EE 5200 - Lecture 19 Mon Oct 14, 2019

Topics for Today:

- Announcements
  - Matlab may be incorporated in upcoming Hmwks.
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems from Ch.6, solutions posted
  - Next: Transmission Lines as 2-port networks
  -

Chapter 5 - Recap: Series Inductance of Transmission Lines

- Review of mutual inductance concepts - recap
- Mutual inductance between 2 conductors
- Inductance matrix for group of conductors
- ATPDraw Line Constants
- Traditional methods for per-phase parameters
  - Geometric mean “averaging” of effective radius and phase spacing. Single-circuit, double-circuit.
  - Use of tables - standard 1-foot phase spacing.

Chapter 6 - Shunt Capacitance Transmission Lines

- Fundamental definition of capacitance, $C = Q/V$
- Capacitance between 2 conductors
- Capacitance to earth, image charges
- Capacitance of 3-phase overhead lines, [C] matrix.
- Geometric mean “averaging” of effective radius and phase spacing. Single-circuit, double-circuit.
- Use of tables - standard 1-foot phase spacing.

EE 5200 Term Project (updated: Oct 4, 2022) Fall 2022

A term **project** shall be done in lieu of a final exam. The project you choose:
- must be of topical interest, and relate to course material of EE5200. 2 persons/team.
- must be new work (not copied from your previous courses or a past student's project).
- must demonstrate a graduate student level of mastery and **application** of the related concepts and theories. [Note: this is not simply a term paper, but a **project**.]
- is sufficiently researched, referenced, and documented, and also includes the in-depth analysis and evaluation of the concepts of the most key journal paper related to this work.
- length of body of report: approximately 10 pages of text (not including figures, tables, or equations). All writing is your own original writing. No plagiarism. No self-plagiarism.

Time line and required submissions are as follows, add'l deliverables contribute to the grade of your term project, i.e. ~15-20% of your course grade. Schedule is:
- Week 6: submit short e-mail with idea(s) requesting instructor feedback.
- Week 7: submit formal outline of project and list of key references.
- **Week 9: submit expanded outline of project and complete reference list** - *.
- Week 11: Submit draft of journal paper analysis (JPA)
- **Week 12: Demo working base case model, submit rough draft of project report** *
- **Week 13: Submit final JPA and .ppt presentation** *
- **Week 14: Submit final report/deliverable** *
- Finals week: be prepared to present/demonstrate project during final exam time-slot.

*** graded milestone**

Report Outline:

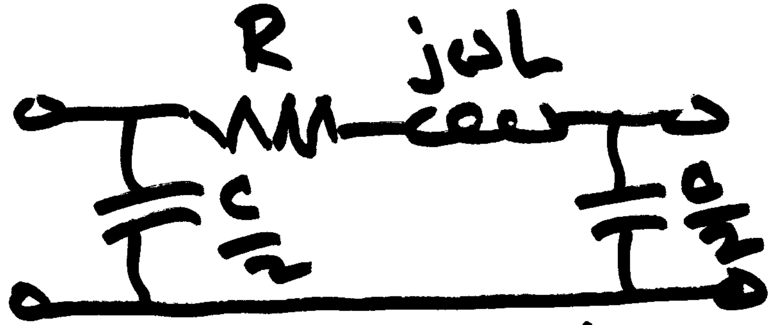
Front Matter:
- Title Page
- Executive Summary (not needed for initial draft)
- Table of Contents (use as "working outline")
- Statement of contributions by each team member, signed in agreement by all.

Body of report (max 10 pages of text, plus figs):
- Introduction (brief overview of project: problem area, motivation, overview of project)
- Background
    - literature search, most important references
    - Presentation of key concepts connected with project
    - Identification of existing voids or weaknesses, and resulting opportunity
- Proposed Approach
    - Overview of basic idea that you will develop and implement
    - Development of applied math details
- Implementation (may be only partially complete in draft versions)
- Results (Expected Results in draft versions)
- Conclusions: salient points, cause-and-effect relationships, sensitivities, etc.
- Recommendations for Continued Work

Supplemental Information:
- Reference List (IEEE format, numbered [1], [2], etc, in order of first author's last name)
- Appendices as required to document details

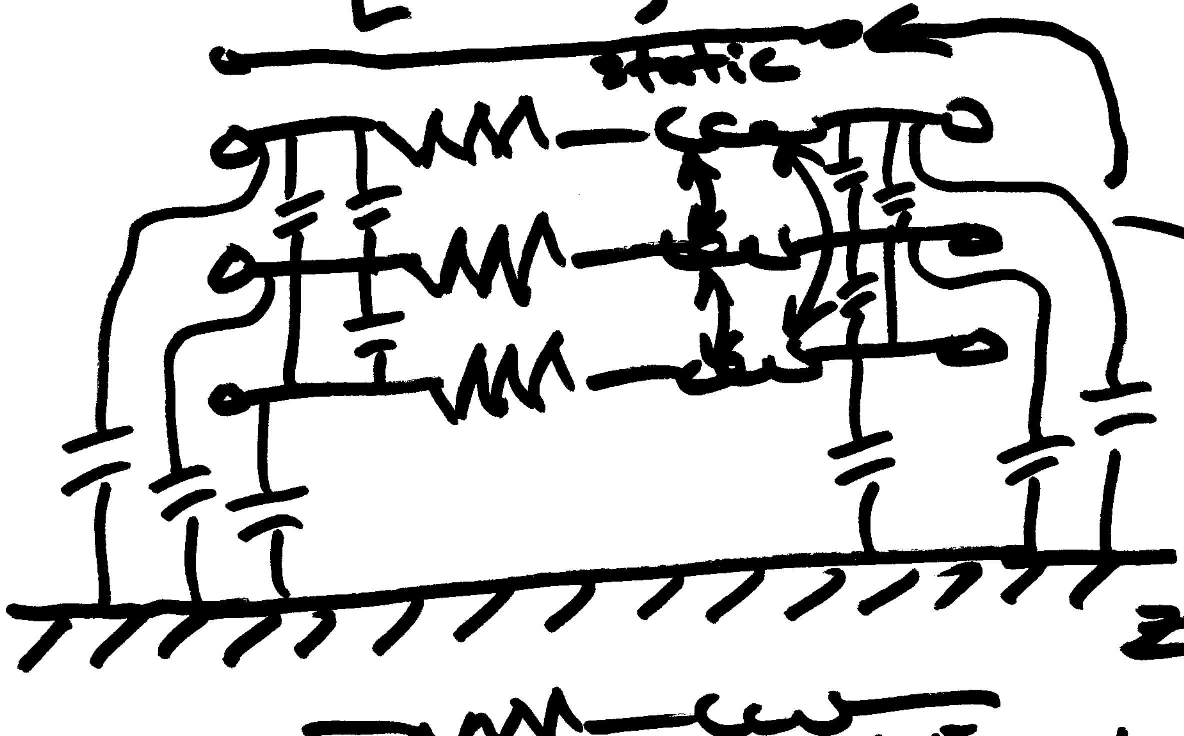
Required format and layout:
- Font: 11-pt CG Times w/1.25-1.5 line spacing; or 10-pt comic or ariel w/1.0-1.25 line space
- Page layout: 1" margins, include page numbering within margin area.
- Use equation editor, number equations, call out references by number [1].

Transmission Lines:



per-phase
ΦA-N

Short-circuit
Load-Flow
Stability



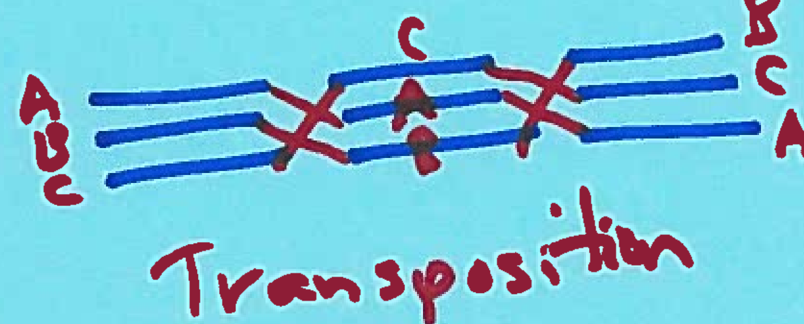
Carson's Formula

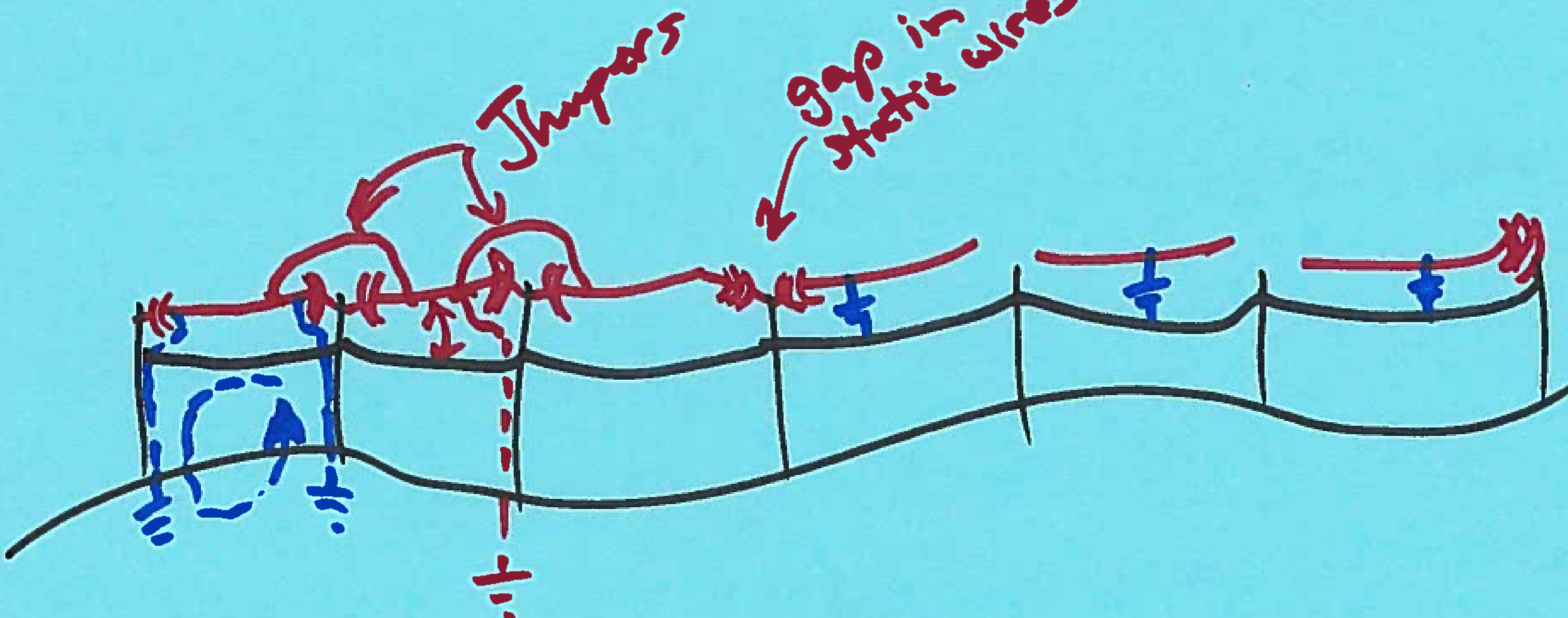
Full 3-phase
Coupled L's
and C's

Return
via
earth,
static
wires

Segmented ground/static "wires"?

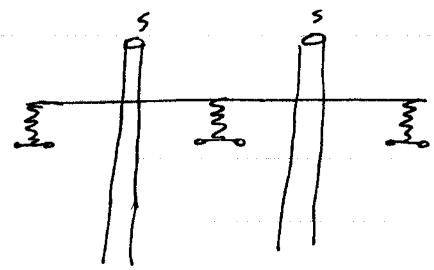
- Segmented Ground





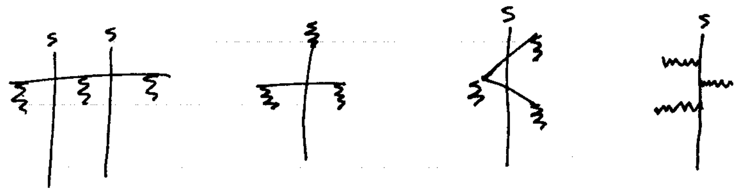
LECTURE NO. 5

Overhead T-line Configurations

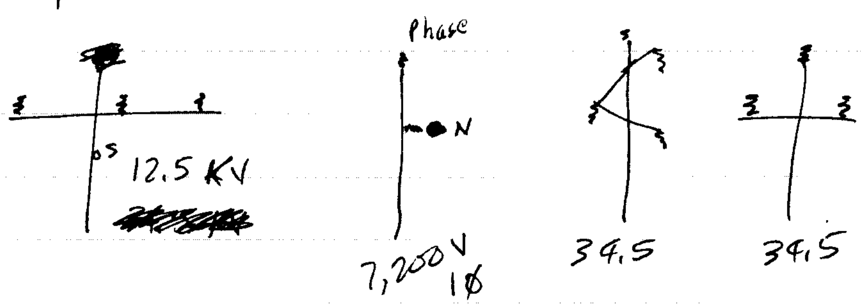


69 KV → 765 KV

SEE "EPRI RED BOOK"



34.5 KV → 115 KV

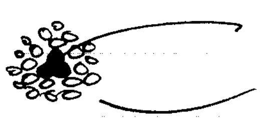


2.4 KV → 34.5 KV

<u>CONDUCTORS</u> - Were ~~aluminum~~ copper but are now aluminum (lighter, cheaper, less corona)

AAC : All aluminum conductor
AAA : All aluminum alloy conductor
ACSR : <u>A</u>luminum <u>C</u>onductor <u>S</u>teel <u>R</u>einforced
ACAR : Aluminum Conductor Alloy Reinforced
AWAC : Aluminum Wound Air Core (Expanded ACSR)

Wire is stranded for strength



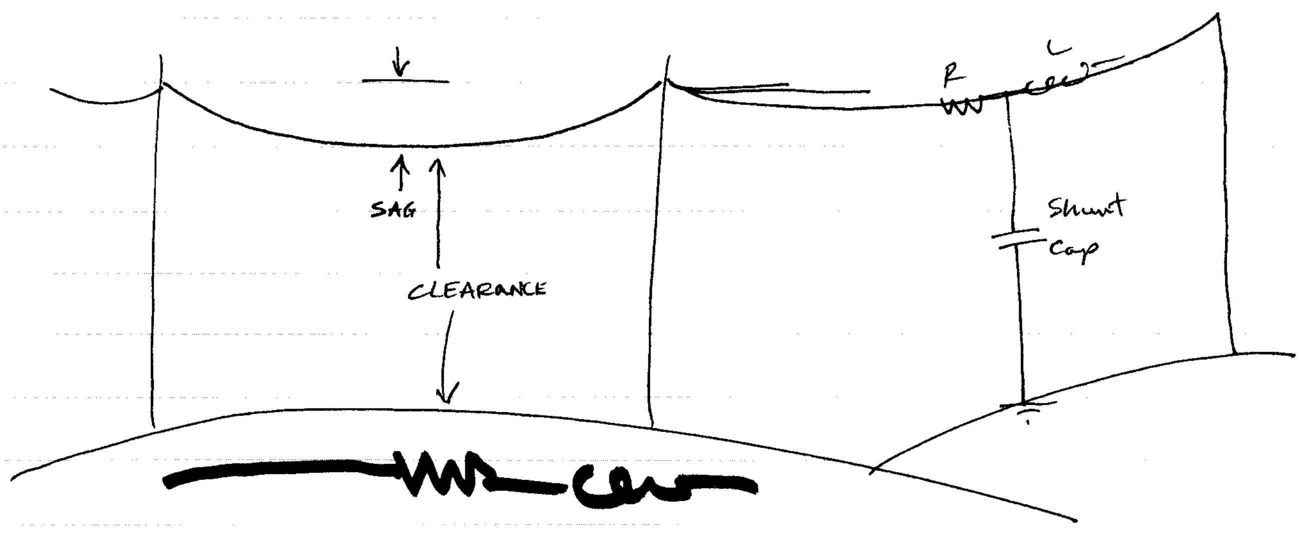
Steel carries mechanical load
Al carries current

Look at table A.1 in Stevenson (pp. 750-753)
($1 \text{ cmil} = .7854 \text{ mil}^2$)

1000 circular mills

266.8	MCM = KCM	WaxWing	18/1	2 layers Al, 1 steel
266.8	MCM	Partrige	26/7	2 layers Al, 2 steel (115-kV OTP)
795	MCM	Cuckoo	24/7	2 layers Al, 2 steel (230-kV OTP)
~~1272~~	~~MCM~~	~~Bittern~~	~~45/7~~	
1272	MCM	Bittern	45/7	3 layers Al, 2 steel (345-kV Minkota)

Total number of strands $= 3x^2 - 3x + 1$ | x = no. of layers



Plus: Mutual Inductance and capacitance between lines.

Lucky to estimate T-Line Params within 5-10%.

RESISTANCE

From Physics $R = \frac{\rho L}{A}$

ρ = Resistivity $\frac{\Omega\text{-}CM^2}{ft}$ (ΩM)
L = Length ft M
A = Area CMIL M^2

$1 \text{ Cmil} = \frac{\pi d^2}{4}$ $d = .001$ in $\ominus^1$

Area in circular mils = (dia in Mils)^2
" " square inches = (area in circular mils)$\frac{4}{\pi}$

$\rho_{Cu} = 10.66\ \Omega \cdot \text{cmil/ft}$
→ $\rho_{AL} = 17\ \Omega \cdot \text{cmil/ft}$
} at 20°C

Corrections to resistance:

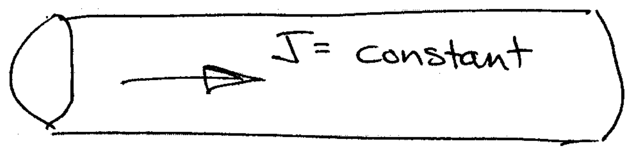
1) Spiraling: $R = \frac{\rho L}{A}(1.02)$ (add 2% to length)

2) Temp Rise: $\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$

R_1 = Res @ t_1 ~~4~~
R_2 = Res @ t_2
T = 241 Cu
T = 228 AL
t_1 & t_2 in °C

3) Resistance & Current Density

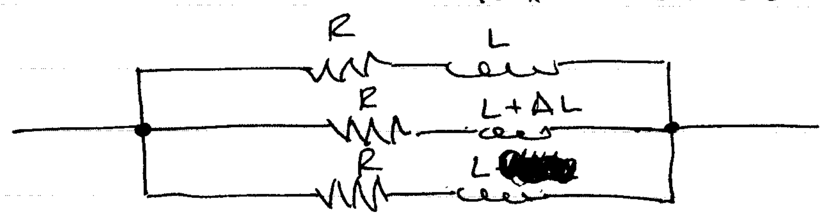
For dc current



@ dc.

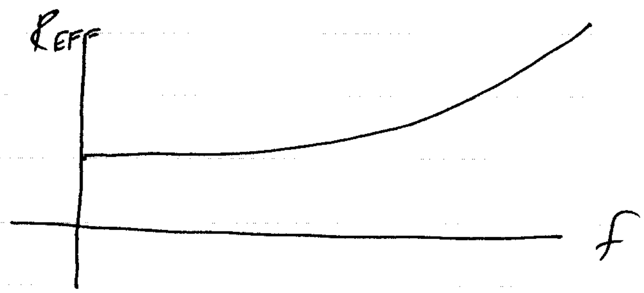
current density is uniform across cross section of conductor.

