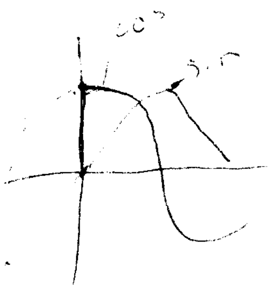
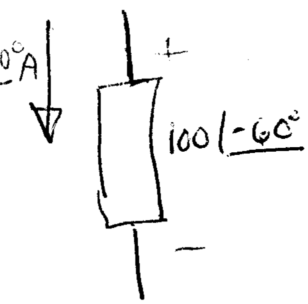
Chap 1 Problem Solutions

1.1 If $v = 141.4\sin(\omega t + 30°)$ V and $i = 11.31\cos(\omega t - 30°)$ A, find for each (*a*) the maximum value, (*b*) the rms value and (*c*) the phasor expression in polar and rectangular form if voltage is the reference. Is the circuit inductive or capacitive?





Solution:

(a) Maximum values:

$$V_{max} = 141.4 \text{ V} \qquad I_{max} = 11.31 \text{ A}$$

(b) rms values:

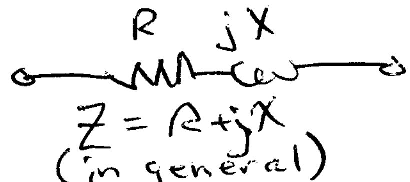
$$|V| = \frac{141.4}{\sqrt{2}} = 100 \text{ V} \qquad |I| = \frac{11.31}{\sqrt{2}} = 8 \text{ A}$$

(c) Phasor expressions in polar and rectangular form:

$$V = 100\angle 0° = 100 + j0 \text{ V}$$
$$I = 8\angle -60° = 4 - j6.93 \text{ A}$$

The circuit is inductive as I lags V.

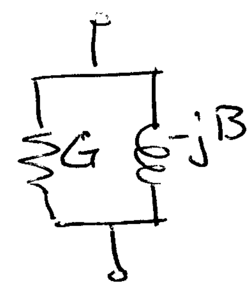
1.2 If the circuit of Prob. 1.1 consists of a purely resistive and a purely reactive element, find R and X, (*a*) if the elements are in series and (*b*) if the elements are in parallel.



Solution:

(a) Elements in series:

$$Z = \frac{100\angle 0°}{8\angle -60°} = 12.5\angle 60° \ \Omega = 6.25 + j10.83 \ \Omega$$
$$R = 6.25 \ \Omega \qquad X_L = 10.83 \ \Omega$$



(b) Elements in parallel:

$$Y = \frac{1}{Z} = 0.08\angle -60° = 0.04 - j0.0693$$
$$R = \frac{1}{0.04} = 25 \ \Omega \qquad X_L = \frac{1}{0.0693} = 14.43 \ \Omega$$

1.3 In a single-phase circuit $V_a = 120\angle 45°$ V and $V_b = 100\angle -15°$ V with respect to a reference node o. Find V_{ba} in polar form.

Solution:

$$V_{ba} = V_{b0} - V_{a0}$$
$$V_{ba} = 100\angle -15° - 120\angle 45° = 96.59 - j25.88 - (84.85 + j84.85)$$
$$= 11.74 - j110.73 = 111.35\angle -83.95° \ \text{V}$$

1.4 A single-phase ac voltage of 240 V is applied to a series circuit whose impedance is $10\angle 60^\circ\ \Omega$. Find R, X, P, Q and the power factor of the circuit.

Solution:

$$\begin{aligned}
R &= 10\cos 60^\circ = 5.0\ \Omega \\
X &= 10\sin 60^\circ = 8.66\ \Omega \\
I &= \frac{240\angle 0^\circ}{10\angle 60^\circ} = 24\angle -60^\circ\ \text{A} \\
P &= (24)^2 \times 5 = 2880\ \text{W} \\
Q &= (24)^2 \times 8.66 = 4988\ \text{var} \\
\text{p.f.} &= \cos\left(\tan^{-1}\frac{4988}{2880}\right) = 0.50 \\
\text{or} \quad & \cos\left(\tan^{-1}\frac{X}{R}\right) = 0.50
\end{aligned}$$

1.5 If a capacitor is connected in parallel with the circuit of Prob. 1.4 and if this capacitor supplies 1250 var, find the P and Q supplied by the 240-V source, and find the resultant power factor.

