Transformer Modeling for Simulation of Low-Frequency Transients

J.A. Martinez, *Member, IEEE*, and B.A. Mork, *Member*, IEEE

Abstract-- **This presentation gives a review of models proposed for representing transformers in low-frequency transients, with the application of interest being ferroresonance. The document presents a classification of the most popular models and discusses guidelines for representation of nonlinear and frequency dependent phenomena associated with transients below the first winding resonance.**

Index Terms— **Transformer Modeling, Ferroresonance, Inrush, Simulation.**

I. INTRODUCTION

T he development of an accurate transformer model can be very complex due to the large number of core designs and to the fact that several transformer parameters are both non-linear and frequency dependent. Physical attributes whose behavior may need to be correctly represented are core and coil configurations, self- and mutual inductances between coils, leakage fluxes, skin effect and proximity effect in coils, magnetic core saturation, hysteresis and eddy current losses in core, and capacitive effects [1]. Models of varying complexity have been developed and implemented in simulation tools to duplicate the transient behavior of transformers. This presentation summarizes the state-of-the-art on transformer models for simulation of low frequency transients, such as ferroresonance, inrush transients, and harmonic interactions.

II. TRANSFORMER MODELS

Transformer models for simulation of low-frequency transients can be classified into three groups, whose main characteristics are summarized below.

1) Matrix representation: The transformer equation for transient calculations can be written in the following form

$$[v] = [R][i] + [L][di/dt] \qquad (1)$$

where [R] and jω[L] are respectively the real and the imaginary part of the branch impedance matrix. In case of a very low excitation current, the transformer should be described by the following equation

$$[di/dt] = [L]^{-1}[v] - [L]^{-1}[R][i] \qquad (2)$$

Both approaches include phase-to-phase couplings and terminal characteristics, but they do not consider differences in core or winding topology; besides these models are linear and theoretically valid only for the frequency at which the nameplate data was obtained, although they are reasonably accurate for frequencies below 1 kHz [2]. For simulation of saturable cores, excitation may be omitted from the matrix description and attached externally at the model terminals in the form of non-linear elements; such core is not always topologically correct, but good enough in many cases.

2) Saturable Transformer Component: A single-phase N-winding transformer model can be based on a star-circuit representation, whose equation has the following form [2]

$$[L]^{-1}[v] = [L]^{-1}[R][i] + [di/dt] \qquad (3)$$

Saturation and hysteresis effects can be modeled by adding an extra non-linear inductor at the star point. This model can be extended to three-phase units through the addition of a zero-sequence reluctance parameter. This model is of limited application, even for single-phase units, since magnetizing inductance and the resistance in parallel are connected to the star point, which is not always the correct topological connecting point.

3) Topology-based models can very accurately represent any type of core design in low-frequency transients if parameters are properly determined. These models can be derived using at least two different approaches.

Duality-based models: The application of the principle of duality results in models that include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects [3] − [6]. In the equivalent magnetic circuit, windings appear as MMF sources, leakage paths appear as linear reluctances, and magnetic cores appear as saturable reluctances. The mesh and node equations of the magnetic circuit are duals of the electrical equivalent node and mesh equations respectively. Winding resistances, core losses, and capacitive coupling effects are not obtained directly from the transformation, but can be added to the equivalent circuit.

Geometric models: Topologically correct models can be based on the following formulation

$$[v] = [R][i] + [d\lambda/dt] \qquad (4)$$

The coupling between magnetic and electrical equations is made taking into account the core topology, see [7], [8].

Juan A. Martinez is with the Dept. d'Enginyeria Elèctrica, Universitat Politècnica de Catalunya, 08028 Barcelona, Spain.

Bruce A. Mork is with the Dept. of Electrical Engineering, Michigan Technological University, Houghton, MI 49931, USA.

III. NONLINEAR AND FREQUENCY-DEPENDENT PARAMETERS

Some transformer parameters are non-linear and/or frequency dependent due to three major effects: saturation, hysteresis and eddy currents. Saturation and hysteresis are included in the representation of the iron core and introduce distortion in waveforms. Excitation losses are caused by hysteresis and eddy current effects, although in modern transformers they are mostly due to eddy current.