For ac, magnetic lines of force in center of conductor force current to outside surface. (Non-uniform flux distribution)



Therefore, $R_{ac} > R_{dc}$ by about 1-10% @ 60Hz

Use tables (p. 750) to get actual measured values



$$R_{EFFECTIVE} = R_{AC} = \frac{P_{LOSS}}{|I|^2}\ \Omega$$

∴ Resistance is no problem - use tables.

INDUCTANCE - NOT AS EASY BUT TABLES CAN BE USED IN MANY CASES.

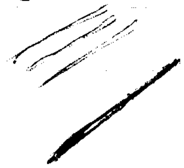
Cases to Consider

1) Self inductance of single line.

2) Single phase groups of wires.

3) Mutual inductance with parallel phone lines, etc.

4) 3ϕ, Bundled conductors



Going back to field theory -

$$v = L\frac{di}{dt} \qquad \tilde{V} = j\omega L \tilde{I}$$

$$L = \frac{\lambda_{\text{LINKED BY } I}}{I}$$

ψ & I are in phase

λ = Flux Linked/M
I = Current (Phasor rms)
L = Inductance/unit length

$$\lambda = \int \vec{B} \cdot \vec{dA}$$

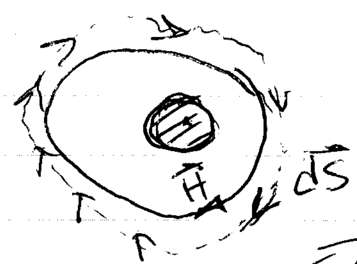
B = Magnetic Flux Density WB/m^2
dA = area in square meters

$B = \mu H = \mu_r \mu_0 H$

H = Magnetic field intensity $\frac{A\text{-}T}{m}$

μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ H

μ_r = relative permeability $= 1$ for air



$\Rightarrow |\vec{H}|\,|d\vec{s}| \cos\theta_{HS}$

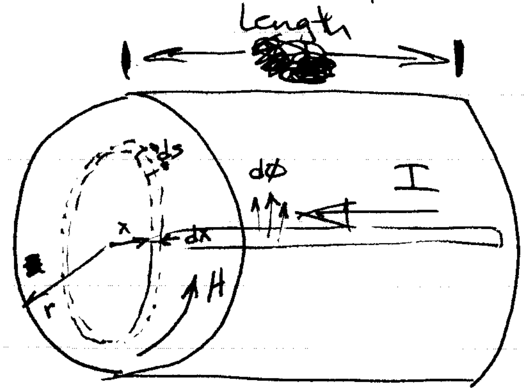
$\oint \vec{H} \cdot d\vec{s} = I_{ENCLOSED}$ (Amperes Law)

$d\vec{s}$ = incremental length (dl or dx)

Procedure to find L: (reversing preceding)

1) Find H given I
2) Find $B = \mu_0 H$
3) Find $\lambda = \psi$ linked (Wb)
4) Determine I
4a) $L = \frac{\lambda}{I}$

EX: Case I: Self (internal) inductance of single wire:



$\oint \vec{H} \cdot d\vec{s} = I_{enclosed}$

since $\vec{H}$ & $d\vec{s}$ are in same direction,

$\oint \vec{H} \cdot d\vec{s} = H_x(2\pi x) = I_x$

where I_x = current enclosed by radius x

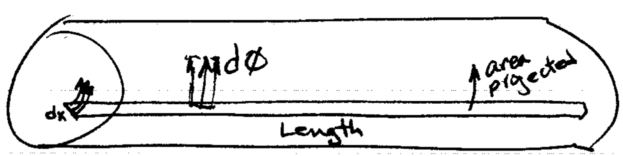
Assuming constant current density,

1) $I_x = \frac{\cancel{\pi} x^2}{\cancel{\pi} r^2} I$ where I = total current in conductor

2) $H_x = \frac{I_x}{2\pi X}$ from previous eqn.

$H = \frac{I x^2}{2\pi x r^2} = \boxed{\frac{I x}{2\pi r^2} \quad \frac{A-T}{m}}$ first part of procedure satisfied.

$B = \mu_0 H = \boxed{\frac{\mu_0 x I}{2\pi r^2}}$ second part of procedure satisfied.



$d\phi$ = magnetic flux/meter Wb/m

$d\phi = \frac{\mu x I}{2\pi r^2} dx$ Wb/m

$\psi_{LINKED_x} = \phi \frac{\pi x^2}{\pi r^2}$ Wb

$d\psi_L = \frac{\mu_0 x I}{2\pi r^2} dx \,(\text{Length})\left(\frac{x^2}{r^2}\right)$

ψ_L = lines of flux × no. of turns linked

$\psi_L = \int_0^r \frac{\mu_0 x^3 I (\text{Length})}{2\pi r^4} dx = \boxed{\frac{\mu_0 I (\text{Length})}{8\pi}}$ third part

$\boxed{L = \frac{\psi_L}{I} = \frac{\mu_0 (\text{Length})}{8\pi}}$ solution.

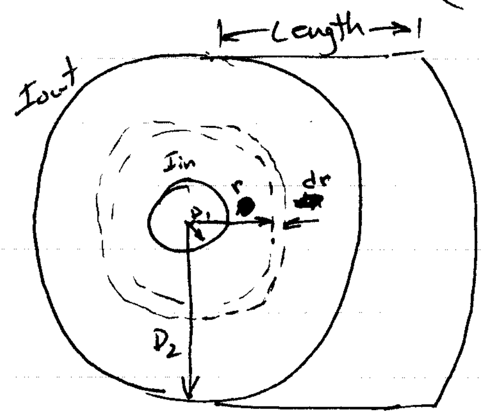
Since $\mu_0 = 4\pi \times 10^{-7}\ H/m$

$$L = \frac{4\pi \times 10^{-7}}{8\pi} = \boxed{\frac{1}{2} \times 10^{-7}\ H/m}\ \text{internal}$$

$$= .05\ \mu H/m$$

Note: Internal INDUCTANCE DOES NOT DEPEND ON THE SIZE OF THE CONDUCTOR

EX: CASE II – COAX (2-WIRE SYSTEM)



Let $I_{in} = I_{out}$

$$\int H \cdot ds = I_{ENCLOSED}$$

$$H\, 2\pi r = I$$

$$\boxed{H = \frac{I}{2\pi r}} \quad \#1$$

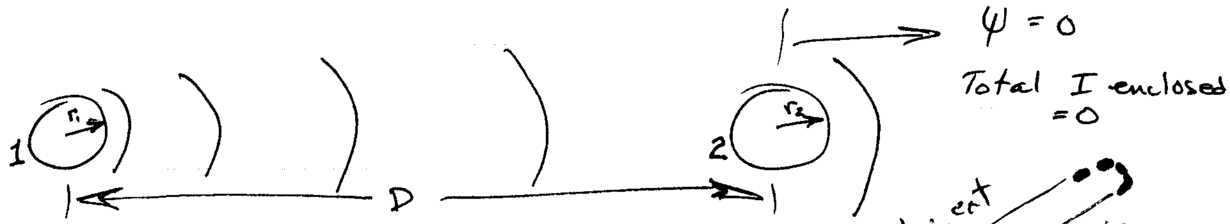
$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r} \quad \#2$$

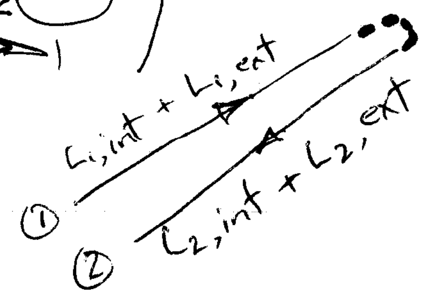
$$\psi = \int_{D_1}^{D_2} B \cdot dA = \text{Length} \int_{D_1}^{D_2} \frac{\mu_0 I}{2\pi r}\, dr$$

$$\psi = \frac{\mu_0 I \text{ length}}{2\pi} \ln \frac{D_2}{D_1} \quad \#3$$

$$L_{12} = \frac{\psi}{I} = \frac{\mu_0 \text{ Length}}{2\pi} \ln \frac{D_2}{D_1} = \boxed{2 \times 10^{-7} \ln \frac{D_2}{D_1}\ H/m}$$ Answer

Two Wire System





Look at flux linked by I_1

$L_1 = L_{INTERNAL} + \underbrace{L_{EXTERNAL}}_{\text{= mutual between 1 \& 2}}$ IF $D \gg r_2$ & r_1

$L_{1\,ext} = 2\times10^{-7} \ln \dfrac{D}{r_1}$ ← mutual effect of cond 2 on cond 1.

$L_{1\,int} = \frac{1}{2}\times10^{-7}$ H/m

$$L_1 = 2\times10^{-7}\ln\frac{D}{r_1} + \frac{1}{2}\times10^{-7}$$

$$= \left(\frac{1}{2} + 2\ln\frac{D}{r_1}\right)\times10^{-7} \quad \text{H/m}$$

$$= 2\times10^{-7}\left(\frac{1}{4} + \ln\frac{D}{r_1}\right)$$

Substitute $\frac{1}{4} = \ln e^{+\frac{1}{4}}$

$$L_1 = (2\times10^{-7})\left(\ln\frac{D}{r_1 e^{-1/4}}\right)$$

$.7788 = e^{-\frac{1}{4}}$

Substitute $r' = r_1 e^{-1/4}$ = effective radius

$$\boxed{L_1 = 2\times10^{-7}\ln\frac{D}{r_1'} \quad \text{H/m}}$$

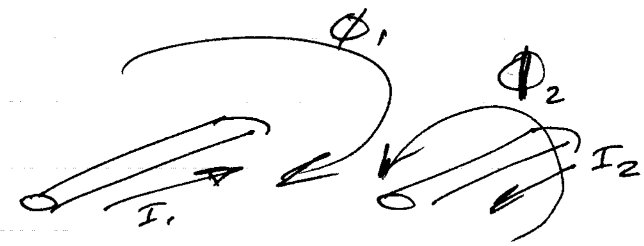
$$\boxed{L_1 = .7411 \log_{10} \frac{D}{r_1'} \quad mH/mile}$$

more common form

r_1' = radius of a ficticious conductor with no internal flux but that has the same total inductance as the original conductor of radius r_1

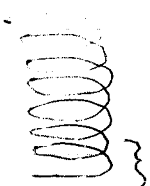
So we multiply $r_1 \times e^{-\frac{1}{4}} = r_1 \times .7788$ to get r_1' - only for solid round Conductors. Note: May have to measure for stranded conductor.

