EE 5200 - Lecture 42 **Friday Dec 15, 2017**

Topics for Today:

- Announcements
  - Term Project final report due Mon Dec 18^{th}
  - Appendices: include journal paper and journal paper review.
  - Final Project .ppt presentations - by Tues Dec 19^{th} 9am
  - All teams prepare .ppt (if not presenting on Weds, embed audiostream).
  - Office: EERC 614. Phone: 906.487.2857

- Project Presentations (on-campus teams)
  - Emphasize your project (Journal paper in Appdx of report)
  - 6 presentations in 2 hrs - ~15-20 mins each including Q&A.
  - Provide .ppt handouts for audience (15 copies).
  - Handouts - landscape, 4 slides/page, double-sided, stapled

- Today:
  - Closing comments on stability
  - Intro to overall system operation (EE5230)
  - Quick look at State Estimation (Ch.15).
  - Intro to Smart Grid Technologies
  - Semester Wrapup

To: ee5200-0a-l@mtu.edu
From: Bruce Mork <bamork@mtu.edu>
Subject: EE5200 wrapup, deadlines & deliverables

EE5200 students:

See this link for a full listing of term projects. The TA and I have pre-identified 6 teams that are invited to present (marked with ?). Let us know by Friday what you intend to do.

http://www.ece.mtu.edu/faculty/bamork/ee5200/ProjList.pdf

Other teams are welcome to volunteer as well. We will choose six that are a combination of timely interesting topics and well-executed projects. Those that present at the final exam timeslot can avoid needing to record audio on all of the ppt slides.

Summary of deadlines and deliverables (as posted on class web page):

- Fri 5pm - Dispatch exercise due.
- Mon 9am - Term project report due. Send via e-mail, include all code and files that make up the project deliverables. No extensions.
- Tues 9am - Term project presentation ppt slides due. No extensions.
- Wed Dec 20th, 12:45pm - Final exam timeslot - all on-campus students will meet for presentations and evaluations. Attendance is mandatory, role shall be taken. (Online students: view the recorded video, evaluate the project presentations, submit to me and TA by Thursday COB.)

I understand that many of you were absent from today's lecture due to a deadline in Dr. Weaver's class. Unfortunately, you missed a detailed guidance on structuring your report and presentations, how to write Executive Summary, how to address Conclusions and Recommendations, etc. This will be on today's recorded video, really important for you to view it.

Dr. Mork
Mani

Time: Finals Week | Wednesday, Dec 20th - 12:45 - 2:45pm | Room: DOW 642

Allocated Time: ~15-20 minutes per presentation; 2-4 mins between

Start Time	Team Members	Topic
?	Zhiyuan Yang Yinqi Wang	Risk-based Contingency Analysis Using Monte Carlo Simulation Method
	Shiena Kundu Amit Sabu	Compensation techniques for reactive power control in transmission systems.
	Mohamed Hassan** Ayodeji Amoo**	Techniques for reducing total harmonic distortion (THD) on low and high voltage systems by analyzing and utilizing active/passive methods.
?	Vivek Vyasraj Sorab Madhur Arun Jagtap	Effects of dynamics of PV generation on Power Systems.
	Kara Eshelman** Mohammed A. Hossain**	Effects of series capacitors in transmission lines
	Apeksha Manekar Vaibhavi Tharval	Power factor correction and reduction of voltage fluctuation in microgrid
?	Eric Monte* Britton Borlace*	Transmission Line Protection through the use of Lighting Arrestors
?	Sreenivasulu Kamma Renu Anvesh Maddi	Active Power Flow Control using the Phase Shift Transformer and TCSC comparison
?	Mohammad Asif Iqbal Khan* Aashay Thatte	Data-driven modeling of a renewable energy powered micro-grid at Michigan Tech Campus
	Harshada Gaikwad Saurabh Singh	Lightning transient effect on transmission line.
	Nikunj Mathukiya Raakesh Raja Ramachandran	To study the analysis of fault current for the selection of the rating of different circuit breakers
	Vishal Arya Ashwin Pathak	Modelling and analysis of Lightning Arrestors for use in Distribution substations
	Yashdeep Datta Rushikesh Kadam	Impact of Distributed Generation over Transmission line & Distribution network (study power system attributes)
?	Praveen Iyer Mudit Kumar	Evaluating the performance of Transmission line with UPFC through simulation
	Narendra Raghav Arun Varkey Malppan	Application of phase shifting transformers(PST) in electrical networks
	Devalkumar Thakar Aakash Revdiwala	Comparison between two fault detection methods in transmission line: Two-Terminal Transmission Line Fault Location Using Traveling Waves and combination of impedance based method and voltage sage matching.
	Micheal Tracy**	Remediation of VAR deficiency on a small islanded grid with limited generation

*Stands for On-Campus On-line Student

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- Quick review of EE4221 + EE4222
- Delve more deeply into
  - Applied math
  - Concepts
  - Software: Matlab, Aspen, ATP
- Global perspective of power system.
- Professional skill development.

SMART GRID

SMART GRID

1) Advanced sensor "system" (fully)
2) Embedded processors
3) Comm & Control - <u>GPS</u> time-stamped
   - Wide Area

5223 - <u>Section ΦB</u> -
5224 - Lab:

- State Estimation (EE5230)