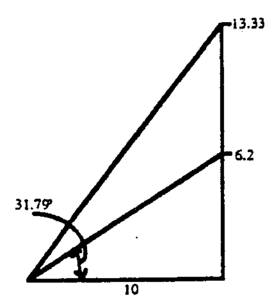
Solution:

$$\begin{aligned}
P &= 2880\ \text{W} \\
Q &= 4988 - 1250 = 3738\ \text{var} \\
\text{p.f.} &= \cos\left(\tan^{-1}\frac{3738}{2880}\right) = 0.61
\end{aligned}$$

1.6 A single-phase inductive load draws 10 MW at 0.6 power factor lagging. Draw the power triangle and determine the reactive power of a capacitor to be connected in parallel with the load to raise the power factor to 0.85.

Solution:

$$\begin{aligned}
\frac{10}{0.6}\sin\left(\cos^{-1}0.6\right) &= 13.33 \\
\cos^{-1}0.85 &= 31.79^\circ \\
10\tan 31.79^\circ &= 6.2\ \text{var} \\
Q_c &= -(13.33 - 6.2) \\
&= -7.13\ \text{Mvar}
\end{aligned}$$



1.7 A single-phase induction motor is operating at a very light load during a large part of every day and draws 10 A from the supply. A device is proposed to "increase the efficiency" of the motor. During a demonstration the device is placed in parallel with the unloaded motor and the current drawn from the supply drops to 8 A. When two of the devices are placed in parallel the current drops to 6 A. What simple device will cause this drop in current? Discuss the advantages of the device. Is the efficiency of the motor increased by the device? (Recall that an induction motor draws lagging current).

Solution:

A capacitor will cause the drop in current in the line because the lagging component of current drawn by the motor will be partially offset by the leading current drawn by the capacitor. The current drawn by the motor, however, will be unchanged if the terminal voltage remains constant. So the motor efficiency will remain the same. Loss in the line supplying the motor will be less due to the lower line current. If the line to the motor from the supply bus is long, the voltage drop in the line will be reduced and this may be desirable.

1.8 If the impendance between machines 1 and 2 of Example 1.1 is $Z = 0 - j5\ \Omega$ determine (a) whether each machine is generating or consuming power, (b) whether each machine is receiving or supplying positive reactor power and the amount, and (c) the value of P and Q absorbed by the impedance.

Solution:

$$\begin{aligned} I &= \frac{100 + j0 - (86.6 + j50)}{-j5} = 10 + j2.68 = 10.35\underline{/15^\circ}\ \text{A} \\ E_1 I^* &= 100\,(10 - j2.68) = 1000 - j268 \\ E_2 I^* &= (86.6 + j50)\,(10 - j2.68) = 1000 + j268 \end{aligned}$$

Machine 1 generates 1000 W, receives 268 var

Machine 2 absorbs 1000 W, receives 268 var

Capacitor in the line supplies $(10.35)^2 \times 5 = 536$ var

1.9 Repeat Problem 1.8 if $Z = 5 + j0\ \Omega$.

Solution:

$$\begin{aligned} I &= \frac{100 + j0 - (86.6 + j50)}{5} = 2.68 - j10 = 10.35\underline{/-75^\circ}\ \text{A} \\ E_1 I^* &= 100\,(2.68 + j10) = 268 + j1000 \\ E_2 I^* &= (86.6 + j50)\,(2.68 + j10) = -268 + j1000 \end{aligned}$$

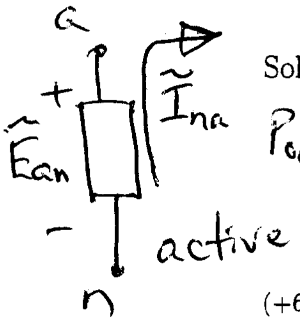
Machine 1 generates 268 W, delivers 1000 var

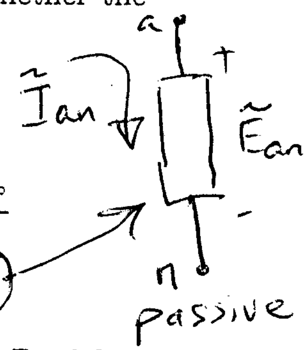
Machine 2 generates 268 W, receives 1000 var

Resistance in the line absorbs $(10.35)^2 \times 5 = 536$ W

Both machines are generators.

1.10 A voltage source $E_{an} = -120\angle 210°$ V and the current through the source is given by $I_{na} = 10\angle 60°$ A. Find the values of P and Q and state whether the source is delivering or receiving each.





Solution:

$$
\begin{aligned}
E_{an}I^*_{na} &= -120\angle 210° \times 10\angle -60° = -1200\angle 150° = 1200\angle -30° \\
P &= 1039 \text{ W delivered} \\
Q &= -600 \text{ var delivered}
\end{aligned}
$$

(+600 var absorbed by source, since I_{na} defines positive current from n to a and E_{an} defines point a at higher potential than n when e_{an} is positive.)