Since the currents are opposite



L_{TOTAL} for this 2 wire loop circuit is the

$$L_{TOTAL} = L_1 + L_2 = L_{1,int} + L_{1,ext} + L_{2,int} + L_{2,ext}$$



where

$$L_2 = 2\times10^{-7} \ln \frac{D}{r_2'}$$

$$L_{TOTAL} = 2\times10^{-7} \left(\ln \frac{D}{r_2'} + \ln \frac{D}{r_1'} \right)$$

$$= 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}} \quad H/m$$

$\leftarrow$ GMR

$$= 1.482 \log_{10} \frac{D}{\sqrt{r_1' r_2'}} \quad mH/mile.$$

EX: 3 mile solid conductor 0.5" DIA
Find X_L @ 60 Hz & 2 ft spacing

$$L = 1.482 \log_{10} \frac{2\text{ ft}}{r'} \frac{\text{mH}}{\text{mile}} (3\text{ miles})$$

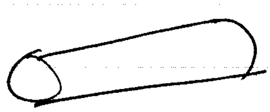
$$r' = \frac{0.5''}{2} e^{-1/4} = .1947''$$

$$r' = (.1947'')\left(\frac{1\text{ ft}}{12''}\right) = .016225\text{ ft}$$

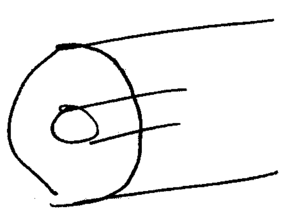
$$L = 1.482(3) \log_{10} \frac{2}{.016225} = \underline{\underline{9.296\text{ mH}}}$$

$$\boxed{\begin{aligned} X_c &= 2\pi 60 L \\ &= 377L = 3.504\Omega \end{aligned}}$$

Summary



wire $L = 2\times10^{-7}$ H/m



coax $L = 2\times10^{-7} \ln \frac{r_2}{r_1}$ H/m
ind between the conductors

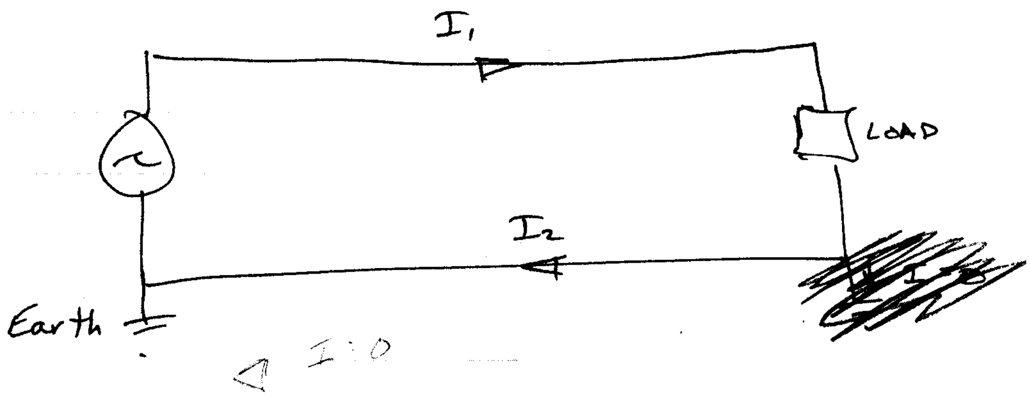
2 wires $L_{TOTAL} = 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}}$ H/m

for solid wires only

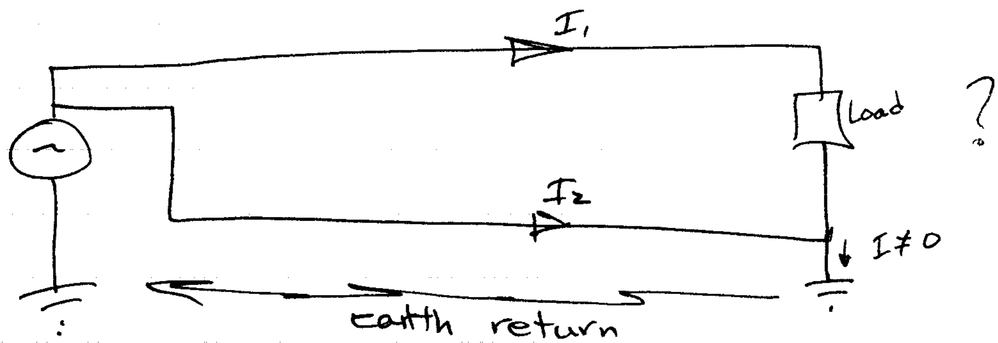
where $r_1' = r_1 e^{-1/4}$

includes self inductance

IMPORTANT - For 2 wire loop
$I_1 = I_2$



IF there is a ground fault,



The above calculated L is incorrect

EX: For 2 wire FLICKER with assumed spacing

$Z = .1945 + j.684$

For 2 wire with earth return (like zero sequence)

$Z = .652 + j2.7455$

So must not have earth return to use the method in the book.

More Detailed Explanation of "Lecture No.5"

Inductance

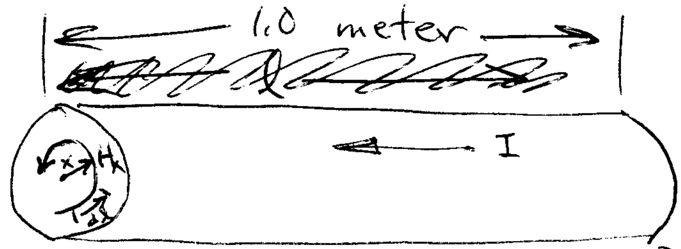
$$L = \frac{\lambda}{i}$$

4 Cases:

1) Self-inductance of single line
2) Single phase groups of wires.
3) Mutual inductances { parallel lines, phone lines }
4) 3ϕ, Bundled Conductors

Field Theory Basics

Current flowing in conductor produces magnetic field H (right hand rule)



$$\vec{B} = \underbrace{\mu_r}_{(1.0 \text{ for air or nonmagnetic mat'ls})} \underbrace{\mu_o}_{4\pi \times 10^{-7}\ H/m} \vec{H}$$

B = Magnetic flux density (Wb/m^2)
H = Magnetic field intensity (A/m)

To find H, use Ampere's Circuital Law

$$\oint \vec{H} \cdot \vec{dl} = I \text{ enclosed} = \int \vec{J} \cdot \vec{dS}$$

Stoke's Theorem

in general $I_{ENCL} = \underbrace{2\pi x}_{\text{circumference}} H_x$

STEP 1

$$\boxed{H_x = \frac{I_x}{2\pi x} \text{ Amps/meter}}$$

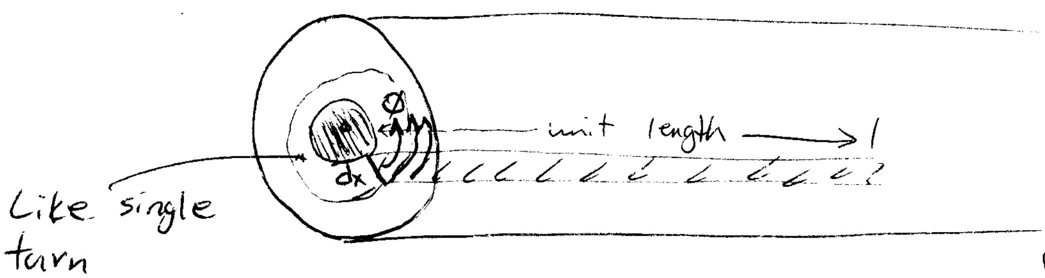
For a nonmagnetic conductor,

$$B_x = \mu_0 H_x = \frac{\mu_0 I_x}{2\pi x} \ \mathrm{Wb/m^2}$$

But $I_x = \left(\frac{x}{r}\right)^2 I$

STEP 2 $$\boxed{B_x = \frac{\mu_0 x I}{2\pi r^2} \ \mathrm{Wb/m^2}}$$ flux density

Looking at flux ϕ flowing thru a differential cross section of conductor



flux crossing strip dx × unit length,

$$d\phi = B_x\,dx \ \mathrm{Wb/m}$$

$$\boxed{\lambda = N\phi_{enclosed}}$$

The flux linked (contained) by concentric flux path of radius x is:

STEP 3 $$d\lambda = \underbrace{\left(\frac{x}{r}\right)^2}_{\text{Portion of area for } r = x} d\phi = \frac{\mu_0 I}{2\pi r^4} x^3\,dx$$

integrating

$$\lambda_{\text{contained by wire}} = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3\,dx$$

$$= \frac{\mu_0 I}{2\pi r^4}\left(\frac{r^4}{4}\right) = \frac{\mu_0 I}{8\pi} \ \frac{\mathrm{Wb{\cdot}T}}{\mathrm{m}}$$

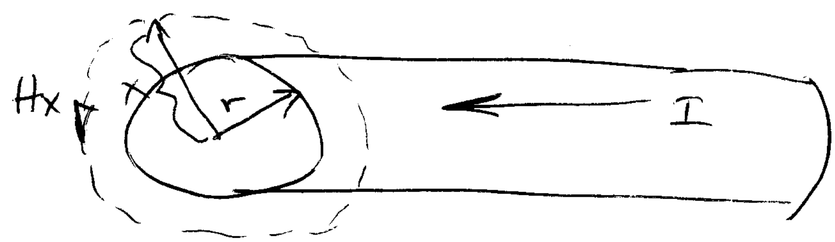
$$\boxed{\lambda = \frac{4\pi \times 10^{-7} I}{8\pi} = \frac{1}{2} \times 10^{-7} I}$$

Then,

$$\boxed{L = \frac{\lambda}{I} = \frac{1}{2} \times 10^{-7} \, H/m} = .05 \, \mu H/m$$

Note: Internal Inductance does not depend on size of conductor.

