EE 5200 - Lecture 37 **Monday Dec 3, 2018**

Topics for Today:

- Announcements
  - Software: online students - if you want ATP/ATPDraw license, verify licensing, fwd to us when you get it, and we will mail you the install CD.
  - Office: EERC 614. Learning Center: 4-6pm Wed and Fri.
  - Recommended problems & all solutions: Ch.13 solns now posted.
  - Homework Syst Op - due this Friday, Dec 7th (extension to Monday 9am).

Ongoing topics...

- Chapter 13 - Power system operation
  - Constrained optimization methods - LaGrange multipliers
  - Optimal Dispatch, Generator Scheduling
    - Economics
    - Other constraints - environmental, contractual, availability
    - System load characteristics
  - Application to lossless system
  - System including losses - use [B] loss coefficient matrix

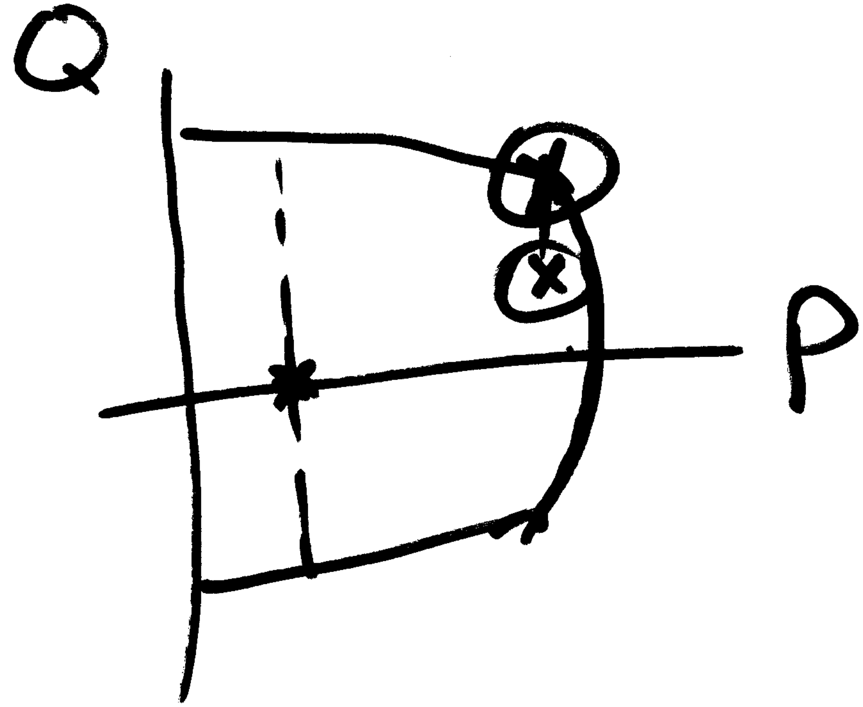
Minimize $f(x)$ (cost function) (objective funct)

$P_{G1}, P_{G2} \cdots P_{GM}$

Subject to: (constraints)

$$
\begin{aligned}
g_1(x) &= 0 \\
g_2(x) &= 0 \\
&\vdots \\
g_m(x) &= 0 \\
g_n(x) &\leq 0 \\
&\vdots \\
g_N(x) &\leq 0
\end{aligned}
$$

