_{out} = V_p I_p = V_s I_s$$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$$

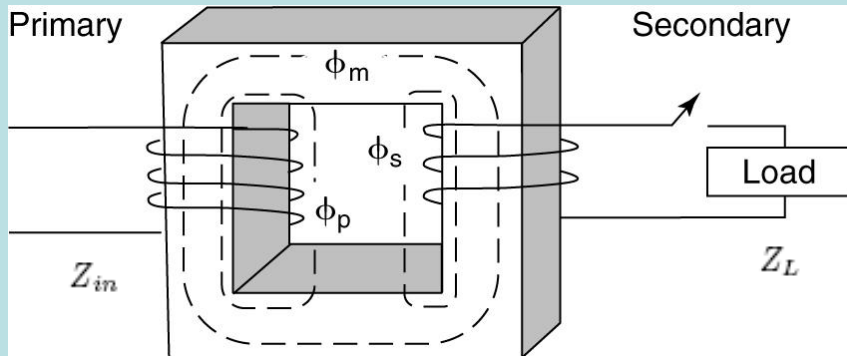
Ratio of the currents is inverse of the voltage ratio or the inverse of the turns ratio.

It makes sense: if we raise the voltage level to a load with a step-up transformer, then the secondary current drawn by the load would have to be less than the primary current, since the apparent power is constant

Example

Impedance

Due to the fact that the transformer changes the voltage and current levels in opposite directions, it also changes the apparent impedance as seen from the two sides of the transformer.



Ohm's law applied at the load:

$$Z_L = \frac{V_s}{I_s}$$

Recollect:

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$$

$$Z_L = \frac{\frac{V_p}{a}}{aI_p} = \frac{V_p}{a^2 I_p} = \frac{Z_{in}}{a^2}$$
$$Z_{in} = a^2 Z_L$$

The Reflected (referred) impedance
(the impedance looking into
the primary side of the
transformer)

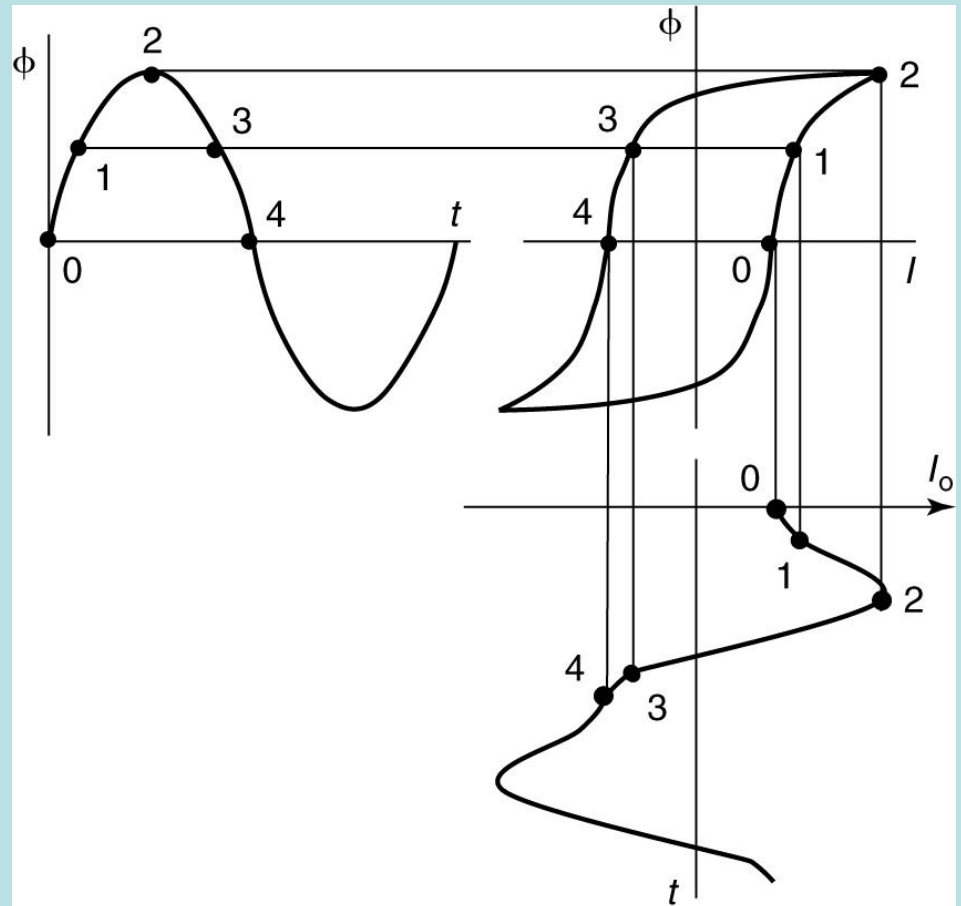
When we move an impedance from the secondary to the primary side of the transformer we multiply by the turns ratio squared. When moving the impedance from the primary to the secondary, we divide it by the turns ratio squared.

This process is called referring the impedance to the side we move it, and allows us use transformers to match impedances between a source and a load

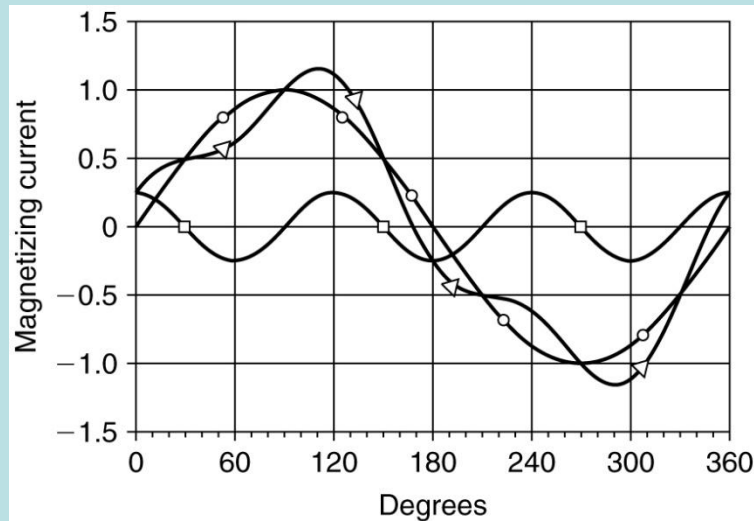
Exciting Current

In real life we deal with real transformers which require current in the primary winding to establish the flux in the core. The current that establishes the flux is called the exciting current. Magnitude of the exciting current is usually about 1%-5% of the rated current of the primary for power transformers but may be much higher for small transformers.

According to Faraday's law if we apply a sinusoidal voltage to the transformer, then the flux will also be sinusoidal, but due to the non-linearity of B-H curve for the iron core, the current will not be sinusoidal even if the flux is sinusoidal and the current will be out of phase with flux.

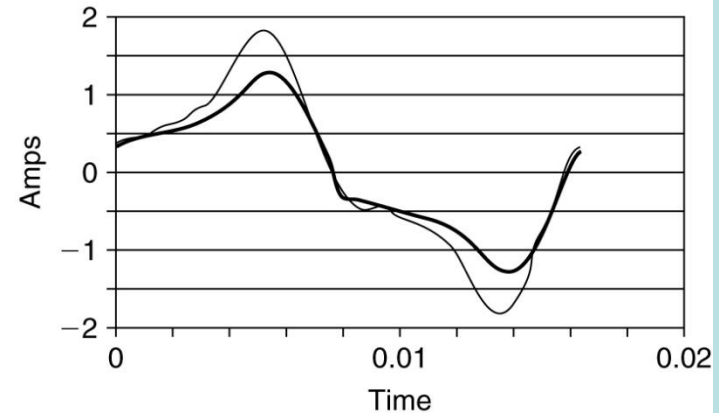


The current is not sinusoidal but it is periodic, thus can be represented by a Fourier series.



○ Fundamental □ Third harmonic △ First and third harmonics

(a)



— 130 volts applied — 120 volts applied

(b)

a. Harmonic content of exciting current.

b. Measured exciting current.

A slightly higher applied voltage causes the transformer to draw a much higher exciting current, which would also increase the core losses.

RMS value of the exciting current is calculated :

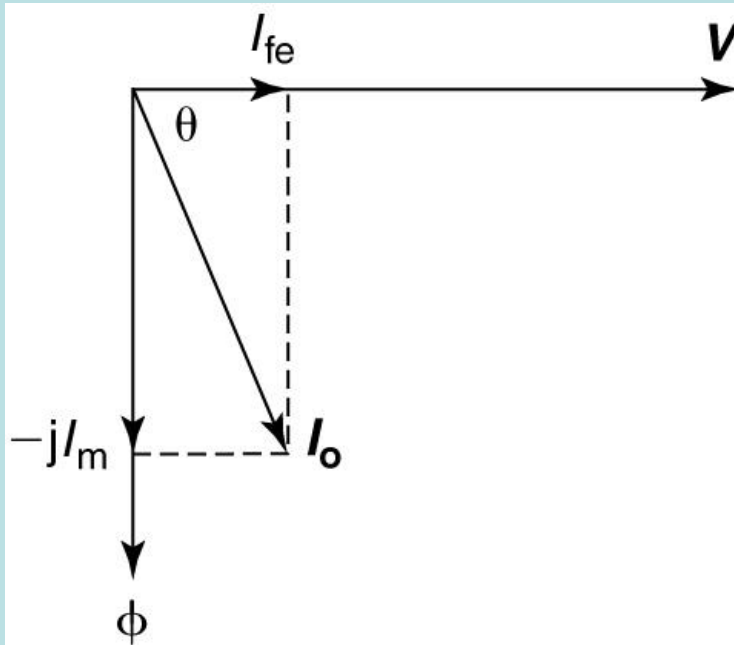
$$I_{0,RMS} = \sqrt{\frac{\int_0^{2\pi} [i_0(wt)]^2 d(wt)}{2\pi}}$$

The exciting current is not in phase with the flux.

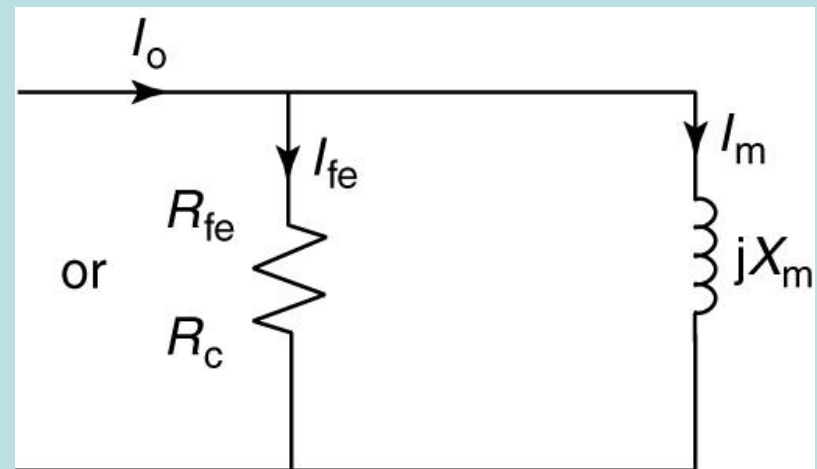
The voltage is 90 degrees ahead of the flux, since voltage is the derivative of the flux.

The exciting current phasor lies between the voltage phasor and the flux phasor, therefore the current can be separated into two components:

- One in phase with the voltage, I_{fe} . Represents real power being consumed and is called: **Core-loss current**
- One in phase with the flux, I_m . Represents reactive power and is called: **Magnetizing current**



Phasor diagram of exciting current



Equivalent circuit of transformer core

The resistance R_c consumes real power corresponding to the core loss of the transformer. The inductive reactance X_m draws the current to create the magnetic field in the transformer core.

BIL (Basic Insulation Level)

A transformer BIL is the peak transient voltage level that the transformer can withstand for a specified time.

