Substituting these values in the equations determined in Prob. 1.25, compute the voltages at the nodes of Fig. 1.23. Numerically determine the corresponding $\mathbf{Z}_{\text{bus}}$ matrix.

Solution:

Using the $\mathbf{Y}_{\text{bus}}$ solution of Problem 1.25, substitute the given admittance values:

$$\begin{bmatrix} -j14.5 & j8.0 & j4.0 & j2.5 \\ j8.0 & -j17.0 & j4.0 & j5.0 \\ j4.0 & j4.0 & -j8.8 & j0 \\ j2.5 & j5.0 & j0 & -j8.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.0\underline{/-90^\circ} \\ 0.68\underline{/-135^\circ} \end{bmatrix}$$

Compute voltages:

$$\begin{aligned} \mathbf{Y}_{\text{bus}}\mathbf{V} &= \mathbf{I} \\ \mathbf{Y}_{\text{bus}}^{-1}\mathbf{Y}_{\text{bus}}\mathbf{V} &= \mathbf{Y}_{\text{bus}}^{-1}\mathbf{I} \\ \text{where } \mathbf{Y}_{\text{bus}}^{-1} &= \mathbf{Z}_{\text{bus}} = \begin{bmatrix} j0.7187 & j0.6688 & j0.6307 & j0.6194 \\ j0.6688 & j0.7045 & j0.6242 & j0.6258 \\ j0.6307 & j0.7045 & j0.6840 & j0.5660 \\ j0.6194 & j0.6258 & j0.5660 & j0.6840 \end{bmatrix} \\ \mathbf{V} &= \mathbf{Y}_{\text{bus}}^{-1}\mathbf{I} \\ \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} &= \begin{bmatrix} j0.7187 & j0.6688 & j0.6307 & j0.6194 \\ j0.6688 & j0.7045 & j0.6242 & j0.6258 \\ j0.6307 & j0.7045 & j0.6840 & j0.5660 \\ j0.6194 & j0.6258 & j0.5660 & j0.6840 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1.0\underline{/-90^\circ} \\ 0.68\underline{/-135^\circ} \end{bmatrix} \\ \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} &= \begin{bmatrix} 0.9285 - j0.2978 \\ 0.9251 - j0.3009 \\ 0.9562 - j0.2721 \\ 0.8949 - j0.3289 \end{bmatrix} = \begin{bmatrix} 0.9750\underline{/-17.78^\circ} \\ 0.9728\underline{/-18.02^\circ} \\ 0.9941\underline{/-15.89^\circ} \\ 0.9534\underline{/-20.18^\circ} \end{bmatrix} \end{aligned}$$

Chapter 2 Problem Solutions

2.1 A single-phase transformer rated 7.2 kVA, 1.2kV/120 V has a primary winding of 800 turns. Determine (*a*) the turns ratio and the number of turns in the secondary winding, (*b*) the currents carried by the two windings when the transformer delivers its rated kVA at rated voltages. Hence, verify Eq. (2.7).

Solution:

(*a*)

$$\begin{aligned} \frac{N_1}{N_2} &= \frac{V_1}{V_2} = \frac{1.2 \times 10^3}{120} = 10 \\ \text{Therefore, } N_2 &= \frac{N_1}{10} = \frac{800}{10} = 80 \end{aligned}$$

(b)

$$\begin{aligned}
S_{rated} &= |V_1|_{rated}\,|I_1|_{rated} = |V_2|_{rated}\,|I_2|_{rated} \\
7.2 \times 10^3 &= 1.2 \times 10^3\,|I_1|_{rated} = 120\,|I_2|_{rated} \\
|I_1|_{rated} &= \frac{7.2 \times 10^3}{1.2 \times 10^3} = 6 \text{ A} \\
|I_2|_{rated} &= \frac{7.2 \times 10^3}{120} = 60 \text{ A} \\
\text{Left-hand side of Eq. (2.7):} \quad & \frac{I_1}{I_2} = \frac{6}{60} = 0.1 \\
\text{Right-hand side of Eq. (2.7):} \quad & \frac{N_2}{N_1} = \frac{1}{10} = 0.1 \\
\text{Left-hand side of Eq. (2.7)} &= \text{Right-hand side of Eq. (2.7)}
\end{aligned}$$

2.2 The transformer of Prob. 2.1 is delivering 6 kVA at its rated voltages and 0.8 power factor lagging. (*a*) Determine the impedance Z_2 connected across its secondary terminals. (*b*) What is the value of this impedance referred to the primary side (i.e. Z_2')? (*c*) Using the value of Z_2' obtained in part (*b*), determine the magnitude of the primary current and the kVA supplied by the source.

Solution:

(a)

$$\begin{aligned}
S_2 &= |S_2|\angle\theta = 6 \times 10^3\angle 36.9^\circ \text{ VA} \\
I_2 &= \left(\frac{S_2}{V_2}\right)^* \\
Z_2 &= \frac{V_2}{I_2} = \frac{V_2}{S_2^*/V_2^*} = \frac{|V_2|^2}{S_2^*} \\
&= \frac{(120)^2}{6 \times 10^3\angle -36.9^\circ}\ \Omega \\
&= 2.4\angle 36.9^\circ\ \Omega = (1.92 + j1.44)\ \Omega
\end{aligned}$$

(b)

$$\begin{aligned}
Z_2' &= \left(\frac{N_1}{N_2}\right)^2 Z_1 = \left(\frac{V_1}{V_2}\right)^2 Z_1 = 100 \times 2.4\angle 36.9^\circ\ \Omega \\
&= 240\angle 36.9^\circ\ \Omega = 192 + j144\ \Omega
\end{aligned}$$

(c)

$$\begin{aligned}
|I_1| &= \frac{|V_1|}{|Z_2'|} = \frac{1.2 \times 10^3}{240}\text{ A} = 5 \text{ A} \\
|S_1| &= |V_1|\,|I_1| = 1.2 \times 10^3 \times 5 \text{ VA} = 6 \text{ kVA}
\end{aligned}$$

2.3 With reference to Fig. 2.2, consider that the flux density inside the center-leg of the transformer core, as a function of time t, is $B(t) = B_m \sin(2\pi f t)$ where B_m is the peak value of the sinusoidal flux density and f is the operating frequency in Hz. If the flux density is uniformly distributed over the cross-sectional area A m^2 of the center-leg, determine

(*a*) the instantaneous flux $\phi(t)$ in terms of B_m, f, A and t,

(*b*) the instantaneous induced-voltage $e_1(t)$, according to Eq. (2.1).

(*c*) Hence show that the rms magnitude of the induced voltage of the primary is given by $|E_1| = \sqrt{2}\pi f N_1 B_m A$.

(*d*) If $A = 100$ cm^2, $f = 60$ Hz, $B_m = 1.5$ T and $N_1 = 1000$ turns, compute $|E_1|$.

