EE 5200 - Lecture 39 **Fri Dec 7, 2018**

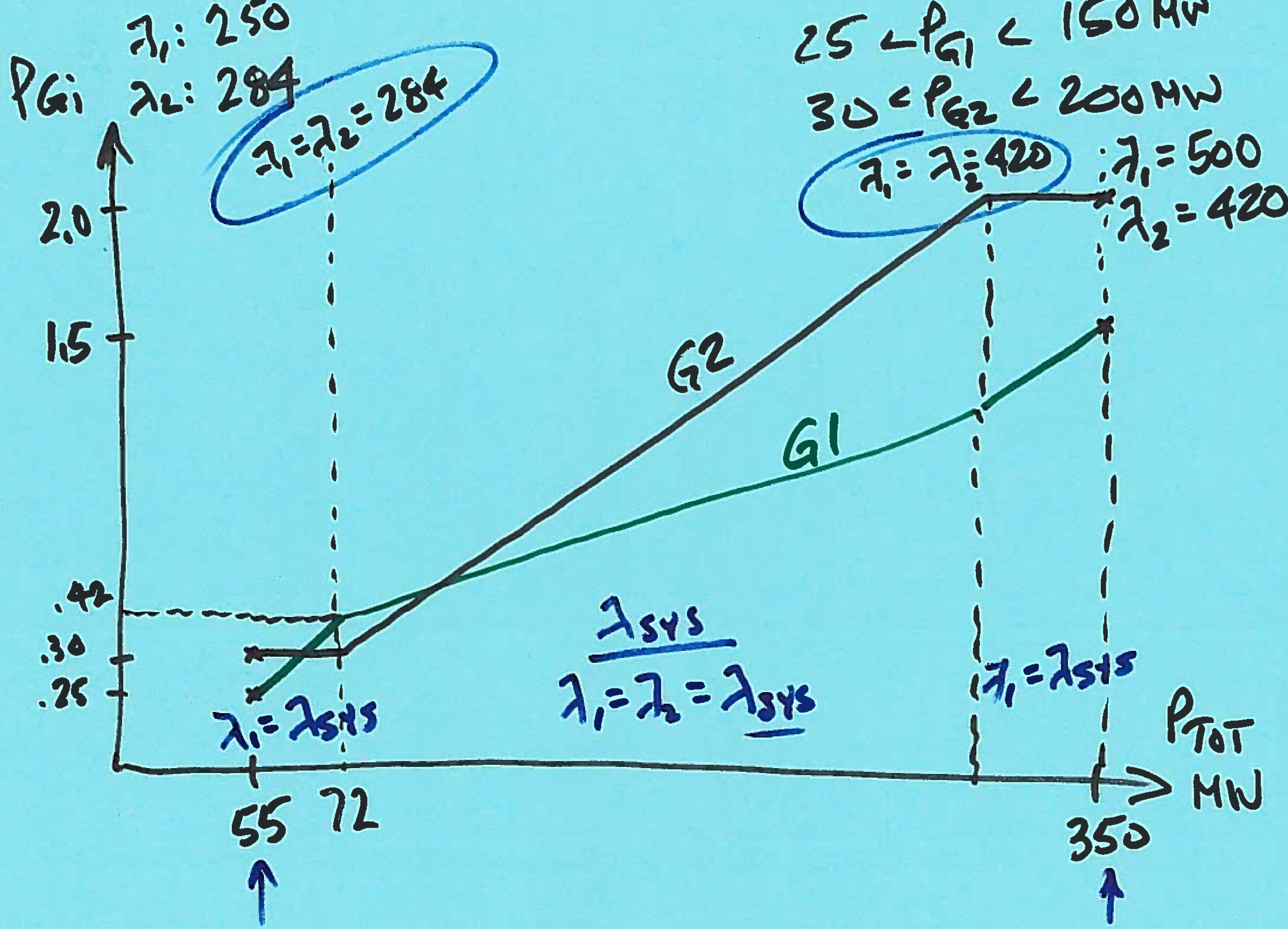
Topics for Today:

- Announcements
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - Office: EERC 614. Learning Center: W, F 4-6pm
  - Recommended problems & all solutions: 13 solns now posted.
  - Homework Syst Op - due latest Dec 10th 9am.

Ongoing topics, wrapping up optimal dispatch...

Chapter 16 - Stability

- Dr. Mork's lecture notes "System Stability" – See Week 13.
- Basic overview. Lead-in to EE6210 (Kundur's taxonomy).
  - Angle stability vs. voltage stability
  - Small disturbance vs. Transient stability
  - H: Stored energy per MW, J: rotational moment of inertia
  - Coherency
  - Swing Equation
  - Equal area review
  - Reclosing strategies



λ_1: 250
P_{Gi}
λ_2: 284
$\lambda_1 = \lambda_2 = 284$
$25 < P_{G1} < 150$ MW
$30 < P_{G2} < 200$ MW
$\lambda_1 = \lambda_2 = 420$
$\lambda_1 = 500$
$\lambda_2 = 420$
2.0
1.5
.42
.30
.25
G2
G1
λ_{sys}
$\lambda_1 = \lambda_{sys}$
$\lambda_1 = \lambda_2 = \lambda_{sys}$
$\lambda_1 = \lambda_{sys}$
P_{TOT}
MW
55
72
350