1.11 Solve Example 1.1 if $E_1 = 100\angle 0°$ V and $E_2 = 120\angle 30°$ V. Compare the results with Example 1.1 and form some conclusions about the effect of variation of the magnitude of E_2 in this circuit.

Solution:

$$
\begin{aligned}
I &= \frac{100 - (103.92 + j60)}{j5} = \frac{-3.92 - j60}{j5} = -12 + j0.78 \\
E_1 I^* &= 100\,(-12 - j0.78) = -1200 - j78 \\
E_2 I^* &= (103.92 + j60)\,(-12 - j0.78) = -1247 - j720 - j81 + 46.8 \\
&= -1200 - j801
\end{aligned}
$$

Machine 1 absorbs 1200 W and 78 var
Machine 2 delivers 1200 W and 801 var
$801 - 78 = 723$ var absorbed by line

In Example 2.1 the line received 536 var, half from each source. Raising $|E_2|$ caused some increase in power transfer and some increase in Q supplied to the line, but the significant fact is that raising $|E_2|$ caused that source to supply not only all the Q absorbed by the line but also 78 var delivered to the $|E_1|$ source.

1.12 Evaluate the following expressions in polar form:

(a) $a - 1$

(b) $1 - a^2 + a$

(c) $a^2 + a + j$

(*d*) $ja + a^2$

Solution:

(*a*) $a - 1 \;=\; -0.5 + j0.866 - 1 \;=\; 1.732\angle 150^\circ$

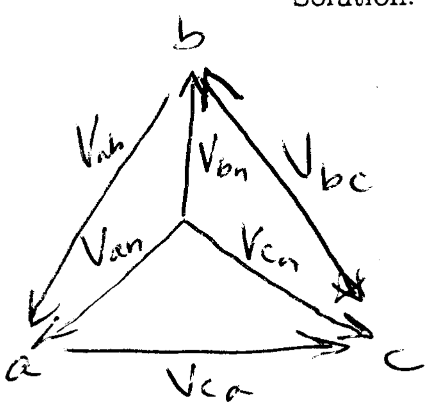
(*b*) $1 - a^2 + a \;=\; 1 - (-0.5 - j0.866) - 0.5 + j0.866 \;=\; 1 + j1.732 = 2.00\angle 60^\circ$

(*c*) $a^2 + a + j \;=\; -0.5 - j0.866 - 0.5 + j0.866 + j1 \;=\; -1 + j1 = 1.414\angle 135^\circ$

(*d*) $ja + a^2 \;=\; 1\angle 210^\circ + 1\angle 240^\circ \;=\; -0.866 - j0.5 - 0.5 - j0.866 \;=\; -1.366 - j1.366$
$\;=\; 1.932\angle 225^\circ$

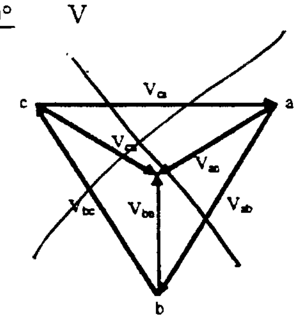
1.13 Three identical impedances of $10\angle -15^\circ$ Ω are Y-connected to balanced three-phase line voltages of 208 V. Specify all the line and phase voltages and the currents as phasors in polar form with V_{ca} as reference for a phase sequence of *abc*.

Solution:



$$V_{an} = 120\angle 210^\circ \text{ V} \qquad V_{ab} = 208\angle 240^\circ \text{ V}$$
$$V_{bn} = 120\angle 90^\circ \text{ V} \qquad V_{bc} = 208\angle 120^\circ \text{ V}$$
$$V_{cn} = 120\angle -30^\circ \text{ V} \qquad V_{ca} = 208\angle 0^\circ \text{ V}$$

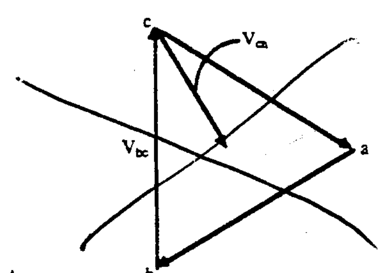
$$I_a = \frac{V_{an}}{Z} = \frac{120\angle 210^\circ}{10\angle -15^\circ} = 12\angle 225^\circ \text{ A}$$
$$I_b = \frac{V_{bn}}{Z} = \frac{120\angle 90^\circ}{10\angle -15^\circ} = 12\angle 105^\circ \text{ A}$$
$$I_c = \frac{V_{cn}}{Z} = \frac{120\angle -30^\circ}{10\angle -15^\circ} = 12\angle -15^\circ \text{ A}$$