The insulation class of a transformer is the maximum RMS working voltage of the transformer. The BIL is between 5 and 30 times the insulation class.

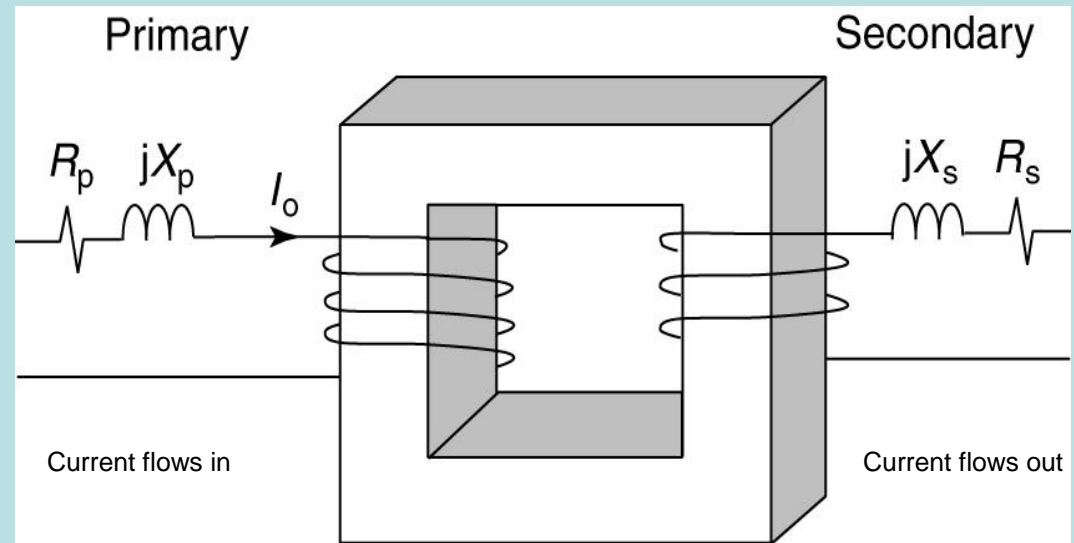
The high voltage withstanding capability is necessary because of lightning and switching transients. Lightning strikes impress very high voltage transients of short duration on transmission and distribution lines that propagate down the lines to the transformers.

If the BIL levels were lower the reliability of the electrical power system would be lower.

Transformer Equivalent Circuits

All non-ideal transformers have:

- Winding resistance
- A core with finite permeability
- Leakage flux
- Hysteresis
- Eddy current losses



$$V_p = R_p I_p + N_p \frac{d\phi}{dt}$$

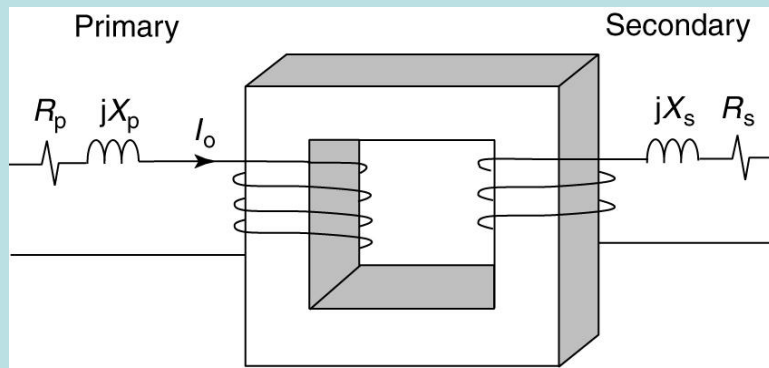
$$V_p = R_p I_p + N_p \frac{d\phi_p}{dt} + N_p \frac{d\phi_m}{dt}$$

Primary leakage
flux

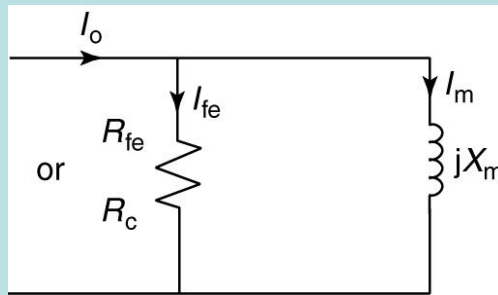
Mutual flux that
links both coils

$$V_s = -R_s I_s + N_s \frac{d\phi}{dt}$$

$$V_s = -R_s I_s - N_s \frac{d\phi_s}{dt} + N_s \frac{d\phi_m}{dt}$$

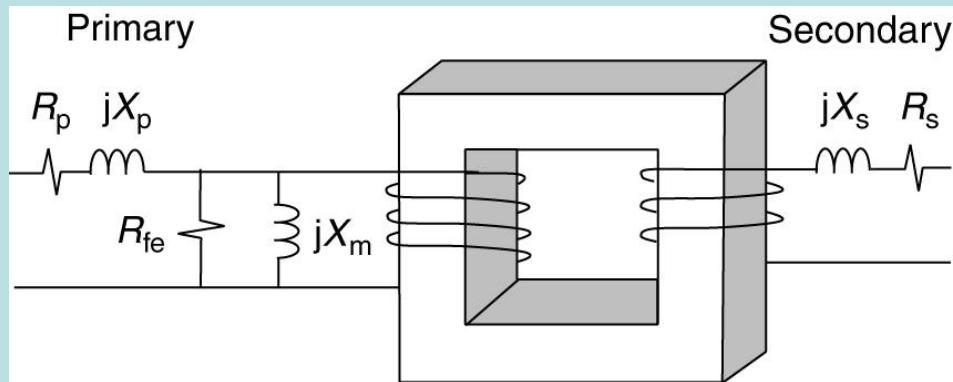


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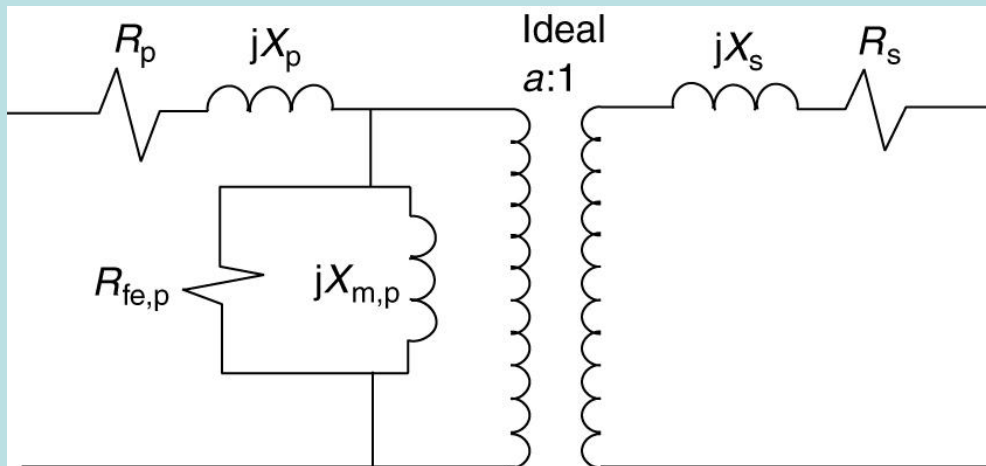
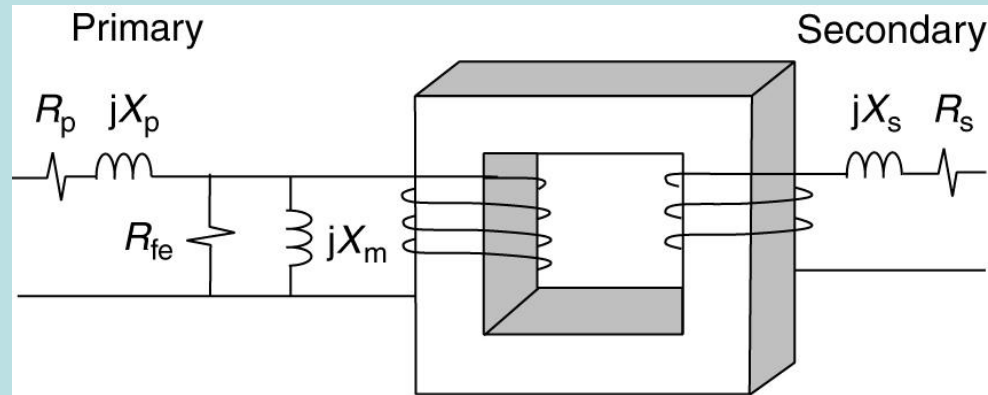


Magnetizing branch

||



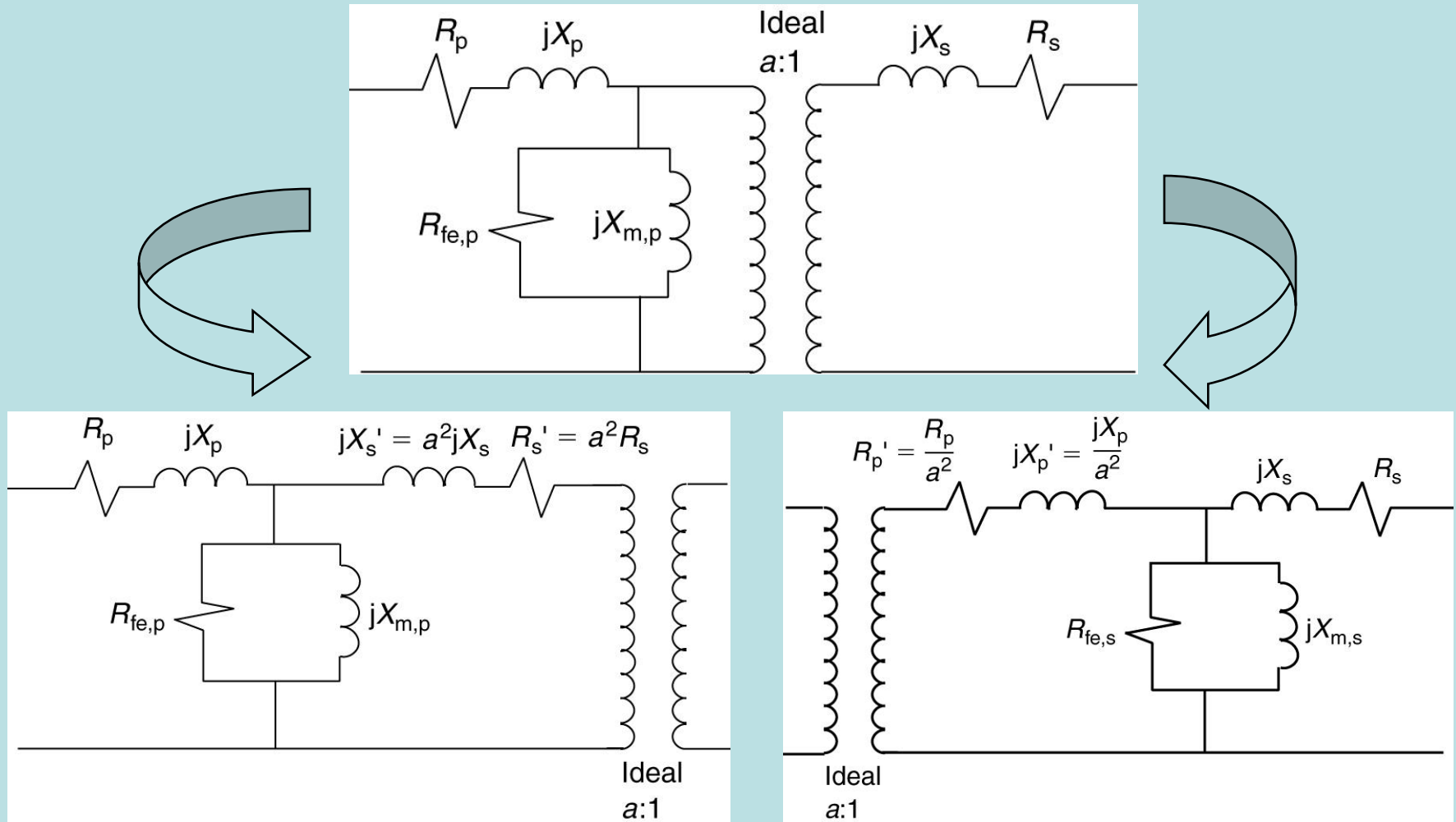
The transformer is now ideal and the circuit elements account for the losses and voltage drops in the real transformer.