Solution:

(*a*)

$$\phi(t) = B(t)\,A = B_m A \sin(2\pi f t)$$

(*b*)

$$e_1(t) = N_1 \frac{d\,\phi(t)}{dt} = N_1 B_m A \frac{d}{dt}\{\sin(2\pi f t)\} = 2\pi f N_1 B_m A \cos(2\pi f t)$$

(*c*)

$$E_1 = \frac{1}{\sqrt{2}}[e_1(t)]_{max} = \frac{2\pi f N_1 B_m A}{\sqrt{2}} = \sqrt{2}\,\pi f N_1 B_m A$$

(*c*) With given values,

$$E_1 = \sqrt{2}\pi \times 60 \times 1000 \times 1.5 \times 100 \times 10^{-4} \text{ V} = 4.0 \text{ kV}$$

2.4 For the pair of mutually coupled coils shown in Fig. 2.4, consider that $L_{11} = 1.9$ H, $L_{12} = L_{21} = 0.9$ H, $L_{22} = 0.5$ H and $r_1 = r_2 = 0$ Ω. The system is operated at 60 Hz.

(*a*) Write the impedance form [Eq. (2.24)] of the system equations

(*b*) Write the admittance form [Eq. (2.26)] of the system equations

(*c*) Determine the primary voltage V_1 and the primary current I_1 when the secondary is

(*i*) open circuited and has the induced voltage $V_2 = 100\angle 0°$ V

(*ii*) short circuited and carries the current $I_2 = 2\angle 90°$ A

Solution:

(*a*) From Eq. (2.22) and (2.23),

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = j120\pi \begin{bmatrix} 1.9 & 0.9 \\ 0.9 & 0.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
$$= j10^2 \begin{bmatrix} 7.163 & 3.393 \\ 3.393 & 1.885 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(*b*) From Eq. (2.25),

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = -j10^{-2} \begin{bmatrix} 7.163 & 3.393 \\ 3.393 & 1.885 \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$= -j10^{-2} \begin{bmatrix} 0.947 & -1.705 \\ -1.705 & 3.600 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(*c*) (*i*)

$$\begin{bmatrix} V_1 \\ 100\angle 0^\circ \end{bmatrix} = 100\angle 90^\circ \begin{bmatrix} 7.163 \\ 3.393 \end{bmatrix} I_1$$
$$\text{hence } I_1 = 0.295\angle -90^\circ \text{ A}$$
$$V_1 = 211.11\angle 0^\circ \text{ V}$$

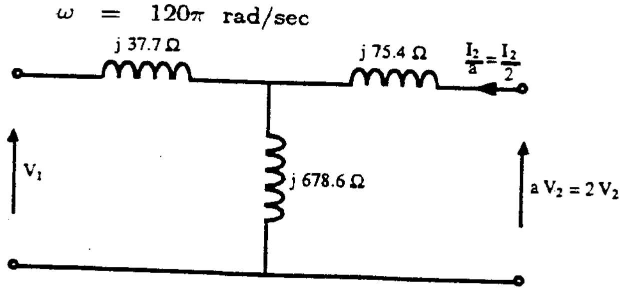
(*ii*)

$$\begin{bmatrix} I_1 \\ 2\angle 90^\circ \end{bmatrix} = 10^{-2} \times 1\angle -90^\circ \begin{bmatrix} 0.947 \\ -1.705 \end{bmatrix} V_1$$
$$\text{hence } V_1 = 117.30\angle 0^\circ \text{ V}$$
$$I_1 = 1.11\angle -90^\circ \text{ A}$$

2.5 For the pair of mutually coupled coils shown in Fig. 2.4, develop an equivalent-T network in the form of Fig. 2.5. Use the parameter values given in Prob. 2.4 and assume that the turns ratio a equals 2. What are the values of the leakage reactances of the windings and the magnetizing susceptance of the coupled coils?

Solution:

$$L_{1l} = L_{11} - aL_{21} = 1.9 - 2 \times 0.9 \text{ H} = 0.1 \text{ H}$$
$$L_{2l} = L_{22} - L_{12}/a = 0.5 - 2 \times 0.9/2 \text{ H} = 0.05 \text{ H}$$
$$a^2 L_{2l} = 4 \times 0.05 \text{ H} = 0.2 \text{ H}$$
$$L_m = aL_{21} = 2 \times 0.9 \text{ H} = 1.8 \text{ H}$$
$$\omega = 120\pi \text{ rad/sec}$$



Leakage reactances: $x_1 = 37.7\ \Omega$

$$
\begin{aligned}
x_2' &= 75.4\ \Omega \\
x_2 &= \frac{75.4}{4}\ \Omega = 18.85\ \Omega \\
\text{Magnetizing susceptance: } B_m &= \frac{1}{\omega L_m} = \frac{1}{120\pi \times 1.8}\ \text{S} \\
&= 1.474 \times 10^{-3}\ \text{S}
\end{aligned}
$$

2.6 A single-phase transformer rated 1.2 kV/120 V, 7.2 kVA has the following winding parameters: $r_1 = 0.8\ \Omega$, $x_1 = 1.2\ \Omega$, $r_2 = 0.01\ \Omega$and $x_2 = 0.01\ \Omega$. Determine

(*a*) the combined winding resistance and leakage reactance referred to the primary side, as shown in Fig. 2.8,

(*b*) the values of the combined parameters referred to the secondary side

(*c*) the voltage regulation of the transformer when it is delivering 7.5 kVA to a load at 120 V and 0.8 power factor lagging.

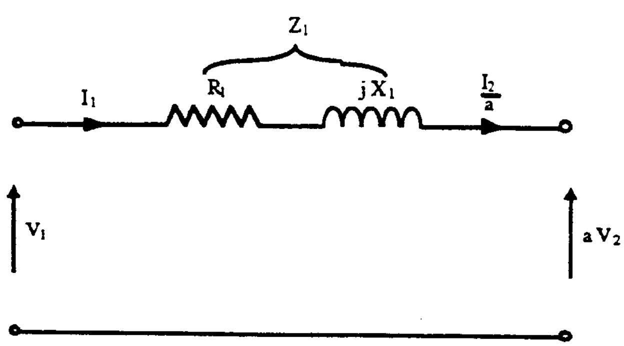
Solution:

(*a*) With turns ratio $a = 1.2 \times 10^3/120 = 10$,

$$
\begin{aligned}
R_1 &= r_1 + a^2 r_2 = 0.8 + 100 \times 0.01\ \Omega = 1.8\ \Omega \\
X_1 &= x_1 + a^2 x_2 = 1.2 + 100 \times 0.01\ \Omega = 2.2\ \Omega
\end{aligned}
$$

(*b*)

$$
\begin{aligned}
R_2 &\triangleq R_1/a^2 = 1.8/100\ \Omega = 0.018\ \Omega \\
X_2 &\triangleq X_1/a^2 = 2.2/100\ \Omega = 0.022\ \Omega
\end{aligned}
$$