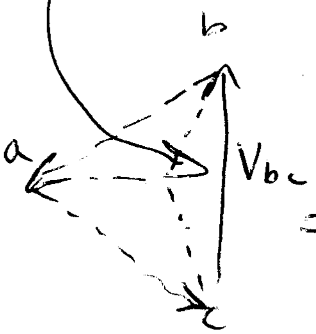
1.14 ~~In a balanced three~~-phase system the Y-connected impedances are $10\angle 30^\circ$ Ω. If $V_{bc} = 416\angle 90^\circ$ V, specify I_{cn} in polar form.

given

Solution:



$$\frac{416}{\sqrt{3}} = 240 \text{ V}$$
$$V_{cn} = 240\angle -60^\circ \text{ V}$$
$$I_{cn} = \frac{240\angle -60^\circ}{10\angle 30^\circ} = 24\angle -90^\circ \text{ A}$$



$\Rightarrow \tilde{V}_{cn} = 240\angle -60^\circ$

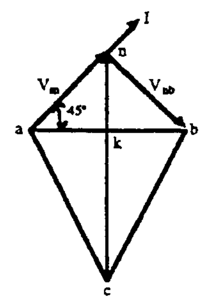
$$\tilde{I}_{cn} = \frac{\tilde{V}_{cn}}{\tilde{Z}_{cn}} = \frac{240\angle -60^\circ}{10\angle 30^\circ} = 24\angle -90^\circ \text{ A}$$

1.15 The terminals of a three-phase supply are labeled a, b and c. Between any pair a voltmeter measures 115 V. A resistor of 100 Ω and a capacitor of 100 Ω at the frequency of the supply are connected in series from a to b with the resistor connected to a. The point of connection of the elements to each other is labeled n. Determine graphically the voltmeter reading between c and n if phase sequence is abc and if phase sequence is acb.

Solution:

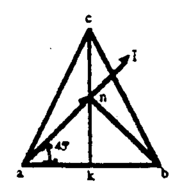
Sequence a-b-c

$$\begin{aligned} \overline{nk} &= \frac{115}{2} = 57.5 \text{ V} \\ \overline{kc} &= 115\sin 60^\circ = 99.6 \text{ V} \\ \text{meter reading} &= 57.5 + 99.6 = 157.1 \text{ V} \end{aligned}$$



Sequence a-c-b

$$\begin{aligned} \overline{kc} &= 99.6 \text{ V} \\ \overline{nk} &= 57.5 \text{ V} \\ \text{meter reading} &= 99.6 - 57.5 = 42.1 \text{ V} \end{aligned}$$



1.16 Determine the current drawn from a three-phase 440-V line by a three-phase 15-hp motor operating at full load, 90% efficiency and 80% power factor lagging. Find the values of P and Q drawn from the line.

Solution:

$$\begin{aligned} |I| &= \frac{15 \times 746}{\sqrt{3} \times 440 \times 0.9 \times 0.8} = 20.39 \text{ A} \\ P &= \sqrt{3} \times 440 \times 20.39 \times 0.8 = 12,431 \text{ W drawn from line} \\ Q &= \sqrt{3} \times 440 \times 20.39 \times 0.6 = 9,324 \text{ var drawn from line} \end{aligned}$$

1.17 If the impendance of each of the three lines connecting the motor of Prob. 1.16 to a bus is $0.3 + j1.0\ \Omega$, find the line-to-line voltage at the bus which supplies 440 V at the motor.

Solution:

$$I = 20.39\,(0.8 - j0.6) = 16.31 - j12.23 \text{ A}$$

When the reference is voltage to neutral of the motor at the terminal where I is calculated, or $440/\sqrt{3} = 254\angle 0^\circ$ V, the supply bus voltage to neutral is

$$254 + j0 + (0.3 + j1.0)(16.31 - j12.23) = 271.1 + j12.64$$

$$\text{Line-to-line voltage } |V| = \sqrt{3}\,|271.1 + j12.64| = 470 \text{ V}$$

1.18 A balanced-Δ load consisting of pure resistances of $15\ \Omega$ per phase is in parallel with a balanced-Y load having phase impedances of $8 + j6\ \Omega$. Identical impedances of $2 + j5\ \Omega$ are in each of the three lines connecting the combined loads to a 110-V three-phase supply. Find the current drawn from the supply and line voltage at the combined loads.