Equivalent T-circuit

Third order circuit. It takes third order differential equation to solve it.

We can refer the impedances of the secondary to the primary side (or vice versa) yielding the equivalent circuits



All resistances and reactances have been referred to the primary side

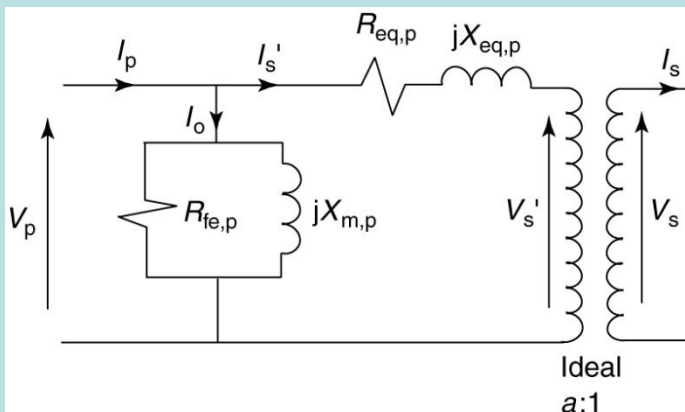
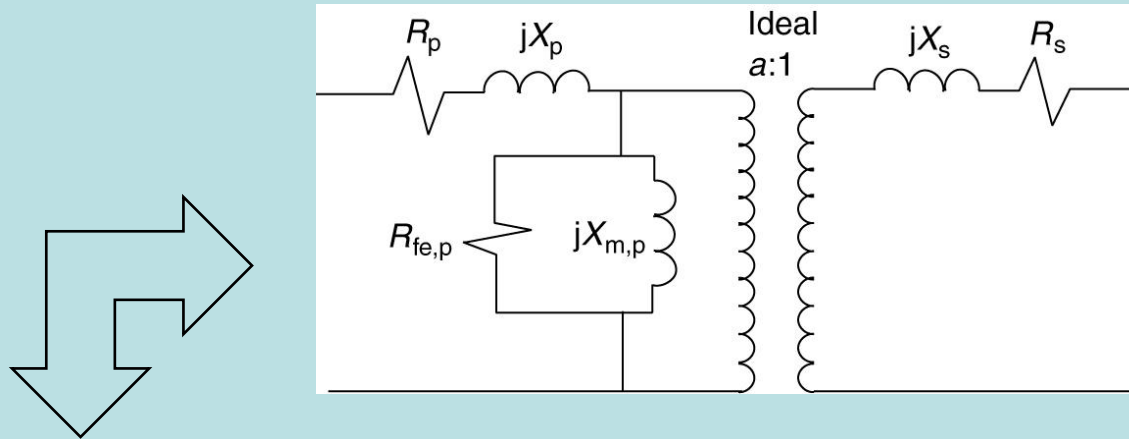
All resistances and reactances have been referred to the secondary side

Cantilever Equivalent Circuit.

The combination of the winding resistances is called **equivalent resistance**.

The combination of the leakage reactances is called the **equivalent reactance**.

The Cantilever Equivalent Circuit Model neglects the voltage drop of the exciting current in the primary coil but makes calculation the exciting current much easier because the primary voltage is applied directly to the magnetizing reactance and the core-loss resistance.

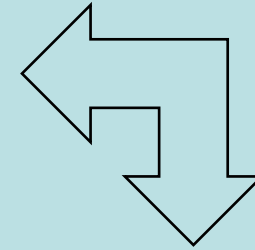
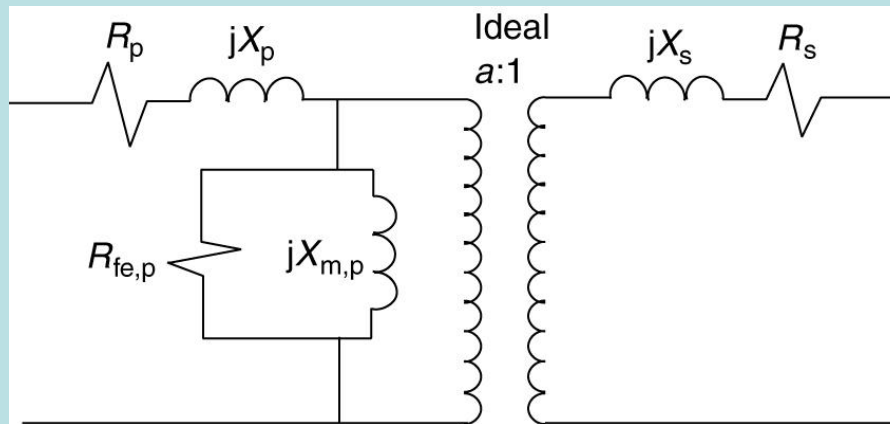


$$R_{eq.primary} = R_p + a^2 R_s = R_p + R'_s$$

$$X_{eq.primary} = X_p + a^2 X_s = X_p + X'_s$$

$$Z_{eq.primary} = R_{eq.primary} + jX_{eq.primary}$$

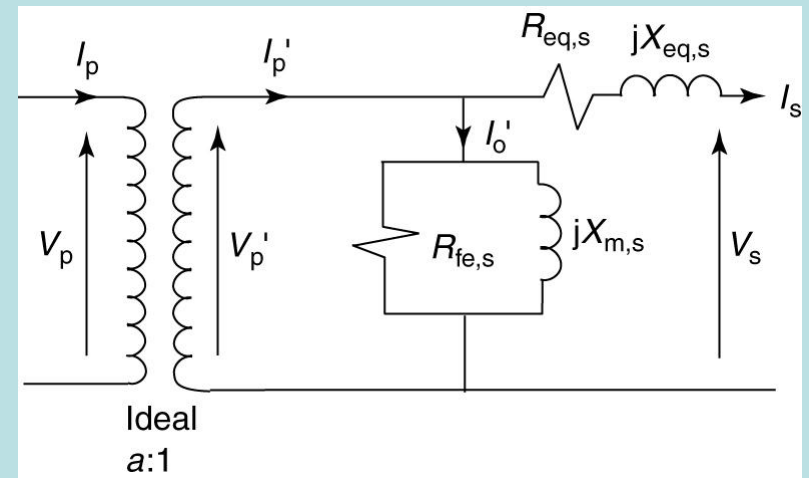
Cantilever circuit referred to the primary.



$$R_{eq.secondary} = \frac{R_p}{a^2} + R_s = R_p + R'_s$$

$$X_{eq.secondary} = \frac{X_p}{a^2} + X_s = X_p + X'_s$$

$$Z_{eq.secondary} = R_{eq.secondary} + jX_{eq.secondary}$$

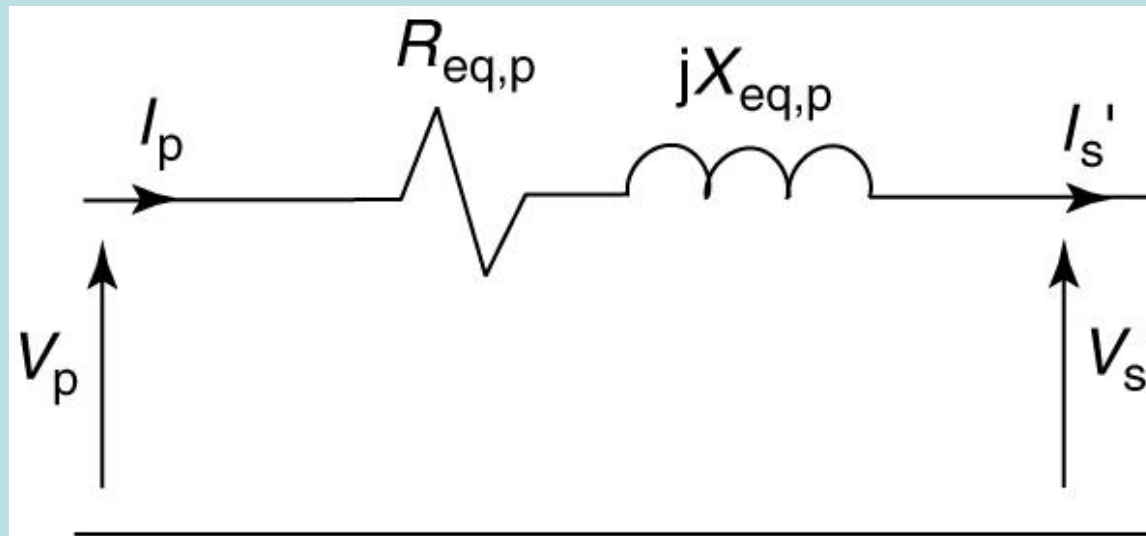


Cantilever circuit referred to the secondary.

Series Equivalent Circuit

Note: In large-scale system studies, even cantilever model becomes too complex, so one final simplification is made.

We completely neglect the magnetizing branch of the transformer model. Only combined winding resistance and leakage reactance are included, resulting in the first order model (takes first order differential equation to solve it).



Series equivalent circuit.

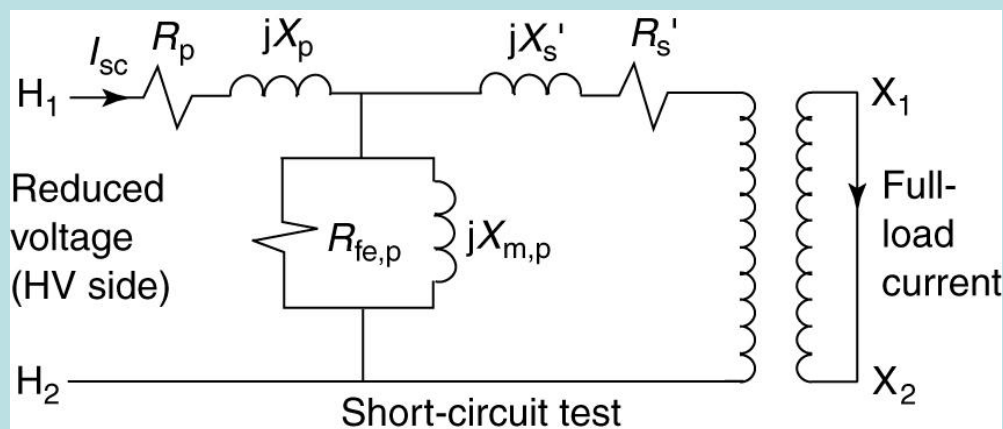
Since there are no shunt elements, the primary and secondary currents are equal to each other.

Determining Circuit Parameters

For developed model to be useful, there must be a way to determine the values of the model parameters.

We use two test to determine this parameters:

Short-circuit test One side of the transformer is shorted, and voltage is applied on the other side until rated current flows in the winding. The applied voltage, winding current, and input power are measured.



Technique: generally, the low-voltage side of the transformer is shorted and voltage is applied to the high-voltage side, because it only takes about 4%-7% of rated voltage to cause rated current to flow in the winding.

The measurements are used to calculate value of R_{eq} and jX_{eq} .