$$P_G = P_L + \underline{\underline{P_{TL}}}$$

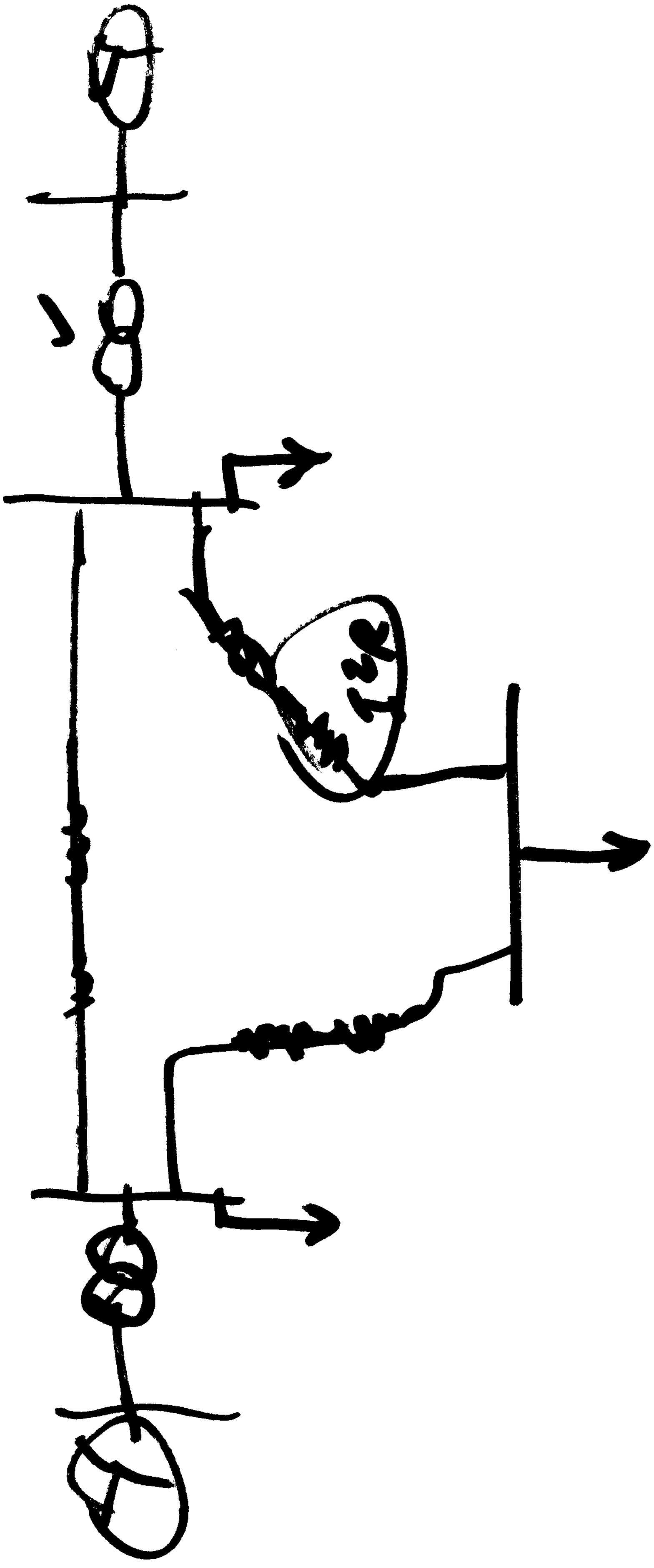
$\Longrightarrow$ Constraint Eqn

$$G = P_G - P_L - \underline{\underline{P_{TL}}} = 0$$

How to find P_{TL}?

$$\underline{\underline{P_{TL}}} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{Gi} B_{ij} P_{Gj}$$

$$= [P_G]^T \underbrace{[B]}_{?} [P_G]$$



Before, without losses: $P_G = \sum_{i=1}^{n} P_{G_i} = P_L$

With losses: $\boxed{P_G = P_L + P_{TL}}$

Constraint Equation: $G = P_G - P_L - P_{TL} = 0$

Main problem is to find P_{TL}.
From Kron, Kirchmeyer, George:

Losses in terms of P_{Gi}:

$$P_{TL} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{Gi} B_{ij} P_{Gj}$$

$$= [P_G]^T [B] [P_G]$$

$$= \begin{bmatrix} P_{G1} & P_{G2} & \cdots & P_{Gn} \end{bmatrix} \begin{bmatrix} B_{11} & & \\ & \ddots & \\ & & B_{nn} \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gn} \end{bmatrix}$$

How to determine B constants?
(Many methods. Each utility to their own.)

1) Empirical Method · "Hill-Stevenson Method"

Run lots of Load flows and examine sensivity of ~~[illegible]~~ all δs to P_{Gi}

Key: Find all $\left.\dfrac{\partial \delta_K}{\partial P_{Gi}}\right|_{K=1,n}$

Refer to method in book.

Once B coefficients are obtained.

Setting up the Lagrangian:

<u>minimize:</u> $C = \sum_{i=1}^{n} C_i$ (objective)

<u>subject to:</u> $P_G - P_L - \underbrace{P_{TL}}_{\text{T-line losses}} = 0$ (constraint)

$$\mathcal{L} = \sum_{i=1}^{n} C_i - \lambda (P_G - P_L - P_{TL})$$

Penalty Factor^{-1}

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial C_i}{\partial P_{Gi}} - \lambda \underbrace{\left(1 - \frac{\partial P_{TL}}{\partial P_{Gi}}\right)}_{L^{-1}} = 0$$

For optimized condition, $\lambda = \lambda_i$
(all λ's equal)

From before, $\frac{\partial C_i}{\partial P_{Gi}} = 2\alpha_i P_{Gi} + \beta_i$

$$\frac{\partial P_{TL}}{\partial P_{Gi}} = 2 \sum_{j=1}^{n} B_{ij} P_{Gj}$$

Constraint Equation:

$$\sum_{i=1}^{n} P_{Gi} - P_L - \sum_{i=1}^{n} \sum_{j=1}^{n} P_{Gi} B_{ij} P_{Gj} = 0$$