For a conductor of radius r, H_x outside the conductor at radius x



Step 1 $$H_x = \frac{I}{2\pi x} \Big| \; x \geq r$$

step 2 $$B_x = \mu_0 H_x = \frac{\overset{4\pi \times 10^{-7}}{\mu_0} I}{2\pi x} = 2 \times 10^{-7} \frac{I}{x} \; wb/m^2$$

~~$d\phi = B_x \, dx = 2 \times 10^{-7} \frac{I}{x} dx$ wb/m~~

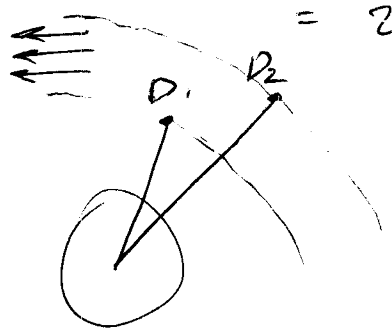
$$d\lambda = B_x \, dx = 2 \times 10^{-7} \frac{I}{x} dx = \frac{\mu_0 I}{2\pi x} dx$$

flux linked between 2 external points at distance D_1 & D_2 from center is then:

$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2\times10^{-7} I \int_{D_1}^{D_2} \frac{dx}{x}$$

$$= 2\times10^{-7} I \left(\ln D_2 - \ln D_1\right)$$

Flux flowing between D_1 & D_2



D_1 is closer (usually $= r$)

D_2 is distance to next conductor.

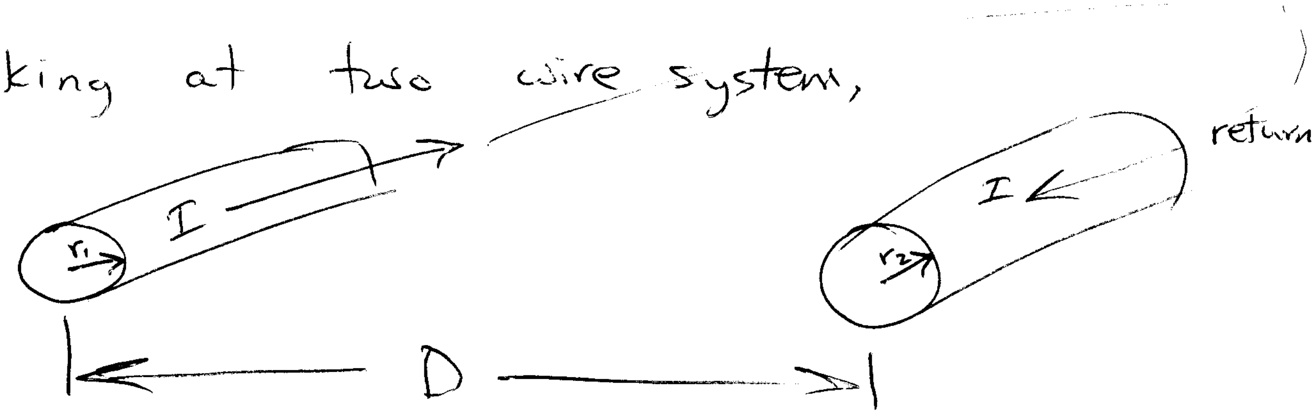
step 3 $$\boxed{\lambda_{12} = 2\times10^{-7} I \ln\left(\frac{D_2}{D_1}\right)} \ \text{wb-T/m}$$

$$\boxed{L_{12} = \frac{\lambda_{12}}{I} = 2\times10^{-7} \ln\left(\frac{D_2}{D_1}\right) \ \text{H}}$$

if $D_1 = r$
and $D_2 = D$ then

$$L_{12} = 2\times10^{-7} \ln\left(\frac{D}{r}\right)$$

Looking at two wire system,



Assumes $D >> r_1$

L due to conductor 1

$$L_1 = L_{INT,1} + L_{EXT,1}$$

$$= \frac{1}{2} \times 10^{-7} \, H/m + 2\times10^{-7} \ln\left(\frac{D}{r_1}\right) H/m$$

Combining,

$$L_1 = 2\times10^{-7} \left[\frac{1}{4} + \ln\left(\frac{D}{r_1}\right)\right] H/m$$

Substitute $\frac{1}{4} = \ln e^{+1/4}$

$$L_1 = 2\times10^{-7} \ln \frac{D}{r_1 e^{-1/4}} H/m$$

$r_1' =$ effective resistance which includes effect of L_{INT}

$$\boxed{L_1 = 2\times10^{-7} \ln \frac{D}{r_1'} H/m \quad \text{or,} \quad L_1 = .7411 \log \frac{D}{r_1'} H/m}$$

$\boxed{r_1' = .7788 \, r_1}$

only for solid conductor. (use tables A-4, A-3 otherwise)

Inductance increases with D
decreases with r

→ Do ex. p. 7

Inductance for conductor 2 is

$$L_2 = 2\times10^{-7} \ln \frac{D}{r_2'} \ H/m$$

for entire circuit,

$$L_{TOTAL} = L_1 + L_2$$

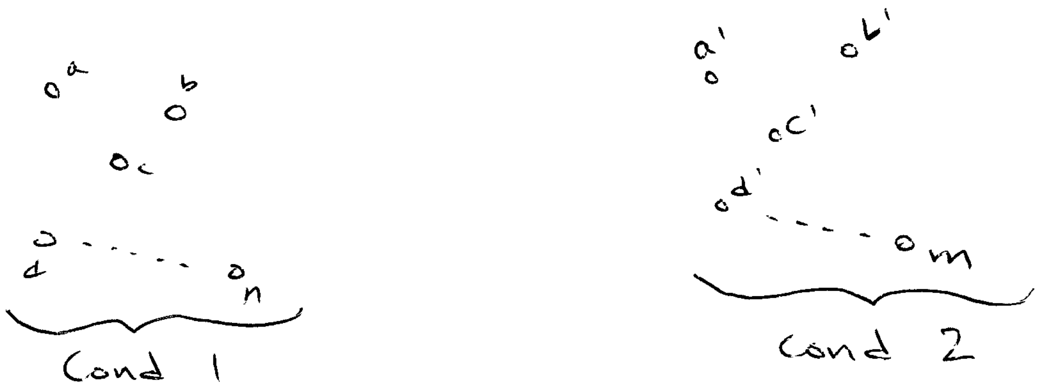
$$= 2\times10^{-7}\left[\ln \frac{D}{r_1'} + \ln \frac{D}{r_2'}\right]$$

$$= 2\times10^{-7} \ln \frac{D^2}{r_1' r_2'}$$

$$\boxed{\underset{\substack{\text{2-wire circuit}\\ \text{Loop Inductance}}}{L} = 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}}} \ H/m \quad = 1.482 \log_{10} \frac{D}{\sqrt{r_1' r_2'}} \ mH/mi$$

Note: Geometric mean radius

IN GENERAL,
for single phase multi-conductor line,



$$L = 4\times10^{-7} \ln \frac{GMD}{GMR}$$

$$GMD = \sqrt[mn]{(D_{aa'} D_{ab'} \cdots D_{am}) \cdots (D_{na'} D_{nb'} \cdots D_{nm})}$$

$$GMR_1 = \sqrt[n^2]{(\underset{\substack{\uparrow \\ r_a'}}{D_{aa}} D_{ab} \cdots D_{am}) \cdots (D_{ma} D_{mb} \cdots \underset{\substack{\uparrow \\ r_m'}}{D_{mm}})}$$

$$GMR_2 = \sqrt[m^2]{(\underset{\substack{\uparrow \\ r_{a'}'}}{D_{a'a'}} D_{a'b'} \cdots D_{a'm}) \cdots (D_{ma'} D_{mb'} \cdots \underset{\substack{\uparrow \\ r_m'}}{D_{mm}})}$$

$$L = 2 \times 10^{-7} \ln \frac{GMD}{GMR_1} + 2 \times 10^{-7} \ln \frac{GMD}{GMR_2}$$

$$L = 4 \times 10^{-7} \ln \frac{GMD}{\underbrace{\sqrt{GMR_1 GMR_2}}_{GMR}}$$

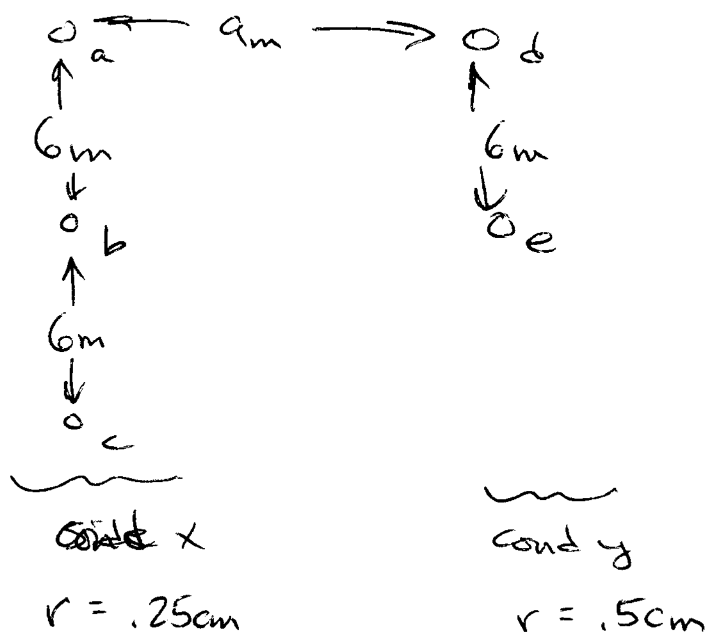
<u>Do examples</u>

<u>Ex:</u> 3 mile solid cond 0.5" DIA, 2-ft spacing.