Solution:

Convert Δ to equivalent Y having $15/3 = 5\ \Omega$/phase

$$\frac{5\,(8 + j6)}{5 + 8 + j6} = \frac{40 + j30}{13 + j6} \times \frac{13 - j6}{13 - j6} = \frac{700 + j50}{205}$$
$$= 3.41 + j0.732 = 3.49\angle 12.1^\circ\ \Omega$$

Current drawn at supply:

$$Z = 2 + j5 + 3.41 + j0.73 = 5.41 + j5.73 = 7.88\angle 46.65^\circ\ \Omega$$
$$|I| = \frac{110/\sqrt{3}}{7.88} = 8.06 \text{ A from supply}$$

Letting V_t equal voltage at the load, line-to-line voltage:

$$V_t = 8.06 \times 3.49 = 28.13 \text{ V to neutral}$$
$$\text{Line-to-line } V_2 = \sqrt{3} \times 28.13 = 48.72 \text{ V}$$

1.19 A three-phase load draws 250 kW at a power factor of 0.707 lagging from a 440-V line. In parallel with this load is a three-phase capacitor bank which draws 60 kVA. Find the total current and resultant power factor.

Solution:

Letting S_1 and S_2 represent the load and capacitor bank, respectively,

$$S_1 = 250 + j250$$

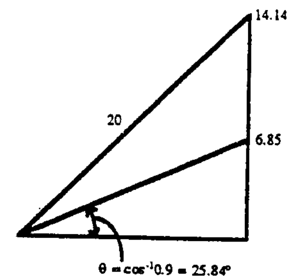
$$\begin{aligned} S_2 &= 0 - j60 \\ \text{where } S_1 + S_2 &= 250 + j190 = 314\angle 37.23^\circ \text{ kW} \\ |I| &= \frac{314,000}{\sqrt{3} \times 440} = 412.0 \text{ A} \\ \text{p.f.} &= \cos 37.23^\circ = 0.796 \text{ lag} \end{aligned}$$

1.20 A three-phase motor draws 20 kVA at 0.707 power factor lagging from a 220-V source. Determine the kilovoltampere rating of capacitors to make the combined power factor 0.90 lagging, and determine the line current before and after the capacitors are added.

Solution:

From the figure,

$$\begin{aligned} \theta &= \cos^{-1} 0.9 = 25.84^\circ \\ 14.14 \tan 25.84^\circ &= 6.85 \\ 14.14 - 6.85 &= 7.29 \text{ kvar} \end{aligned}$$



Without capacitors:

$$|I| = \frac{20,000}{\sqrt{3} \times 220} = 52.5 \text{ A}$$

With capacitors:

$$|I| = \frac{|14.14 + j6.85| \times 1000}{\sqrt{3} \times 220} = 41.2 \text{ A}$$

1.21 A coal mining "drag line" machine in an open-pit mine consumes 0.92 MVA at 0.8 power factor lagging when it digs coal, and it generates (delivers to the electric system) 0.10 MVA at 0.5 power factor leading when the loaded shovel swings away from the pit wall. At the end of the "dig" period, the change in supply current magnitude can cause tripping of a protective relay which is constructed of solid-state circuitry. Therefore it is desired to minimize the change in current magnitude. Consider the placement of capacitors at the machine terminals and find the amount of capacitive correction (in kvar) to eliminate the change in steady-state current magnitude. The machine is energized from a 36.5 kV, three-phase supply. Start the solution by letting Q be the total three-phase megavars of the capacitors connected across the machine terminals, and write an expression for the magnitude of the line current *drawn by* the machine in terms of Q for both the digging and generating operations.

Solution:

Assume line-to-line voltage $|V|$ is constant. Then constant current magnitude $|I|$ means constant $|S|$ where $|S| = \sqrt{3}\ |V|\ |I^*| \times 10^{-6}$ MVA.

Dig period:

$$
\begin{aligned}
|S| &= |0.92\,(0.8 + j0.6) - jQ| \\
&= |0.736 + j0.552 - jQ| \\
|S|^2 &= 0.542 + 0.305 - 1.104Q + Q^2 \;=\; 0.847 - 1.104Q + Q^2
\end{aligned}
$$

Swing period:

$$
\begin{aligned}
|S| &= |-0.1\angle{-60^\circ} - jQ| \;=\; |-0.05 + j0.0866 - jQ| \\
|S|^2 &= (-0.05)^2 + (0.0866 - Q)^2 \;=\; 0.0025 + 0.0075 - 0.1732Q + Q^2 \\
&= 0.01 - 0.1732Q + Q^2
\end{aligned}
$$

and equating $|S|^2$ for the dig and swing periods, we have

$$
\begin{aligned}
0.847 - 1.104Q + Q^2 &= 0.01 - 0.1732Q + Q^2 \\
0.937Q &= 0.837 \\
Q &= 0.899 \text{ Mvar or } 899 \text{ kvar}
\end{aligned}
$$

1.22 A generator (which may be represented by an emf in series with an inductive reactance) is rated 500 MVA, 22 kV. Its Y-connected windings have a reactance of 1.1 per unit. Find the ohmic value of the reactance of the windings.