Input impedance to the transformer is the primary winding in series with the parallel combination of the secondary winding and the exciting branch:

$$Z_{in} = R_p + jX_p + [(R_s' + jX_s') || R_{fe,primary} || jX_{m,primary}]$$

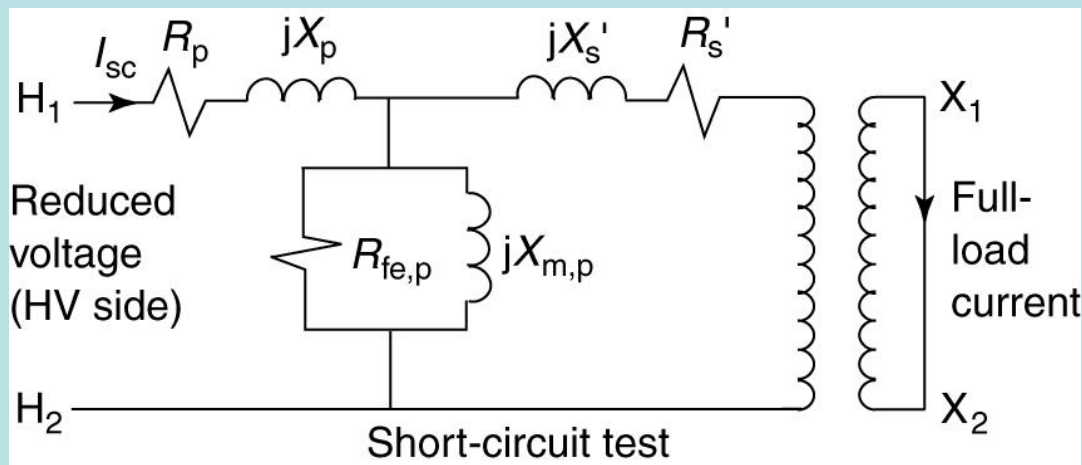
The core branch elements are much larger than the winding impedance:

$$R_{fe,primary} \gg R_s'$$

$$X_{m,primary} \gg X_s'$$

$$Z_{in} = R_p + jX_p + (R_s' + jX_s')$$

$$Z_{in} = R_{eq,primary} + jX_{eq,primary}$$



To conduct the short-circuit test, measure the voltage applied to the transformer high side V_{sc} , the short-circuit current in the high-side winding I_{sc} , and the power into the transformer P_{sc} . Having these parameters we calculate the magnitude of input Impedance:

$$|Z_{eq,primary}| \approx |Z_{sc}| = \frac{V_{sc}}{I_{sc}}$$

$$R_{eq,primary} \approx \frac{P_{sc}}{I_{sc}^2}$$

$$X_{eq,primary} = \sqrt{|Z_{eq,primary}|^2 - R_{eq,primary}^2}$$

Once we have values for the equivalent winding resistance and reactance we can apportion them to the two sides by assuming the windings have equal resistance and reactance when referred to the same side:

$$R_{primary} = R'_s = \frac{1}{2} R_{eq,primary}$$

$$X_{primary} = X'_s = \frac{1}{2} X_{eq,primary}$$

Measurements are very similar for three-phase transformers.

Three-phase power, line current, and line voltage are usually measured. The calculations are done on a per phase basis whether wye or delta.

Recall that delta impedance is three times wye impedance.

To obtain phase values:

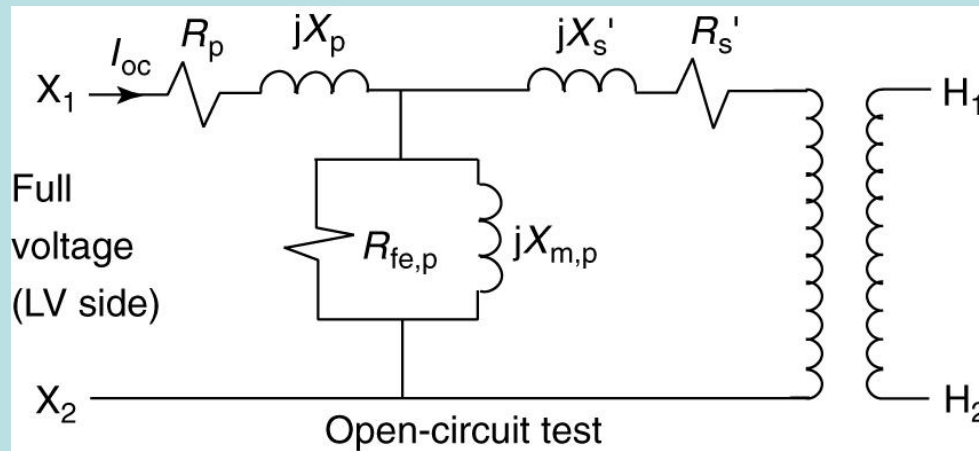
$$P_P = \frac{P_{SC}}{3}$$

$$R_{eq} = \frac{P_P}{I_P^2}$$

$$Z_{eq} = \frac{V_P}{I_P}$$

$$Z_{eq} = \sqrt{Z_P^2 - R_P^2}$$

Open-circuit test



High-voltage side is opened and rated voltage is applied to the low-voltage side (primary side).

The low-voltage side is used to avoid high-voltage measurements.

With no load current, only exciting current I_o flows and because the impedance of the primary side is small, the voltage across the magnetizing branch is approximately equal to the applied voltage.

Procedure: - Measure the voltage applied to the transformer low side V_{oc} , the open-circuit current in the low-side winding I_{oc} and the power into the transformer during open circuit test P_{oc} .
- Calculate resistance and reactance.

The input impedance during open-circuit test is the primary winding in series with the exciting branch:

$$Z_{in} = R_p + jX_p + (R_{fe,primary} || jX_{m,primary})$$

$$\begin{aligned} R_{fe,primary} &\gg R_{primary} \\ X_{m,primary} &\gg jX_{primary} \end{aligned}$$

$$Z_{in} = R_{fe,primary} || jX_{m,primary}$$

Now we have the voltage applied to $R_{fe,primary}$ and the power dissipated, so we can calculate:

$$R_{fe,primary} = \frac{V_{oc}^2}{P_{oc}}$$

Because the magnetizing reactance is in parallel with $R_{fe,primary}$ we first need to find the reactive power to find reactance:

$$|S_{oc}| = V_{oc} \times I_{oc}$$

$$Q_{oc} = \sqrt{|S_{oc}|^2 - P_{oc}^2}$$

$$X_{m,primary} = \frac{V_{oc}^2}{Q_{oc}}$$

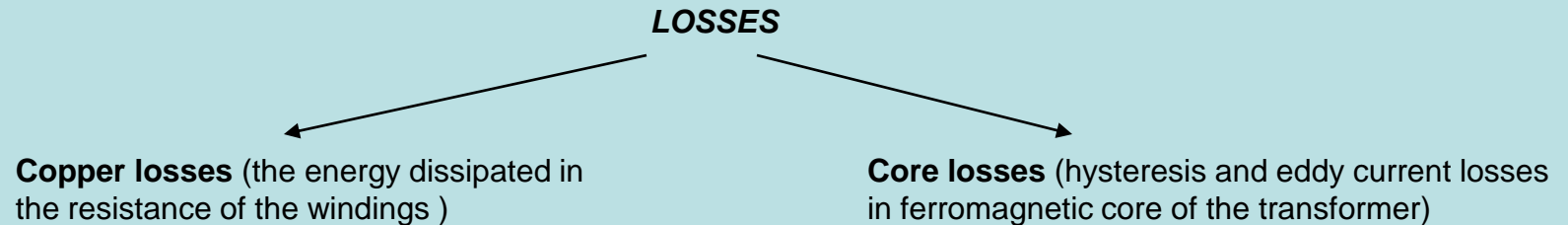
Transformer Efficiency

Efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}}$$

Input is the output plus losses:

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{out} + P_{loss}}$$



Losses in the transformer

Winding resistance

Current flowing through the windings causes resistive heating of the conductors.

Eddy currents

Ferromagnetic materials are also good conductors, and a solid core made from such a material also constitutes a single short-circuited turn throughout its entire length. Induced eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material.

Hysteresis losses

Each time the magnetic field is reversed, a small amount of energy is lost to hysteresis within the magnetic core, the amount being dependant on the particular core material.

Mechanical losses

The alternating magnetic field causes fluctuating electromagnetic forces between the primary and secondary windings, that induce vibrations.

Stray losses

Not all the magnetic field produced by the primary is intercepted by the secondary. A portion of the [leakage flux](#) may induce eddy currents within nearby conductive objects, such as the transformer's support structure, and be converted to heat.

Cooling system

Large power transformers may be equipped with [cooling fans](#), [oil pumps](#) or water-cooled [heat exchangers](#) designed to remove heat.

The power used to operate the cooling system is typically considered part of the losses of the transformer.

K-Factor-Rated Transformers

The overheating effect of nonlinear loads on wiring systems, transformers, motors, and generators can be severe.

K-Factor-Rated transformers are specifically designed to handle nonlinear loads.

They are typically constructed of thinner steel laminations, lower-loss steel, and larger conductors. This allows them to provide harmonic currents without overheating.

The K-factor rating of the transformer is an indication of the transformer's ability to deliver power to a nonlinear load without exceeding the transformer's specified operating temperature limits.

K-factor is defined as:

$$K = \sum h^2 \left(\frac{I_h}{I_{RMS}} \right)^2$$

where h is the harmonic number, I_h is the RMS value of the harmonic current, and I_{RMS} is the total RMS current.

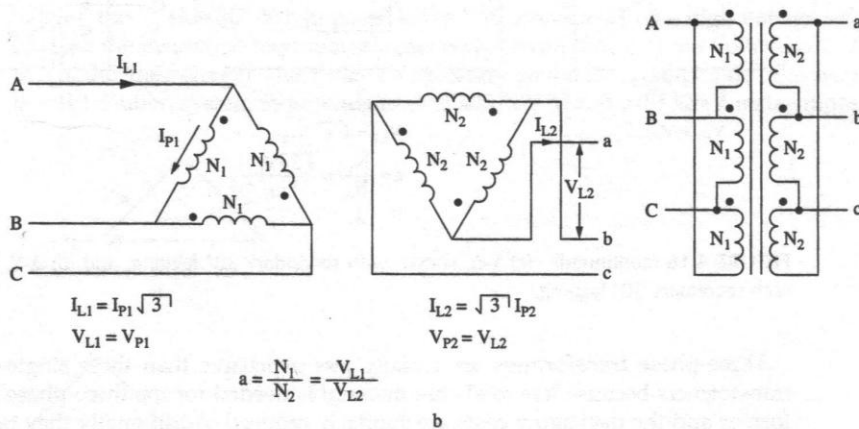
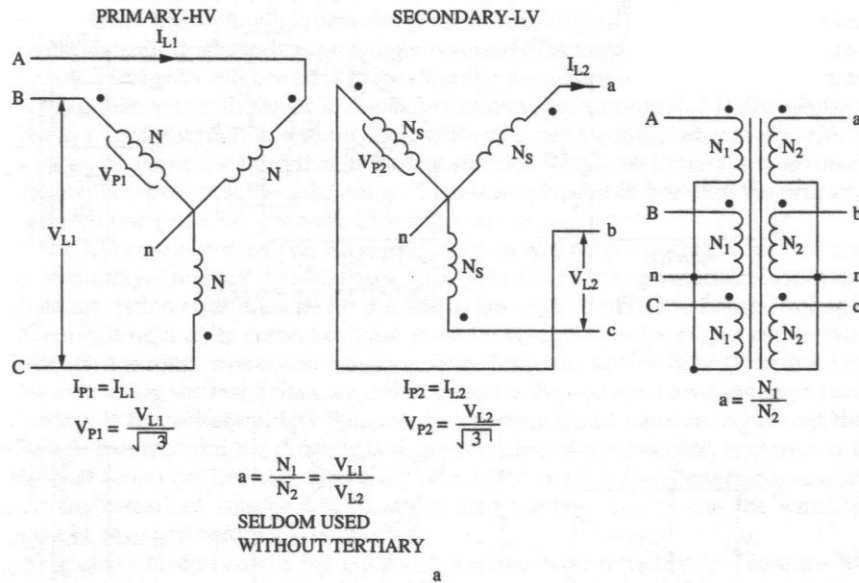
Three-phase transformer connections

There are four major three-phase transformer connections :

1. Y-Y
2. Delta-Delta
3. Y-Delta
4. Delta-Y

Three-phase transformers are less expensive than 3-single-phase transformers because less total core material is needed for the three-phase transformer and the packaging cost is reduced.