(*c*)

$$
\begin{aligned}
z_1 &= (1.8 + j2.2)\ \Omega \\
I_{2,\text{FL}} &= |S_2/V_2| \angle{-\theta} = \frac{7200}{120}\angle{-36.9^\circ}\ \text{A} = 60\angle{-36.9^\circ}\ \text{A} \\
I_{1,\text{FL}} &= \frac{I_{2,\text{FL}}}{a} = 6.0\angle{-36.9^\circ}\ \text{A} \\
aV_{2,\text{FL}} &= 1200\ \text{V} \\
V_{1,\text{FL}} &= aV_{2,\text{FL}} + I_{1,\text{FL}} Z_1
\end{aligned}
$$

$$
\begin{aligned}
&= 1200 + 6.0\angle{-36.9^\circ}\,(1.8 + j2.2)\ \text{V} = 1216.57\angle{0.19^\circ}\ \text{V} \\
|V_{2.\text{FL}}| &= 120\ \text{V} \\
|V_{2.\text{NL}}| &= V_{1.\text{FL}}/a = 121.66\ \text{V} \\
\%\ \text{Regulation} &= (121.66 - 120)/120 = 1.38\ \%
\end{aligned}
$$

2.7 A single-phase transformer is rated 440/220 V, 5.0 kVA. When the low-voltage side is short circuited and 35 V is applied to the high-voltage side, rated current flows in the windings and the power input is 100 W. Find the resistance and reactance of the high- and low-voltage windings if the power loss and ratio of reactance to resistance is the same in both windings.

Solution:

$$
\begin{aligned}
\text{Rated } I &= \frac{5000}{220} = 22.73\ \text{A (low voltage)} \\
&= \frac{5000}{440} = 11.36\ \text{A (high voltage)} \\
R &= \frac{100}{11.36^2} = 0.774\ \Omega \\
Z &= \frac{35}{11.36} = 3.08\ \Omega\ \ (R,\ Z,\ X \text{ high-voltage}) \\
X &= \sqrt{3.08^2 - 0.774^2} = 2.98\ \Omega \qquad \frac{X}{R} = \frac{2.98}{0.774} = 3.85
\end{aligned}
$$

For equal loss in high- and low-voltage windings,

$$
\begin{aligned}
\text{High voltage: } r &= \frac{0.774}{2} = 0.387\ \Omega \\
x &= 3.85 \times 0.387 = 1.49\ \Omega \\
\text{Low voltage: } r &= 0.387 \times \left(\frac{220}{440}\right)^2 = 0.097\ \Omega \\
x &= 1.49\left(\frac{220}{440}\right)^2 = 0.373\ \Omega
\end{aligned}
$$

2.8 A single-phase transformer rated 1.2 kV/120 V, 7.2 kVA yields the following test results:

Open-Circuit Test (Primary Open)

Voltage $V_2 = 120$ V; Current $I_2 = 1.2$ A; Power $W_2 = 40$ W

Short-Circuit Test (Secondary Shorted)

Voltage $V_1 = 20$ V; Current $I_1 = 6.0$ A; Power $W_1 = 36$ W

Determine

(a) the parameters $R_1 = r_1 + a^2 r_2$, $X_1 = x_1 + a^2 x_2$, G_c and B_m referred to the primary side, Fig. 2.7

(b) the values of the above parameters referred to the secondary side

(c) the efficiency of the transformer when it delivers 6 kVA at 120 V and 0.9 power factor.

Solution:

(a) From open-circuit test,

$$\begin{aligned} G'_c &= W_2/V_2^2 = 40/120^2 \text{ S} = 2.78 \times 10^{-3} \text{ S} \\ |Y'_m| &= I_2/V_2 = 1.2/120 \text{ S} = 0.01 \text{ S} \\ B'_m &= \sqrt{|Y'_m|^2 - {G'_c}^2} = 9.606 \times 10^{-3} \text{ S} \\ a &= 1.2 \times 10^3/120 = 10 \end{aligned}$$

Therefore,

$$\begin{aligned} G_c &= G'_c/a^2 = 2.78 \times 10^{-5} \text{ S} \\ B_m &= B'_m/a^2 = 9.606 \times 10^{-5} \text{ S} \end{aligned}$$

From the short-circuit test,

$$\begin{aligned} R &= W_1/I_1^2 = 36/6.0^2 \ \Omega = 1.0 \ \Omega \\ |Z| &= V_1/I_1 = 20/6.0 \ \Omega = 3.33 \ \Omega \\ X &= \sqrt{|Z|^2 - R^2} = 3.18 \ \Omega \end{aligned}$$

(b)

$$\begin{aligned} R' &= R/a^2 = 0.01 \ \Omega & X' &= X/a^2 = 0.0318 \ \Omega \\ G'_c &= 2.78 \times 10^{-3} \text{ S} & B'_m &= 9.606 \times 10^{-3} \text{ S} \end{aligned}$$

(c) When $S_2 = 6.0$ kVA and $V_2 = 120$ V,

$$I_2 = \frac{6 \times 10^3}{120} \text{ A} = 50 \text{ A}$$

$$\begin{aligned} \text{Core loss at } V_2 &= 120 \text{ V} = 40 \text{ W} \\ \text{Winding loass at } I_2 &= 50 \text{ A} = |I_2|^2 R' = 50^2 \times 0.01 \text{ W} = 25 \text{ W} \\ \text{Power output at } S_2 &= 6.0 \text{ kVA at 0.9 p.f.} = 6 \times 10^3 \times 0.9 \text{ W} = 5400 \text{ W} \\ \eta &= \frac{5400}{5400 + 40 + 25} = 98.81 \ \% \end{aligned}$$

2.9 A single-phase transformer rated 1.2 kV/120 V, 7.2 kVA has primary-referred parameters $R_1 = r_1 + a^2 r_2 = 1.0\ \Omega$ and $X_1 = x_1 + a^2 x_2 = 4.0\ \Omega$. At rated voltage its core loss may be assumed to be 40 W for all values of the load current.

(a) Determine the efficiency and regulation of the transformer when it delivers 7.2 kVA at $V_2 = 120$ V and power factor of (i) 0.8 lagging, (ii) 0.8 leading.

(b) For a given load voltage and power factor it can be shown that the efficiency of a transformer attains its maximum value at the kVA load level which makes the I^2R winding losses equal to the core loss. Using this result, determine the maximum efficiency of the above transformer at rated voltage and 0.8 power factor, and the kVA load level at which it occurs.