↓

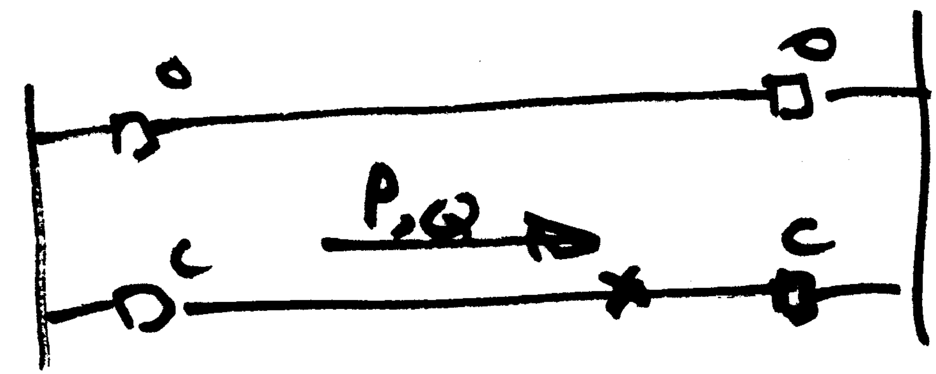
Synchro-Phasors

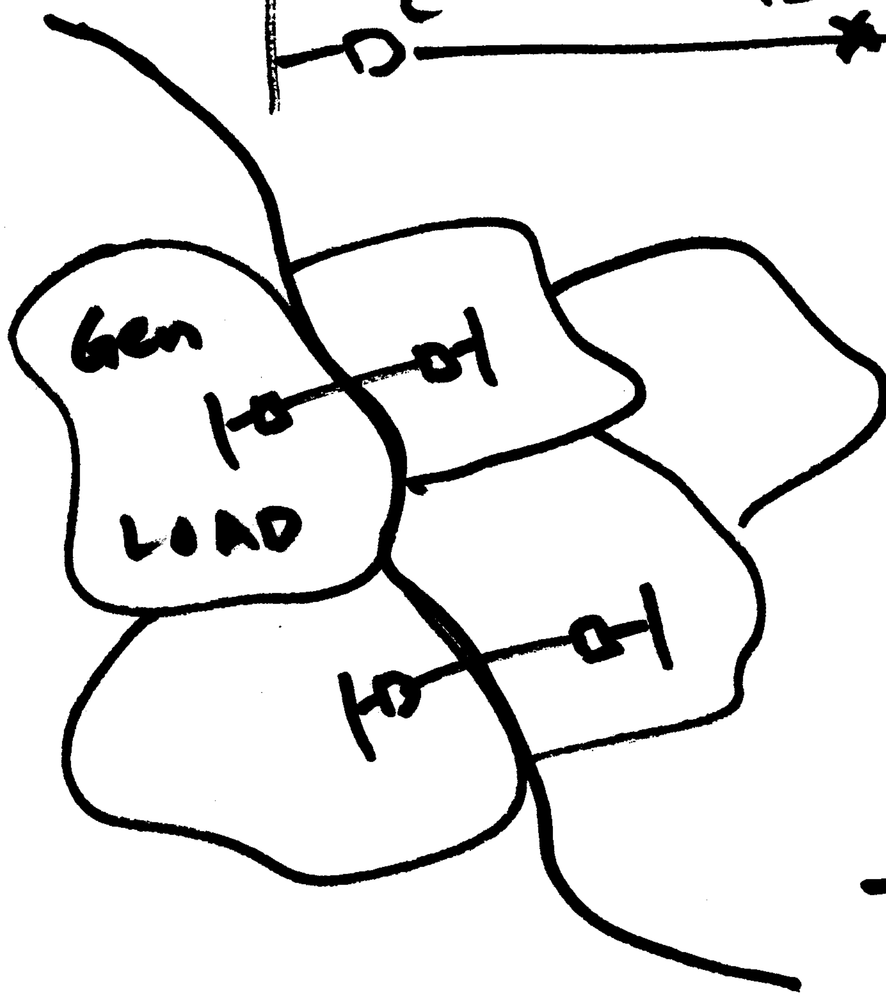
SMART GRID

PMU

↓

WAM / WAC / WAMPAC





"Separate System"

→ Islands.

