Dove is the conductor for bundling:

$$
\begin{aligned}
D_s &= 0.0314\,(2.54 \times 12 \times 10^{-2}) = 0.00957 \text{ m} \\
D_s^b &= \sqrt{0.00957 \times 0.4} = 0.0619 \text{ m} \\
X &= 2 \times 10^{-7} \times 10^3 \times 377 \ln \frac{13.86}{0.0619} = 0.408\ \Omega/\text{km}
\end{aligned}
$$

4.22 Calculate the inductive reactance in ohms per kilometer of a bundled 60-Hz three-phase line having three ACSR *Rail* conductors per bundle with 45 cm between conductors of the bundle. The spacing between bundle centers is 9, 9 and 18 m.

Solution:

$$D_{eq} = \sqrt[3]{9 \times 9 \times 18} = 11.34 \text{ m}$$

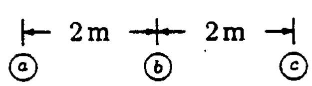
Rail:

$$
\begin{aligned}
D_s &= 0.0386 \text{ ft} = 0.0386\,(2.54 \times 12 \times 10^{-2}) = 0.0118 \text{ m} \\
D_s^b &= \sqrt[3]{0.0118 \times 0.45 \times 0.45} = 0.1337 \text{ m} \\
X &= 2 \times 10^{-7} \times 10^3 \times 377 \ln \frac{11.34}{0.1337} = 0.3348\ \Omega/\text{km}
\end{aligned}
$$

Chapter 5 Problem Solutions

5.1 A three-phase transmission line has flat horizontal spacing with 2 m between adjacent conductors. At a certain instant the charge on one of the outside conductors is 60 μC/km, and the charge on the center conductor and on the other outside conductor is -30 μC/km. The radius of each condutor is 0.8 cm. Neglect the effect of the ground and find the voltage drop between the two identically charged conductors at the instant specified.

Solution:



$$
\begin{aligned}
q_a &= 60 \times 10^{-6} \text{ C/km} \\
q_b &= q_c = -30 \times 10^{-6} \text{ C/km} \\
V_{bc} &= \frac{10^{-6}}{2\pi k}\left(60 \ln \frac{4}{2} - 30 \ln \frac{2}{r} - 30 \ln \frac{r}{2}\right) \quad \text{where } r \text{ is in meters} \\
&= \frac{10^{-6} \times 60}{2\pi \times 8.85 \times 10^{-9}} = 744.5 \text{ V}
\end{aligned}
$$

5.2 The 60-Hz capacitive reactance to neutral of a solid conductor, which is one conductor of a single-phase line with 5 ft spacing , is 196.1 kΩ-mi. What value of reactance would be specified in a table listing the capacitive reactance in ohm-miles to neutral of the conductor at 1-ft spacing for 25 Hz? What is the cross-sectional area of the conductor in circular mils?

Solution:

At 5-ft spacing,

$$
\begin{aligned}
X_C &= 2.965 \times 10^4 \ln\frac{5}{r} = 196,100\ \Omega \cdot \text{mi} \\
\ln\frac{5}{r} &= 6.614 \\
r &= 0.00670 \text{ ft, or } 0.0805 \text{ in} \\
&= (2 \times 0.0805 \times 1000)^2 = 25,992 \text{ circ mils}
\end{aligned}
$$

From Eq. (5.12), at 1-ft spacing and 25 Hz,

$$
X_C = \frac{1.779}{25} \times 10^6 \ln\frac{1}{0.00670} = 356,200\ \Omega \cdot \text{mi}
$$

5.3 Solve Example 5.1 for 50 Hz operation and 10 ft spacing.

Solution:

$$
\begin{aligned}
X_C &= \frac{1.779 \times 10^6}{50} \ln\frac{10}{0.0268}\ \Omega \cdot \text{mi} = 0.2115\ \text{M}\Omega \cdot \text{mi} \\
B_C &= \frac{1}{X_C} = 4.728\ \mu\text{S/mi} \\
X_a' &= \frac{60}{50} \times 0.1074\ \text{M}\Omega \cdot \text{mi} \\
X_d' &= \frac{60}{50} \times 0.0683\ \text{M}\Omega \cdot \text{mi} \\
X_C &= \frac{60}{50}(0.1074 + 0.0683)\ \text{M}\Omega \cdot \text{mi} = 0.2109\ \text{M}\Omega \cdot \text{mi} \\
B_C &= 4.742\ \mu\text{S/mi}
\end{aligned}
$$

5.4 Using Eq. (5.23), determine the capacitance to neutral (in μF/km) of a three-phase line with three *Cardinal* ACSR conductors equilaterally spaced 20 ft apart. What is the charging current of the line (in A/km) at 60 Hz and 100 kV line to line?