Solution:

(a) (i) $\cos\theta = 0.8$, lagging $\theta = -36.9°$

$$\begin{aligned} V_2 &= 120\angle 0° \ \text{V} \\ I_2 &= \frac{7200}{120}\angle -36.9° = 60\angle -36.9° \ \text{A} \\ \text{Total loses} &= 40 + 60^2 \times \frac{1.0}{100} \ \text{W} = 76 \ \text{W} \\ \text{Output power} &= 7.2 \times 10^3 \times 0.8 \ \text{W} = 5760 \ \text{W} \\ \eta &= \frac{5760}{5760 + 76} = 98.698 \ \% \end{aligned}$$

$$\begin{aligned} V_{1,\text{FL}} &= aV_{2,\text{FL}} + \frac{I_{2,\text{FL}}}{a}(R_1 + jX_1) \\ V_{1,\text{FL}} &= 120 \times 10\angle 0° + \frac{60}{10}\angle -36.9° \ (1.0 + j4.0) \ \text{V} = 1219.3\angle 0.73° \\ |V_{2,\text{FL}}| &= 120 \ \text{V} \qquad |V_{2,\text{FL}}| = |V_{1,\text{FL}}|/a = 121.93 \ \text{V} \\ \% \ \text{Regulation} &= \frac{121.93 - 120}{120} = 1.61 \ \% \end{aligned}$$

(ii) $\cos\theta = 0.8$, leading $\theta = 36.9°$
$\eta = 98.698\%$ because it does not depend on whether θ is leading or lagging.

$$\begin{aligned} V_{1,\text{FL}} &= aV_{2,\text{FL}} + \frac{I_{2,\text{FL}}}{a}(R_1 + jX_1) \\ V_{1,\text{FL}} &= 120 \times 10\angle 0° + \frac{60}{10}\angle 36.9° \ (1.0 + j4.0) \ \text{V} = 1190.6\angle 1.1° \\ \% \ \text{Regulation} &= \frac{119.06 - 120}{120} = -0.78 \ \% \end{aligned}$$

(b) Load current at which η is maximum is given by

$$|I_2^*|^2 \frac{R}{a^2} = P_{\text{core}}$$

Therefore,

$$\begin{aligned} |I_2^*| &= \sqrt{\frac{40}{1.0/100}} \ \text{A} = 63.245 \ \text{A} \\ \text{Winding loss at } |I_2^*| &= 40 \ \text{W} \end{aligned}$$

$$\text{Output} = 120 \times 63.245 \times 0.8 \text{ W} = 6071.57 \text{ W}$$

$$\eta_{max} = \frac{6071.57}{6071.57 + 40 + 40} = 98.700 \%$$

$$\text{Corresponding kVA level} = 120 \times 63.245 \text{ VA} = 7.589 \text{ kVA}$$

2.10 A single-phase system similar to that shown in Fig. 2.10 has two transformers *A-B* and *B-C* connected by a line *B* feeding a load at the receiving end *C*. The ratings and parameter values of the components are

Transformer *A-B*: 500 V/1.5 kV, 9.6 kVA, leakage reactance = 5%
Transformer *B-C*: 1.2 kV/120 V, 7.2 kVA, leakage reactance = 4%
Line *B*: series impedance = $(0.5 + j3.0)\ \Omega$
Load *C*: 120 V, 6 kVA at 0.8 power factor lagging

(*a*) Determine the value of the load impedance in ohms and the actual ohmic impedances of the two transformers referred to both their primary and secondary sides.

(*b*) Choosing 1.2 kV as the voltage base for circuit *B* and 10 kVA as the systemwide kVA base, express all system impedances in per unit.

(*c*) What value of sending-end voltage corresponds to the given loading conditions?

Solution:

(*a*) Ohmic impedances

Transformer *A-B*

Primary: $$\frac{500^2}{9.6 \times 10^3} \times j0.05 = j1.302\ \Omega$$

Secondary: $$\frac{1.5^2 \times 10^6}{9.6 \times 10^3} \times j0.05 = j11.719\ \Omega$$

Transformer *B-C*

Primary: $$\frac{1.2^2 \times 10^6}{7.2 \times 10^3} \times j0.04 = j8.0\ \Omega$$

Secondary: $$\frac{120^2}{7.2 \times 10^3} \times j0.04 = j0.08\ \Omega$$

Load: $$\frac{|V|^2}{|S|} \angle \theta = \frac{120^2}{6 \times 10^3} \angle \cos^{-1} 0.8 = 2.4 \angle 36.9^\circ\ \Omega$$

(*b*) Impedance bases

Circuit *B*: $$\frac{1.2^2 \times 10^6}{10 \times 10^3}\ \Omega = 144\ \Omega$$

Circuit *C*: $$\frac{120^2}{10 \times 10^3}\ \Omega = 1.44\ \Omega$$

Subject:

Per unit impedances on new bases:

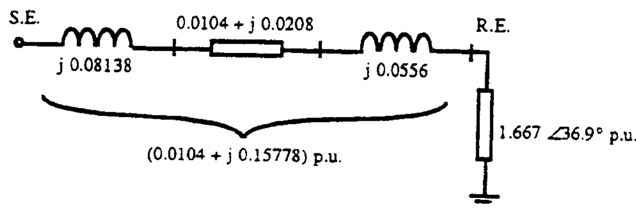
$$\text{Transformer } A\text{-}B: \quad j\frac{11.719}{144} = j0.08138 \text{ per unit}$$

$$\text{Transformer } B\text{-}C: \quad j\frac{8}{144} = j0.0556 \text{ per unit}$$

$$\text{Line } B: \quad \frac{(1.5 + j3.0)}{144} = 0.0104 + j0.0208 \text{ per unit}$$

$$\text{Load: } \quad \frac{2.4}{1.44}\angle 36.9^\circ = 1.667\angle 36.9^\circ \text{ per unit}$$

(c) Sending-end voltage calculations



$$V_R = 120 \text{ V} = 1.0 \text{ per unit}$$

$$V_S = 1.0 \times \frac{1.667\angle 36.9^\circ + (0.0104 + j0.15778)}{1.667\angle 36.9^\circ} = 1.0642 \text{ per unit}$$

The sending-end voltage base is

$$V_{S,\,\text{base}} = \frac{500}{1.5 \times 10^3} \times 1.2 \times 10^3 = 400 \text{ V}$$

Therefore, the required sending-end voltage is

$$V_S = 400 \times 1.0642 = 425.69 \text{ V}$$

2.11 A balanced Δ-connected resistive load of 8000 kW is connected to the low-voltage, Δ-connected side of a Y-Δ transformer rated 10,000 kVA, 138/13.8 kV. Find the load resistance in ohms in each phase as measured from line to neutral on the high-voltage side of the transformer. Neglect transformer impedance and assume rated voltage is applied to the transformer primary.

Solution:

$$|I_{\text{line}}| = \frac{8,000}{\sqrt{3} \times 138} = 33.47 \text{ A}$$

$$R = \frac{138,000/\sqrt{3}}{33.47} = 2380 \ \Omega$$

2.12 Solve Prob. 2.11 if the same resistances are reconnected in Y.

Solution:

If the Δ-connected resistors are reconnected in Y, then the resistance to neutral will be three times as great and

$$R = 3 \times 2380 = 7140 \ \Omega$$

2.13 Three transformers, each rated 5 kVA, 220 V on the secondary side, are conected Δ-Δ and have been supplying a balanced 15 kW purely resistive load at 220 V. A change is made which reduces the load to 10 kW, still purely resistive and balanced. Someone suggests that, with two-thirds of the load, one transformer can be removed and the system can be operated open-Δ. Balanced three-phase voltages will still be supplied to the load since two of the line voltages (and thus also the third) will be unchanged.