Once $[B]$ is known:

$$PF_i = \frac{1}{1 - 2\sum_{j=1}^{n} B_{ij} P_{Gj}} = \frac{\partial P_{TL}}{\partial P_{Gi}} \text{ Penalty Factor}$$

$$\lambda_i = \frac{\dfrac{\partial C_i}{\partial P_{Gi}}}{1 - \dfrac{\partial P_{TL}}{\partial P_{Gi}}} \qquad \frac{\partial C_i}{\partial P_{Gi}} = 2\alpha_i P_{Gi} + \beta_i$$

$$PF_i = \frac{1}{1 - \dfrac{\partial P_{TL}}{\partial P_{Gi}}}$$

$$\lambda_i = \left(\frac{\partial C_i}{\partial P_{Gi}}\right)\left(PF_i\right)$$

$$\left(1 - \frac{\partial P_{TL}}{\partial P_{Gi}}\right)\lambda = \frac{\partial C_i}{\partial P_{Gi}}$$

For optimum, $\lambda = \lambda_i$ (all λ's the same)

$$\lambda_i = \left(\frac{\partial C_i}{\partial P_{Gi}}\right)\left(PF_i\right) = \lambda_{(SYS)}$$

Note: Error on p. 320 of book. $\lambda_1 = \dfrac{2\alpha_1 P_{G1} + \beta_1}{PF_1}$

$$\lambda_i = \frac{2\,\alpha_i P_{Gi} + \beta_i}{1 - 2\sum_{j=1}^{n} B_{ij} P_{Gj}}$$

<u>Ex 8.3</u>

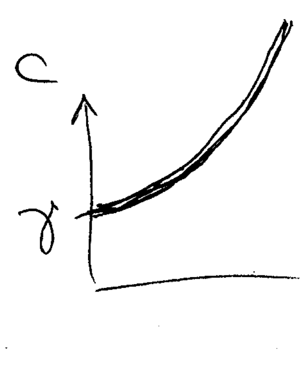
$$[B] = \begin{bmatrix} .001694 & -.004940 \\ -.004940 & 0.014406 \end{bmatrix}$$

$$\lambda_1 = \frac{2\alpha_1 P_{G1} + \beta_1}{1 - 2\left(B_{11} P_{G1} + B_{12} P_{G2}\right)}$$

$\alpha_1 = 100$
$\beta_1 = 200$
$\gamma = ?$

$$\lambda_2 = \frac{2\,\alpha_2 P_{G2} + \beta_2}{1 - 2\left(B_{21} P_{G1} + B_{22} P_{G2}\right)}$$

$\alpha_2 = 40$
$\beta_2 = 260$



$$\lambda_1 = \frac{200 P_{G1} + 200}{1 - 2\left[(.001694) P_{G1} + (.00494) P_{G2}\right]}$$

$$\lambda_2 = \frac{80 P_{G2} + 260}{1 - 2\left[(-.00494) P_{G1} + (.014406) P_{G2}\right]}$$

$$P_{TL} = B_{11} P_{G1}^2 + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

$$P_{G1} + P_{G2} - P_{TL} = 2.82$$

Solve trial & error (or could set up N-R)

(Could use starting values from case where line losses are neglected.) i.e. - starting values of P_{G1} & P_{G2}

Start: $P_{G1} = 1.02$ p.u. $P_{G2} = 1.80$ p.u.

then calculate λ_1, λ_2, P_{TL}.

If $\lambda_2 > \lambda_1$ then decrease P_{G2}, increase P_{G1}

Calculate P_{TL} & $P_{G1} + P_{G2} - P_{TL}$.

If $P_L < 2.82$, increase both P_{G1} & P_{G2}.

See table on page 320.

Hand calculation recommended to give students a feel for problem & interrelationships.

Gross

8.2 Economic Dispatch Considering Losses

In section 8.1, the economic dispatch problem was solved neglecting transmission losses. Experience has shown that in some cases, this approximation produces results that are in serious error. We recall the power balance equation

$$P_G = P_L + P_{TL} \tag{8.7a}$$

The revised equation of constraint, considering losses, is then

$$G = P_G - P_L - P_{TL} = 0 \tag{8.14}$$

Recall that the problem's variables are the P_{G_i}'s. Therefore, it is necessary to formulate the losses using the P_{G_i}'s as variables.

Let us consider the losses of the components of the transmission system, specifically, transformers and lines. Transformers have two types of losses: copper and iron. The iron, or magnetic, losses vary with core flux density, which in turn varies with voltage. Since the transformer voltage varies indirectly, and very little, with load, the variation of core loss with load is quite small and not important. However, the copper loss varies as I^2R; the load current varies directly with power loading; and the series resistance loss varies with the *square* of this current. Thus, the transformer copper loss should be included in the transmission losses. Likewise, the I^2R losses in the series element of the transmission line constitute transmission losses. The challenge is to functionally relate these I^2R losses to the P_{G_i} variables.

Many investigators, including Kron, Kirchmeyer, and George have worked on this problem and proposed a loss equation that formulates the loss as a quadratic function of the P_{G_i}'s;†

$$P_{TL} = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j} \tag{8.15a}$$

$$= \tilde{\mathbf{P}}_{G_t}[\mathbf{B}]\tilde{\mathbf{P}}_G \tag{8.15b}$$

where

$$\tilde{\mathbf{P}}_G = \begin{bmatrix} P_{G_1} \\ P_{G_2} \\ \vdots \\ P_{G_n} \end{bmatrix} = (n \times 1) \qquad \text{vector of generated powers.}$$

$[\mathbf{B}] = (n \times n)$ array with general entry B_{ij}.