WAM - Wide Area Meas.
WAC - Wide Area Control

PMU - Phasor Measurement Unit $\rightarrow$ SCADA

$|\tilde{V}|$ δ

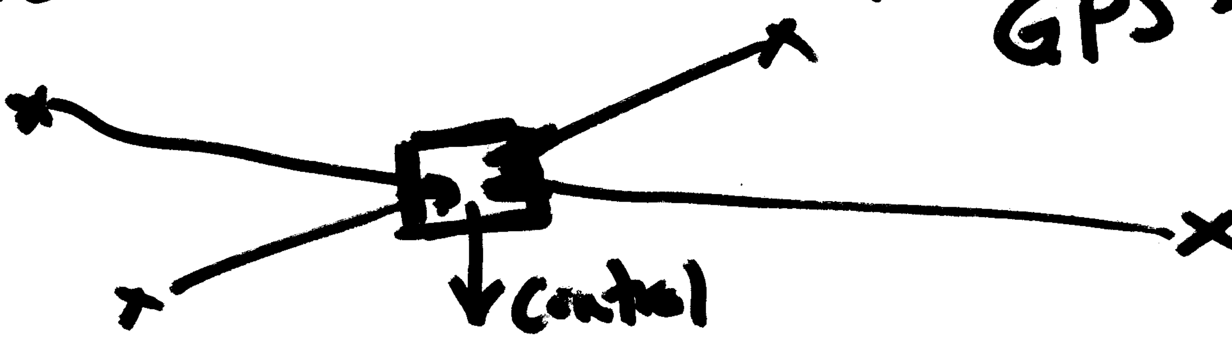
$|\tilde{I}|$ θ

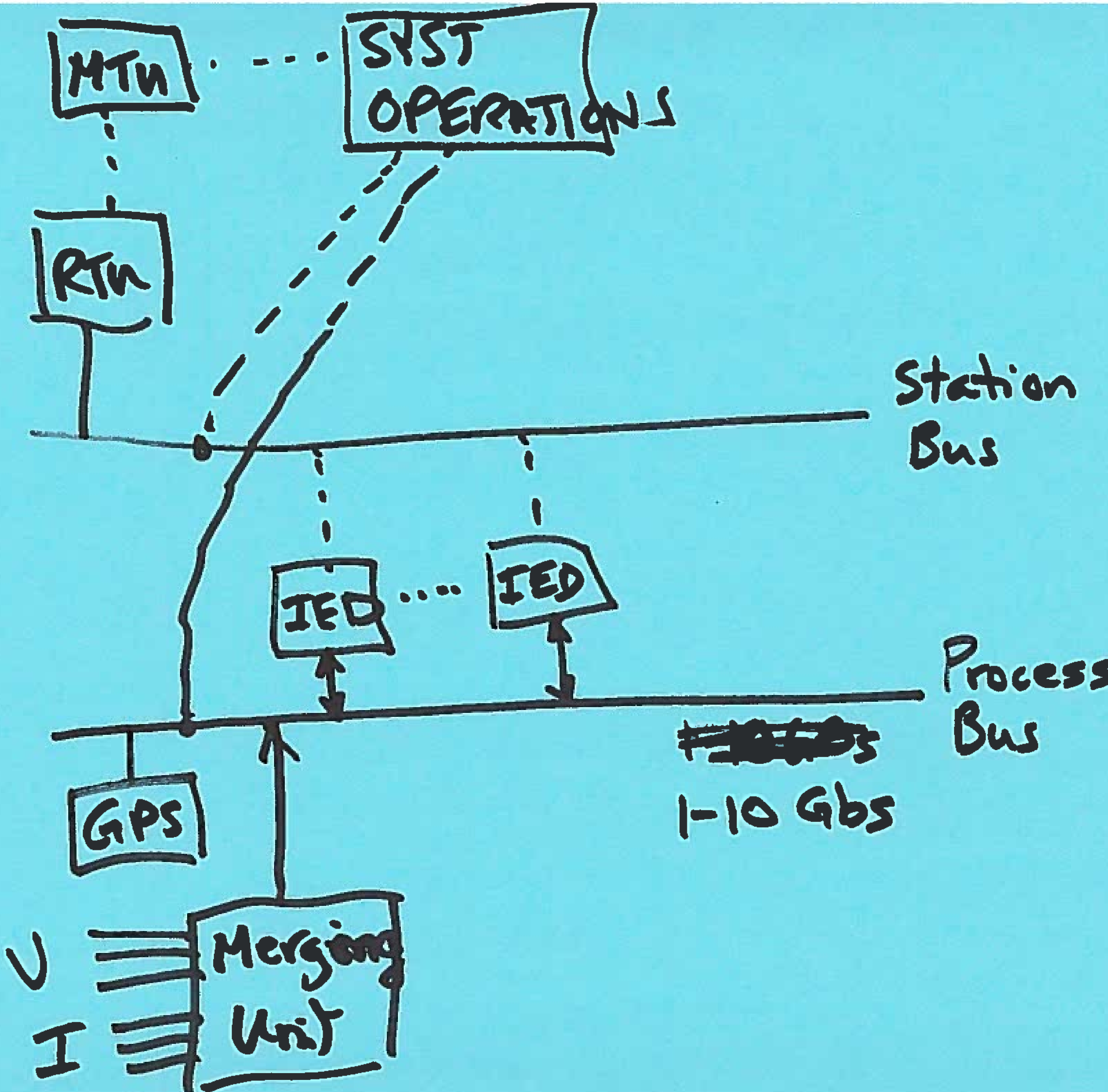
1-2 s updates

* Synchrophasors
 - High-speed updates

* IEC 61850 - Inter-operability.
 - High-speed intranet (10 GBs)

* IEEE 1588 - Data Synchronization
GPS time-tag




MTU
SYST OPERATIONS
RTU
Station Bus
IED
IED
Process Bus
1-10 Gbs
GPS
Merging Unit
V
I

- EE6210 – Stability
- EE5220 – transients
- EE5223 – Relaying
- EE5230 – Sys. Operations.

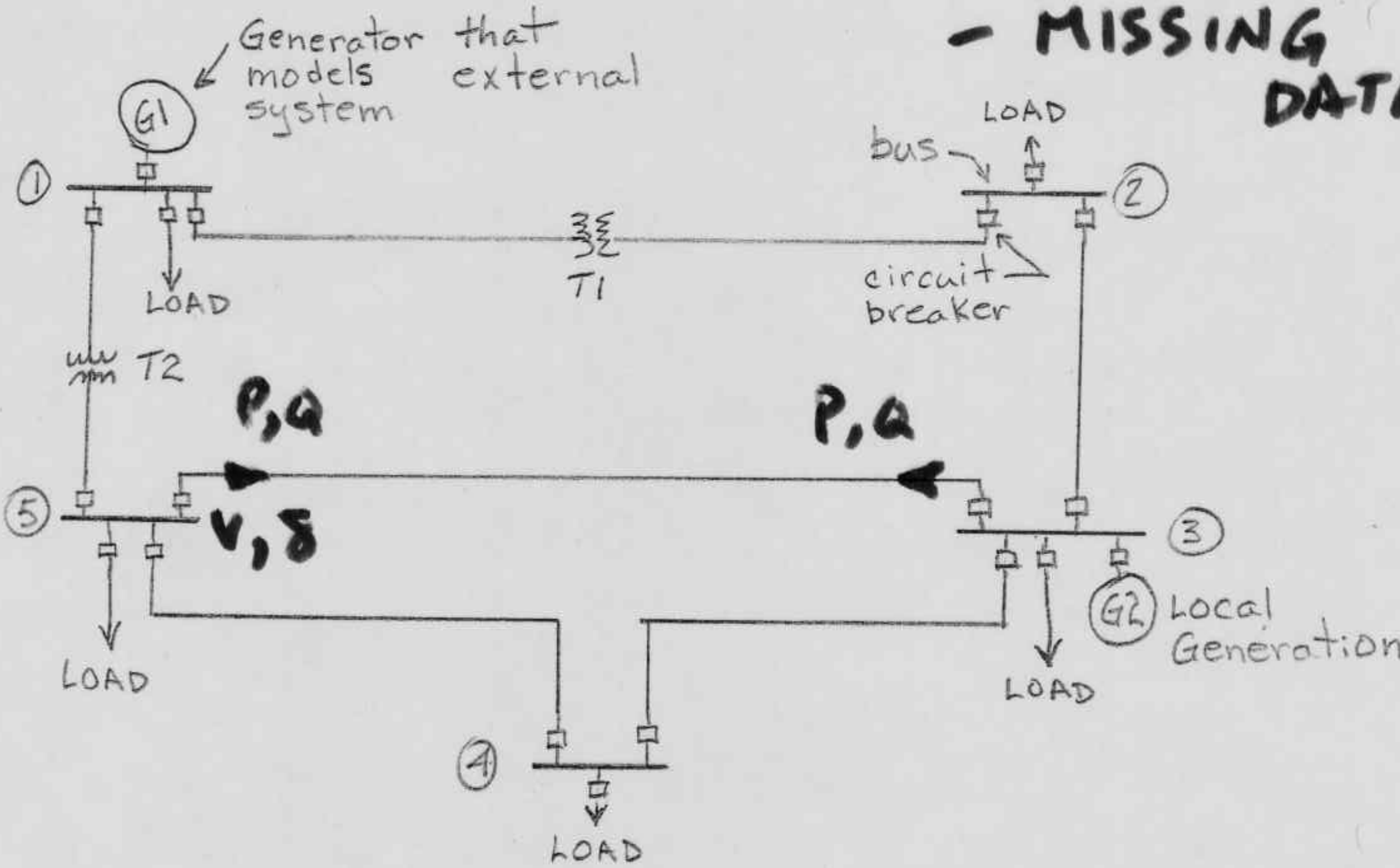
→ $\boxed{\text{State Est.}}$ – __Ch. 16__

– 2003, Aug.

WAM – time-tag (GPS)

TYPICAL BASIC POWER SYSTEM

- BAD DATA
- MISSING DATA



MAJOR COMPONENTS	LOCALLY CONTROLLABLE QTY
1. Generators	Voltage Power (Voltage Angle)
2. Transformers	Power Transfer (REAL & IM)
3. Transmission Lines	IN or OUT
4. Circuit Breakers	OPEN or CLOSED
5. Loads	Limits only - loads fluctuate periodically.