To investigate further the suggestion

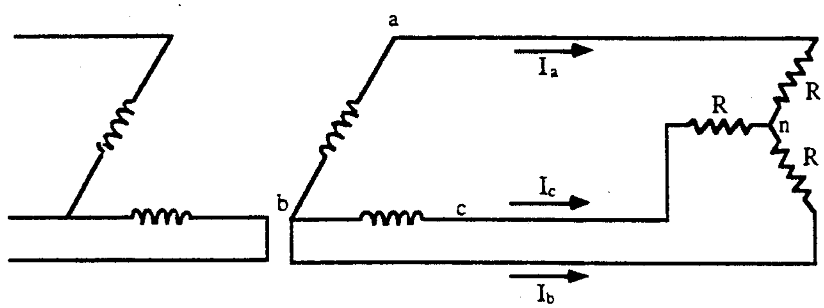
(*a*) Find each of the line currents (magnitude and angle) with the 10 kW load and the transformer between a and c removed. (Assume $V_{ab} = 220\angle 0^\circ$ V, sequence $a\ b\ c$.)

(*b*) Find the kilovoltamperes supplied by each of the remaining transformers.

(*c*) What restriction must be placed on the load for open-Δ operation with these transformers?

(*d*) Think about why the individual transformer kilovoltampere values include a Q component when the load is purely resistive.

Solution:



(*a*) V_{ab} and V_{bc} remain the same after removing the third transformer, so V_{ca} is also the same and we have a three-phase supply, and these voltages are: $V_{ab} = 220\angle 0^\circ$ V, $V_{bc} = 220\angle 240^\circ$ V and $V_{ca} = 220\angle 120^\circ$ V. Then, $V_{an} = 127\angle -30^\circ$ V, $V_{bn} = 127\angle 210^\circ$ V and $V_{cn} = 127\angle 90^\circ$ V. The line currents are

$$I_a = \frac{10,000}{\sqrt{3} \times 220}\angle -30^\circ = 26.24\angle -30^\circ \text{ A}$$
$$I_b = 26.24\angle 210^\circ \text{ A}$$
$$I_c = 26.24\angle 90^\circ \text{ A}$$

(*b*) $\text{kVA}_{\text{supplied}} = 220 \times 26.24 \times 10^{-3} = 5.772$ kVA

(*c*) The load must be reduced to $(5.0/5.772) \times 100 = 86.6\%$ or 4.33 kW for each transformer.

(*d*) The current and voltage in each of the remaining two transformers are not in phase. Output of each transformer before the reduction in load is,

$$S_1 = V_{ab}I_a^* = 220\angle 0^\circ \times 26.24\angle 30^\circ = 5000 + j2886 \text{ VA}$$
$$S_2 = V_{cb}I_c^* = 220\angle 60^\circ \times 26.24\angle 270^\circ = 5000 - j2886 \text{ VA}$$

Note that Q is equal in magnitude but opposite in sign. There is no Q output from the open delta. After the load reduction,

$$\begin{aligned} S_1 &= 4333 + j2500 \text{ VA} \\ S_2 &= 4333 - j2500 \text{ VA} \end{aligned}$$

2.14 A transformer rated 200 MVA, 345Y/20.5Δ kV connects a balanced load rated 180 MVA, 22.5 kV, 0.8 power factor lag to a transmission line. Determine

(*a*) the rating of each of three single-phase transformers which when properly connected will be equivalent to the above three-phase transformer and

(*b*) the complex impedance of the load in per unit in the impedance diagram if the base in the transmission line is 100 MVA, 345 kV.

Solution:

(*a*) Each single-phase transformer is rated 200/3 = 66.7 MVA. Voltage rating is $(345/\sqrt{3})\,/20.5$ or 199.2/20.5 kV.

(*b*)

$$\text{Load } Z = \frac{(22.5)^2}{180}\angle \cos^{-1} 0.8 = 2.81\angle 36.87^\circ \ \Omega \ \text{(low-voltage side)}$$

At the load,

$$\begin{aligned} \text{Base } V &= 20.5 \text{ kV} \\ \text{Base } Z &= \frac{(20.5)^2}{100} = 4.20 \ \Omega \\ \text{Load } Z &= \frac{2.81}{4.20}\angle 36.87^\circ = 0.669\angle 36.87^\circ \text{ per unit} \end{aligned}$$

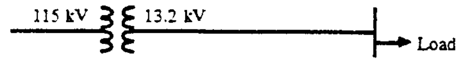
2.15 A three-phase transformer rated 5 MVA, 115/13.2 kV has per-phase series impedance of $(0.007 + j0.075)$ per unit. The transformer is connected to a short distribution line which can be represented by a series impedance per phase of $(0.02 + j0.10)$ per unit on a base of 10 MVA, 13.2 kV. The line supplies a balanced three-phase load rated 4 MVA, 13.2 kV, with lagging power factor 0.85.

(*a*) Draw an equivalent circuit of the system indicating all impedances in per unit. Choose 10 MVA, 13.2 kVA as the base at the load.

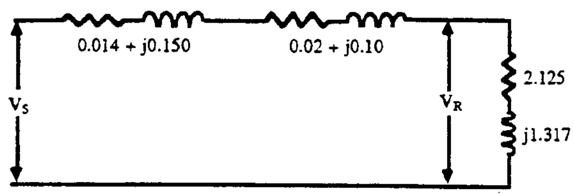
(*b*) With the voltage at the primary side of the transformer held constant at 115 kV, the load at the receiving end of the line is disconnected. Find the voltage regulation at the load.

Solution:

(a) Base voltages are shown on the single-line diagram.



$$
\begin{aligned}
\text{Transformer } Z &= \frac{10}{5}(0.007 + j0.075) \;=\; 0.014 + j0.150 \;\text{ per unit} \\
V_S &= 1.0 \;\text{ per unit} \\
\text{Line } Z &= 0.02 + j0.10 \;\text{ per unit} \\
\text{Load } |Z| &= \frac{(13.2)^2 \times 1000}{3400/0.85} \;=\; 43.56 \;\Omega \\
\text{Base } Z \text{ at load} &= \frac{(13.2)^2}{10} \;=\; 17.42 \;\Omega \\
\text{Load } Z &= \frac{43.56}{17.42}\angle \cos^{-1} 0.85 \;=\; 2.50\angle 31.8^\circ \\
&= 2.125 + j1.317 \;\text{ per unit}
\end{aligned}
$$



(values are in per unit)

(b) Voltage regulation calculations

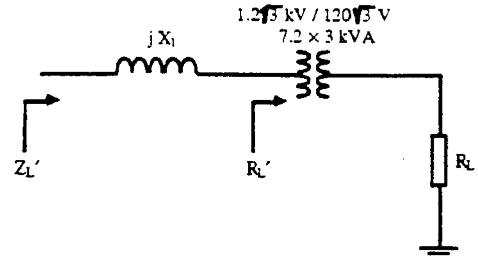
$$
\begin{aligned}
I &= \frac{1.0}{0.014 + 0.02 + 2.125 + j(0.150 + 0.10 + 1.317)} \;=\; \frac{1.0}{2.668\angle 35.97^\circ} \\
&= 0.375\angle -35.97^\circ \;\text{ per unit} \\
V_{R,FL} &= 0.375\angle -35.97^\circ \times 2.5\angle 31.8^\circ \;=\; 0.937\angle -4.17^\circ \;\text{ per unit} \\
V_{R,NL} &= V_S \;=\; 1.0 \\
\text{V.R.} &= \frac{1 - 0.937}{0.937} \times 100 \;=\; 6.72\;\%
\end{aligned}
$$

2.16 Three identical single-phase transformers, each rated 1.2 kV/120 V, 7.2 kVA and having a leakage reactance of 0.05 per unit, are connected together to form a three-phase bank. A balanced Y-connected load of 5 Ω per phase is connected across the secondary of the bank. Determine the Y-equivalent per-phase impedance (in ohms and in per unit) seen from the primary side when the transformer bank is connected (a) Y-Y, (b) Y-Δ, (c) Δ-Y and (d) Δ-Δ. Use Table 2.1.