The $[\mathbf{B}]$ array is symmetrical, such that $B_{ij} = B_{ji}$. Computing the B_{ij} values, called the B constants, can be implemented by several techniques. One approach, called the Hill–Stevenson method, calculates B constants using partial derivatives,

† A more general version is

$$P_{TL} = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j} + \sum_{i=1}^{n} B_i P_{G_i} + B_0$$

some of which are evaluated numerically using data obtained from a series of load flow runs. A description of the method follows.
Observe that

$$\frac{\partial P_{TL}}{\partial P_{G_k}} = \frac{\partial}{\partial P_{G_k}}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j}\right] \tag{8.16a}$$

$$= 2\sum_{j=1}^{n} B_{kj} P_{G_j} \tag{8.16b}$$

Furthermore,

$$\frac{\partial^2 P_{TL}}{\partial P_{G_m}\partial P_{G_k}} = 2B_{km} = 2B_{mk} \tag{8.17a}$$

But the subscript notation is arbitrary, so that

$$B_{ij} = \frac{1}{2}\left[\frac{\partial^2 P_{TL}}{\partial P_{G_i}\partial P_{G_j}}\right] \tag{8.17b}$$

Consider the general power system as we viewed it for power flow in Figure 7.2. The total power injected into the transmission network is†

$$P_{TL} = \mathrm{Re}\left[\sum_{i=1}^{b} \bar{V}_i \bar{I}_i^*\right] \tag{8.18a}$$

If the transmission network were lossless, P_{TL} would be zero (power in = power out); in general, this summation would equal the total transmission system loss P_{TL}.

$$P_{TL} = \mathrm{Re}\left[\sum_{i=1}^{b} \bar{V}_i\left(\sum_{j=1}^{b} \bar{V}_j \bar{y}_{ij}\right)^*\right] \tag{8.18b}$$

$$= \sum_{i=1}^{b}\sum_{j=1}^{b} V_i V_j y_{ij}\cos(\delta_i - \delta_j - \gamma_{ij}) \tag{8.18c}$$

Now, we compute from equation (8.18c)‡:

$$\frac{\partial P_{TL}}{\partial \delta_k} = 2\sum_{i=1}^{b} V_i V_k g_{ik}\sin(\delta_i - \delta_k) \tag{8.19}$$

Now, by the chain rule, consider that

$$\frac{\partial P_{TL}}{\partial P_{G_j}} = \sum_{k=1}^{b} \frac{\partial P_{TL}}{\partial \delta_k}\frac{\partial \delta_k}{\partial P_{G_j}} \tag{8.20}$$

†Here, b is the total number of buses, and n, generators. In Chapter 7, n was the total number of buses.

‡The simplification is complicated and relegated to an exercise for the student. See problem 8-7.

T.. .oublesome terms are the $\partial\delta_k/\partial P_{G_j}$ factors. The Hill–Stevenson method ..ides a clever way of evaluating these partial derivatives numerically. A base l..d flow case is established that hopefully approximates the ultimate allocation of .eneration. One approach is to solve the lossless economic dispatch problem as a first approximation. Then increase all bus loads proportionally by a small amount (say, 10%), and allow generator 1 alone to pick up the increased load. The load flow solves for the phase (δ_k) at all b buses, from which the change in phase ($\Delta\delta_k$) can be calculated. Therefore,

$$\frac{\partial \delta_k}{\partial P_{G_1}} \cong \frac{\Delta\delta_k}{\Delta P_{G_1}} = A_{k1}$$

This partial derivative is reasonably constant over a rather wide range of generation and load values. We repeat the process n times, allowing each generator to act alone to pick up the inorease in load, such that

$$\frac{\partial \delta_k}{\partial P_{G_j}} \cong \frac{\Delta\delta_k}{\Delta P_{G_j}} = A_{kj} \qquad \begin{matrix} k = 1, 2, \dots, b \\ j = 1, 2, \dots, n \end{matrix} \tag{8.21}$$

The partial derivatives so approximated are represented as the A constants. Let us now compute a second partial derivative

$$\begin{aligned} \frac{\partial}{\partial P_{G_i}}\left(\frac{\partial P_{TL}}{\partial P_{G_j}}\right) = \frac{\partial^2 P_{TL}}{\partial P_{G_i}\partial P_{G_j}} &= \frac{\partial}{\partial P_{G_i}}\left[\sum_{k=1}^{b} \frac{\partial P_{TL}}{\partial \delta_k}\left(\frac{\partial \delta_k}{\partial P_{G_j}}\right)\right] \\ &= \sum_{m=1}^{b} \frac{\partial}{\partial \delta_m}\left[\sum_{k=1}^{b} \frac{\partial P_{TL}}{\partial \delta_k}\left(\frac{\partial \delta_k}{\partial P_{G_j}}\right)\right]\left(\frac{\partial \delta_m}{\partial P_{G_i}}\right) \\ &= \sum_{m=1}^{b}\sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k}\left(\frac{\partial \delta_k}{\partial P_{G_j}}\right)\left(\frac{\partial \delta_m}{\partial P_{G_i}}\right) \end{aligned} \tag{8.22a}$$

or

$$\frac{\partial^2 P_{TL}}{\partial P_{G_i}\partial P_{G_j}} = \sum_{m=1}^{b}\sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} A_{kj} A_{mi} \tag{8.22b}$$

Recall that

$$\frac{\partial P_{TL}}{\partial \delta_k} = 2\sum_{i=1}^{b} V_i V_k g_{ik} \sin(\delta_i - \delta_k) \tag{8.19}$$

We wish to compute

$$\frac{\partial}{\partial \delta_m}\left(\frac{\partial P_{TL}}{\partial \delta_k}\right) = \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} \tag{8.23a}$$

where δ_m is one specific δ_i. We determine that

$$\frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} = 2V_m V_k g_{mk} \cos(\delta_m - \delta_k) \qquad m \neq k \tag{8.23b}$$

$$= -2\sum_{\substack{i=1 \\ i\neq m}}^{b} V_i V_m g_{im} \cos(\delta_i - \delta_m) \qquad m = k \tag{8.23c}$$