System is modelled with R's, L's, C's and ac voltage or current sources. System is decidedly nonlinear and analysis done with complex variables either in polar or rectangular form.

SYSTEM ANALYSIS

I. LOADFLOW CALCULATIONS: Use Newton-Raphson method adapted to complex variables. Solution converges in 3-7 iterations regardless of system size.

Solution gives voltage magnitude and angle at each bus, complex power flow in each bus connection, and generator outputs.

II. SYSTEM STABILITY: Numerical integration of 2nd order differential equations. Use loadflow to determine initial conditions of system. Solve for internal torque angle of each generator vs. time following a system disturbance. Can build up knowledge base for different system configurations (contingencies)

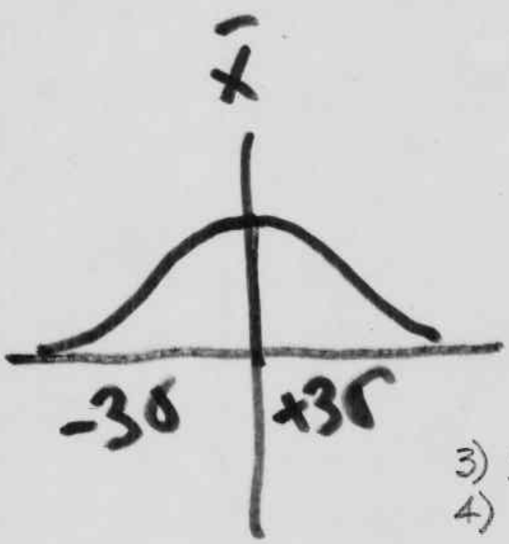
NEITHER OF ABOVE CAN BE PERFORMED ON REAL TIME BASIS, BUT CAN BE USED TO BUILD UP A KNOWLEDGE BASE FOR SYSTEM A.I. IMPLEMENTATIONS.

REAL TIME MONITORING & ANALYSIS

STATE ESTIMATION: Measurement devices (transducers) may fail or suffer a drift in calibration. Communication links may fail.

Fixup:
1) Substitute "psuedo-measurements" for unknown data based on available related measurements.
2) Perform WEIGHTED LEAST SQUARES ESTIMATION of all state variables.

$$\text{Minimize} \sum_{i=1}^{m} \underbrace{\frac{(z_i - f_i(x))^2}{\sigma_i^2}}_{\text{measurement residual (normalized)}}$$



z_i = measured value
$f_i(x)$ = estimated value
σ_i : 1/3 device tolerance (STD)
($\pm 3\sigma_i \rightarrow$ 99% of range)
Solve using Newton-Raphson

3) If $|z_i - f_i(x)| > 3\sigma \rightarrow$ BAD MEASUREMENT
4) Pass on estimated values to:

$$\left.\begin{array}{l}\text{CONTINGENCY ANALYSIS PROGRAM} \\ \text{CORRECTIVE ACTION PROGRAM}\end{array}\right\} \text{A.I.}$$

MAJOR OPERATING PROBLEMS

1) Alarm Processing
2) Bad Data Elimination } STATE ESTIMATION
3) Missing Data }
4) Fault Diagnosis
5) Contingency Selection & Security Analysis
6) Normal Control
7) Preventative control
8) Emergency Control
9) Restorative Control
10) Unit Commitment
11) Maintenance Scheduling
12) Fuel Scheduling & Contracts
13) Power exchanges and Prices
14) Operator Training
15) Demand Management
16) Automated Manuals

2 POWER SYSTEM STATE ESTIMATION

introduced in Chapter 11, the problem of monitoring the power flows and tages on a transmission system is very important in maintaining system urity. By simply checking each measured value against its limit, the power tem operators can tell where problems exist in the transmission system—and, s hoped, they can take corrective actions to relieve overloaded lines or :-of-limit voltages.

Many problems are encountered in monitoring a transmission system. These blems come primarily from the nature of the measurement transducers and m communications problems in transmitting the measured values back to operations control center.

Transducers from power system measurements, like any measurement device, l be subject to errors. If the errors are small, they may go undetected and cause misinterpretation by those reading the measured values. In addition, nsducers may have gross measurement errors that render their output less. An example of such a gross error might involve having the transducer nected up backward; thus, giving the negative of the value being measured. ally, the telemetry equipment often experiences periods when communi- ions channels are completely out; thus, depriving the system operator of any rmation about some part of the power system network.

It is for these reasons that power system state estimation techniques have been eloped. A state estimator, as we will see shortly, can "smooth out" small dom errors in meter readings, detect and identify gross measurement errors, l "fill in" meter readings that have failed due to communications failures. To begin, we will use a simple DC load flow example to illustrate the nciples of state estimation. Suppose the three-bus DC load flow of Example were operating with the load and generation shown in Figure 12.1. The only rmation we have about this system is provided by three MW power flow ters located as shown in Figure 12.2.

Only two of these meter readings are required to calculate the bus phase gles and all load and generation values fully. Suppose we use M_{13} and M_{32} l further suppose that M_{13} and M_{32} give us perfect readings of the flows on ir respective transmission lines.

$$M_{13} = 5\ \text{MW} = 0.05\ \text{pu}$$
$$M_{32} = 40\ \text{MW} = 0.40\ \text{pu}$$

en, the flows on lines 1-3 and 3-2 can be set equal to these meter readings.

$$f_{13} = \frac{1}{x_{13}}(\theta_1 - \theta_3) = M_{13} = 0.05\ \text{pu}$$

$$f_{32} = \frac{1}{x_{23}}(\theta_3 - \theta_2) = M_{32} = 0.40\ \text{pu}$$

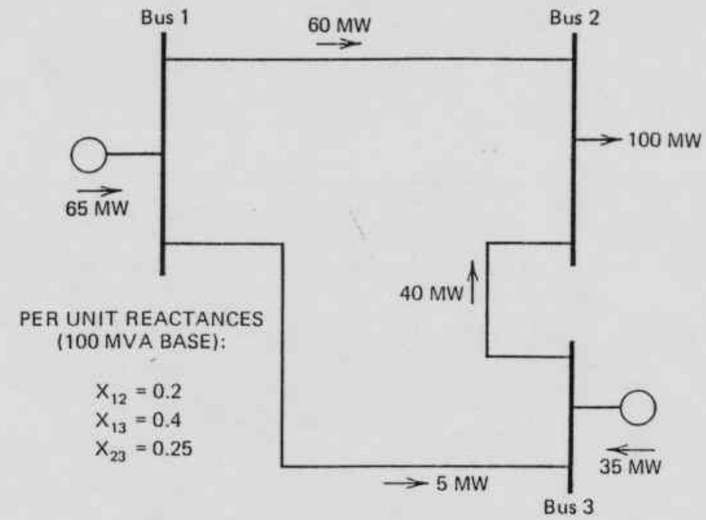


FIG. 12.1 Three-bus system from Example 4B.