Solution:

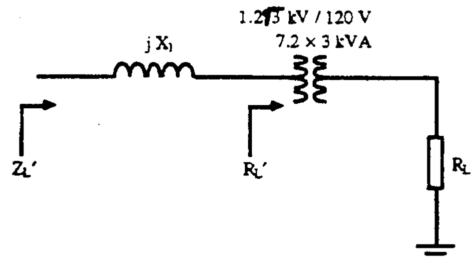
(a) Y-Y connection:

$$|V_{LL}| = 1.2 \times 10^3 \times \sqrt{3}\ \text{V}$$
$$|V_{ll}| = 120\sqrt{3}\ \text{V}$$
$$R'_L = 5 \times \left(\frac{1200\sqrt{3}}{120\sqrt{3}}\right)^2 = 500\ \Omega$$
$$Z_b = \frac{(1.2\sqrt{3})^2 \times 10^6}{7.2 \times 10^3 \times 3} = 200\ \Omega$$
$$X_l = 0.05\ \text{per unit} = 200 \times 0.05\ \Omega = 10\ \Omega$$
$$Z'_L = (500 + j10)\ \Omega$$



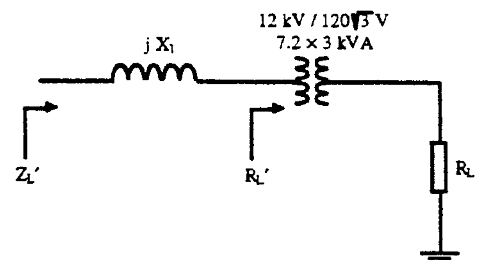
(b) Y-Δ connection:

$$|V_{LL}| = 1200 \times \sqrt{3}\ \text{V}$$
$$|V_{ll}| = 120\ \text{V}$$
$$R'_L = 5 \times \left(\frac{1200\sqrt{3}}{120}\right)^2 = 1500\ \Omega$$
$$X_l = 10\ \Omega \text{ from part } (a)$$
$$Z'_L = (1500 + j10)\ \Omega$$



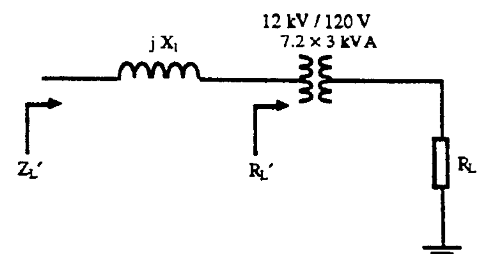
(c) Δ-Y connection:

$$|V_{LL}| = 1200\ \text{V}$$
$$|V_{ll}| = 120\sqrt{3}\ \text{V}$$
$$R'_L = 5 \times \left(\frac{1200}{120\sqrt{3}}\right)^2 = \frac{500}{3} = 166.67\ \Omega$$
$$Z_b = \frac{1200^2}{7.2 \times 3 \times 10^3} = 66.67\ \Omega$$
$$X_l = 0.05\ \text{per unit} = 66.67 \times 0.05\ \Omega = 3.33\ \Omega$$
$$Z'_L = (166.67 + j3.33)\ \Omega$$



(d) Δ-Δ connection:

$$|V_{LL}| = 1200\ \text{V}$$
$$|V_{ll}| = 120\ \text{V}$$
$$R'_L = 5 \times \left(\frac{1200}{120}\right)^2 = 500\ \Omega$$
$$X_l = 3.33\ \Omega \text{ from part } (c)$$
$$Z'_L = (500 + j3.33)\ \Omega$$



2.17 Figure 2.17a shows a three-phase generator supplying a load through a three-phase transformer rated 12 kVA/600 V Y, 600 kVA. The transformer has per-phase leakage reactance of 10%. The line-to-line voltage and the line current at the generator terminals are 11.9 kV and 20 A, respectively. The power factor seen by the generator is 0.8 lagging and the phase sequence of supply is ABC.

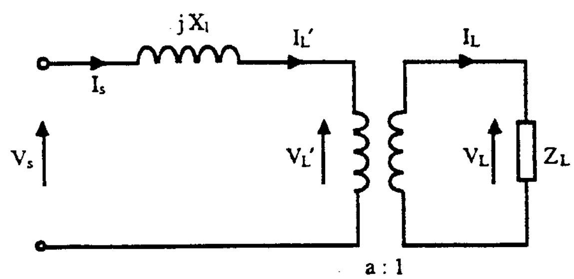
(*a*) Determine the line current and the line-to-line voltage at the load, and the per-phase (equivalent-Y) impedance of the load.

(*b*) Using the line-to-neutral voltage V_A at the transformer primary as reference, draw complete per-phase phasor diagrams of all voltages and currents. Show the correct phase relations between primary and secondary quantities.

(*c*) Compute the real and reactive power supplied by the generator and consumed by the load.

Solution:

(*a*)



$$\text{Voltage ratio} = a = \frac{12 \times 10^3}{600}\angle 30^\circ = 20\angle 30^\circ$$

$$\text{Current ratio} = \frac{1}{a^*} = 0.05\angle 30^\circ$$

$$X_l = \frac{(12 \times 10^3)^2}{600 \times 10^3} \times 0.1 = 24.0 \ \Omega$$

$$\text{Let } V_S = \frac{11.9}{\sqrt{3}}\angle 0^\circ \text{ kV} = 6.87 \text{ kV}$$

$$\text{Then, } I_S = I_L' = 20\angle -36.9^\circ \text{ A}$$

$$I_L = I_L' a^* = 20 \times 20\angle -36.9^\circ - 30^\circ \text{ A} = 400\angle -66.9^\circ \text{ A}$$

$$V_L' = V_S - jX_l I_S = 6.87\angle 0^\circ - \left(\frac{24.0\angle 90^\circ \times 20\angle -36.9^\circ}{1000}\right) \text{ kV}$$

$$= 6.593\angle -3.34^\circ \text{ kV}$$

$$V_L = V_L'/a = \frac{6.593\angle -3.34^\circ}{20\angle 30^\circ} \text{ kV} = 329.65\angle -33.34^\circ \text{ V}$$