A. Modeling of Iron Cores

Iron core behavior is usually represented by a relationship between the magnetic flux density B and the magnetic field intensity H. To characterize the material behavior fully, a model has to be able to plot numerous associated curves (major and minor loops). Hysteresis loops usually have a negligible influence on the magnitude of the magnetizing current, although hysteresis losses and the residual flux can have a major influence on some transients, e.g., inrush currents. Magnetic saturation of an iron core can be represented by the anhysteretic curve, the B–H relationship that would be obtained if there was no hysteresis effect in the material. The saturation characteristic can be modeled by a piecewise linear inductance with two slopes, since increasing the number of slopes does not significantly improve the accuracy. However, there are some cases, e.g. ferroresonance, for which a more detailed representation of the saturation characteristic is usually required. The specification of such inductor requires a curve relating the flux linkage, λ, to the current, i. The information usually available is the rms voltage as a function of the rms current.

B. Modeling of Eddy Current Effects

Several physical phenomena, known as eddy current effects, occur simultaneously in a loaded transformer that result in a nonuniform distribution of current in the conductors, and manifest themselves as an increase in the effective resistance and winding losses with respect to those for direct current. Eddy current effects in transformer windings can be modeled by Foster equivalent circuits. These circuits must be of infinite order to exactly reproduce the impedance at all frequencies. However, a computationally efficient circuit can be derived by fitting only at certain pre-established frequencies [9]. A series model of order equal or less than 2 is adequate for low-frequency transients.

A change in the magnetic field induces also eddy currents in the iron. As a consequence of this, the flux density will be lower than that given by the normal magnetization curve. As frequency changes, flux distribution in the iron core lamination changes. For high frequencies the flux is confined to a thin layer close to the lamination surface, whose thickness decreases as the frequency increases. This indicates that inductances representing iron path magnetization and resistances representing eddy current losses are frequency dependent. Efficient models intended for simulation of frequency dependent magnetizing inductances have been derived by synthesizing Cauer equivalent circuits to match the equivalent impedance of either a single lamination or a coil wound around a laminated iron core limb [10], [11]. Inductive components of these models represent the magnetizing reactances and have to be made non-linear to account for the hysteresis and saturation effects. Since the high frequency components do not contribute appreciably to the flux in the transformer core, it can be assumed that only low frequency components are responsible for driving the core into saturation. It may, therefore, be justifiable to represent as non-linear only the first section of the model, so for low frequency transients a equivalent circuit with order equal or less than 2 may suffice.

IV. PARAMETER DETERMINATION

Data usually available for any power transformer are: power rating, voltage rating, excitation current, excitation voltage, excitation losses, short-circuit current, short-circuit voltage, short-circuit losses, saturation curve, capacitances between terminals and between windings. Excitation and short-circuit currents, voltages and losses must be provided from both direct and homopolar measurements.

The specification of some parameters can be a bottleneck due to the lack of reliable procedures for their determination, since their calculation cannot be performed from standard measurements, and additional information is usually required. See [12] for the calculation of leakage inductances; [5], [6], [13] for the calculation of parameters to be specified in duality-based models; [14] for a study on the influence of eddy current losses and the determination of resistances as a function of frequency; and [15], [16] for the determination of saturation characteristic and hysteresis parameters.

V. CONCLUSIONS

This presentation summarizes the most important issues related to transformer modeling for simulation of low-frequency-transients. Although much effort has been dedicated to the development of transformer models, there is no consensus on the most adequate models. The most important difficulties are the great variety of core designs, the non-linear and frequency dependent behavior of many transformer parameters, and the inadequacy of procedures for acquisition and determination of some transformer parameters.

VI. REFERENCES

[1] IEEE Slow Transients TF, "Modeling and analysis guidelines for slow transients – Part III: The study of ferroresonance," *IEEE Trans. on Power Delivery*, vol. 15, no. 1, pp. 255-265, January 2000.