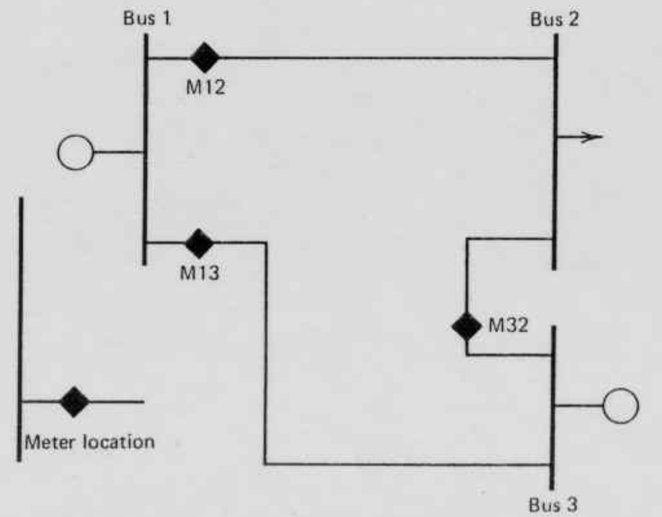


FIG. 12.2 Meter placement.

Since we know that $\theta_3 = 0$ rad, we can solve the f_{13} equation for θ_1, and the f_{32} equation for θ_2, resulting in

$$\theta_1 = 0.02\ \text{rad}$$
$$\theta_2 = -0.10\ \text{rad}$$

We will now investigate the case where all three meter readings have slight errors. Suppose the readings obtained are

$$M_{12} = 62 \text{ MW} = 0.62 \text{ pu}$$
$$M_{13} = 6 \text{ MW} = 0.06 \text{ pu}$$
$$M_{32} = 37 \text{ MW} = 0.37 \text{ pu}$$

If we use only the M_{13} and M_{32} readings, as before, we will calculate the phase angles as follows:

$$\theta_1 = 0.024 \text{ rad}$$
$$\theta_2 = -0.0925 \text{ rad}$$
$$\theta_3 = 0 \text{ rad (still assumed to equal zero)}$$

This results in the system flows as shown in Figure 12.3. Note that the predicted flows match at M_{13} and M_{32}, but the flow on line 1-2 does not match the reading of 62 MW from M_{12}. If we were to ignore the reading on M_{13} and use M_{12} and M_{32}, we could obtain the flows shown in Figure 12.4.

All we have accomplished is to match M_{12}, but at the expense of no longer matching M_{13}. What we need is a procedure that uses the information available from all three meters to produce the best estimate of the actual angles, line flows, and bus load and generations.

Before proceeding, let's discuss what we have been doing. Since the only thing we know about the power system comes to us from the measurements,

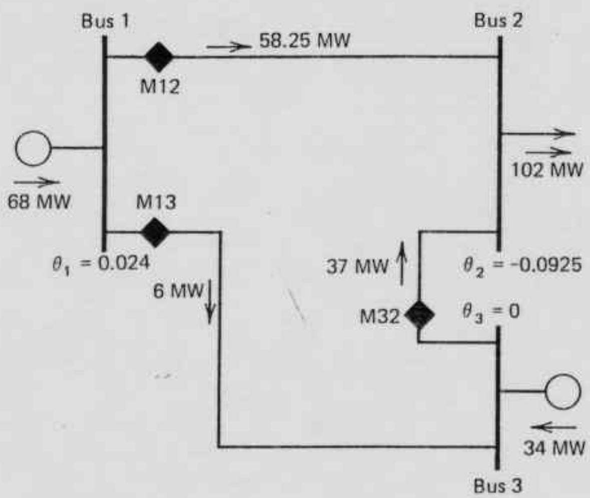


FIG. 12.3 Flows resulting from use of meters M_{13} and M_{32}.

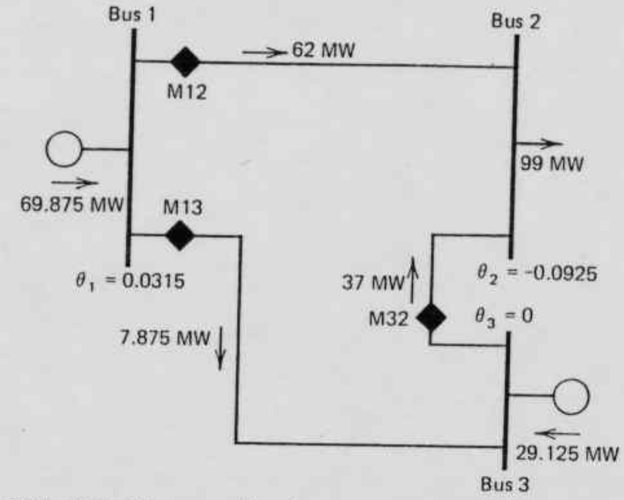


FIG. 12.4 Flows resulting from use of meters M_{12} and M_{32}.

we must use the measurements to estimate system conditions. Recall that i each instance the measurements were used to calculate the bus phase angles a bus 1 and 2. Once these phase angles were known, all unmeasured power flow: loads, and generations could be determined. We call θ_1 and θ_2 the *state variable* for the three-bus system since knowing them allows all other quantities to b calculated. In general, the state variables for a power system consist of the bu voltage magnitude at all buses and the phase angles at all but one bus. Th swing or reference bus phase angle is usually assumed to be zero radians. Not that we could use real and imaginary components of bus voltage if desired. I we can use measurements to estimate the "states" (i.e., voltage magnitudes an phase angles) of the power system, then we can go on to calculate any powe flows, generation, loads, and so forth that we desire. This presumes that th network configuration (i.e., breaker and disconnect switch statuses) is know and that the impedances in the network are also known. Automatic load ta changing autotransformers or phase angle regulators are often included in network, and their tap positions may be telemetered to the control as measured quantity. Strictly speaking, the transformer taps and phase angl regulator positions should also be considered as states since they mus be known in order to calculate the flows through the transformers an regulators.