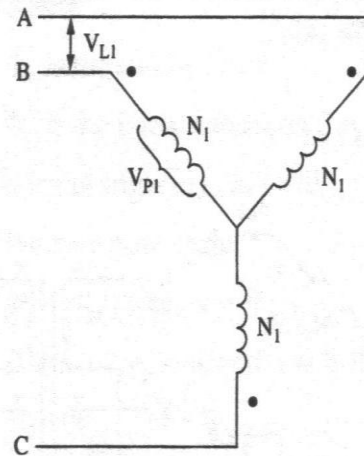
Additionally they take up less space, are lighter, require less on site external wiring for installation, and more efficient than three single-phase transformers.



The **Y-Y** connection has significant third harmonic content on the secondary lines (unless the neutral point is grounded)

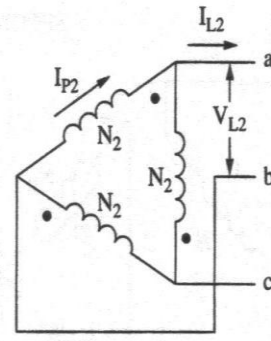
There is no phase shift between the primary and secondary of a Y-Y connected transformer.

The **Delta-Delta** connection has no harmonic problem and no phase shift from primary to secondary. The only disadvantage with respect to a Y connection is that the delta insulation class must be for the line to line instead of line to neutral voltage.



$$I_{P1} = I_{L1}$$

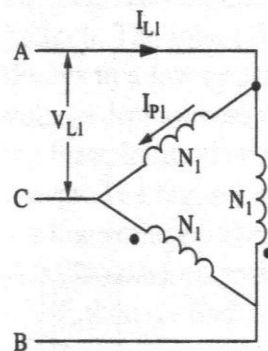
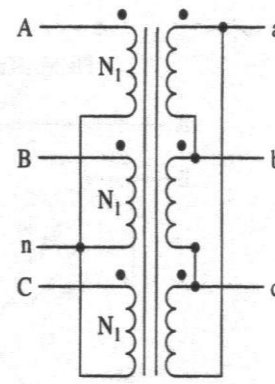
$$V_{P1} = \frac{V_{L1}}{\sqrt{3}}$$



$$I_{L2} = \sqrt{3} I_{P2}$$

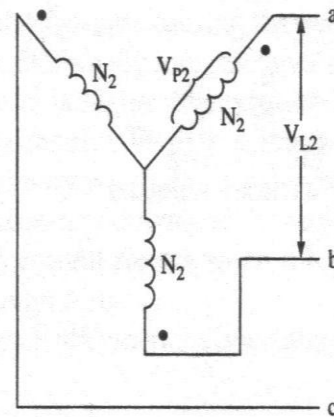
$$V_{L2} = V_{P2} = a = \frac{N_1}{N_2} = \frac{V_{L1}}{\sqrt{3}}$$

c



$$I_{L1} = \sqrt{3} I_{P1}$$

$$V_{L1} = V_{P1}$$

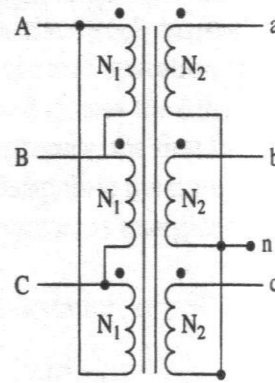


$$I_{L2} = I_{P2}$$

$$V_{L2} = \sqrt{3} V_{P2}$$

$$a = \frac{N_1}{N_2} = \frac{\sqrt{3} V_{L1}}{V_{L2}}$$

d



There is a 30 degrees phase shift in both connections. United States industry convention is to connect the secondary so it lags the high voltage primary by 30 degrees.

When possible the Y is connected to the high voltage side because the insulation requirements are lower (recall that Y phase voltage is $1/\sqrt{3}$ that of the line voltage).

The Y may be necessary on the low voltage side because of the distribution system requirements as in 480/277 V and 208/120 V installations.

Transformer per-unit system

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

$$P_{\text{base}1\phi} = Q_{\text{base}1\phi} = S_{\text{base}1\phi}$$

$$I_{\text{base}} = \frac{S_{\text{base}1\phi}}{V_{\text{baseLN}}}$$

$$Z_{\text{base}} = R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{baseLN}}^2}{I_{\text{base}}^2} = \frac{V_{\text{baseLN}}^2}{S_{\text{base}1\phi}^2}$$

$$Y_{\text{base}} = G_{\text{base}} = B_{\text{base}} = \frac{1}{Z_{\text{base}}}$$

The subscripts LN and 1 ϕ denote "line-to-neutral" and "per-phase" respectively, for three-phase circuits (these equations are also valid for single-phase circuits)

By convention, we adopt the following two rules for base quantities:

1. The value of $S_{\text{base}1\phi}$ is the same for the entire power system of concern .
2. The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.

With these two rules a per-unit impedance remains unchanged when referred from one side of a transformer to the other.

By properly specifying base quantities, the transformer equivalent circuit can be simplified.

The ideal transformer winding can be eliminated, such that voltages, currents, and external impedances and admittances expressed in per-unit do not change when they are referred from one side of a transformer to the other.

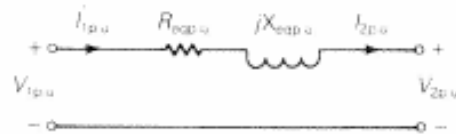
This can be a significant advantage even in a power system of moderate size, where hundreds of transformers may be encountered.

The per-unit system allows us to avoid the possibility of making serious calculation errors when referring quantities from one side of a transformer to the other.

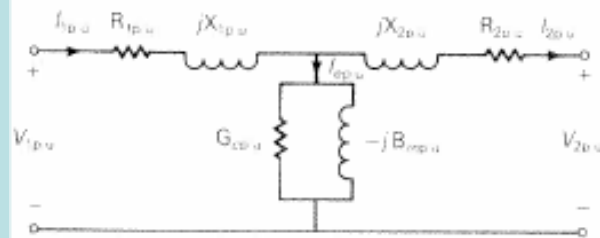
Manufacturers usually specify the impedances of machines and transformers in per-unit or percent of nameplate rating.



(a) Ideal transformer



(b) Neglecting exciting current



(c) Complete representation

$$E_{1p.u.} = E_{2p.u.}, \text{ and } I_{1p.u.} = I_{2p.u.}$$

$$E_{1p.u.} = \frac{E_1}{V_{base1}} = \frac{N_1}{N_2} \times \frac{E_2}{V_{base1}}$$

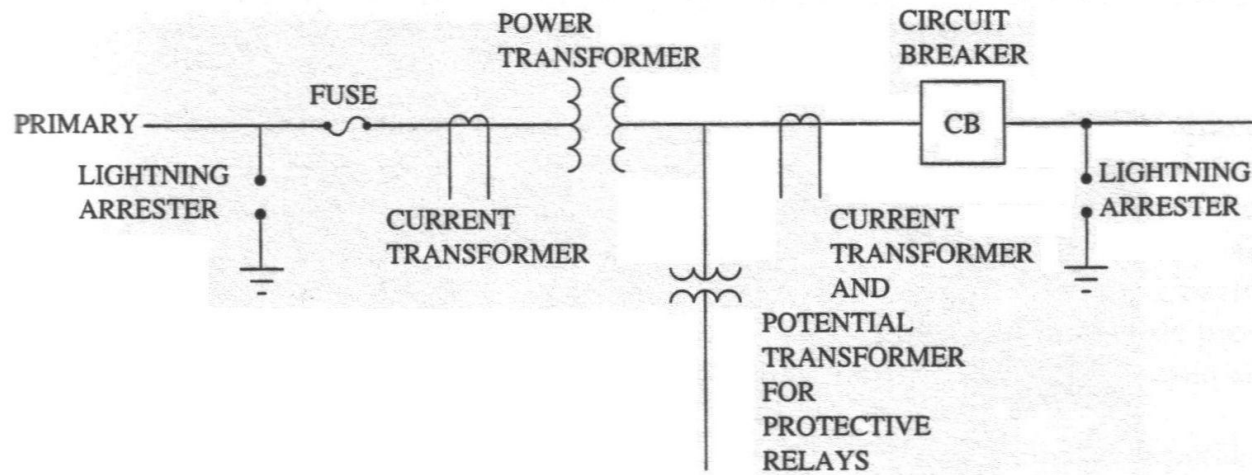
$$V_{base1}/V_{base2} = V_{rated1}/V_{rated2} = N_1/N_2,$$

$$E_{1p.u.} = \frac{N_1}{N_2} \frac{E_2}{\left(\frac{N_1}{N_2}\right) V_{base2}} = \frac{E_2}{V_{base2}} = E_{2p.u.}$$

$$I_{1p.u.} = \frac{I_1}{I_{base1}} = \frac{N_2}{N_1} \frac{I_2}{I_{base1}}$$

$$I_{1p.u.} = \frac{N_2}{N_1} \frac{I_2}{\left(\frac{N_2}{N_1}\right) I_{base2}} = \frac{I_2}{I_{base2}} = I_{2p.u.}$$

Transformer protection



Power transformer one-line drawing with protection

Transformers, especially power transformers, are expensive. The transformer **must be protected** from overcurrent due to faults on its secondary circuit, and from over voltage, which is usually caused by lightning.

The protective devices:

Lightning protection is provided by **lightning arresters**. Zinc oxide lightning arresters are the most popular now. The voltage at which the lightning arresters begins conducting, absorbing power, and preventing further voltage rise on the line is set at a voltage below the maximum insulating voltage of the transformer and above the maximum operating voltage of the transformer. Lightning arresters are used on both the primary and secondary side of the power transformers because lightning can strike on either side.

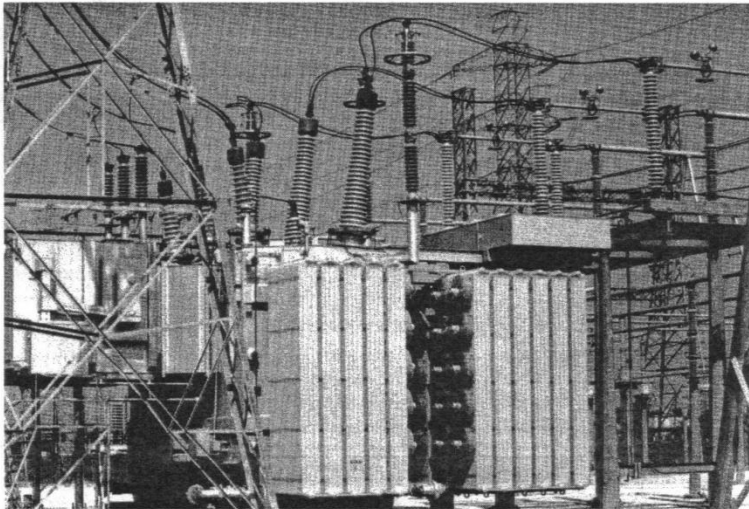
Overcurrent protection is provided by **circuit breakers** and their associated protective **relays, and fuses**. A **fuse** opens on overcurrent. **Circuit breakers** open electrical contacts when they receive a trip signal from one of their associated relays. The opening is done by driving the contacts apart with powerful springs. Circuit breakers are used on the secondary side of the transformer with a fuse back up on the primary side.