$$L = 3\left[1.482 \log_{10} \frac{2 \text{ feet}}{\underbrace{\left(\frac{0.5}{2}\right)\left(\frac{1}{12}\right)\left(e^{-1/4}\right)}_{r' \text{ in feet}}}\right] = \underline{9.296 \text{mH}}$$

$$X_c = 2\pi 60 L = 377L = \underline{\underline{3.504\Omega}}$$

Ex:



$$GMD = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

$$D_{ad} = D_{be} = 9m$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2+9^2} = \sqrt{177}\ m$$

$$D_{cd} = \sqrt{\underset{3}{9^2}+\underset{4}{12^2}} = \underset{5}{15m}$$

$$GMD = \sqrt[6]{9^2 \cdot 177^{3/2} \cdot 15} = \underline{10.743m}$$

$$GMR_1 = \sqrt[9]{(.0025\ e^{-1/4})^3 \times 6^4 \times 12^2} = \underline{\underline{0.481m}}$$

$$GMR_2 = \sqrt[4]{(.005\ e^{-1/4})^2 \times 6^2} = \underline{\underline{0.153m}}$$

$$L_{xy} = 4\times10^{-7} \ln \frac{10.743}{\sqrt{(.481)(.153)}} = 1.472\ \mu H/m$$

$$(= 2.37\ mH/mi)$$

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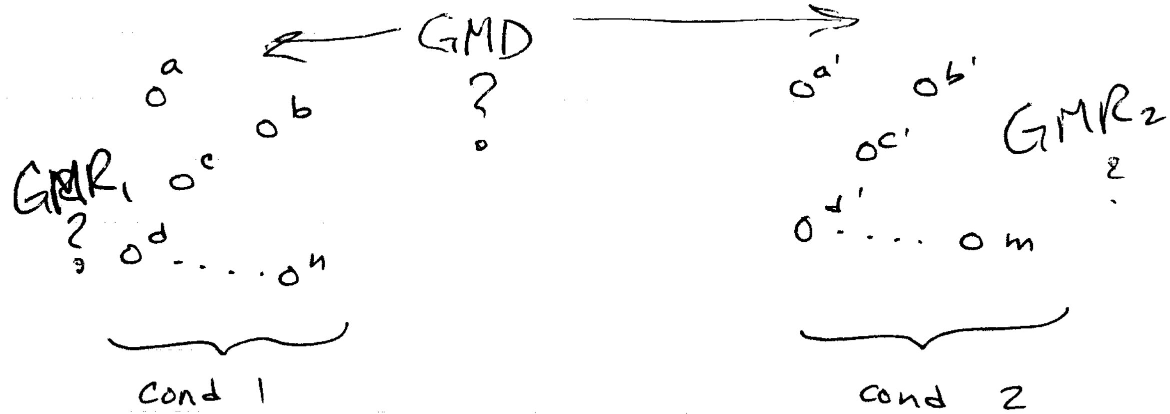
~~BUNDLED CONDUCTORS~~

Grouped conductors (Bundled)

What Happens if:



Before, we needed D = distance between conductors

$L_1 = 2\times10^{-7} \ln \frac{D}{r'}$ r' = radius of conductor

$$L_1 = 2\times10^{-7} \ln \frac{GMD}{GMR} = 2\times10^{-7} \ln \frac{D_M}{D_S}$$

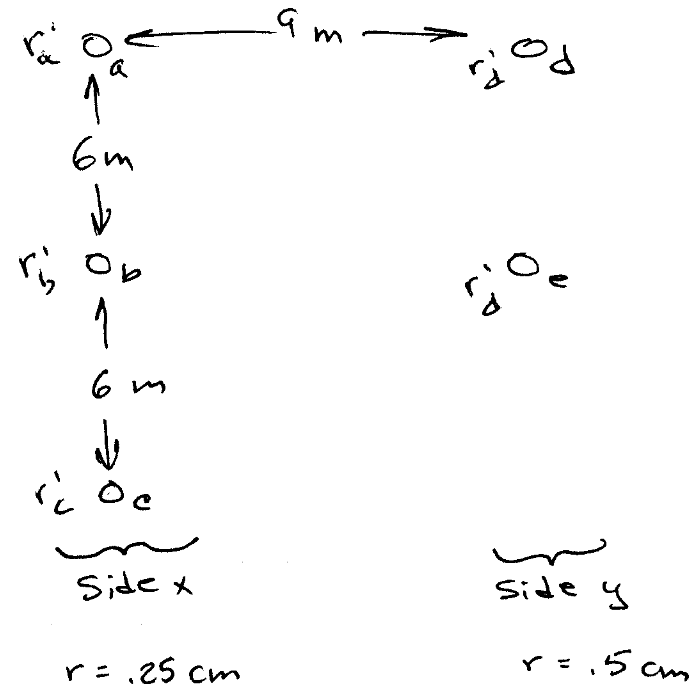
$$GMD = \sqrt[mn]{(D_{aa'} D_{ab'} D_{ac'} \cdots D_{am'})(D_{ba'} D_{bb'} D_{bc'} \cdots D_{bm})(\cdots)(D_{na'} D_{nb'} \cdots D_{nm})}$$

$$GMR_1 = \sqrt[n^2]{(D_{aa} D_{ab} D_{ac} \cdots D_{an})(D_{ba} D_{bb} \cdots D_{bn})(\cdots)(D_{na} D_{nb} D_{nc} \cdots D_{nn})}$$

D_M = GMD = Geometric Mean ~~Diameter~~ Distance

D_S = GMR = Geometric Mean Radius $\boxed{D_{aa} = r'_a}$

Ex:



$$D_m = GMD = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

Same for sides x & y

$$D_{ad} = D_{be} = 9\,m$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2+9^2} = \sqrt{117}\,m$$

$$D_{cd} = \sqrt{9^2+12^2} = 15\,m$$

$$D_M = \sqrt[6]{9^2 \times 15 \times 117^{3/2}} = \underline{10.743\,m}$$

Side x

$$D_s = GMR = \sqrt[9]{(D_{aa} D_{ab} D_{ac})(D_{ba} D_{bb} D_{bc})(D_{ca} D_{cb} D_{cc})}$$

$$= \sqrt[9]{(.25 \times .7788 \times 10^{-2})^3 \times 6^4 \times 12^2} = \underline{0.481\,m}$$

Side y

$$D_s = GMR = \sqrt[4]{(.5 \times .7788 \times 10^{-2})^2 \times 6^2} = \underline{0.153\,m}$$

$$L_x = 2 \times 10^{-7} \ln \frac{10.743}{.481} = \boxed{6.212 \times 10^{-7}\ H/m}$$

$$L_y = 2\times10^{-7} \ln \frac{10.743}{.153} = \boxed{8.503\times10^{-7} \ H/m}$$

$$\boxed{L_{xy} = L_x + L_y = 14.715\times10^{-7} \ H/m = 2.37 \ mH/mi}$$

tighter bundling gives higher L
decreased phase spacing gives lower L

Tables

Usually we want inductance in ohms per mile

$$X_L = 2\pi f L = 2\pi f L \times 2\times10^{-7} \ln \frac{D_m}{D_s} \ H/m$$

$$= 2.022 \times10^{-3} f \ln \frac{D_m}{D_s} \ \text{ohms/mile}$$

where D_s is listed in table A.1 for 60 Hz

$$\boxed{X_L = .2794 \log_{10} \frac{D_m}{D_s} \ \Omega/mi \quad \text{for } 60 \text{ Hz}}$$

$$X_L = .2794 \log_{10} \frac{1}{D_s} + .2794 \log_{10} \frac{D_m}{1} \ \Omega/\text{mile}$$

From Table A4

or

$$X_L = X_a + X_d$$

From Table A.2

X_a: inductive reactance at one foot spacing

X_d: inductive reactance spacing factor

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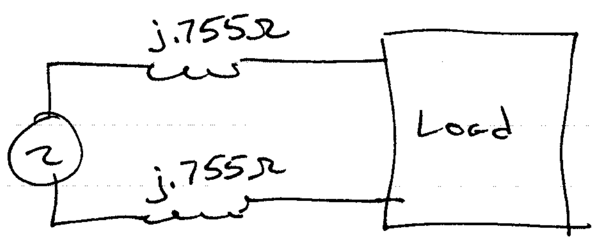
Ex: Waxwing at 10 ft Spacing - single phase

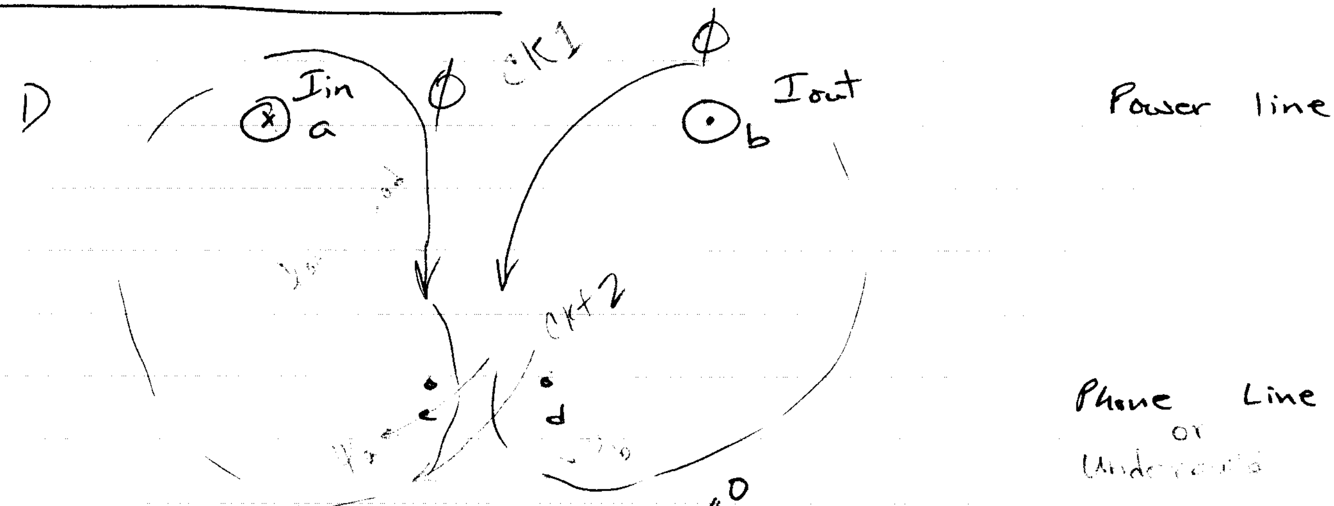
$X_L = .476 + .2794\ \Omega/\text{mile}$