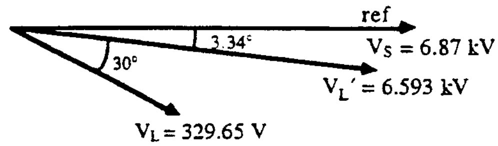
$$\text{Line voltage at the load} = \sqrt{3}|V_L| = 571 \text{ V}$$

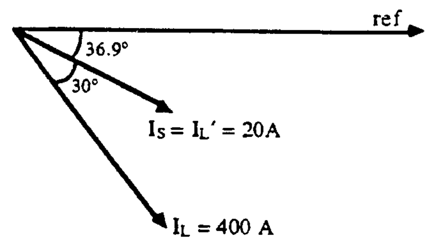
$$\text{Line current at the load} = |I_L| = 400 \text{ A}$$

$$\text{Load impedance - } Z_L = V_L/I_L = \frac{329.65\angle -33.34^\circ}{400\angle -66.9^\circ} \ \Omega$$

$$= 0.824\angle 33.6^\circ \ \Omega$$

(*b*)





(*c*) $P_g + jQ_g$ from the generator is $3V_S I_S^*$, where

$$\begin{aligned} 3V_S I_S^* &= 3 \times 6.87\angle 0^\circ \times 20\angle 36.9^\circ \text{ kVA} = 412.2\angle 36.9^\circ \text{ kVA} \\ &= 329.8 \text{ kW} + j247.3 \text{ kvar} \end{aligned}$$

$P_L + jQ_L$ by the load is $3V_L I_L^*$, where

$$\begin{aligned} 3V_L I_L^* &= \frac{3 \times 329.65\angle -33.34^\circ \times 400\angle 66.9^\circ}{1000} \text{ kVA} = 395.6\angle 33.56^\circ \text{ kVA} \\ &= 329.7 \text{ kW} + j218.7 \text{ kvar} \end{aligned}$$

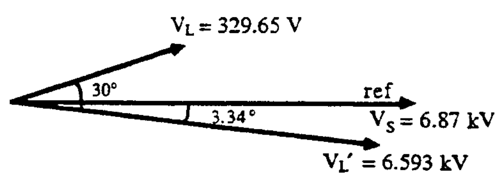
2.18 Solve Prob. 2.17 with phase sequence ACB.

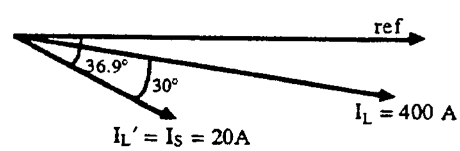
Solution:

(*a*) Final answers remain the same except for the following intermediate results:

$$\begin{aligned} a &= 20\angle -30^\circ \qquad 1/a^* = 0.05\angle -30^\circ \\ I_L &= I_L' a^* = 400\angle -36.9^\circ + 30^\circ \text{ A} = 400\angle -6.9^\circ \text{ A} \\ V_L &= V_L'/a = 329.65\angle -3.34^\circ + 30^\circ \text{ V} = 329.65\angle 26.7^\circ \text{ V} \end{aligned}$$

(*b*)





(*c*) Same results as in Problem 2.17.

2.19 A single-phase transformer rated 30 kVA, 1200/120 V is connected as an autotransformer to supply 1320 V from a 1200 V bus.

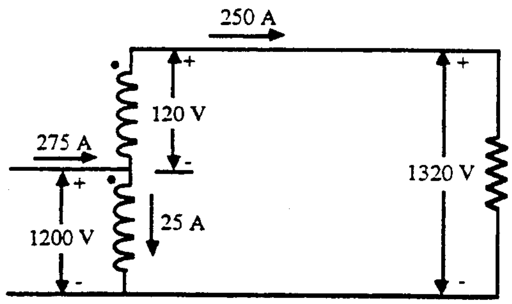
(*a*) Draw a diagram of the transformer connections showing the polarity marks on the windings and directions chosen as positive for current in each winding so that the currents will be in phase.

(*b*) Mark on the diagram the values of rated current in the windings and at the input and output.

(*c*) Determine the rated kilovoltamperes of the unit as an autotransformer.

(*d*) If the efficiency of the transformer connected for 1200/120 V operation at rated load unity power factor is 97%, determine its efficiency as an autotransformer with rated current in the windings and operating at rated voltage to supply a load at unity power factor.

Solution:



$$\text{rated } I_{\text{HV}} = \frac{30,000}{1200} = 25 \text{ A}$$

$$\text{rated } I_{\text{LV}} = \frac{30,000}{120} = 250 \text{ A}$$

Connected for 1200/120-V operation (regular transformer),

$$P_{out} = 30,000 \text{ W} \qquad P_{in} = 30,928 \text{ W}$$

$$\text{Loss} = 928 \text{ W}$$

Loss remains the same in the autotransformer because current in the windings and voltage across the windings are unchanged. For the autotransformer,

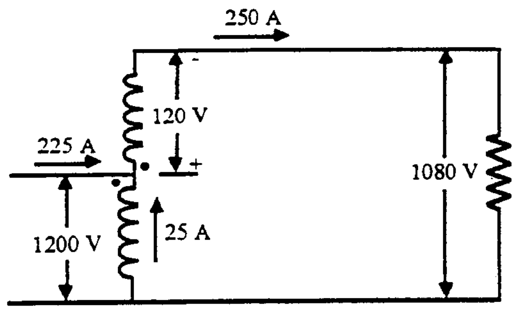
$$P_{out} = 250 \times 1320 = 330,000 \text{ W} \qquad P_{in} = 330,928 \text{ W}$$

$$\eta = \frac{330,000}{330,928} \times 100 = 99.7\% \qquad \text{Rated kVA} = 330,000$$

Note that, once we consider loss, we no longer have an ideal transformer; and both winding resistance and reactance as well as magnetizing current and core loss must be considered. The applied voltage and input current will be greater than the values shown to achieve rated output, in which case the equivalent circuit corresponding to Fig. 2.7 would be used.

2.20 Solve Prob. 2.19 if the transformer is to supply 1080 V from a 1200 V bus.

Solution:



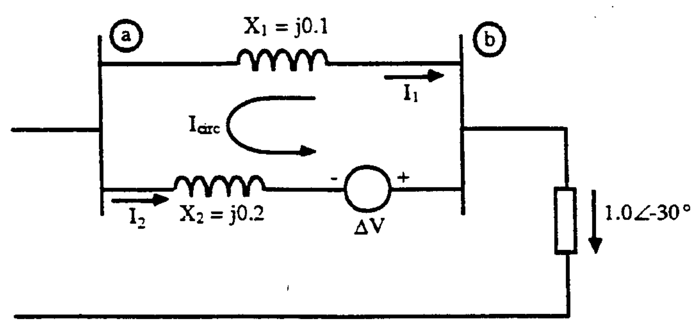
As in Prob. 2.19, Loss = 928 W. As an autotransformer,

$$P_{out} = 250 \times 1080 = 270,000 \text{ W}$$
$$P_{in} = 270,928 \text{ W}$$
$$\eta = \frac{270,000}{270,928} \times 100 = 99.7 \%$$

Rated kVA = 270,000, but see the note which accompanies the solution of Problem 2.19.