To return to the three-bus DC power flow model, we have three meter providing us with a set of redundant readings with which to estimate the tw states θ_1 and θ_2. We say that the readings are redundant since, as we saw earlier only two readings are necessary to calculate θ_1 and θ_2, the other reading i always "extra." However, the "extra" reading does carry useful informatio and ought not to be discarded summarily.

This simple example serves to introduce the subject of *static-state estimation*, which is the art of estimating the exact system state given a set of imperfect measurements made on the power system. We will digress at this point to develop the theoretical background for static-state estimation. We will return to our three-bus system in Section 12.3.4.

12.3 MAXIMUM LIKELIHOOD WEIGHTED LEAST-SQUARES ESTIMATION

12.3.1 Introduction

Statistical estimation refers to a procedure where one uses samples to calculate the value of one or more unknown parameters in a system. Since the samples (or measurements) are inexact, the estimate obtained for the unknown parameter is also inexact. This leads to the problem of how to formulate a "best"estimate of the unknown parameters given the available measurements.

The development of the notions of state estimation may proceed along several lines, depending on the statistical criterion selected. Of the many criteria that have been examined and used in various applications, the following three are perhaps the most commonly encountered.

1. *The maximum likelihood criterion*, where the objective is to maximize the probability that the estimate of the state variable, $\hat{\mathbf{x}}$, is the true value of the state variable vector, $\mathbf{x}$ (i.e., maximize $P(\hat{\mathbf{x}}) = \mathbf{x}$).
2. *The weighted least-squares criterion*, where the objective is to minimize the sum of the squares of the weighted deviations of the estimated measurements, $\hat{\mathbf{z}}$, from the actual measurements, $\mathbf{z}$.
3. *The minimum variance criterion*, where the object is to minimize the expected value of the sum of the squares of the deviations of the estimated components of the state variable vector from the corresponding components of the true state variable vector.

When normally distributed, unbiased meter error distributions are assumed, each of these approaches results in identical estimators. This chapter will utilize the maximum likelihood approach because the method introduces the measurement error weighting matrix $[R]$ in a straightforward manner.

The maximum likelihood procedure asks the following question: "What is the probability (or likelihood) that I will get the measurements I have obtained?" This probability depends on the random error in the measuring device (transducer) as well as the unknown parameters to be estimated. Therefore, a reasonable procedure would be one that simply chose the estimate as the value that maximizes this probability. As we will see shortly, the maximum likelihood estimator assumes that we know the probability density function (PDF) of the random errors in the measurement. Other estimation

schemes could also be used. The "least-squares" estimator does not require th: we know the probability density function for the sample or measurement error However, if we assume that the probability density function of sample (measurement error is a normal (Gaussian) distribution, we will end up wit the same estimation formula. We will proceed to develop our estimatio formula using the maximum likelihood criterion assuming normal distributio for measurement errors. The result will be a "least-squares" or more precise a "weighted least-squares" estimation formula, even though we will develo the formulation using the maximum likelihood criteria. We will illustrate th method with a simple electrical circuit and show how the maximum likelihoo estimate can be made.

First, we introduce the concept of *random measurement error*. Note that w have dropped the term "sample" since the concept of a measurement is muc more appropriate to our discussion. The measurements are assumed to be i error: that is, the value obtained from the measurement device is close to th true value of the parameter being measured but differs by an unknown erro Mathematically, this can be modeled as follows.

Let z^{meas} be the value of a measurement as received from a measurement device. Let z^{true} be the true value of the quantity being measured. Finally, let η be the random measurement error. We can then represent our measured value as

$$z^{\text{meas}} = z^{\text{true}} + \eta \tag{12.1}$$

The random number, η, serves to model the uncertainty in the measurement: If the measurement error is unbiased, the probability density function of η i usually chosen as a normal distribution with zero mean. Note that othe measurement probability density functions will also work in the maximur likelihood method as well. The probability density function of η is

$$\text{PDF}(\eta) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\eta^2/2\sigma^2) \qquad (12.2$$

where σ is called the standard deviation and σ^2 is called the variance of th random number. PDF(η) describes the behavior of η. A plot of PDF(η) is show in Figure 12.5. Note that σ, the standard deviation, provides a way to mode the seriousness of the random measurement error. If σ is large, the measuremen is relatively inaccurate (i.e., a poor-quality measurement device), whereas : small value of σ denotes a small error spread (i.e., a higher-quality measuremen device). The normal distribution is commonly used for modeling measuremen errors since it is the distribution that will result when many factors contribut to the overall error.

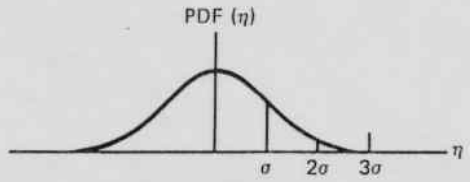


FIG. 12.5 The normal distribution.

12.3.2 Maximum Likelihood Concepts

The principle of maximum likelihood estimation is illustrated by using a simple DC circuit example as shown in Figure 12.6. In this example, we wish to estimate the value of the voltage source, x^{true}, using an ammeter with an error having a known standard deviation. The ammeter gives a reading of z_1^{meas}, which is equal to the sum of z_1^{true} (the true current flowing in our circuit) and η_1 (the error present in the ammeter). Then we can write

$$z_1^{\text{meas}} = z_1^{\text{true}} + \eta_1 \tag{12.3}$$

Since the mean value of η_1 is zero, we then know that the mean value of z_1^{meas} is equal to z_1^{true}. This allows us to write a probability density function for z_1^{meas} as

$$\text{PDF}(z_1^{\text{meas}}) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[\frac{-(z_1^{\text{meas}} - z_1^{\text{true}})^2}{2\sigma_1^2}\right] \tag{12.4}$$

where σ_1 is the standard deviation for the random error η_1. If we assume that the value of the resistance, r_1, in our circuit is known, then we can write

$$\text{PDF}(z_1^{\text{meas}}) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[\frac{-\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)^2}{2\sigma_1^2}\right] \tag{12.5}$$

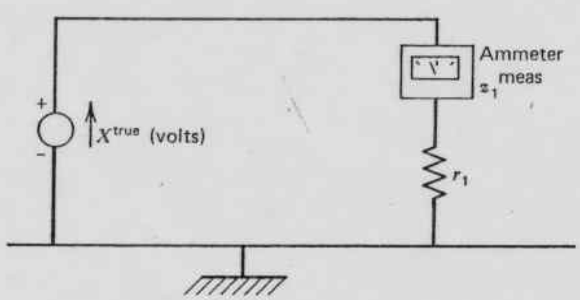


FIG. 12.6 Simple DC circuit with current measurement.