Types of distribution transformers

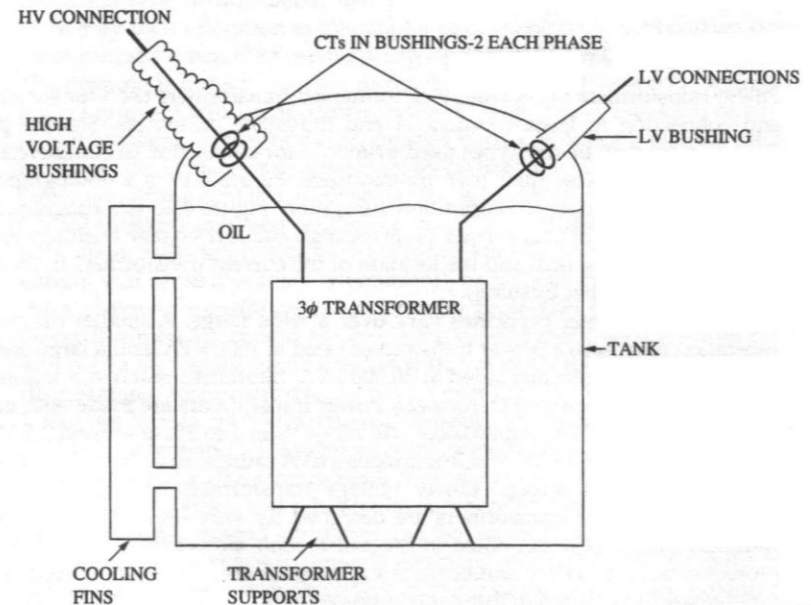
Transformers used in distribution include: power transformers; autotransformers, which may also be power transformers; distribution transformers; and instrument transformers.

1. Power transformers

Power transformers are normally oil immersed transformers used for substations and connection to large commercial and industrial customers. Smaller power transformers may be dry types used primarily for connection to commercial customers who purchase their own transformers.



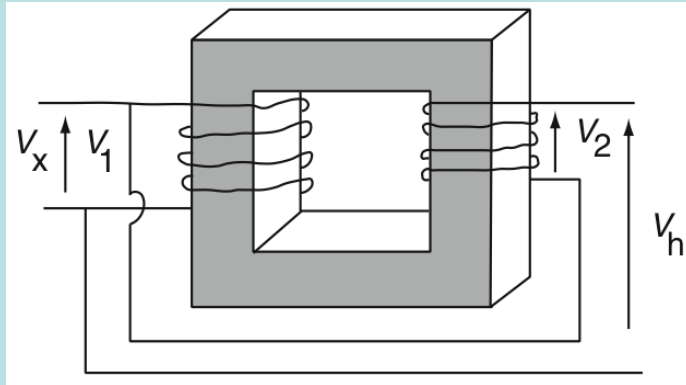
Photograph of power transformer (Courtesy of Houston Lighting and Power)



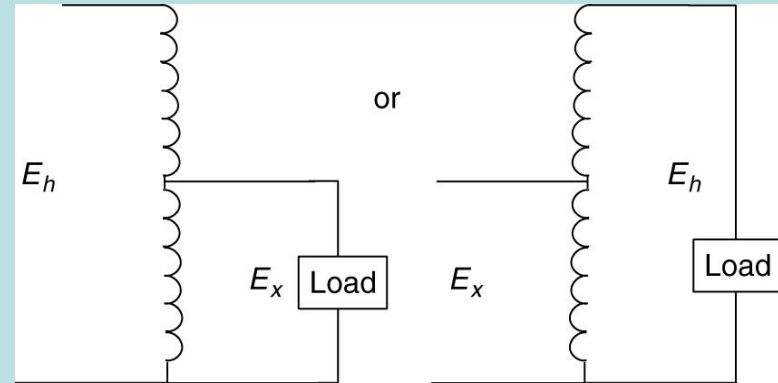
Power transformers are designed for very high efficiency, and are routinely maintained. Power transformers are well protected both for safety and economy. Almost all larger power transformers are three-phase as opposed to three single-phase units.

Power transformer capacities vary over a wide range. A smaller distribution substation may have a power transformer rated at 1000 kVA, and a large one may have a power transformer rated at 30,000 kVA.

Autotransformers



Elementary autotransformer.



Autotransformer schematic.

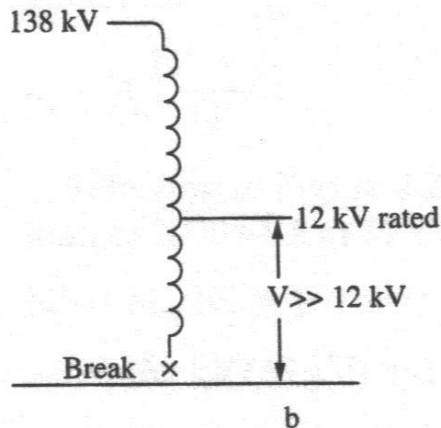
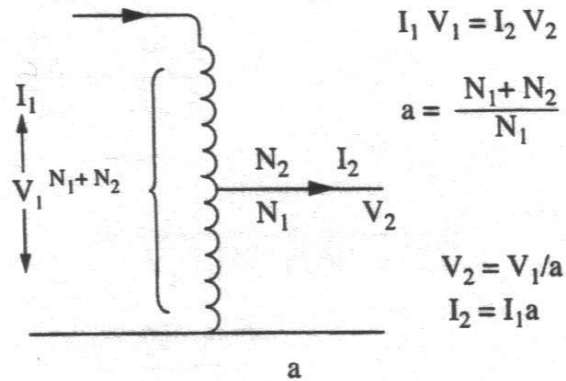
The low-voltage coil is essentially placed on the top of the high-voltage coil and called the **series coil**. This connection is called **autotransformer** and can be used as a step-up or a step-down transformer.

Advantages of an autotransformer:

1. Higher power rating
2. Cheaper
3. More efficient
4. Low exciting current
5. Better voltage regulation

Disadvantages of an autotransformer:

1. Larger short-circuit currents available
2. No isolation between the primary and secondary
3. Most useful for relatively small voltage changes



Autotransformers are one winding transformers that are often used in transmission and sub transmission substations.

Note that the primary and secondary sides are not isolated from each other. So if the secondary neutral, which is common to the primary neutral, is opened for any reason the full primary voltage could appear on the secondary side with disastrous results.

Unsafe conditions would result for people as well as damage to the low voltage side protective devices and equipment.

For this reason autotransformers are never used as the final transformer in distribution substations.

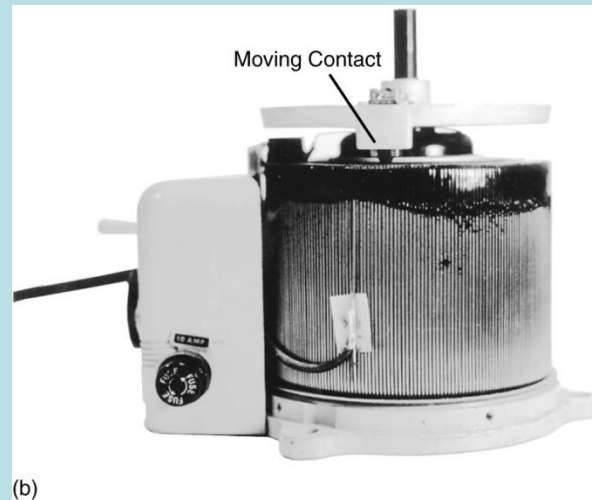
The ideal transformer relationships are approximately true for autotransformers as well as isolation (conventional) transformers.

Practical applications of autotransformers:

1. Connecting transmission lines of slightly different voltages (F.E 115kV and 138kV)
2. Compensating for voltage drop on long feeder circuits
3. Providing variable voltage control in the laboratory
4. Changing 208V to 240V or vice versa
5. Adjusting the output voltage of a transformer to keep the system voltage constant as the load varies.



(a)

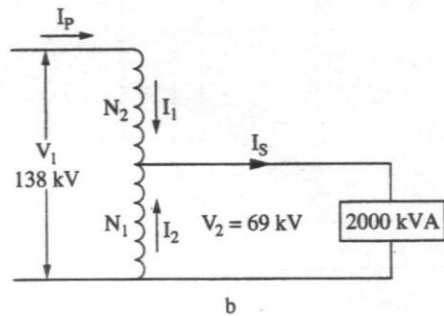
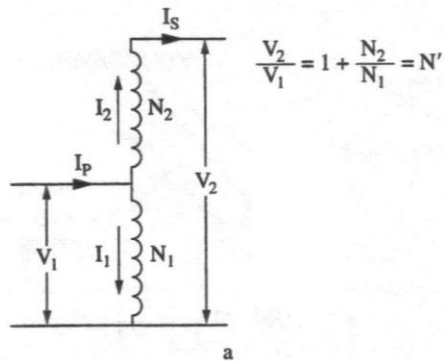


(b)

Photograph of an autotransformer for variable-voltage control in the lab.

- a. Laboratory autotransformer.
- b. Cover removed to show coil and sliding contact.

The Autotransformer Rating Advantage



Autotransformer diagram for discussion of (a) ratio and (b) rating

$$S_I = \text{VA rating as an isolation transformer} = I_p V_1$$

$$S_A = \text{VA rating as an autotransformer} = I_1 V_1$$

$$\frac{S_A}{S_I} = \frac{I_1 V_1}{I_p V_1} = \frac{N' - 1}{N'}$$

$$S_A = S_I \frac{N' - 1}{N'}$$

Distribution Transformers

Distribution transformers are those that are used to provide the final link with the customer.

The distribution voltage is brought down to a level that is safe to use on the customer's premises.

The primary voltage is between 34.5 kV and 2.3 kV, single or three phase depending on the customer size,
and the secondary is normally either 480Y /277 V, 208Y /120 V three phase or 240/120 V single phase.

Distribution transformers may be dry type, in which the windings are not immersed in oil. These are usually used in moderate to small commercial installations where cost is the primary consideration.

Instrument Transformers

Instrument transformers provide line current and voltage information to protective relays and control systems at low power levels.

Two types of transformers are in this class: current transformers and potential transformers.

A **current transformer (CT)** is a type of instrument [transformer](#) designed to provide a [current](#) in its secondary winding proportional to the current flowing in its primary.

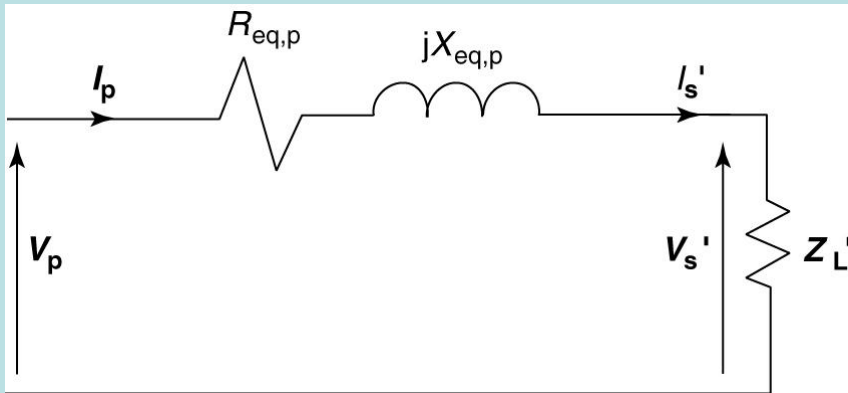
They are commonly used in metering and [protective relaying](#) in the [electrical power industry](#) where they facilitate the safe measurement of large currents, often in the presence of [high voltages](#).

The current transformer safely isolates measurement and control circuitry from the high voltages typically present on the circuit being measured.

Potential transformers (PT) step line voltage down to the 0-150 V range for which most instruments are designed.

Potential transformers are very large for their VA rating because of the amount of insulation required for the high voltage side.