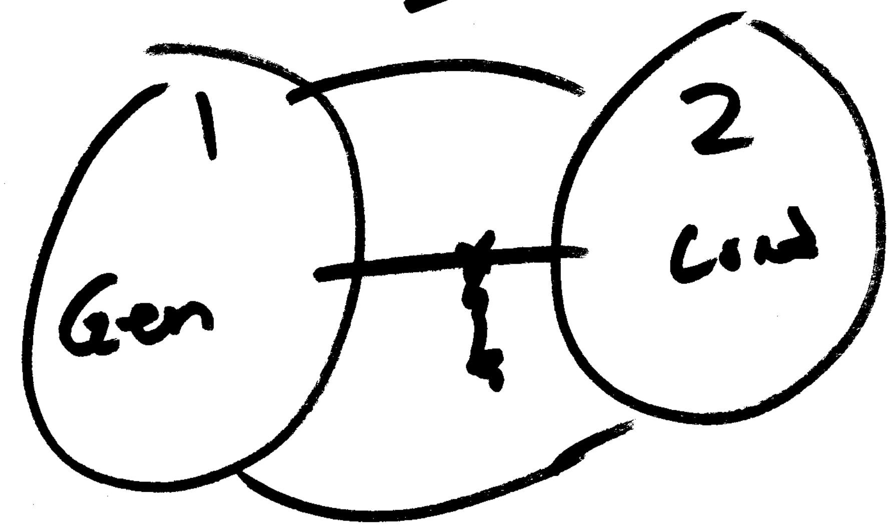
2003 Blackout



Contingency

N-2

N-1



Economic Dispatch - Optimum allocation of generation among system generators.

Goal: Maximize system efficiency
Minimize system losses (can't bill customers)

Specifics: Control voltage/vars
- Adjust generator exciter
- Reactors, caps (shunt)
- Tap-Changining transformers

Control Power Flow
- Control P_{gen} at each generator
- Phase-shifting transformers
- Line Switching

Frequency - (later)
- Prime mover control (droop controller)
- Load management.

$$P_{GEN} = P_{LOADS} + P_{\substack{TRANS/DIST \\ LOSSES}}$$

$$P_G = \sum_{i=1}^{n} P_{Gi}$$

How should P_G be divvied up among the n units?

Constraints:

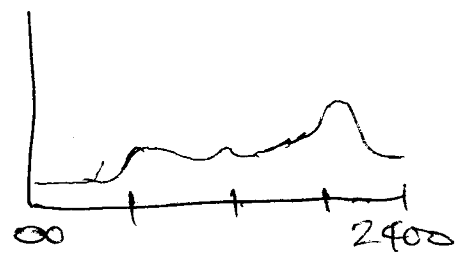
UNIT COMMITMENT
- On-line time reqmnts - Coal 8 hrs+
- Some units down for maintenance
- Should have rolling/spinning reserve in case units fail.
- $P_{min_i} < P_{G_i} < P_{max_i}$

Therme constraints of turbine. (P_{min_i})

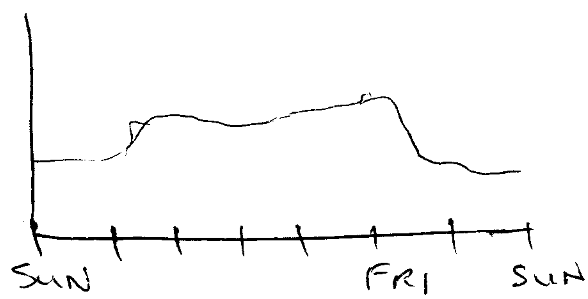
I^2R of stator (P_{max_i})

Lingo - Load Characteristics:

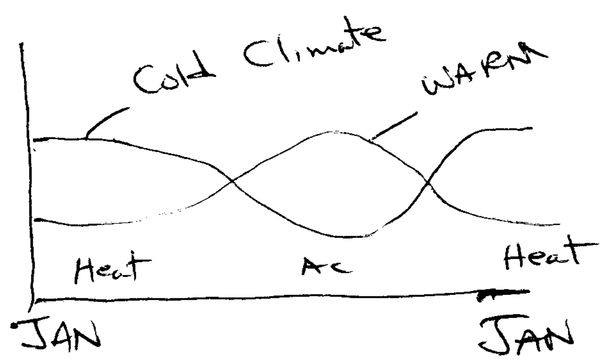
A) Daily -



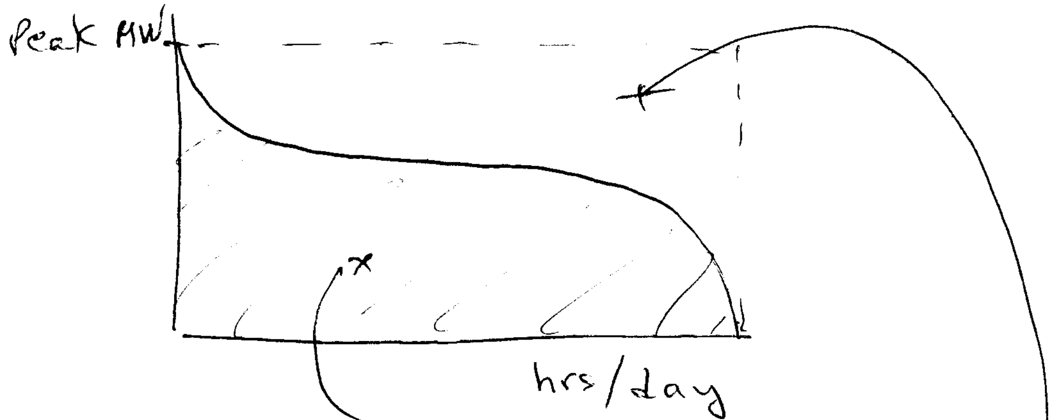
B) Weekly -



C) ~~Monthly~~ -
Annual



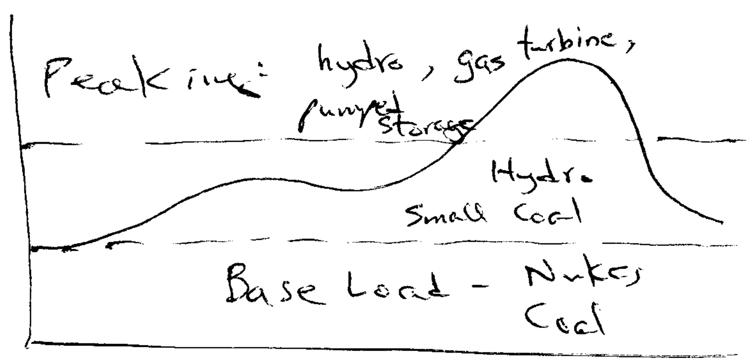
Load Duration Curve

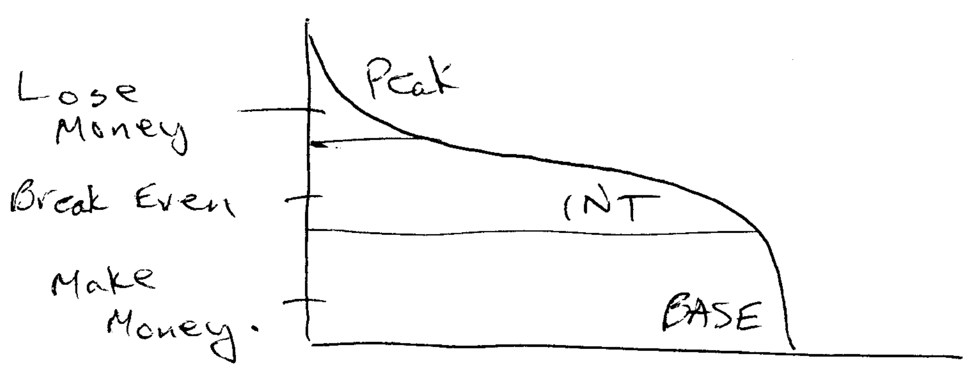


Load Factor: $LF = \dfrac{\text{Energy used}}{\text{Peak Power} \times \text{hrs}}$

0.4 - Bad
0.85 - Good

Strategy:



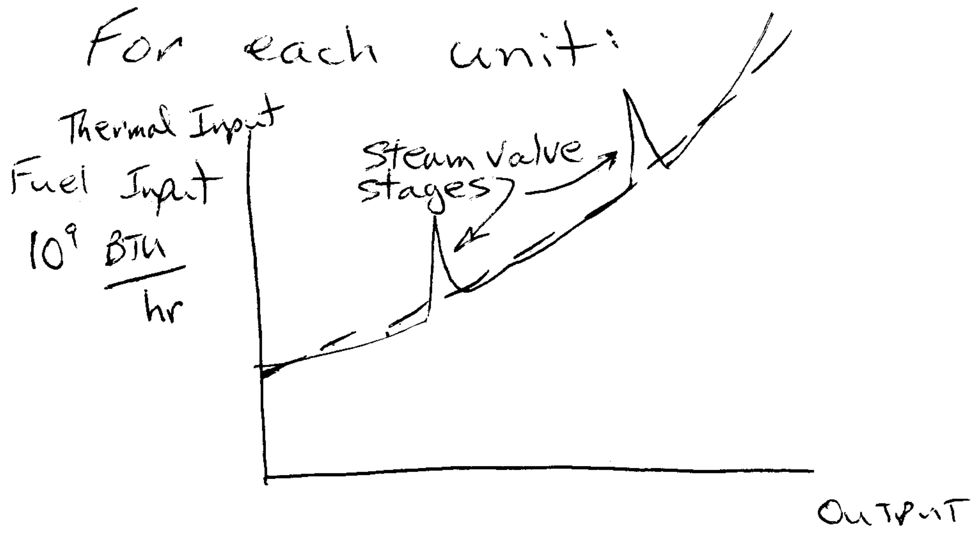


Ways a Utility can make money:

- Raise rates PUC/PSC must approve
- Sell more Base Load MWH
- Reduce Peak load (Load management)
  - Interruptable loads - water heaters, etc
  - Time of day rates

- Increase efficiency
  - Reduce Aux use in plant (10-15%)
  - Improve thermal efficiency (Net Heat Rate)
  - * Economic Dispatch

For each unit:



$$HR = \frac{\text{Input Thermal power, BTU/hr}}{\text{Electrical output}}$$

Typical: 10.5×10^6 BTU/MWH

Recognize form as $1/\eta$

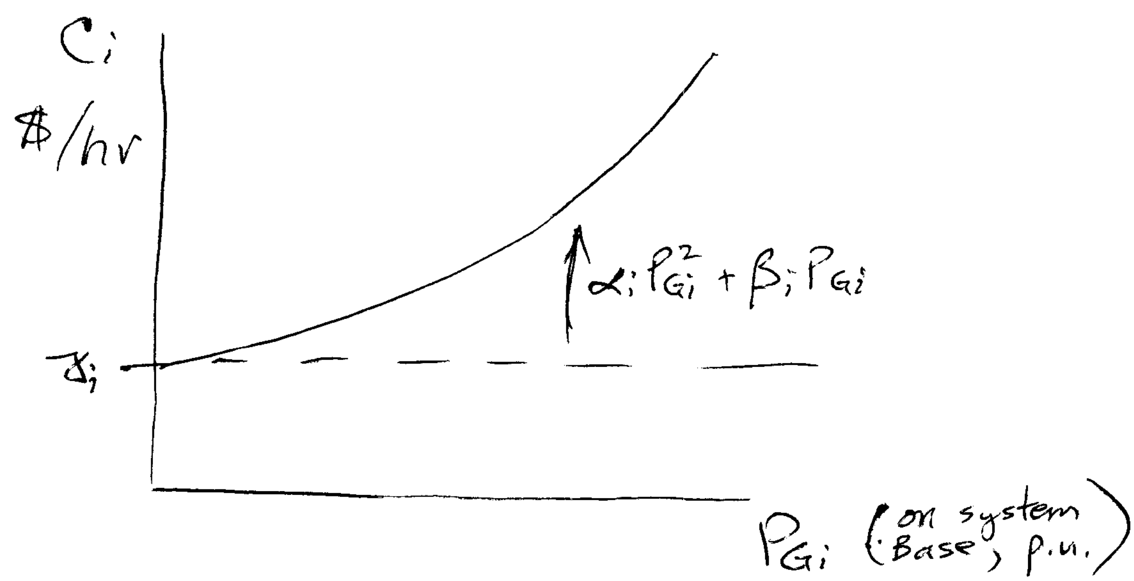
But one $BTU/hr = 0.293\ W$

$$\eta = \frac{1}{HR \times 0.293\times 10^{-6}} = \boxed{\frac{3.413\times 10^{6}}{HR}}$$

Operating cost of unit i

$$C_i = F_i P_i$$

- P_i: Input power in MBTU
- F_i: Fuel cost in \$/MBTU (Order of Mag. \$1.50) (+ labor, supplies, maint).



Empirically, $C_i = \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i$

α, β, γ in \$/hr

~~The curve is empirically described as:~~

$$\cancel{C_i = \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i}$$

Again,
$$P_G = P_L + P_{TL}$$

Problem: solve for n P_{Gi}'s subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

~~Example~~

Objective function:

$$\text{Min} \quad F(x_1, x_2, x_3 \cdots x_n)$$

Constraints:
$$G_1(x_1, x_2 \cdots x_n) = 0$$
$$\vdots$$
$$G_m(x_1, x_2 \cdots x_n) = 0$$

Usually (for our purposes) $m = 1$

1) Form the Lagranian:

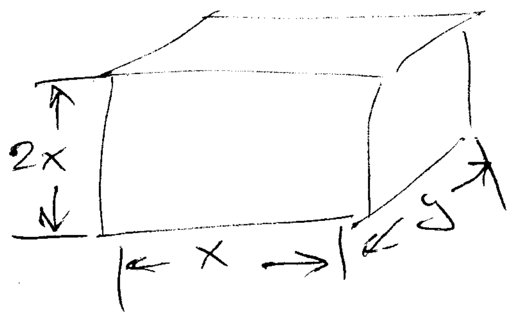
$$\mathcal{L} = F(x_1, x_2 \cdots x_n) - \lambda_1 G(x_1, x_2 \cdots x_n)$$

2) Find all partial dercivates of $\mathcal{L}$ wrt $x_1, x_2 \cdots x_n$ and set them $= 0$

3) Solve for $(x_1, x_2 \cdots x_n)$, λ from partial derivatives & $G(x_1, x_2 \cdots x_n)$

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)
Min if pos definite.
Local Max if neg def, Saddle if indef.

ex: 1 Box of dimensions x, y, z



Maximize Volume for $S = 432\ cm^2$

Objective function: $V = xyz$
$= 2x^2y$

Constraint:
$$2(xy + 2x^2 + 2xy) - 432 = 0$$

$$\mathcal{L} = 2x^2y - \lambda(4x^2 + 6xy - 432)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 4x - 8\lambda x \overset{\text{neg}}{\cancel{+}} 6y = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2x^2 - 6\lambda x = 0$$

Constraint: $2(xy + 2x^2 + 2xy) - 432 = 0$

solve simultaneously.
$\lambda = 2$
$x = 6\,cm$
$y = 8\,cm$
$V = 576\,cm^3$

Applying to Economic Dispatch:

Objective: $C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i$

Constraints: $G = P_G - P_L = 0 = \sum_{i=1}^{n} \underset{\text{gen}}{P_{Gi}} - \underset{\text{Total Loads.}}{P_L}$

(ignore line losses for now)

① $\mathcal{L} = C - \lambda \left(\sum_{i=1}^{n} P_{Gi} - P_L \right)$

② Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial C}{\partial P_{Gi}} - \lambda (1) = \frac{\partial C}{\partial P_{Gi}} - \lambda$$

since λ is the same in every term, one way to satisfy conditions is:

$$\frac{\partial C}{\partial P_{G1}} - \lambda = 0, \quad \frac{\partial C}{\partial P_{G2}} - \lambda = 0 \quad \cdots \quad \frac{\partial C}{\partial P_{Gn}} - \lambda = 0$$