Solution:

For *Cardinal* conductors,

$$
\begin{aligned}
r &= \frac{1.196}{2} = \frac{1}{12} \text{ ft} \\
C_n &= \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\frac{20}{(1.196)/24}} \text{ F/m} = 9.276 \times 10^{-12} \text{ F/m} = 9.276 \times 10^{-3}\ \mu\text{F/km}
\end{aligned}
$$

$$I_{\text{chg}} = 2\pi \times 60 \times 9.276 \times 10^{-9} \times \frac{100 \times 10^3}{\sqrt{3}} \text{ A/km} = 0.202 \text{ A/km}$$

5.5 A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 25 ft and the third is 42 ft. The conductors are ACSR *Osprey*. Determine the capacitance to neutral in microfarads per mile and the capacitive reactance to neutral in ohm-miles. If the line is 150 mi long, find the capacitance to neutral and capacitive reactance of the line.

Solution:

$$\begin{aligned}
\textit{Osprey}\text{ diam.} &= 0.879 \text{ in} \\
D_{eq} &= \sqrt[3]{25 \times 25 \times 42} = 29.72 \text{ ft} \\
C_n &= \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \frac{29.72 \times 12}{(0.879)/2}} \text{ F/m} \\
&= 8.301 \times 10^{-12} \text{ F/m} = 8.301 \times 10^{-6} \times 1.609 \ \mu\text{F/m} = 0.01336 \ \mu\text{F/mi} \\
X_C &= \frac{10^6}{377 \times 0.01336} = 0.1985 \times 10^6 \ \Omega \cdot \text{mi}
\end{aligned}$$

From Table A.3, $X_a' = 0.0981$. Interpolation from Table A.4 yields $X_d' = 0.0999 + 0.72(0.1011 - 0.0999) = 0.1006$. From Table A.4, $X_C = 0.1987 \times 10^6 \ \Omega \cdot \text{mi}$.

For 150 miles,

$$\begin{aligned}
C_n &= 150 \times 0.01336 = 2.004 \ \mu\text{F} \\
X_C &= \frac{0.1987}{150} \times 10^6 = 1325 \ \Omega
\end{aligned}$$

5.6 A three-phase 60-Hz line has flat horizontal spacing. The conductors have an outside diameter of 3.28 cm with 12 m between conductors. Determine the capacitive reactance to neutral in ohm-meters and the capacitive reactance of the line in ohms if its length is 125 mi.

Solution:

$$\begin{aligned}
D_{eq} &= \sqrt[3]{12 \times 12 \times 24} = 15.12 \text{ m} \\
r &= 0.0328/2 = 0.0164 \\
X_C &= \frac{2.862}{60} \times 10^9 \ln \frac{15.12}{0.0164} = 3.256 \times 10^8 \ \Omega \cdot \text{m}
\end{aligned}$$

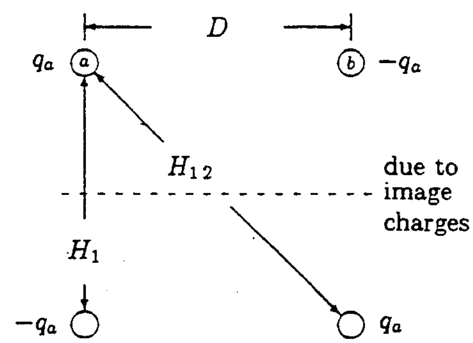
For 125 miles,

$$X_C = \frac{3.256 \times 10^8}{125 \times 1609} = 1619 \ \Omega$$

5.7 (*a*) Derive an equation for the capacitance to neutral in farads per meter of a single-phase line, taking into account the effect of ground. Use the same nomenclature as in the equation derived for the capacitance of a three-phase

line where the effect of ground is represented by image charges.
(b) Using the derived equation, calculate the capacitance to neutral in farads per meter of a single-phase line composed of two solid circular conductors each having a diameter of 0.229 in. The conductors are 10 ft apart and 25 ft above ground. Compare the result with the value obtained by applying Eq. (5.10).

Solution:



(a) Due to charges on a, b:

$$V_{ab} = \frac{1}{2\pi k}\left[q_a \ln\frac{D}{r} - q_a \ln\frac{r}{D}\right]$$

Due to image charges:

$$V_{ab} = \frac{1}{2\pi k}\left[-q_a \ln\frac{H_{12}}{H_1} + q_a \ln\frac{H_1}{H_{12}}\right]$$

Due to image and actual charges:

$$V_{ab} = \frac{q_a}{2\pi k}\left[\ln\frac{D^2}{r^2} - \ln\frac{H_{12}^2}{H_1^2}\right] = \frac{q_a}{\pi k}\left[\ln\frac{D}{r} - \ln\frac{H_{12}}{H_1}\right]$$

$$C_{an} = 2C_{ab} = \frac{2\pi k}{\ln\frac{D}{r} - \ln\frac{H_{12}}{H_1}} \text{ F/m}$$