Voltage Regulation



The transformer windings have impedance so there will be a voltage drop across them that changes with current.

The secondary voltage will vary as the load changes.

Voltage regulation is a measure of the change in secondary voltage from no-load to full-load and is usually expressed as a percentage of the full-load voltage.

If there were **no load** on the transformer, the **current** would be **zero** and the referred secondary voltage would be equal to the primary voltage.

The **no-load voltage** (referred to the primary) of the transformer is the **primary voltage**.

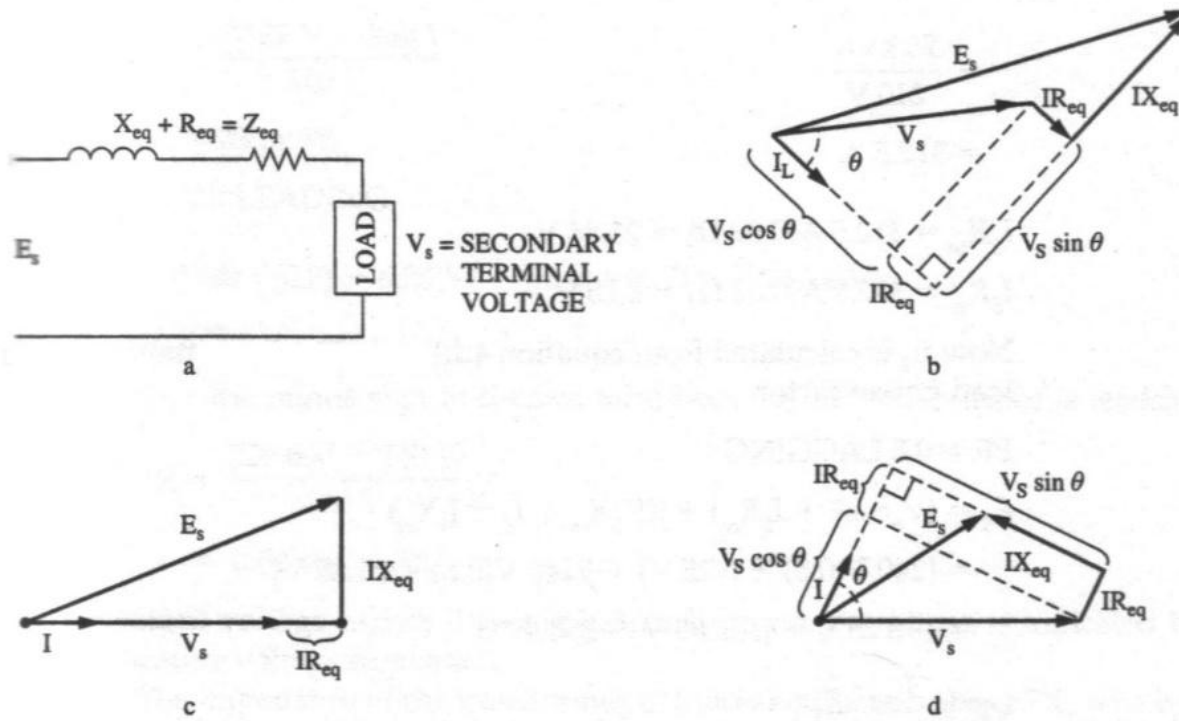
As the load increases to **full load**, current flows in the windings of the transformer and there is a voltage drop across the transformer, and the referred value of the secondary voltage is no longer equal to the primary voltage.

Voltage regulation:

$$VR = \frac{|V_{nl}| - |V_{fl}|}{|V_{fl}|} \times 100$$

Transformer Voltage Regulation Calculation

We wish to find a reasonably easy and accurate method of calculating the secondary voltage as the load changes.



We hold the secondary voltage constant for our calculations, in other words V_s is the reference phasor.

The induced secondary voltage, $E_s = V_p/a$ is the variable voltage.

The equivalent circuit referred to the secondary is shown in "a". "b" shows the phasor diagram of the secondary circuit with a lagging power factor load.

The IR_{eq} voltage drop is in phase with the load current, and the IX_{eq} voltage drop is 90 percent ahead of the load current. The dotted lines dropping to the dotted line extending at the same angle as the load current make the phasor diagram into a right triangle.

From the right triangle:

$$E_s = (V_s \cos \theta + IR_{eq}) + j(V_s \sin \theta + IX_{eq})$$

A general equation where last term is + for a lagging power factor, and – for a leading power factor is:

$$E_s = (V_s \cos \theta + IR_{eq}) + j(V_s \sin \theta \pm IX_{eq})$$

There are some applications, however, where poor regulation is desired.



1. **Discharge lighting**, where a step-up transformer is required to initially generate a high voltage (necessary to "ignite" the lamps), then the voltage is expected to drop off once the lamp begins to draw current. This is because discharge lamps' voltage requirements tend to be much lower after a current has been established through the arc path.

In this case, a step-up transformer with poor voltage regulation suffices nicely for the task of conditioning power to the lamp.



2. **Current control for AC arc welders**, which are nothing more than step-down transformers supplying low-voltage, high-current power for the welding process.

A high voltage is desired to assist in "striking" the arc (getting it started), but like the discharge lamp, an arc doesn't require as much voltage to sustain itself once the air has been heated to the point of ionization.

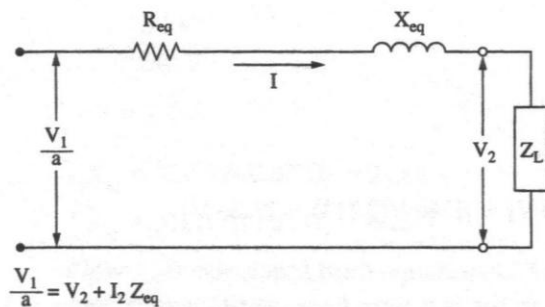
Transformer Voltage Regulation Per unit Concept

Using Kirchoff's law:

$$\frac{V_1}{a} = V_2 + I_2 Z_{eq}$$

Dividing both sides by the base voltage of the secondary side, V_{2B} we obtain:

$$\frac{V_1}{a V_{2B}} = \frac{V_2}{V_{2B}} - \frac{I_2 Z_{eq}}{V_{2B}}$$



Voltage regulation calculation, per unit values

$$V_{2B} = V_{1B} \quad V_{2B} = I_{2B} Z_{2B}$$

$$V_{1pu} = V_{2pu} + I_{2pu} Z_{eqpu}$$

Let V_{2pu} be the reference phasor as before, then

$$I_2 = I_{2B} \quad I_{2pu} = 1$$

The voltage regulation equation can now be written

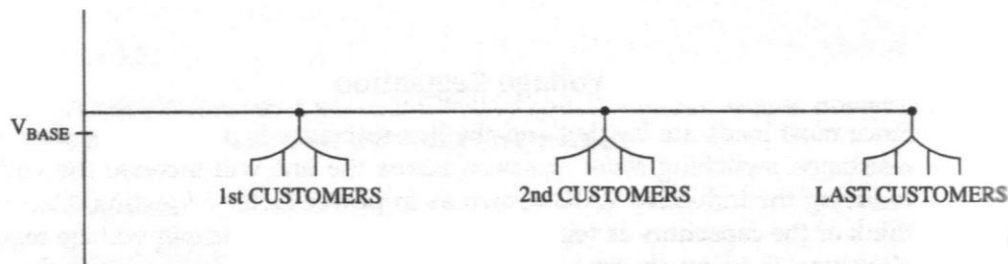
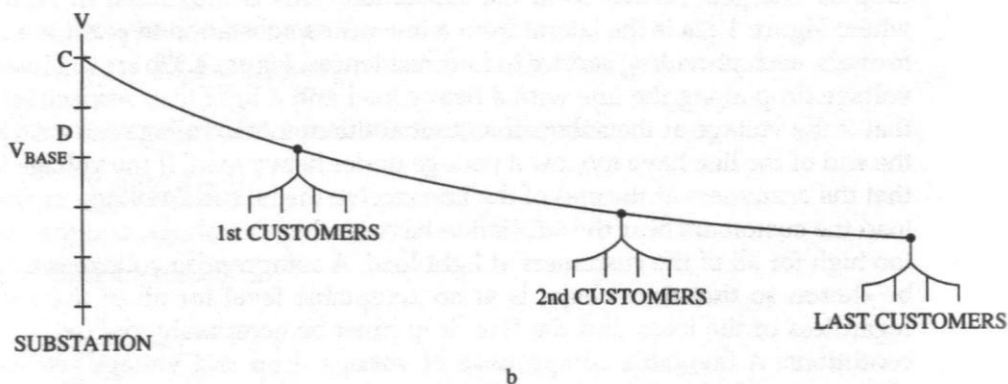
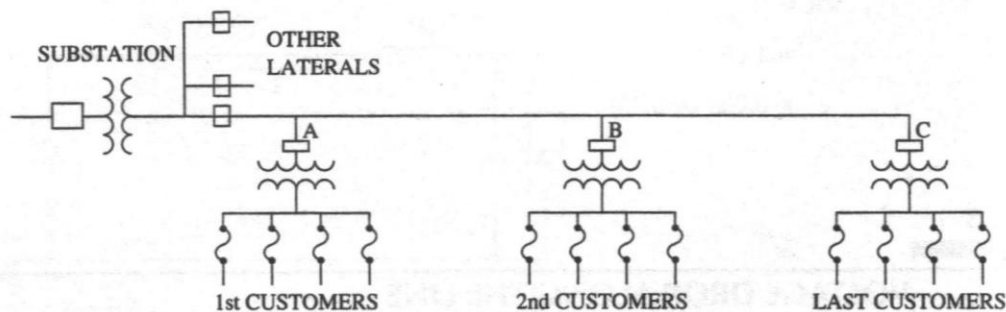
$$VR = \frac{\left| \frac{V_1}{a V_{2B}} \right| - V_{2B}}{V_{2B}}$$

$$V_{1pu} = 1 \angle 0^\circ + 1 \angle \pm \theta(Z_{eqpu})$$

Dividing by results in

$$VR = \frac{|V_1|}{|V_{1B}|} - 1 = |V_{1pu}| - 1$$

VOLTAGE DROP ALONG THE LINE



Voltage drop along line (a) Representation of distribution circuit (b) line voltage drop with heavy load and (c) line voltage drop with light load.

If the voltage at the substation is set at the nominal voltage the customers at the end of the line have too low a voltage under heavy load.

If the voltage is set so that the customers at the end of the line receive the nominal voltage under heavy load the customers near the substation have too high a voltage, and the voltage is too high for all of the customers at light load.

A compromise voltage setting must be chosen so that the voltage is at an acceptable level for all of the customers regardless of the load, and the line drop must be acceptably low under all load conditions.

A favorable compromise of voltage drop and voltage setting is not always possible for all load conditions, so other means of voltage regulation have been devised for such conditions.

Methods for Voltage Regulation

Capacitors for voltage regulation

Switch shunt capacitors used across the line to increase the voltage by reducing the inductive VARs drawn as in power factor correction.

Shunt capacitors are only used for lagging load power factors and their main goal is to correct for load power factor, and their only current is from the VARs.

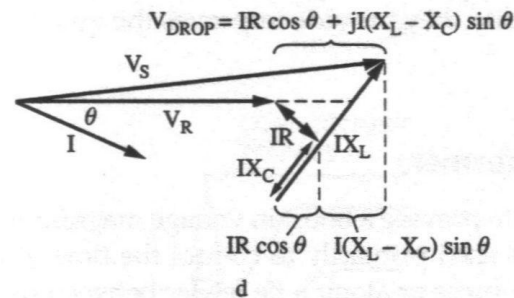
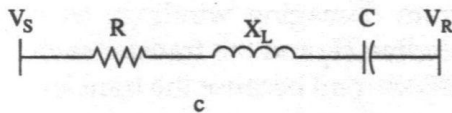
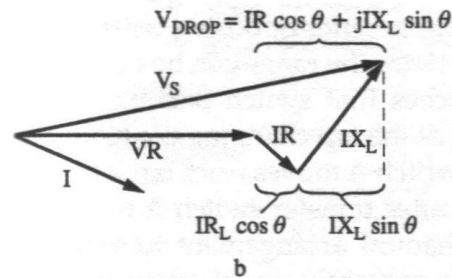
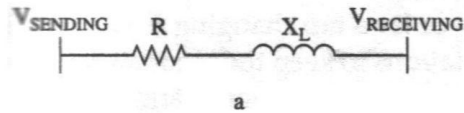
Switched capacitor banks are expensive because they must have sensing equipment to monitor the line voltage and control equipment activate the proper switching.