To relate this work to the B constants, recall that

$$B_{ij} = \frac{1}{2}\left(\frac{\partial^2 P_{TL}}{\partial P_{G_i}\partial P_{G_j}}\right) \tag{8.24}$$

Finally,

$$B_{ij} = \frac{1}{2}\left(\sum_{m=1}^{b}\sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} A_{mi} A_{kj}\right) \tag{8.25}$$

We now return to our main concern, that is, solving the economic dispatch problem considering losses.

$$C = \sum_{i=1}^{n} C_i \quad \text{(objective function)} \tag{8.6}$$

$$P_G - P_L - P_{TL} = 0 \quad \text{(equation of constraint)} \tag{8.14}$$

$$\mathscr{L} = \sum_{i=1}^{n} C_i - \lambda[P_G - P_L - P_{TL}] \quad \text{(the Lagrangian)} \tag{8.26}$$

The general partial derivative with respect to P_{G_i} is

$$\frac{\partial \mathscr{L}}{\partial P_{G_i}} = \frac{\partial C_i}{\partial P_{G_i}} - \lambda\left(1 - \frac{\partial P_{TL}}{\partial P_{G_i}}\right) = 0 \tag{8.27}$$

or, we redefine λ_i such that

$$\lambda_i = \frac{\partial C_i/\partial P_{G_i}}{1 - (\partial P_{TL}/\partial P_{G_i})} = \lambda \tag{8.28a}$$

where

$$\frac{\partial C_i}{\partial P_{G_i}} = 2\alpha_i P_{G_i} + \beta_i \tag{8.28b}$$

and

$$\frac{\partial P_{TL}}{\partial P_{G_i}} = 2\sum_{j=1}^{n} B_{ij} P_{G_j} \tag{8.28c}$$

and the equation of constraint requires that

$$\left(\sum_{i=1}^{n} P_{G_i}\right) - P_L - \left(\sum_{i=1}^{n}\sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j}\right) = 0 \qquad (8.29)$$

Thus, the condition for economically optimum operation requires the *weighted* incremental cost functions for all units to be equal (to each other and λ)

$$\lambda_i = \lambda \qquad (8.30)$$

The weighting factor is sometimes called the penalty factor of generator $i(PF_i)$

Li $$PF_i = \frac{1}{1-(\partial P_{TL}/\partial P_{G_i})} = \frac{1}{1 - 2\sum_{j=1}^{n} B_{ij} P_{G_j}} \qquad (8.31)$$

Example 8.3 is useful for illustrative purposes.

Example 8.3

A single-line diagram for the system in example 8.2 is shown in Figure 8.4. A base case load flow study on the system provides the following results:

$V_1 = 1\angle 0°$ $\quad P_{G_1} = 1.0313$ $\quad P_{L_1} = 2.1000$

$V_2 = 1\angle 6.616°$ $\quad P_{G_2} = 1.8200$ $\quad P_{L_2} = 0.7200$

$P_{TL} = 0.0313$

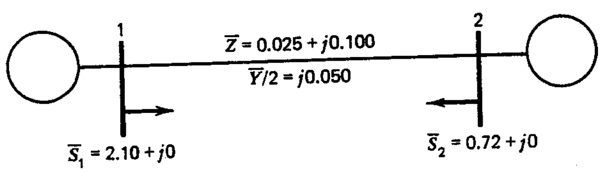


Figure 8.4. System for example 8.3.

(a) Calculate the A constants. The load at each bus was increased 10%. If unit 1 picks up the load,

$$\Delta\delta_1 = 0 \text{ (using bus 1 as phase reference)}$$

$$\Delta\delta_2 = 6.187 - 6.616 = -0.429° \ (-0.007487 \text{ rad})$$

$$\Delta P_{G_1} = 1.3094 - 1.0313 = 0.2781$$

$$A_{11} = 0 \qquad A_{21} = \frac{-0.007487}{0.278100} = -0.026924$$

If unit 2 picks up the load,

$$\Delta\delta_1 = -7.947 + 6.616 = -1.331° \ (-0.02323 \text{ rad})$$

$$\Delta\delta_2 = 0 \text{ (using bus 2 as phase reference)}$$

$$\Delta P_{G_2} = 2.1159 - 1.8200 = 0.2959$$

$$A_{12} = \frac{-0.02323}{0.29590} = -0.078507 \qquad A_{22} = 0$$

(b) Calculate the B constants

$$[Y] = \begin{bmatrix} 2.353 - j9.362 & -2.353 + j9.412 \\ -2.353 + j9.412 & 2.353 - j9.362 \end{bmatrix}$$

$$g_{11} = g_{22} = 2.353$$

$$g_{12} = g_{21} = -2.353$$

$m = k$

$$\tfrac{1}{2}(\partial^2 P_{TL}/\partial\delta_m\partial\delta_k) = -\sum_{\substack{i=1\\ i\neq m}}^{2} V_i V_m g_{im}\cos(\delta_i - \delta_m)$$

$$= -(1)(1)(g_{12})\cos(0 - 6.616°)$$

$$= +2.337$$

$m \neq k$

$$\tfrac{1}{2}(\partial^2 P_{TL}/\partial\delta_m\partial\delta_k) = V_m V_k g_{mk}\cos(\delta_m - \delta_k)$$

$$= (1)(1)(-2.337) = -2.337$$

Finally,

$$B_{ij} = \frac{1}{2}\sum_{m=1}^{2}\sum_{k=1}^{2} \frac{\partial^2 P_{TL}}{\partial\delta_m\partial\delta_k} A_{mi}A_{kj}$$

$$= 2.337(A_{1i}A_{1j} - A_{1i}A_{2j} - A_{2i}A_{1j} + A_{2i}A_{2j})$$

$$B_{11} = 2.337[(-0.026924)^2] = 0.001694$$

$$B_{12} = 2.337[-(-0.026924)(-0.078507)] = -0.004940$$

$$B_{22} = 2.337[+(-0.078507)^2] = 0.014406$$

Check P_{TL}

$$P_{TL} = B_{11}P_{G_1}^2 + 2B_{12}P_{G_1}P_{G_2} + B_{22}P_{G_2}^2$$

$$= (0.001694)(1.0313)^2 - 2(0.004940)(1.0313)(1.82) + (0.014406)(1.82)^2$$

$= 0.0310$ (compared with 0.0313; the small error is due to the fact that the small angle changes were in the order of the load flow convergence criteria).