Solution:

$$
\begin{aligned}
\text{Base } Z &= \frac{(22)^2}{500} = 0.968\ \Omega \\
X &= 1.1 \times 0.968 = 1.065\ \Omega
\end{aligned}
$$

1.23 The generator of Prob. 1.22 is in a circuit for which the bases are specified as 100 MVA, 20 kV. Starting with the per-unit value given in Prob. 1.22, find the per-unit value of reactance of the generator windings on the specified base.

Solution:

1.1 0.2662 p.u.

$$
X = \sout{1.065}\left(\frac{100}{500}\right)\left(\frac{22}{20}\right)^2 = \sout{0.2577} \text{ per unit}
$$

1.24 Draw the single-phase equivalent circuit for the motor (an emf in series with inductive reactance labeled Z_m) and its connection to the voltage supply described in Probs. 1.16 and 1.17. Show on the diagram the per-unit values of the line impedance and the voltage at the motor terminals on a base of 20 kVA, 440 V. Then using per-unit values find the supply voltage in per unit and convert the per-unit value of the supply voltage to volts.

Solution:

Per-unit base calculations:

$$\text{Base } Z = \frac{(0.44)^2 \times 1000}{20} = 9.68 \text{ per unit}$$
$$R = \frac{0.3}{9.68} = 0.031 \text{ per unit}$$
$$X = \frac{1.0}{9.68} = 0.1033 \text{ per unit}$$
$$\text{Base } I = \frac{20,000}{\sqrt{3} \times 440} = 26.24 \text{ A}$$
$$I = \frac{20.39}{26.24} = 0.777 \text{ per unit}$$

Voltage calculations:

$$\begin{aligned} V &= 1.0 + 0.777\,(0.8 - j0.6)\,(0.031 + j0.1033) \\ &= 1.0 + 0.777 \times 0.1079\underline{/\,36.43^\circ} \\ &= 1.0 + 0.0674 + j0.0498 = 1.0686\underline{/\,2.97^\circ} \text{ per unit} \\ |V_{LL}| &= 1.0686 \times 440 = 470 \text{ V} \end{aligned}$$

1.25 Write the two nodal admittance equations, similar to Eqs. (1.57) and (1.58), for the voltages at nodes ② and ④ of the circuit of Fig. 1.23. Then arrange the nodal admittance equations for all four independent nodes of Fig. 1.23 into the $\mathbf{Y}_{\text{bus}}$ form of Eq. (1.61).

Solution:

$$\begin{aligned} &\text{bus ②} \quad (V_2 - V_3)\,Y_b + (V_2 - V_1)\,Y_d + (V_2 - V_4)\,Y_e = 0 \\ &\text{bus ④} \quad V_4 Y_g + (V_4 - V_1)\,Y_f + (V_4 - V_2)\,Y_e = I_4 \end{aligned}$$

Rearranging equations for bus ② and bus ④ yields

$$\begin{aligned} &\text{bus ②} \quad -V_1 Y_d + V_2\,(Y_b + Y_d + Y_e) - V_3 Y_b - V_4 Y_e = 0 \\ &\text{bus ④} \quad -V_1 Y_f - V_2 Y_e + V_4\,(Y_g + Y_f + Y_e) = I_4 \end{aligned}$$

The $\mathbf{Y}_{\text{bus}}$ form is

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} (Y_c + Y_d + Y_f) & -Y_d & -Y_c & -Y_f \\ -Y_d & (Y_b + Y_d + Y_e) & -Y_b & -Y_e \\ -Y_c & -Y_b & (Y_a + Y_b + Y_c) & 0 \\ -Y_f & -Y_e & 0 & (Y_e + Y_f + Y_g) \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_3 \\ I_4 \end{bmatrix}$$

1.26 The values for the parameters of Fig. 1.23 are given in per unit as follows:

$Y_a = -j0.8 \quad Y_b = -j4.0 \quad Y_c = -j4.0 \quad Y_d = -j8.0 \quad Y_e = -j5.0$
$Y_f = -j2.5 \quad Y_g = -j0.8 \quad I_3 = 1.0\underline{/-90^\circ} \quad I_4 = 0.68\underline{/-135^\circ}$

Substituting these values in the equations determined in Prob. 1.25, compute the voltages at the nodes of Fig. 1.23. Numerically determine the corresponding $\mathbf{Z}_{\text{bus}}$ matrix.