$= .7554\ \Omega/\text{mile}$ (one cond only)

For 10 miles of line, (1ϕ)



Mutual Inductance



$I_a = -I_b$

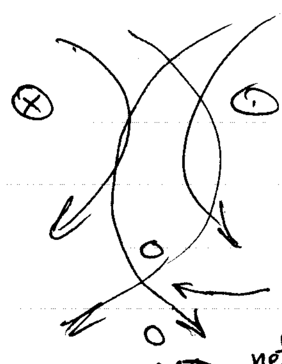
$V_{cd} = j\omega M I_a$ want = 0

I_a & I_b produce same flux (same I)

$\sqrt{K_{12}K_{21}}\,L_1L_2 = M_{12} = \dfrac{\psi_{12}}{I_1} = 2 \times .7411 \log_{10} \dfrac{D_{ad}}{D_{ac}}\ \mu H/mi$

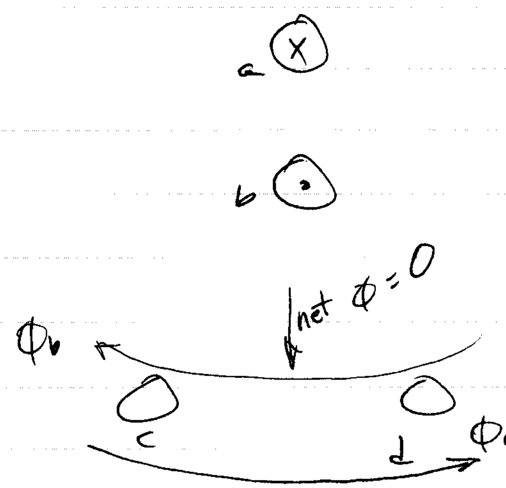
~~Dimension~~
~~Integer Alph1, Alph2~~
~~Data ALPH1//~~

2) Cancel out



cancel M = 0

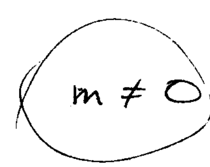
3)

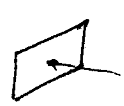


m = 0

each one cancels itself out

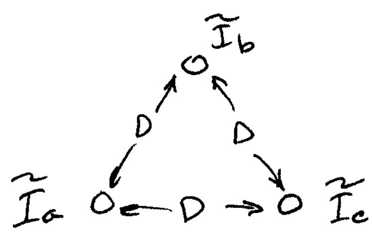
4)





ϕ pass through loop to induce voltage.

3 PHASE EQUIL SPACING



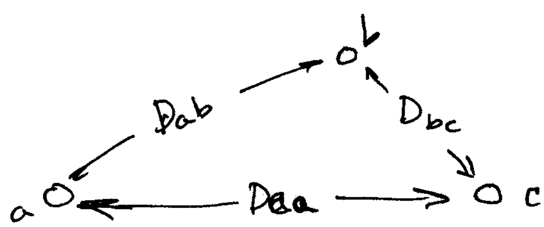
$$\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = 0$$

$$\tilde{I}_a = -(\tilde{I}_b + \tilde{I}_c)$$

So same situation as before w/ 2wire circuit

$$\boxed{L = .7411 \log_{10} \frac{D}{D_s} \ \mathrm{mH/mi} \quad \text{(per phase)}}$$

3 PHASE UNSYMM SPACING



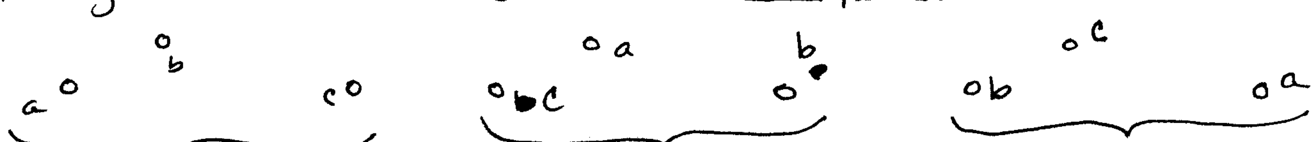
$\tilde{I}_a + \tilde{I}_b + I_c = 0$
But flux cut by $I_a \neq$ Flux cut by $I_b \neq$ flux cut by I_c

$\boxed{\text{But close enough}}$

$$\boxed{L_a \cong L_b \cong L_c \cong .7411 \log_{10} \frac{D_{eq}}{D_s} \ \mathrm{mH/mi}}$$

where $D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$

To get $L_a = L_b = L_c$ we transpose the lines



First 10 miles Next 10 miles Next 10 miles

The unbalance then averages out.

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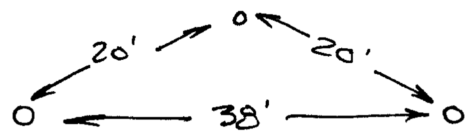
EX:

3.4



60 Hz
ACSR DRAKE

Find the inductive reactance per mile per phase

From table A.1 D_s = GMR = 0.0373 ft

$$D_{eq} = \sqrt[3]{20 \times 20 \times 38} = 24.8 \text{ ft}$$

$$L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \ \text{H/m} = .7411 \log_{10} \frac{D_{eq}}{D_s} \ \text{mH/mi}$$

$$= 13 \times 10^{-7} \ \text{H/m} = 2.092 \ \text{mH/mile}$$

$$X_L = 377L = 377(2.092) \times 10^{-3} \ \Omega/\text{mile}$$

$$\boxed{X_L = .789 \ \Omega/\text{mile}}$$

From Tables $X = X_a + X_d$

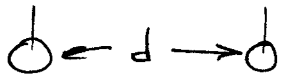
$X_a = 0.399 \ \Omega/\text{mi}$

$X_d = .2794 \log_{10} D_{eq} = .389 \ \Omega/\text{mi}$

$$\boxed{X = X_a + X_d = .399 + .389 = .788 \ \Omega/\text{mi}}$$

Go To Tables - interpolate from A.2 for 24.8 ft.

BUNDLED CONDUCTORS

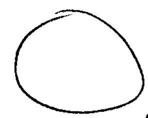


Used at 345-KV and above to reduce corona

For same voltage on a wire, say 100 KV

ρ_s is large

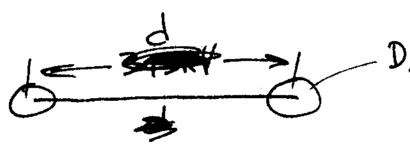
ρ_s = charge density



ρ_s is small

E field is less

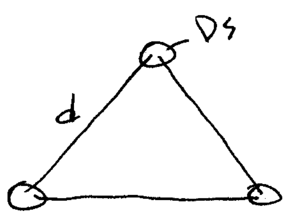
Corona loss - blue haze around conductors
bundling reduces corona



345-KV

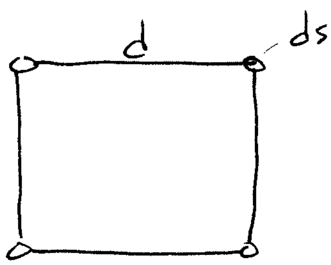
$$D_s^b = GMR = \sqrt[4]{(D_s d)^2} = \sqrt{D_s d}$$

D_s from tables



500 KV

$$\boxed{D_s^b = GMR = \sqrt[9]{(D_s d d)^3} = \sqrt[3]{D_s d^2}}$$



765 KV

$$\boxed{D_s^b = GMR = \sqrt[16]{(D_s d d \sqrt{2} d)^4} = 1.09 \sqrt[4]{D_s \times d^3}}$$

Bundled Conductors

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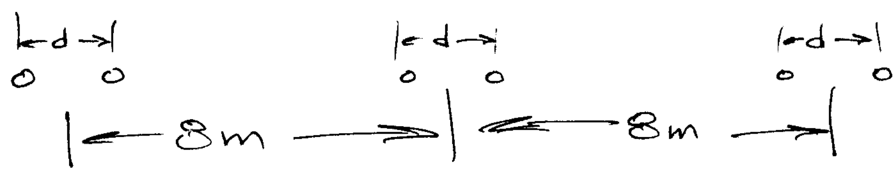
EXAMPLE 3.5

