

Objectives: Constrained minimization via LaGrange multipliers, lossless optimal dispatch, correcting unacceptable bus voltages with shunt capacitor banks or shunt reactors.

- 1) Using the method of LaGrange multipliers, minimize the function  $f(x,y) = x^2 + y^2$ , subject to the constraint  $y = 2x + 8$ . Sketch out the function and the constraint, and explain in practical terms what you've done.
- 2) After running a load flow simulation, the voltage magnitude at a load bus is found to be 1.10 per unit. (Hint: since you will need to address 2 cases here and you may need to do this calculation again in the future, consider writing a Matlab program to do this calculation.)
  - a) What type of shunt impedance should be added at this bus to pull the voltage down to 1.0 per unit?
  - b) If the Thevenin impedance at this bus is  $Z = 4.0 + j6.0$  per unit, calculate the value of this shunt impedance. If your calculation indicates more than one mathematical solution, indicate which one you would select and why.
  - c) Repeat parts a) and b) for the case where a bus voltage of 0.85 per unit is to be raised to 1.03 per unit.
  - d) Write a short explanation of how this technique can be used for planning studies. What bit of information is needed that might not be directly available to someone who is running the load flow studies?
- 3) Two power plants in a system are operating under economic dispatch constraints. For this simplistic case, line losses are neglected. Minimum output of each unit is 0.5 per unit; maximum is 5.0 per unit. Incremental operating costs of the plants in per unit on a 100 MVA base are:

$$\frac{\partial C_1}{\partial P_{G1}} = 200P_{G1} + 900$$

$$\frac{\partial C_2}{\partial P_{G2}} = 300P_{G2} + 800$$

$\lambda_{sys}$	$\lambda_1$	$P_{G1}$	$\lambda_2$	$P_{G2}$	$P_{G1} + P_{G2}$
					1.0 p.u.
				2.75 p.u.	
					5.0 p.u.
					10.0 p.u.

- a) Determine how to optimally schedule the generation for the 4 cases given in the table above. Show your calculations. (The lambda value asked for is the "operative system lambda.")
- b) Plot  $P_{G1}$  and  $P_{G2}$  for system load over the range 1.0 to 10.0 per unit. Clearly label the regions where the inequality constraints are "active" (i.e. minimum or maximum power output limits of an individual generator). Also label the region where only the equalities apply.
- c) What exactly is lambda? Write a short explanation of what it means. Contrast between what the operative "system lambda" is and the lambdas of individual generators.