Solution:

Using the $\mathbf{Y}_{\text{bus}}$ solution of Problem 1.25, substitute the given admittance values:

$$\begin{bmatrix} -j14.5 & j8.0 & j4.0 & j2.5 \\ j8.0 & -j17.0 & j4.0 & j5.0 \\ j4.0 & j4.0 & -j8.8 & j0 \\ j2.5 & j5.0 & j0 & -j8.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.0\angle{-90^\circ} \\ 0.68\angle{-135^\circ} \end{bmatrix}$$

Compute voltages:

$$\begin{aligned} \mathbf{Y}_{\text{bus}}\mathbf{V} &= \mathbf{I} \\ \mathbf{Y}_{\text{bus}}^{-1}\mathbf{Y}_{\text{bus}}\mathbf{V} &= \mathbf{Y}_{\text{bus}}^{-1}\mathbf{I} \\ \text{where } \mathbf{Y}_{\text{bus}}^{-1} &= \mathbf{Z}_{\text{bus}} = \begin{bmatrix} j0.7187 & j0.6688 & j0.6307 & j0.6194 \\ j0.6688 & j0.7045 & j0.6242 & j0.6258 \\ j0.6307 & j0.7045 & j0.6840 & j0.5660 \\ j0.6194 & j0.6258 & j0.5660 & j0.6840 \end{bmatrix} \\ \mathbf{V} &= \mathbf{Y}_{\text{bus}}^{-1}\mathbf{I} \end{aligned}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} j0.7187 & j0.6688 & j0.6307 & j0.6194 \\ j0.6688 & j0.7045 & j0.6242 & j0.6258 \\ j0.6307 & j0.7045 & j0.6840 & j0.5660 \\ j0.6194 & j0.6258 & j0.5660 & j0.6840 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1.0\angle{-90^\circ} \\ 0.68\angle{-135^\circ} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0.9285 - j0.2978 \\ 0.9251 - j0.3009 \\ 0.9562 - j0.2721 \\ 0.8949 - j0.3289 \end{bmatrix} = \begin{bmatrix} 0.9750\angle{-17.78^\circ} \\ 0.9728\angle{-18.02^\circ} \\ 0.9941\angle{-15.89^\circ} \\ 0.9534\angle{-20.18^\circ} \end{bmatrix}$$

Chapter 2 Problem Solutions

2.1 A single-phase transformer rated 7.2 kVA, 1.2kV/120 V has a primary winding of 800 turns. Determine (*a*) the turns ratio and the number of turns in the secondary winding, (*b*) the currents carried by the two windings when the transformer delivers its rated kVA at rated voltages. Hence, verify Eq. (2.7).

Solution:

(*a*)

$$\begin{aligned} \frac{N_1}{N_2} &= \frac{V_1}{V_2} = \frac{1.2 \times 10^3}{120} = 10 \\ \text{Therefore, } N_2 &= \frac{N_1}{10} = \frac{800}{10} = 80 \end{aligned}$$

To: ee5200-l@mtu.edu
Subject: sign convention, conjugate, cosine reference

Glad to see this kind of exchange on the e-mail list,
that is what I have been hoping for.

Some comments that could help with the Ch.1 review problems:

1) As I mentioned in the first lecture when we discussed Euler's identity, it is standard practice to define phasors according to the cosine (real) component and this is termed "cosine reference." Therefore, when converting from time domain to phasor domain, we must first convert all sinusoidal functions to equivalent cos functions. By sketching out a sine and a cosine function, it becomes clear that a sine is just a cosine that has been delayed by 90°. Therefore, sin(wt) = cos(wt -90°) or cos(wt) = sin(wt + 90°).

2) The negative sign associated with I is most likely due to how I is defined on the circuit, i.e. the assumed reference direction of current flow that is marked on the circuit.

3) To correctly calculate complex power consumed by (or flowing in to) a circuit element, Sin = VI* = P + jQ, where V and I have reference polarity/direction according to passive sign convention.

4) Careful with conjugates: remember that the conjugate of a complex number has the same magnitude, but the sign of its angle is changed. For example, if I = $10\angle 30^\circ$ A, then I* = $10\angle -30^\circ$ A. Thus, negating a complex number is not the same as taking its conjugate.

Thanks for the comments on the Ch.1 problems, I will try to go thru these and then issue any corrections that may be required.

Looks like we are off to a good start, good to be thinking about these details and doing some review/refreshing.

Dr. Mork

To: ee5200-l@mtu.edu
Subject: Solutions, Probs 1.1 and 1.10

Let's take a look at the first couple of items in question:

> 1.1, 1.2: v(t) is given as v(t)=141.4sin(wt+30) and i(t) is given as i(t)=11.31cos(wt-30). However, don't you need to put v(t) in terms of cosine, which becomes v(t)=141.4cos(wt+30-90)? Using v(t) as the reference, this give V=100@0 and I=8@30. Current is now leading the voltage and the circuit is capacitive. Does this sound right?

Yes, this is correct, we use cosine as the common basis/reference for expressing all phasor angles. Peak values must be divided by sqrt(2) to get RMS values.

In prob. 1.2, the calculation method is correct, just update the values according to the solution of problem 1.1.