Therefore, each plant must be at same incremental cost, λ_i $(\lambda_1 = \lambda_2 = \cdots \lambda_n = \lambda)$

For each unit,

$$\boxed{\lambda_i = \frac{\partial C_i}{\partial P_{Gi}} = 2\alpha_i P_{Gi} + \beta_i}$$

Example 8.2

Unit 1: $25\text{MW} < P_{G1} < 150\text{MW}$

$$C_1 = \underset{\alpha_1\ \$/\text{hr}/\text{MW}^2}{0.01}\,P_{G1}^2 + \underset{\beta_1\ \$/\text{hr}/\text{MW}}{2}\,P_{G1} + \underset{\gamma_1\ \$/\text{hr}}{100}$$

Unit 2: $30\text{MW} < P_{G2} < 200\text{MW}$

$$C_2 = \underset{\alpha_2}{0.004}\,P_{G2}^2 + \underset{\beta_2}{2.6}\,P_{G2} + \underset{\gamma_2}{80}$$

How to divide P_{G1} & P_{G2} within range $55\text{MW} \le P_L \le 350\text{ MW}$?

For ex, $\underline{P_L = 282\ \text{MW}}$

First, select $S_{BASE} = 100\text{MVA}$ & convert data to p.u.

$$\alpha_1 = (100)^2(.01) = 100 \qquad \alpha_2 = 40$$
$$\beta_1 = (100)(2) = 200 \qquad \beta_2 = 260$$
$$\gamma_1 = 100 \qquad \gamma_2 = 80$$

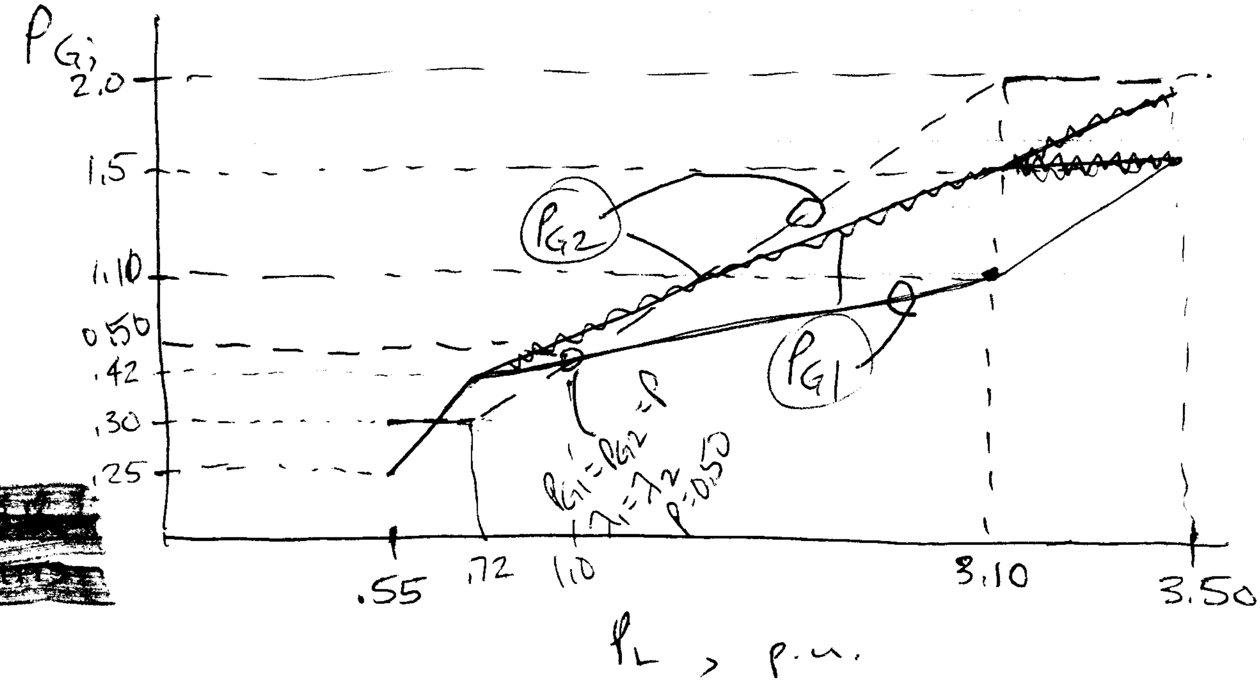
$$0.25 \le P_{G1} \le 1.50\ \text{p.u}$$
$$0.30 \le P_{G2} \le 2.00\ \text{p.u.}$$
$$0.55 \le P_L \le 3.50\ \text{p.u.}$$