(c) Solve for the economically optimum division of load considering losses. The penalty factors are

$$PF_1 = \frac{1}{1 - (\partial P_{TL}/\partial P_{G_1})} = \frac{1}{1 - 0.003388P_{G_1} + 0.009881P_{G_2}}$$

$$PF_2 = \frac{1}{1 + 0.009881P_{G_1} - 0.028811P_{G_2}}$$

$$\lambda_1 = \frac{200(P_{G_1} + 1)}{1 - 0.003388P_{G_1} + 0.009881P_{G_2}}$$

$$\lambda_2 = \frac{80P_{G_2} + 260}{1 + 0.009881P_{G_1} - 0.02881P_{G_2}}$$

The problem is solved by trial and error using a programmable calculator. The results of three trials are

P_{G_1}	P_{G_2}	λ_1	λ_2	P_L
1.0313	1.8200	400.4	423.5	2.820
1.1100	1.7400	416.4	415.5	2.823
1.1060	1.7410	415.6	415.6	2.820

The general problem is too complex to solve by hand. A flow chart for computer implementation is shown in Figure 8.5. The problem may also be formulated and solved by Newton–Raphson methods.

The foregoing discussion applies basically to an all-thermal system; that is, one where all generating units can be assigned an operation cost function (C_i). If hydro units are available, there is no fuel cost associated with the unit output and hence a negligible C_i function. However, this does not mean that there are no restrictions on hydro output whatsoever. The typical restriction involves the volume of water that may be removed from a reservoir in a specific time period. Consider

$$P_H = \eta H g \rho \frac{dv}{dt} \tag{8.32a}$$

where

P_H = hydraulic turbine power in W.
η = hydraulic efficiency of turbine and penstock.
g = acceleration of gravity = 9.8 m/s^2.
H = difference in elevation of reservoir surface and turbine (head) in m.
ρ = mass density of water = 1000 kg/m^3

$\frac{dv}{dt}$ = volume of water flow through turbine in m^3/s.

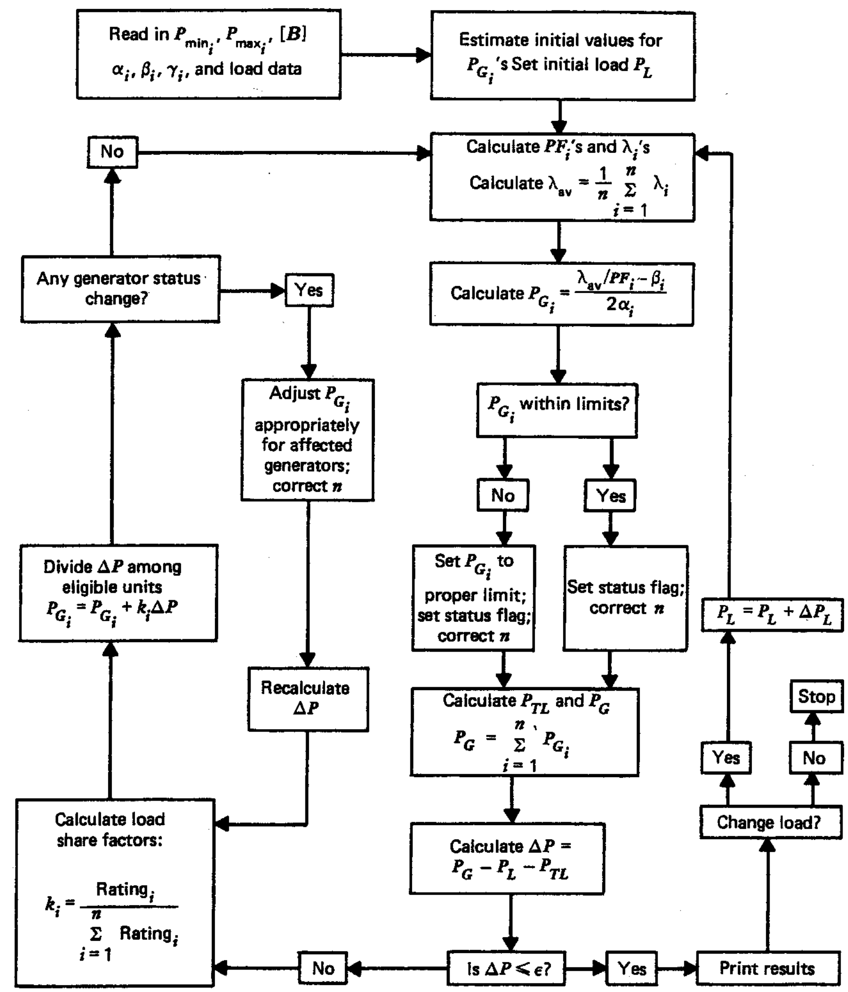


Figure 8.5. Flow chart for economic dispatch considering transmission losses.