ACSR Pheasant $\quad d = 45$ cm



From table A.1 $\qquad D_s = .0466$

$$D_s^b = \sqrt{D_s d} = \sqrt{(.0466)(45)(}$$

$$= \sqrt{(.0466\ \text{ft})\left(\frac{.3048\ \text{m}}{\text{ft}}\right)(.45)}$$

$$\underline{\underline{D_s^b = .08\ \text{m}}}$$

$$D_{eq} = \sqrt[3]{(8)(8)(16)} = 10.08\ \text{m}$$

$$L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s^b} = 2\times10^{-7} \ln \frac{10.08}{.08} = 0.967\ \mu\text{H/m}$$

$$X_L = (377)(.967\times10^{-6})\ \text{H/m} = .365\ \Omega/\text{Km}\ \left(\frac{1.609\ \text{Km}}{\text{m}}\right)$$

$$= \boxed{.587\ \Omega/\text{mi}}$$

Find p.u. series reactance if Line = 160 Km and Base is 100 MVA ~~345~~ KV

$$X_L = (160\ \text{Km})\left(\frac{.365\ \Omega}{\text{Km}}\right) = 58.4\ \Omega$$

$$I_{BASE} = \frac{100\times10^3\ \text{KVA}}{\sqrt{3}(345\ \text{KV})} = 167.35\ \text{A}$$

$$= \frac{345{,}000/\sqrt{3}}{167.35}$$

$$Z_{BASE} = 1190\ \Omega$$

$$\boxed{X_L = \frac{58.4}{1190} = .049\ \text{p.u.}}$$

Ex:

For line in previous example, if no Bundling

$$X_L = (377)(2\times10^{-7}) \ln \frac{10.08}{.0466}\,(160{,}000\,m)\ \Omega$$

$$\boxed{= 64.86\ \Omega}$$

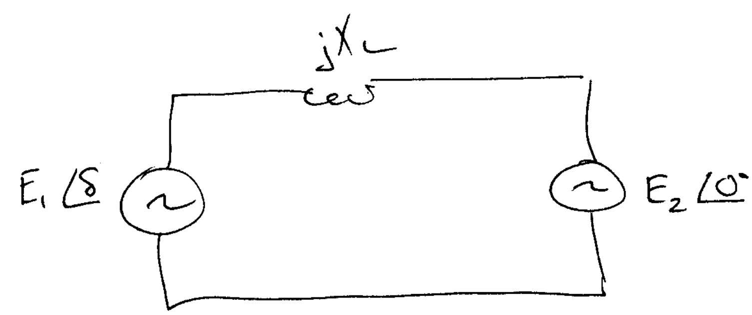
Bundled

$$P_{MAX} = \frac{E_1 E_2}{X}\cancel{\sin\delta}^{\,1} = \frac{E^2}{58.4} = .017$$

$\boxed{11.2\% \text{ more power}}$

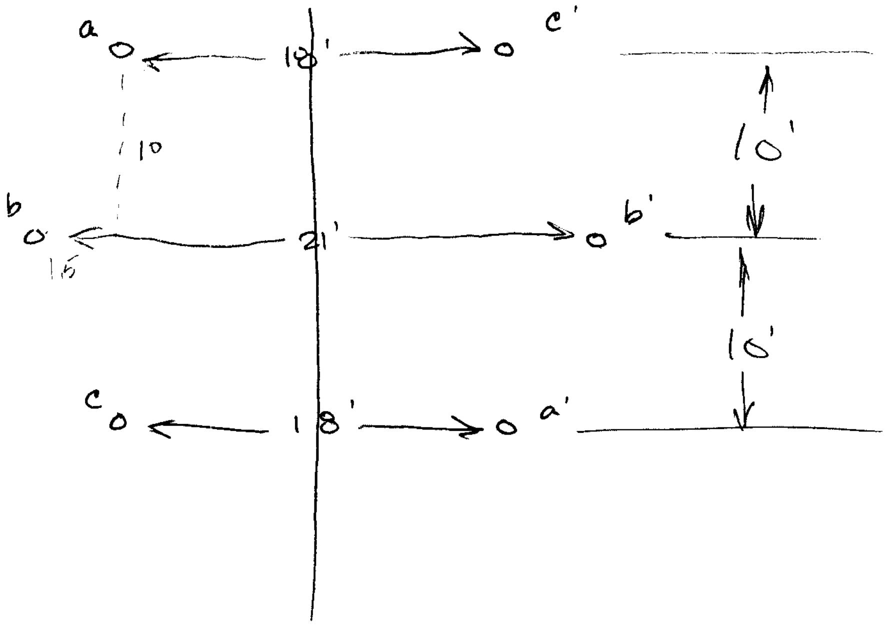
Not Bundled

$$P_{MAX} = \frac{E_1 E_2}{64.86} = \frac{E^2}{64.86} = .0154$$



$$P = \frac{E_1 E_2}{X}\sin\delta$$

DOUBLE CIRCUIT LINES - 3ϕ



300 MCM

OSTRICH 26/7

$$L = .7411 \log_{10} \frac{GMD}{GMR} \ \frac{mH}{mi}$$

FIND ~~GMR~~ Deq using GMD method

$$D_{eq} = \sqrt[3]{D^p_{ab} D^p_{bc} D^p_{ca}}$$

where $D^p_{ab} = \sqrt[4]{d_{ab}\, d_{ab'}\, d_{a'b}\, d_{a'b'}}$

$D^p_{bc} = \sqrt[4]{d_{bc}\, d_{bc'}\, d_{b'c}\, d_{b'c'}}$

$D^p_{ca} = \sqrt[4]{d_{ca}\, d_{ca'}\, d_{c'a}\, d_{c'a'}}$

$d_{ab} = \sqrt{10^2 + 1.5^2} = 10.11$ ft

$d_{ab'} = \sqrt{10^2 + 19.5^2} = 21.91$ ft

$$D^p_{ab} = \sqrt[4]{(10.11)^2 (21.91)^2} = 14.88'$$

$$D^p_{bc} = \sqrt[4]{(10.11)^2 (21.91)^2} = 14.88'$$

$$D^p_{ca} = \sqrt[4]{(20)^2 (18)^2} = 18.97'$$

$$GMD = \boxed{D_{eq} = \sqrt[3]{(14.88)^2 (18.97)} = 16.13'}$$

GMR = ? Like bundled conductors

$D_s = .0229$

$d_{aa'} = \sqrt{18^2 + 20^2} = 26.91'$

$GMR_{aa'} = \sqrt{(d_{aa'})D_s}$

$GMR_{aa'} = \sqrt{(.0229)(26.91)} = \underline{.785 \text{ ft}}$

$GMR_{bb'} = \sqrt{(.0229)(21)} = \underline{.693 \text{ ft}}$

$GMR_{cc'} = \sqrt{(.0229)(26.91)} = .785 \text{ ft} = GMR_{aa'}$

$$GMR = \boxed{D_s^P = \sqrt[3]{(.785)^2(.693)} = .753 \text{ ft}}$$

$$L = .7411 \log_{10} \frac{D_{eq}^P}{D_s^P} = .7411 \log_{10} \frac{16.13}{.753}$$

$= .986 \text{ mH/mi}$ per phase

$$\boxed{X_L = (377)(.986 \times 10^{-3}) = .372 \ \Omega/\text{mile per phase}}$$

SUMMARY OF INDUCTANCE CALCULATIONS

Three Phase

$$L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s} \quad H/m \quad \text{per phase}$$

$$L = .7411 \log_{10} \frac{D_{eq}}{D_s} \quad mH/mi \quad \text{per phase}$$

$$X_L = .0754 \ln \frac{D_{eq}}{D_s} \quad \Omega/Km \quad \text{per phase}$$

$$X_L = .1213 \ln \frac{D_{eq}}{D_s} \quad \Omega/mi \quad \text{per phase}$$

D_{eq} & D_s in same units

Single conductors: D_s from table A.1
D_{eq} = GMD = $\sqrt[3]{\text{prod of sep. distances}}$

Bundled Conductors: $GMR = \sqrt{d_{aa}\, d_{bundling}} = D_s^b$

GMD = D_{eq} between centers of bundles $= D_{eq}^b$

Parallel Circuits: GMR: do like bundled by take geometric mean of the three phases $= D_s^p$

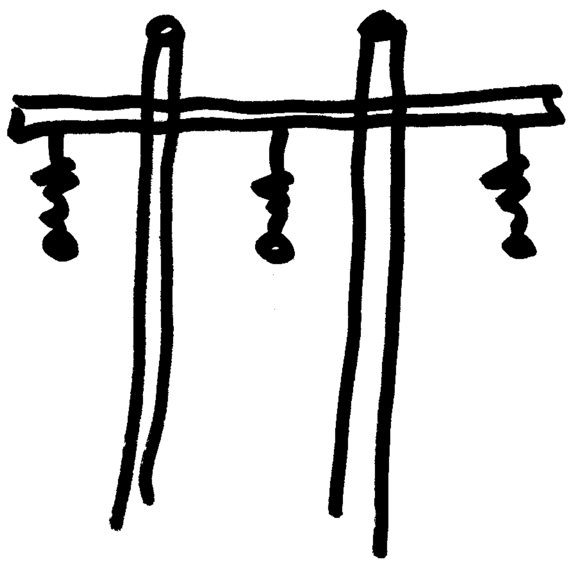
GMD: = ~~Geometric mean~~ Geometric mean of the Geometric means distances between conductors $= D_{eq}^p$

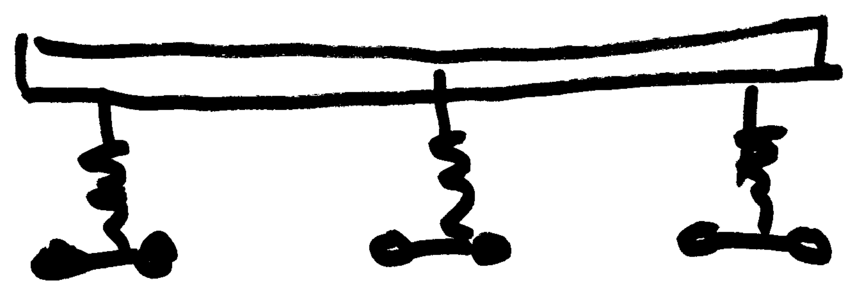
Inductances:

$$L \propto \ln \frac{D}{r} \quad \text{or} \quad \propto \ln \frac{GMD}{GMR}$$

As phase spacing $\uparrow$, $L \uparrow$

As conductor radius $\uparrow$, $L \downarrow$





For same phase spacing, bundled conductors will result in <u>smaller</u> L.