$$\boxed{\begin{aligned} \lambda_1 &= \frac{\partial C_1}{\partial P_{G1}} = 200P_{G1} + 200 \\ \lambda_2 &= \frac{\partial C_2}{\partial P_{G2}} = 80P_{G2} + 260 \\ P_{G1} &+ P_{G2} = 2.82\ \text{p.u.} \end{aligned}}$$

Solving, setting $\lambda_1 = \lambda_2 = \lambda$

$P_{G1} = 1.02$ p.u. (102 MW)
$P_{G2} = 1.80$ p.u. (180 MW)

Looking at complete range, $55\,MW \le P_L \le 350\,MW$,



[redacted]

@ .55 p.u. $P_{G1} = 0.25$, $P_{G2} = 0.30$
$\lambda_1 = 250$, $\lambda_2 = 284$

→ ∴ must increase unit 1 first, until $\lambda_1 = 284$. This happens at $P_{G1} = \frac{284-200}{200} = \underline{0.42\,p.u.}$

→ Then λ_1 & λ_2 can be equal until one unit hits P_{max}. @ 3.5 p.u.,
$\lambda_1 = 500$ @ $P_{G1} = 1.5$ p.u. $\lambda_2 = 420$ @ $P_{G2} = 2.0$ p.u.
∴ P_{G2} limits out first. $P_{G1} = \frac{420-200}{200} = 1.1$ p.u.

→ From $P_L = 3.10$ and up, only P_{G1} increases.

Solution – Gross

We first select $S_{3\phi_{base}} = 100$ MVA; then convert all data into per-unit.

$$\alpha_1 = (S_{3\phi_{base}})^2 0.01 = 100 \qquad \alpha_2 = 40$$
$$\beta_1 = (S_{3\phi_{base}})2.00 = 200 \qquad \beta_2 = 260$$
$$\gamma_1 = 100 \qquad \gamma_2 = 80$$
$$0.25 \le P_{G_1} \le 1.50 \text{ pu}$$
$$0.30 \le P_{G_2} \le 2.00 \text{ pu}$$
$$0.55 \le P_L \le 3.50 \text{ pu}$$
$$\lambda_1 = \frac{\partial C_1}{\partial P_{G_1}} = 200P_{G_1} + 200$$
$$\lambda_2 = \frac{\partial C_2}{\partial P_{G_2}} = 80P_{G_2} + 260$$

Let us tabulate and plot results as we develop them, as shown in Figure 8.3. We start by calculating λ_1 and λ_2 for minimum-generation conditions (point 1). Observe that $\lambda_2 > \lambda_1$. Since we wish to make the λ's equal, the strategy is to load unit 1 first. We do this until $\lambda_1 = 284$, which occurs at

$$P_{G_1} = \frac{284 - 200}{200} = 0.42 \text{ (point 2)}$$

Now, calculate λ_1 and λ_2 at the maximum-generation condition (point 3). Observe