> 1.10: I also assumed S=Ean(-Ian)* and got a different answer than the solutions.

The meaning of S=VI* really hinges on whether active or passive sign convention is used. Use of double subscripts makes it much easier to explain and to understand. The general equation S = VI* is clearly being implemented here using ACTIVE sign convention, the subscripts clearly tell us.

Active sign convention (generator convention) calculates S produced:
==
Sout = (Ean)(Ina)* = (Ean)(-Ian)* = Pout + jQout

Passive sign convention (load convention) calculated S consumed:
==
Sin = (Ean)(Ian)* = (Ean)(-Ina)* = Pin + jQin.

The author's solution is therefore correct. From this we might agree that it is indeed important it is to know the difference between active and passive sign convention... Working at MS level, we need to understand the concepts and details. These seemingly simple problems bring that out.

Dr. Mork

To: ee5200-l@mtu.edu
Subject: Chap 1 - problems 1.8,9,11 and 1.23

> 1.8, 1.9, 1.11: I agree that they should use S1 = E1(-I)* like example 1.1. Also, check the P & Q being delivered versus the P & Q being absorbed/received, in 1.8 for example, where the Z=0-j5. The solution shows S1=1000-j268 and S2=1000+j268. If Z=0-j5, which is purely capacitive, how can all the Vars from machine 1 be transferred to machine 2, while both machines are absorbing 1000 Watts? If you use S1 = E1(-I)*, the transfer of P & Q makes more sense.

Here, I think that if you are careful with active and passive sign conventions, discussed in the previous e-mail,
you will calculate the correct Ps and Qs and all of the P and Q that are generated and consumed will balance out. Let me know if you have any subsequent questions.

> 1.23: In problem 1.22, you are given a per unit reactance of 1.1pu. On the given bases, this gives an actual reactance of 1.065 Ohms. In 1.23 you are asked to find the per-unit reactance using a change of base, which is illustrated in equation 1.56 on page 29. However, the solution uses the actual impedance of 1.065 Ohms from 1.22 in the equation and gets .2557pu as the answer. To get a per-unit reactance as the answer, wouldn't you need to use 1.1pu as Z-old in the equation?

yes, there is a typo, he should have started with 1.1 pu, not the value in ohms.

The "canned" equation given by Eqn. 1.56 is not a very intuitive one, and you have to be careful how you apply it. It is more intuitive if you think of a two-step process:

a) multiply the given p.u. value by its Zbase to obtain the actual ohms.
b) divide the actual ohms by the new Zbase to get the new p.u. impedance.

Therefore, it is more intuitive to express the equation as:

Znew,pu = Zgiven,pu (kVbase,given^2 / MVAbase,given) <== step a)

/ (kVbase,new^2 / MVAbase,new) <== step b)

If you rearrange these terms, you end up with what is given in Eqn 1.56.
To the author's credit, he suggests this two-step approach in the 2nd paragraph on p. 30.

A philosophical observation:
Studying at the MS level, our goal is not only to learn more advanced "stuff" but to also improve our understanding of the fundamentals and concepts. Actually, encountering these errors and confusions in these review problems may have taught us more than if the solutions had all been totally correct -- we had to stop and question what's going on, go back to basic concepts,

and figure it out.

Any more points of uncertainty or possible errors? Please go ahead and start the discussion here, hopefully this is helpful.

See you all in class tomorrow morning, we will go through some more per unit things.

Dr Mork

To: ee5200-l@mtu.edu
Subject: Chapter one problem 1.7

At 12:02 PM 9/3/2003 -0500, you wrote:

> 1.7: I believe that they drew the correct conclusion about the efficiency but for the wrong reason. Anybody care to comment?

The author's rationalization seems to be sound:

Adding shunt capacitors (shunt compensation) reduces the inductive component of the current being drawn from the mains, i.e. flowing down the line, thus reducing the net current flowing in the line. This reduces the I^2 R line losses. The current flowing into the motor, however, is unchanged (assuming the capacitor placement has not changed the terminal voltage).

This is essentially a power factor correction situation, no internal changes have been made to the motor, it is still operating in the same way and with the same efficiency. Since efficiency is related only to real power P, the Q that is produced by the caps has no effect on motor efficiency.

There are some devices, in cyclic loading applications, that increase overall motor efficiency by reducing the source voltage to the motor when the mechanical load on the motor is removed/reduced, and then restores full voltage when the motor is loaded down again. Not restoring full voltage, or operating a loaded induction motor at reduced voltage will draw excessive current, resulting in a very low efficiency and extreme I^2 R heating of the armature windings, thus burning it out. One basic type of motor protection is thus to trip the motor off line if the voltage is too low and/or the current is too high, and/or if the winding temperature gets too high.

Dr. Mork