For a constant-head, constant-efficiency situation,

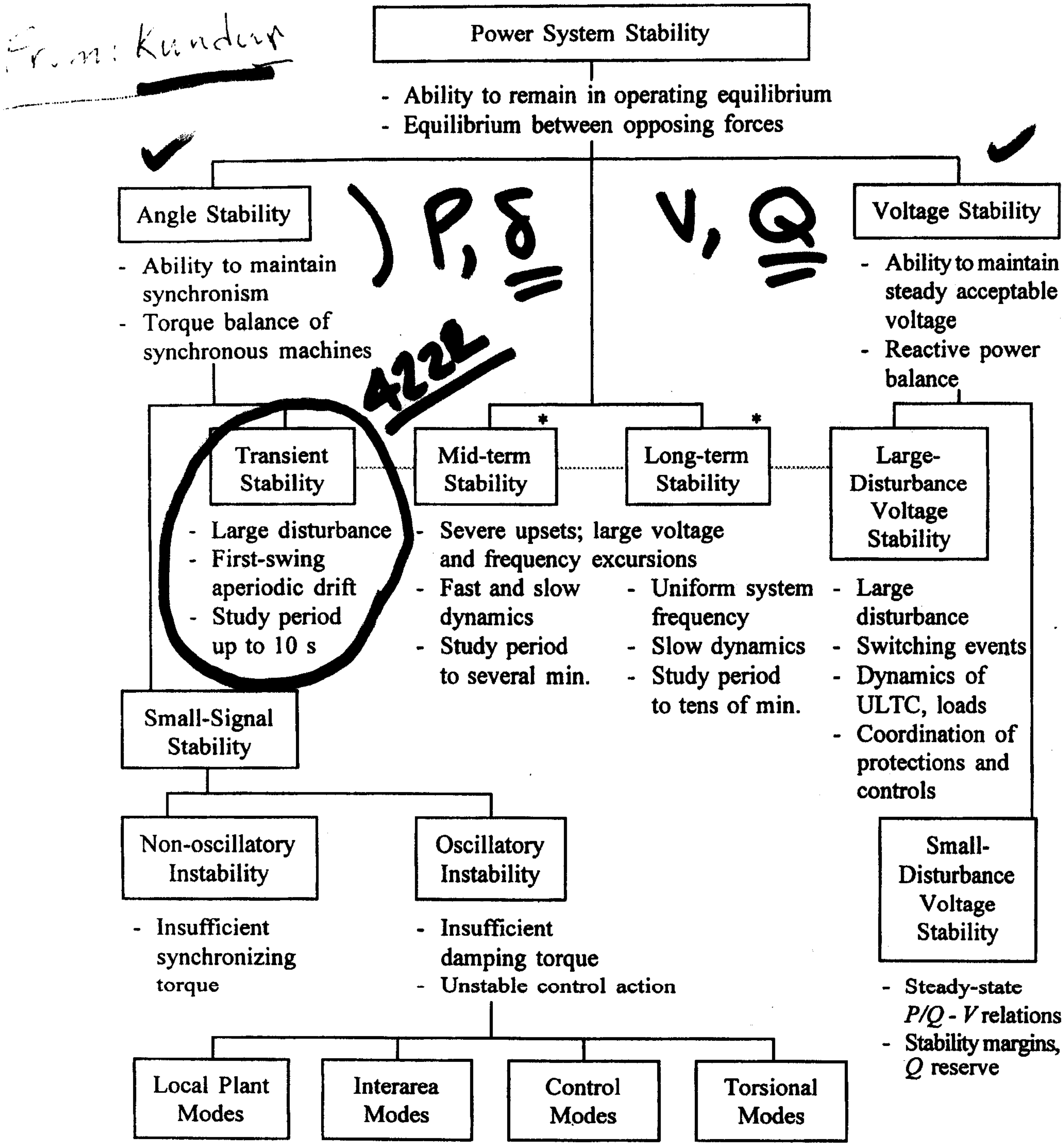
$$P_H = K\frac{dv}{dt} \tag{8.32b}$$

$$K = \eta H g \rho \tag{8.32c}$$

so that

$$\int dv = \frac{1}{K}\int P_H\, dt \tag{8.33a}$$

EE 6210



* With availability of improved analytical techniques providing unified approach for analysis of fast and slow dynamics, distinction between mid-term and long-term stability has become less significant.

Figure 2.9 Classification of power system stability

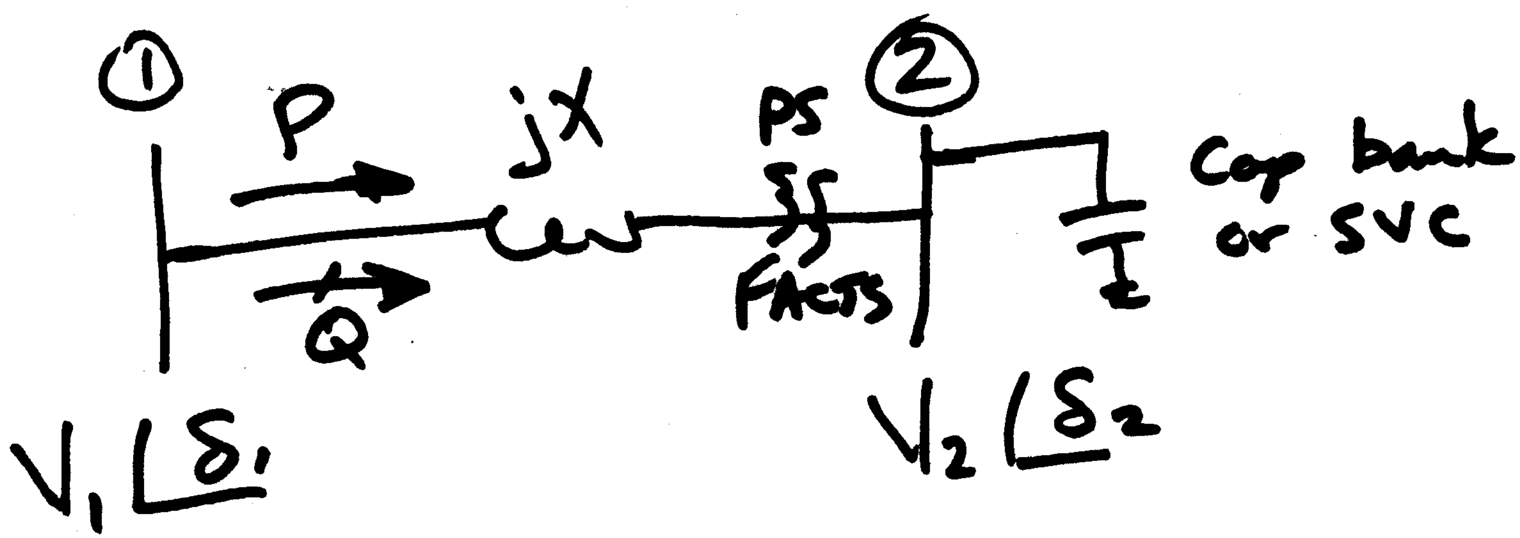
$$P_{1\to 2} = \frac{V_1 V_2}{X} \sin(\underline{\underline{\delta_1}} - \underline{\underline{\delta_2}})$$