$\lambda_1 = \lambda_2 = \lambda_{sys}$

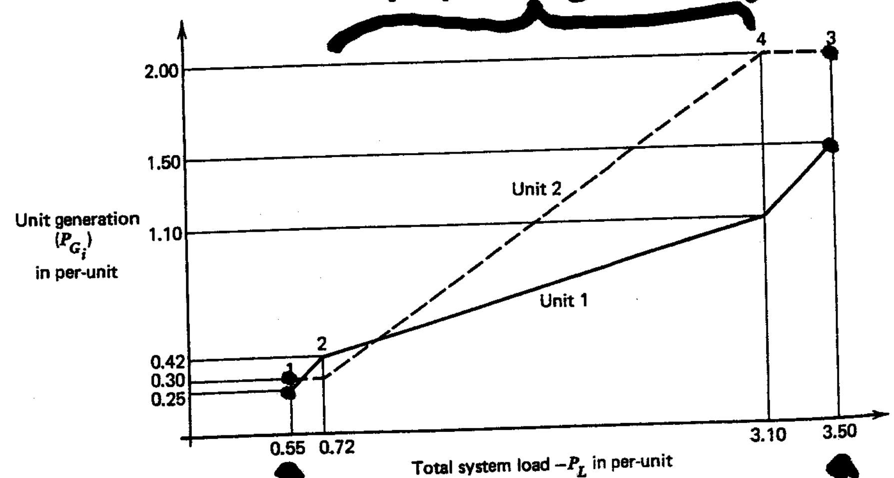


Figure 8.3. Results plotted for example 8.2.

now that $\lambda_1 > \lambda_2$, suggesting that we unload unit 1 first until we bring λ_1 down to $\lambda_2 = 420$. This happens at

$$P_{G_1} = \frac{420 - 200}{200} = 1.10 \text{ (point 4)}$$

Observe that for $0.72 \le P_L \le 3.10$, it is possible to maintain equal λ's. The equations are

$$\lambda_1 = \lambda_2$$
$$200P_{G_1} + 200 = 80P_{G_2} + 260$$

and

$$P_{G_1} + P_{G_2} = P_L$$

These linear relations are plotted in Figure 8.3. For $P_L = 282$ MW $= 2.82$ pu,

$$P_{G_2} = 2.82 - P_{G_1}$$
$$P_{G_1} = 0.4P_{G_2} + 0.3$$
$$= 1.128 - 0.40P_{G_1} + 0.3$$
$$1.4P_{G_1} = 1.428; \quad P_{G_1} = 1.02 \qquad \text{(102 MW)}$$
$$P_{G_2} = 2.82 - 1.02 = 1.80 \qquad \text{(180 MW)}$$

Results are presented in Table 8.1.

Table 8.1. Results for Example 8.2.

Point	P_{G_1}	P_{G_2}	P_L	λ_1	λ_2
1	0.25	0.30	0.55	250	284
2	0.42	0.30	0.72	284	284
3	1.50	2.00	3.50	500	420
4	1.10	2.00	3.10	420	420

In the case where the incremental cost functions (λ_i) are linearized, a simple straightforward general solution is possible

$$2\alpha_i P_{G_i} - \lambda = -\beta_i \qquad i = 1, 2, \ldots, n \tag{8.13a}$$
$$P_{G_1} + P_{G_2} + \cdots + P_{G_n} = P_L \tag{8.13b}$$
$$\begin{bmatrix} 2\alpha_1 & 0 & 0 & \ldots & 1 \\ 0 & 2\alpha_2 & 0 & \ldots & 1 \\ 0 & 0 & 2\alpha_3 & \ldots & 1 \\ \vdots & & & & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{bmatrix} \begin{bmatrix} P_{G_1} \\ P_{G_2} \\ P_{G_3} \\ \vdots \\ -\lambda \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ \\ \vdots \\ P_L \end{bmatrix} \tag{8.13c}$$

Solve the linear set for the P_{G_i}'s and λ.