Coming back to our definition of a maximum likelihood estimator, we nov wish to find an estimate of x (called x^{est}) that maximizes the probability tha the observed measurement z_1^{meas} would occur. Since we have the probabilit density function of z_1^{meas}, we can write

$$\begin{aligned}\text{prob}(z_1^{\text{meas}}) &= \int_{z_1^{\text{meas}}}^{z_1^{\text{meas}} + dz_1^{\text{meas}}} \text{PDF}(z_1^{\text{meas}})\, dz_1^{\text{meas}} \quad \text{as } dz_1^{\text{meas}} \to 0 \\ &= \text{PDF}(z_1^{\text{meas}})\, dz_1^{\text{meas}}\end{aligned} \tag{12.6}$$

The maximum likelihood procedure then requires that we maximize th value of $\text{prob}(z_1^{\text{meas}})$, which is a function of x. That is,

$$\max_x \text{prob}(z_1^{\text{meas}}) = \max_x \text{PDF}(z_1^{\text{meas}})\, dz_1^{\text{meas}} \tag{12.7}$$

One convenient transformation that can be used at this point is to maximiz the natural logarithm of $\text{PDF}(z_1^{\text{meas}})$ since maximizing the Ln of $\text{PDF}(z_1^{\text{meas}})$ will also maximize $\text{PDF}(z_1^{\text{meas}})$. Then we wish to find

$$\max_x \text{Ln}[\text{PDF}(z_1^{\text{meas}})]$$

or

$$\max_x \left[-\text{Ln}(\sigma_1\sqrt{2\pi}) - \frac{\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)^2}{2\sigma_1^2}\right]$$

Since the first term is constant, it can be ignored. We can maximize the function in brackets by minimizing the second term since it has a negative coefficient, that is,

$$\max_x \left[-\text{Ln}(\sigma_1\sqrt{2\pi}) - \frac{\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)^2}{2\sigma_1^2}\right]$$

is the same as

$$\min_x \left[\frac{\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)^2}{2\sigma_1^2}\right] \tag{12.8}$$

The value of x that minimizes the right-hand term is found by simply taking the first derivative and setting the result to zero:

$$\frac{d}{dx}\left[\frac{\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)^2}{2\sigma_1^2}\right] = \frac{-\left(z_1^{\text{meas}} - \frac{1}{r_1}x\right)}{r_1\sigma_1^2} = 0 \tag{12.9}$$

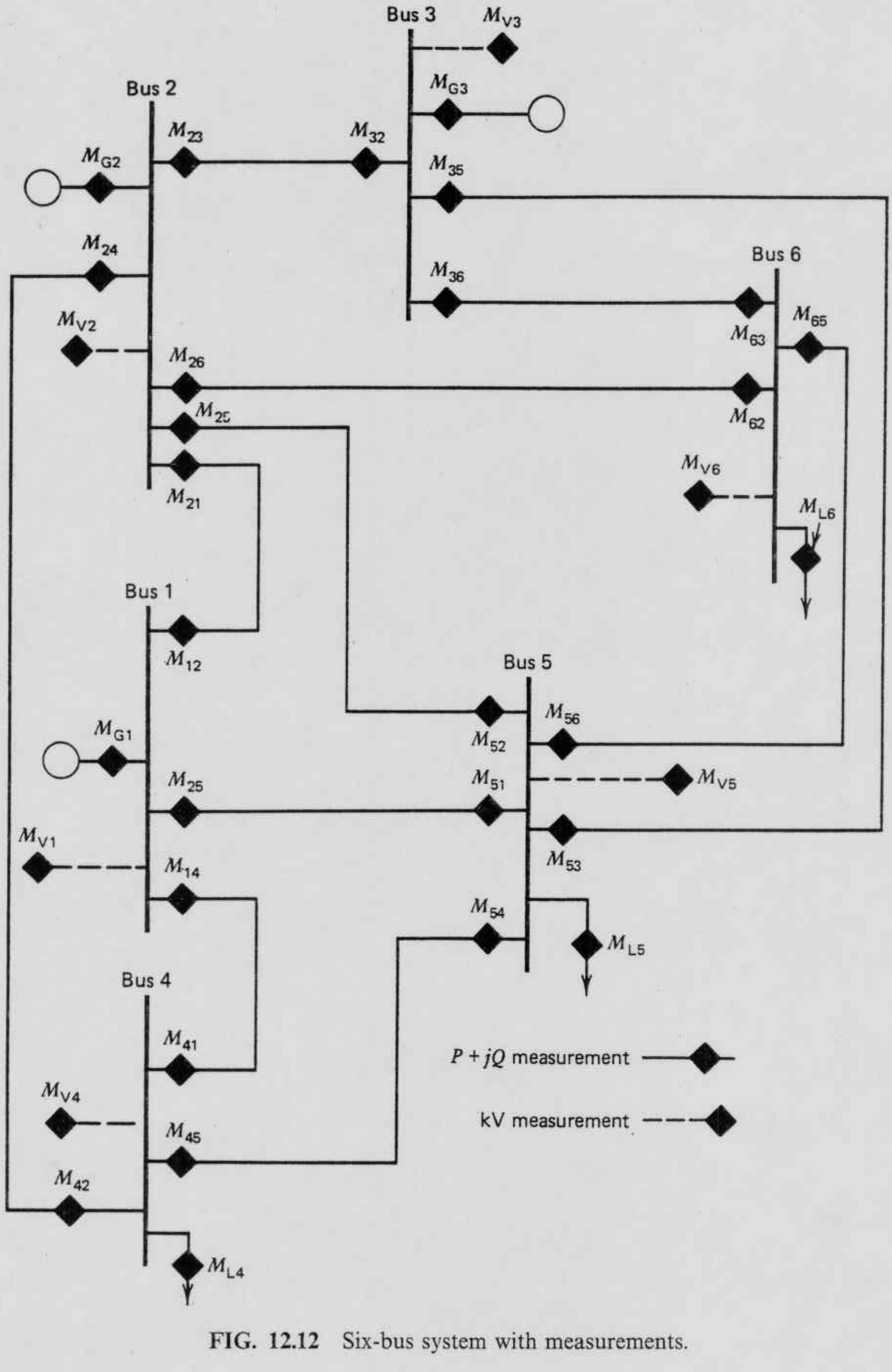


FIG. 12.12 Six-bus system with measurements.