[2] H.W. Dommel, *EMTP Theory Book*, Bonneville Power Administration, Portland, August 1986.

[3] C.M. Arturi, "Transient simulation and analysis of a five-limb generator step-up transformer following an out-of-phase synchronization," *IEEE Trans. Power Delivery*, vol. 6, no. 1, pp. 196-207, January 1991.

[4] F. de León and A. Semlyen, "Complete transformer model for electromagnetic transients," *IEEE Trans. on Power Delivery*, vol. 9, no. 1, pp. 231-239, January 1994.

[5] A. Narang and R. H. Brierley, "Topology based magnetic model for steady -state and transient studies for three phase core type transformers," *IEEE Trans. on Power Systems*, vol. 9, no. 3, pp. 1337-1349, August. 1994.

[6] B.A. Mork, "Five-legged wound-core transformer model: Derivation, parameters, implementation, and evaluation," *IEEE Trans. on Power Delivery*, vol. 14, no. 4, pp. 1519-1526, October 1999.

[7] N.D. Hatziargyriou, J.M. Prousalidis and B.C. Papadias, "Generalised transformer model based on the analysis of its magnetic core circuit," *IEE Proc.-C*, vol. 140, no. 4, pp. 269-278, July 1993.

[8] X. Chen, "A three-phase multi-legged transformer model in ATP using the directly-formed inverse inductance matrix," *IEEE Trans. on Power Delivery*, vol. 11, no. 3, pp. 1554-1562, July 1996.

[9] F. de León and A. Semlyen, "A simple representation of dynamic hysteresis losses in power transformers," *IEEE Trans. on Power Delivery*, vol. 10, no. 1, pp. 315-321, January 1995.

[10] F. de León and A. Semlyen, "Time domain modeling of eddy current effects for transformer transients," *IEEE Trans. on Power Delivery*, vol. 8, no. 1, pp. 271-280, January 1993.

[11] E.J. Tarasiewicz, A.S. Morched, A. Narang and E.P. Dick, "Frequency dependent eddy current models for nonlinear iron cores," *IEEE Trans. on Power Systems*, vol. 8, no. 2, pp. 588-597, May 1993.

[12] F. de León and A. Semlyen, "Efficient calculation of elementary parameters of transformers," *IEEE Trans. on Power Delivery*, vol. 7, no. 1, pp. 376-383, January 1992.

[13] D. L. Stuehm, "Final report - Three phase transformer core modeling," Bonneville Power Administration Award no. DE-BI79-92BP26700, Feb. 28, 1993.

[14] E.E. Fuchs, D. Yildirim and W.M. Grady, "Measurement of eddy-current loss coefficient PEC-R, derating of single-phase transformers, and comparison with K-factor approach.," *IEEE Trans. on Power Delivery*, vol. 15, no. 1, pp. 148-154, January 2000. See Correction in *IEEE Trans. on Power Delivery*, vol. 15, no. 4, pp. 1357, October 2000.

[15] W.L.A. Neves and H.W. Dommel, "On modeling iron core nonlinearities," *IEEE Trans. on Power Systems*, vol. 8, no. 2, pp. 417-425, May 1993.

[16] D.C. Jiles, J.B. Thoelke and M.K. Devine, "Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis," *IEEE Trans. on Magnetics*, vol. 28, no. 1, pp. 27-35, January 1992.

VII. Biographies

Juan A. Martinez was born in Barcelona (Spain). He is Profesor Titular at the Departament d'Enginyeria Elèctrica of the Universitat Politècnica de Catalunya. His teaching and research interests include Transmission and Distribution, Power System Analysis and EMTP applications.

Bruce A. Mork was born in Bismarck, ND. He received the B.S. degree in Mechanical Engineering and the M.S. and Ph.D. degrees in Electrical Engineering from North Dakota State University in 1979, 1981 and 1992 respectively. In September 1992, he joined the faculty of Michigan Technological University, where he is an Associate Professor of Electrical Engineering.