2.21 Two buses ⓐ and ⓑ are connected to each other through impedances $X_1 = 0.1$ and $X_2 = 0.2$ per unit in parallel. Bus b is a load bus supplying a current $I = 1.0\angle{-30^\circ}$ per unit. The per-unit bus voltage V_b is $1.0\angle{0^\circ}$. Find P and Q into bus b through each of the parallel branches (a) in the circuit described, (b) if a regulating transformer is connected at bus b in the line of higher reactance to give a boost of 3% in voltage magnitude toward the load ($a = 1.03$), and (c) if the regulating transformer advances the phase 2° $\left(a = \epsilon^{j\pi/90}\right)$. Use the circulating-current method for parts (b) and (c), and assume that V_a is adjusted for each part of the problem so that V_b remains constant. Figure 2.26 is the single-line diagram showing buses a and b of the system with the regulating transformer in place. Neglect the impedance of the transformer.

Solution:



(a) Thru X_1 the current is $I_1 = \frac{2}{3} \times 1.0\angle{-30^\circ} = 0.577 - j0.333$ and thru X_2 the current is $I_2 = \frac{1}{3} \times 1.0\angle{-30^\circ} = 0.289 - j0.167$. Into bus ⓑ thru X_1,

$$P + jQ = V_b I_1^* = 0.577 + j0.333 \text{ per unit}$$

and into bus ⓑ thru X_2,

$$P + jQ = V_b I_2^* = 0.289 + j0.167 \text{ per unit}$$

(b) $\Delta V = 0.03$; $I_{circ} = \dfrac{0.03}{j0.3} = -j0.1$

$$I_1 = 0.577 - j0.333 - (-j0.1) = 0.577 - j0.233$$
$$I_2 = 0.289 - j0.167 + (-j0.1) = 0.289 - j0.267$$

Into bus ⓑ thru X_1,

$$P + jQ = 0.577 + j0.233 \text{ per unit}$$

and into bus ⓑ thru X_2,

$$P + jQ = 0.289 + j0.267 \text{ per unit}$$

(c)

$$\Delta V = 1.0\angle 2^\circ - 1.0 = 0.9994 + j0.0349 - 1.0 = -0.0006 + j0.0349$$
$$I_{circ} = \frac{-0.0006 + j0.0349}{j0.3} = 0.116 + j0.002$$
$$I_1 = 0.577 - j0.333 - (0.116 + j0.002) = 0.461 - j0.335$$
$$I_2 = 0.289 - j0.167 + 0.116 + j0.002 = 0.405 - j0.165$$

Into bus ⓑ thru X_1,

$$P + jQ = V_b I^* = 0.461 + j0.335 \text{ per unit}$$

and into bus ⓑ thru X_2,

$$P + jQ = V_b I^* = 0.405 + j0.165 \text{ per unit}$$

Note: Compare P and Q found in parts (b) and (c) with part (a).

2.22 Two reactances $X_1 = 0.08$ and $X_2 = 0.12$ per unit are in parallel between two buses ⓐ and ⓑ in a power system. If $V_a = 1.05\angle 10^\circ$ and $V_b = 1.0\angle 0^\circ$ per unit, what should be the turns ratio of the regulating transformer to be inserted in series with X_2 at bus ⓑ so that no vars flow into bus ⓑ from the branch whose reactance is X_1? Use the circulating-current method, and neglect the reactance of the regulating transformer. P and Q of the load and V_b remain constant.

Solution:

In reactance X_1,

$$I_{ab} = \frac{1.05\angle 10^\circ - 1.0}{j0.08} = \frac{1.034 + j0.1823 - 1.0}{j0.08} = 2.279 - j0.425$$

To eliminate vars to bus ⓑ thru X_1, we need in the X_2 branch

$$I_{ab,circ} = -j0.425$$
$$\frac{\Delta V}{j0.8 + j0.12} = -j0.425$$
$$a - 1 = \Delta V = -j0.425(j0.08 + j0.12) = 0.0850$$
$$a = 1.085 \text{ turns ratio}$$

2.23 Two transformers each rated 115Y/13.2Δ kV operate in parallel to supply a load of 35 MVA, 13.2 kV at 0.8 power factor lagging. Transformer 1 is rated 20 MVA with $X = 0.09$ per unit, and transformer 2 is rated 15 MVA with $X = 0.07$ per unit. Find the magnitude of the current in per unit through each transformer, the megavoltampere output of each transformer, and the megavoltamperes to which the total load must be limited so that neither transformer is overloaded. If the taps on transformer 1 are set at 111 kV to give a 3.6% boost in voltage toward the low-voltage side of that transformer compared to transformer 2, which remains on the 115 kV tap, find the megavoltampere output of each transformer for the original 35 MVA total load and the maximum megavoltamperes of the total load which will not overload the transformers. Use a base of 35 MVA, 13.2 kV on the low-voltage side. The circulating-current method is satisfactory for this problem.

Solution:

Converting to the chosen base,

$$X_1 = 0.09 \times (35/20) = 0.1575 \text{ per unit}$$
$$X_2 = 0.07 \times (35/15) = 0.1633 \text{ per unit}$$
$$|S_1| = \frac{0.1633}{0.1575 + 0.1633} \times 35 = 17.8 \text{ MVA}$$
$$|S_2| = \frac{0.1575}{0.1575 + 0.1633} \times 35 = 17.2 \text{ MVA}$$

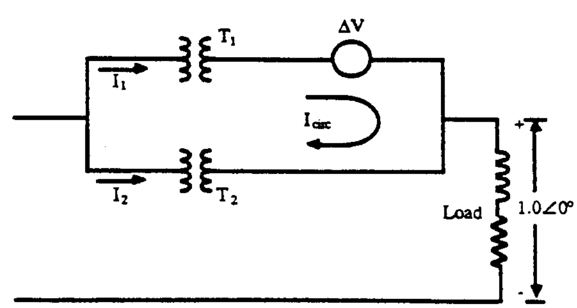
Unit #2 is overloaded, and therefore reduce load to $(15/17.2) \times 35 = 30.5$ MVA.

Currents with 35-MVA load;

$$I_1 = \frac{17.8}{35}(0.8 - j0.6) = 0.407 - j0.305$$
$$I_2 = \frac{17.2}{35}(0.8 - j0.6) = 0.393 - j0.295$$

With 3.6% magnitude boost,

$$I_{circ} = \frac{0.036}{j(0.1633 + 0.1575)} = -j0.112$$
$$|I_1 + I_{circ}| = |0.407 - j0.417| = 0.583$$
$$|I_2 - I_{circ}| = |0.393 - j0.185| = 0.434$$
$$|S_1| = 0.583 \times 1.0 \times 35 = 20.4 \text{ MVA}$$
$$|S_2| = 0.434 \times 1.0 \times 35 = 15.19 \text{ MVA}$$
$$\frac{20.4}{20} = 1.020 \qquad \frac{15.19}{15} = 1.013$$



Reduce load to $(35/1.02) = 34.3$ MVA.