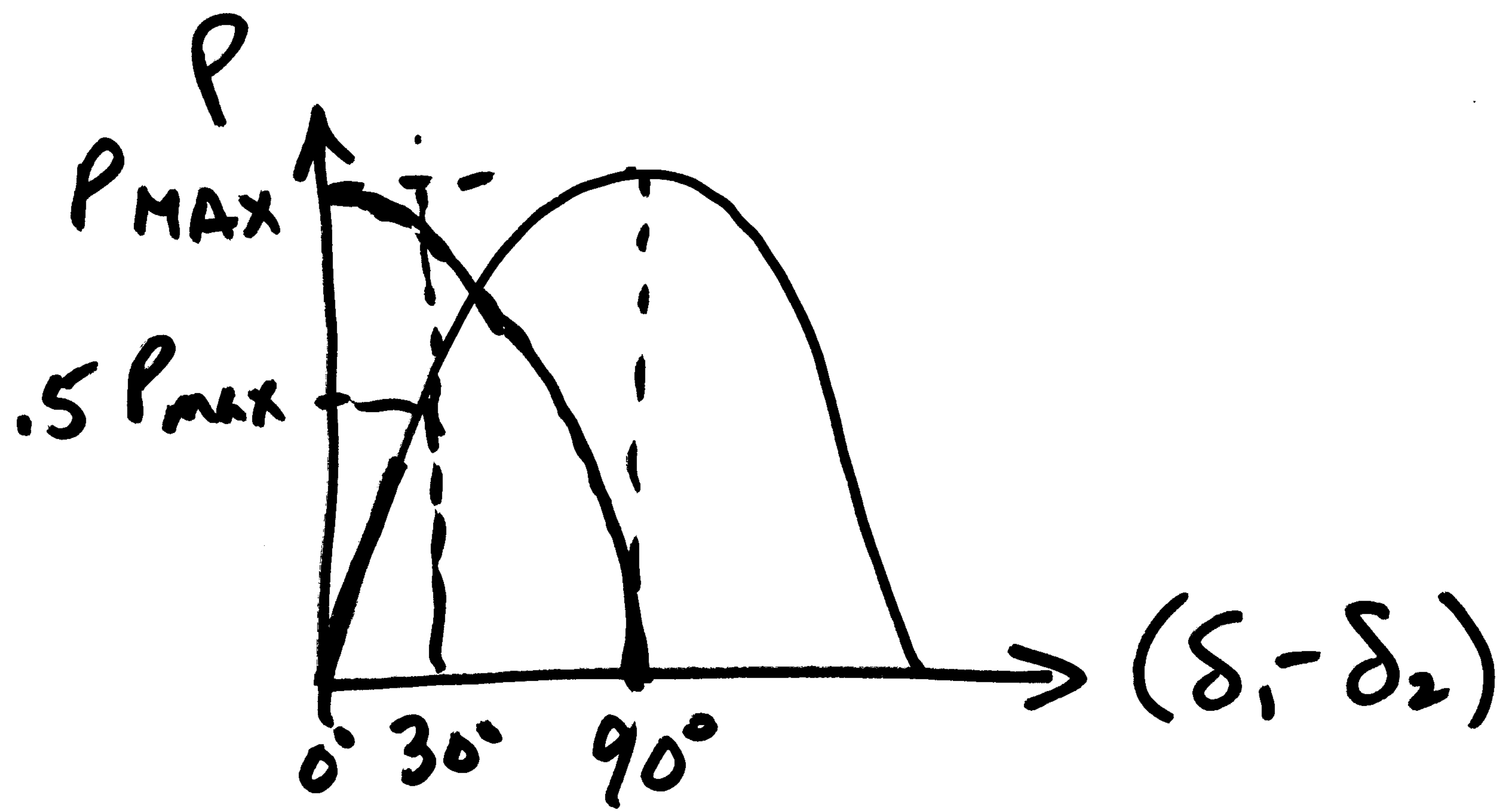
P: most sensitive to δ.

$$Q_{1\to 2} = \frac{V_1 V_2}{X} \underline{\underline{\cos}}(\delta_1 - \delta_2) - \frac{V_2^2}{X}$$

Q: most sensative to V.

Stability Swing: typically $\underline{\underline{1-2}}$ Hz.





P
PMAX
.5 PMAX
0° 30° 90°
(δ1 - δ2)