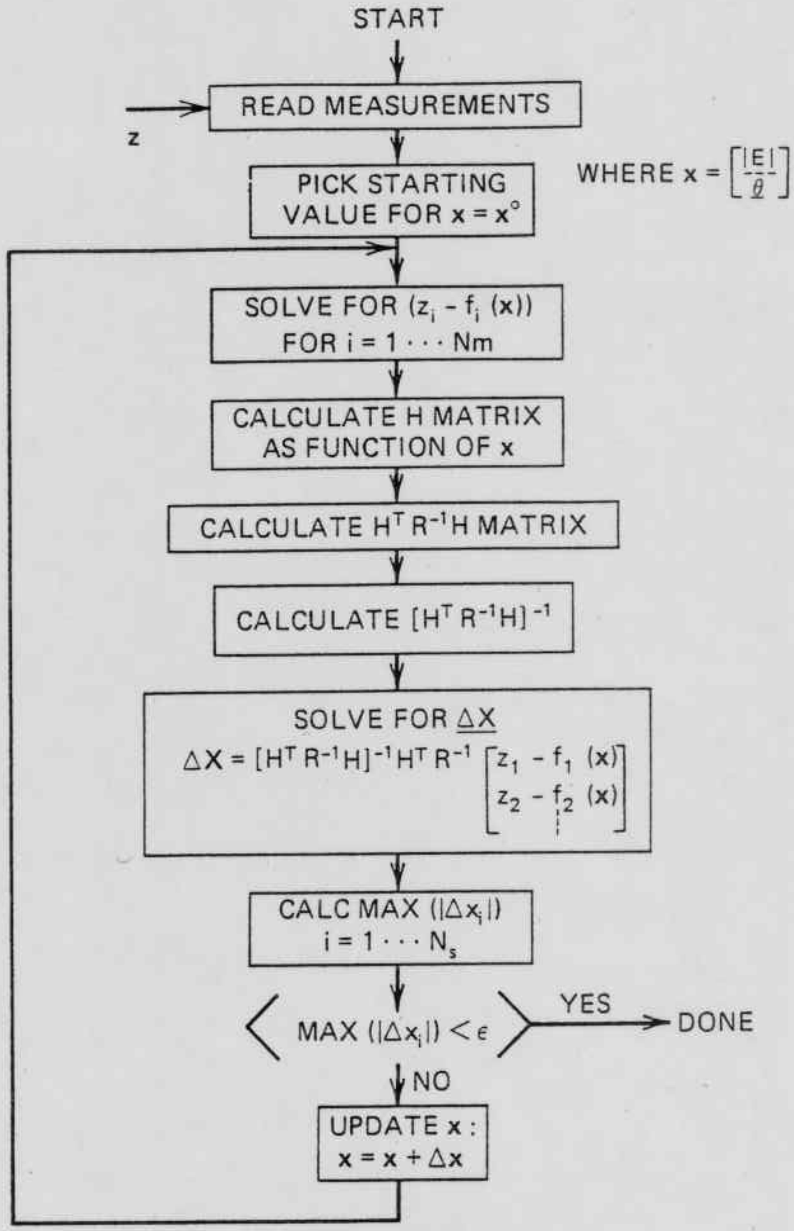


FIG. 12.11 State estimation solution algorithm.

for the bus-voltage magnitudes and phase angles given the measurements shown in Table 12.2. The procedure took three iterations with $\mathbf{x}^0$ initially being set to 1.0 pu and 0 rad for the voltage magnitude and phase angle at each bus, respectively. At the beginning of each iteration, the sum of the measurement residuals, $J(\mathbf{x})$ (see Eq. 12.30), is calculated and displayed. At the end of each iteration, the maximum $\Delta|E|$ and the maximum $\Delta\theta$ are calculated and displayed. The iterative steps for the six-bus system used here produced the results given in Table 12.3.

The value of $J(\mathbf{x})$ at the end of the iterative procedure would be zero if all measurements were without error or if there were no redundancy in the measurements. When there are redundant measurements with errors, the value of $J(\mathbf{x})$ will not normally go to zero. Its value represents a measure of the overall

WAM

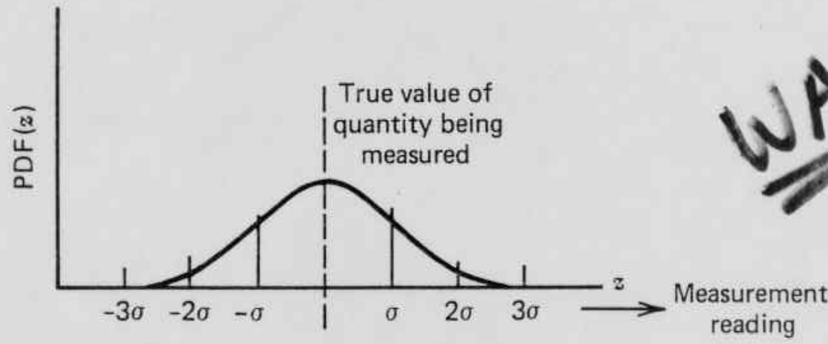


FIG. 12.8 Normal distribution of meter errors.

The formula developed in the last section for the weighted least-squares estimate is given in Eq. 12.23, which is repeated here.

$$\mathbf{x}^{\text{est}} = [[H]^T[R^{-1}][H]]^{-1}[H]^T[R^{-1}]\mathbf{z}^{\text{meas}}$$

where

$\mathbf{x}^{\text{est}}$ = vector of estimated state variables

$[H]$ = measurement function coefficient matrix

$[R]$ = measurement covariance matrix

$\mathbf{z}^{\text{meas}}$ = vector containing the measured values themselves

For the three-bus problem we have

$$\mathbf{x}^{\text{est}} = \begin{bmatrix} \theta_1^{\text{est}} \\ \theta_2^{\text{est}} \end{bmatrix} \tag{12.27}$$

To derive the $[H]$ matrix, we need to write the measurements as a function of the state variables θ_1 and θ_2. These functions are written in per unit as

$$\begin{aligned} M_{12} &= f_{12} = \frac{1}{0.2}(\theta_1 - \theta_2) = 5\theta_1 - 5\theta_2 \\ M_{13} &= f_{13} = \frac{1}{0.4}(\theta_1 - \theta_3) = 2.5\theta_1 \\ M_{32} &= f_{32} = \frac{1}{0.25}(\theta_3 - \theta_2) = -4\theta_2 \end{aligned} \tag{12.28}$$

The reference-bus phase angle, θ_3, is still assumed to be zero. Then

$$[H] = \begin{bmatrix} 5 & -5 \\ 2.5 & 0 \\ 0 & -4 \end{bmatrix}$$