All capacitor banks require protection: fuses, circuit breakers etc.

Series capacitors are connected in series with the line and carry full line current.

The capacitive reactance of the series capacitance is used to cancel the inductive reactance of the line to reduce the voltage drop along the line.

Series capacitors operation



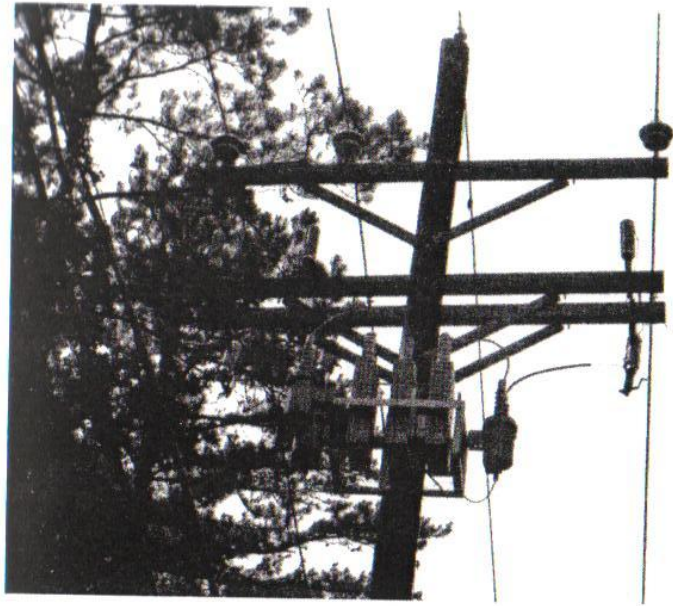
Series capacitor. (a) Line equivalent circuit (b) Phasor diagram with lagging power factor load (c) Equivalent circuit with capacitor and (d) Phasor diagram with capacitor (same V_S as in b).

“a”: line equivalent circuit

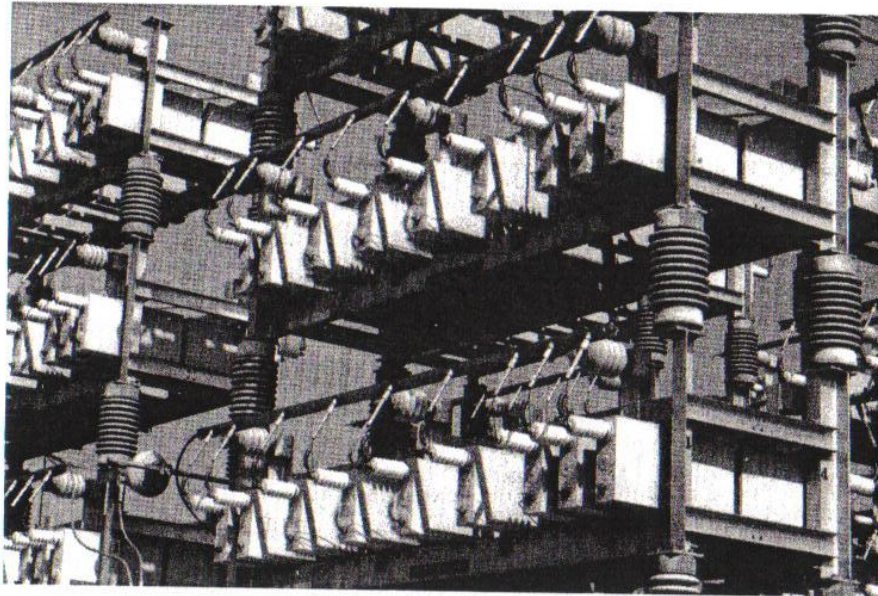
“b”: phasor diagram with lagging power factor load

“c”: the capacitive reactance cancels a portion of the line inductive reactance causing the receiving voltage to rise

Series capacitors can be switched or fixed and are protected the same way as are shunt capacitors

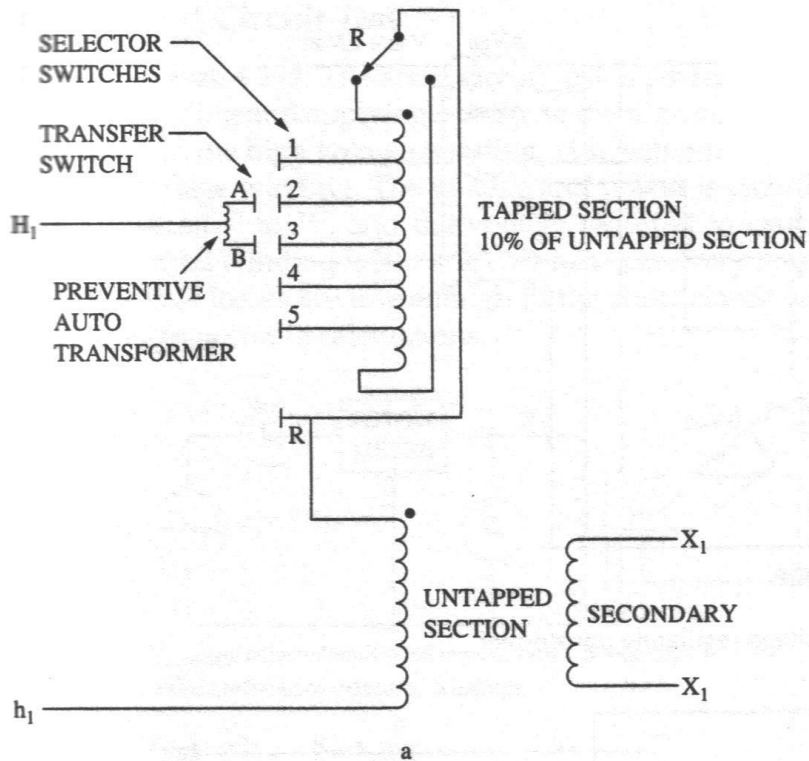


a



b

Tap Changers



Taps are connections on a transformer winding that change the turns ratio slightly (10%-5%)

Tap changers can be manual or automatic.

Manual tap changers are mostly used in distribution substations so that added load can be compensated for, such as new shop in a shopping center.

Motor driven automatic tap changers are used for VR with widely fluctuating loads.

Summary

1. Transformers are electrical machines that work by electromagnetic induction.
2. Two types of transformers: dry and oil immersed. The additional cooling and insulation properties of the oil result in greater power handling capabilities for the oil immersed transformers.
3. The magnetic flux is expanding and collapsing in the primary coil couples to the secondary coil inducing the voltage in the secondary in direct proportion to the turns ratio of the transformer.
4. The losses in a transformer include copper losses, power dissipated in winding resistance, hysteresis losses, power dissipated in work done reversing the magnetic field in the core once every half cycle, and eddy current losses
5. When power is first applied to a transformer the core must be magnetized before the transformation can occur. Energizing a transformer result in high initial current, called inrush current, to establish the initial magnetic field.
6. The change in the secondary voltage from no load to full load is called voltage regulation.

Summary

7. Autotransformers are efficient and less expensive transformers constructed with a tapped single winding for each phase.
8. Transformer reactance and efficiency can be calculated using the results of the short and open circuit test.

■ IMPORTANT NUMBERED EQUATIONS FROM THIS CHAPTER

$$E_{\text{rms}} = 4.44fN\phi_{\text{max}} = 4.44fNB_{\text{max}}A_c \quad (5-6)$$

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a \quad (5-7)$$

$$\frac{|I_p|}{|I_s|} = \frac{|V_s|}{|V_p|} = \frac{N_s}{N_p} = \frac{1}{a} \quad (5-8)$$

$$Z_L = \frac{V_s}{I_s}$$

$$Z_{\text{in}} = a^2 Z_L$$

$$|S_o| = 4.44fNA_cB_{\text{max}}\ell_cH_{\text{rms}}$$

$$R_{\text{eq,p}} = R_p + a^2 R_s = R_p + R'_s$$

$$X_{\text{eq,p}} = X_p + a^2 X_s = X_p + X'_s$$

$$R_{\text{eq,s}} = \frac{R_p}{a^2} + R_s = R'_p + R_s \quad (5-20)$$

$$X_{\text{eq,s}} = \frac{X_p}{a^2} + X_s = X'_p + X_s$$

$$Z_{\text{eq}} = R_{\text{eq}} + jX_{\text{eq}} \quad (5-21)$$

$$|Z_{\text{eq,p}}| \approx |Z_{\text{sc}}| = \frac{V_{\text{sc}}}{I_{\text{sc}}} \quad (5-22)$$

$$R_{\text{eq,p}} \approx \frac{P_{\text{sc}}}{I_{\text{sc}}^2} \quad (5-23)$$

$$X_{\text{eq,p}} = \sqrt{|Z_{\text{eq,p}}|^2 - R_{\text{eq,p}}^2} \quad (5-24)$$

$$X_{\text{eq,p}} \approx \frac{Q_{\text{sc}}}{I_{\text{sc}}^2} \quad (5-25)$$

$$R_p = R'_s = \frac{1}{2} R_{\text{eq,p}} \quad (5-26)$$

$$X_p = X'_s = \frac{1}{2} X_{\text{eq,p}}$$

$$R_{\text{fe,p}} = \frac{V_{\text{oc}}^2}{P_{\text{oc}}} \quad (5-27)$$

$$|S_{\text{oc}}| = V_{\text{oc}} \times I_{\text{oc}} \quad \text{and} \quad (5-28)$$

$$Q_{\text{oc}} = \sqrt{|S_{\text{oc}}|^2 - P_{\text{oc}}^2}$$

$$X_{\text{m,p}} = \frac{V_{\text{oc}}^2}{Q_{\text{oc}}} \quad (5-29)$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \quad (5-30)$$

$$\eta = \frac{P_{\text{in}} - P_{\text{loss}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}}$$

$$= 1 - \frac{P_{\text{loss}}}{P_{\text{out}} + P_{\text{loss}}}$$

$$\text{Volt Reg} = \frac{|V_{\text{nl}}| - |V_{\text{fl}}|}{|V_{\text{fl}}|} \times 100\%$$

$$V'_{\text{nl}} = V_p$$

$$V'_s = V'_{\text{fl}} = V_p - I'_{\text{fl}} Z_{\text{eq,p}}$$

$$V_{\text{fl}} = V_s$$

$$V_{\text{nl}} = V'_p = V_s + I_{\text{fl}} Z_{\text{eq,s}}$$

$$V'_{\text{fl}} = V'_s$$

$$V'_{\text{nl}} = V_p = V'_s + I'_{\text{fl}} Z_{\text{eq,p}}$$

$$K = \sum h^2 \left(\frac{I_h}{I_{\text{rms}}} \right)^2$$

$$S_{\text{auto}} = V_h I_2 \left(\frac{V_2}{V_2} \right) = V_2 I_2 \left(\frac{V_h}{V_2} \right)$$

$$S_{\text{auto}} = S_{\text{rated}} \left(\frac{V_1 + V_2}{V_2} \right)$$

$$= S_{\text{rated}} (a + 1)$$

$$I_{\text{S/C}} = \frac{(I_S)}{Z_{\text{PU}}}$$