(b) By Eq. (5.10),

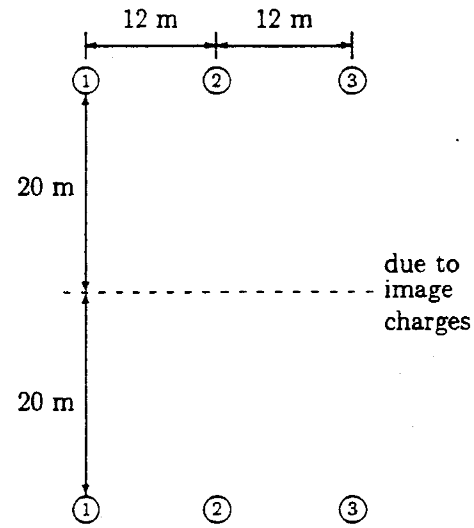
$$C_n = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{10\times 12}{0.229/2}\right)} = 7.996 \times 10^{-12} \text{ F/m}$$

And from part (a) above,

$$C_n = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{10\times 12}{0.229/2}\right) - \ln\left(\frac{\sqrt{50^2+10^2}}{50}\right)} = 8.018 \times 10^{-12} \text{ F/m}$$

5.8 Solve Prob. 5.6 while taking account of the effect of ground. Assume that the conductors are horizontally placed 20 m above ground.

Solution:



$$
\begin{aligned}
H_1 &= H_2 = H_3 = 40 \text{ m} \\
H_{12} &= H_{23} = \sqrt{40^2 + 12^2} \text{ m} = 41.761 \text{ m} \\
H_{31} &= \sqrt{40^2 + 24^2} \text{ m} = 46.648 \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
D_{eq} &= 15.12 \text{ m} \quad \text{and} \quad r = 0.0164 \text{ m} \\
X_C &= \frac{2.862}{60} \times 10^9 \left[\ln \frac{D_{eq}}{r} - \frac{1}{3} \ln \frac{H_{12}H_{23}H_{31}}{H_1 H_2 H_3} \right] \ \Omega \cdot \text{m} \\
&= 4.77 \times 10^7 \left[\ln \frac{15.12}{0.0164} - \frac{1}{3} \ln \frac{41.761 \times 41.761 \times 46.648}{40 \times 40 \times 40} \right] \ \Omega \cdot \text{m} \\
&= 3.218 \times 10^8 \ \Omega \cdot \text{m}
\end{aligned}
$$

For 125 miles,

$$
X_C = \frac{3.218 \times 10^8}{125 \times 1609.34} \ \Omega = 1.60 \text{ k}\Omega
$$

5.9 A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spaicng of 11 m between adjacent conductors. Compare the capacitive reactance in ohm-kilometers per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and the 11 m spacing measured between bundles. The spacing between conductors in the bundle is 40 cm.

Solution:

$$
D_{eq} = \sqrt{11 \times 11 \times 22} = 13.86 \text{ m}
$$

$$
\begin{aligned}
\textit{Bluejay}\colon r &= 1.259 \times 2.54/2 \times 10^{-2} = 0.016 \text{ m} \\
X_C &= 4.77 \times 10^4 \ln \frac{13.86}{0.016} = 322,650 \ \Omega \cdot \text{km}
\end{aligned}
$$

For *Dove*, 2-conductor bundle,

$$r = 0.927 \times 2.54/2 \times 10^{-2} = 0.01177$$
$$D_{sC}^{b} = \sqrt{rd} = \sqrt{0.0177 \times 0.4} = 0.0842 \text{ m}$$
$$X_C = 4.77 \times 10^4 \ln \frac{13.86}{0.0842} = 243,440\ \Omega \cdot \text{km}$$

5.10 Calculate the capacitive reactance in ohm-kilometers of a bundled 60-Hz three-phase line having three ACSR *Rail* conductors per bundle with 45 cm between conductors of the bundle. The spacing between bundle centers is 9, 9 and 18 m.

Solution:

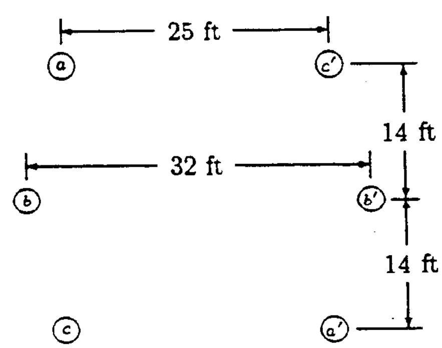
$$D_{eq} = \sqrt{9 \times 9 \times 18} = 11.34 \text{ m}$$
$$r = 1.165 \times 2.54/2 \times 10^{-2} = 0.0148 \text{ m}$$
$$D_{sC}^{b} = \sqrt[3]{0.0148 \times 0.45^2} = 0.1442 \text{ m}$$
$$X_C = 4.77 \times 10^4 \ln \frac{11.34}{0.1442} = 208,205\ \Omega \cdot \text{km}$$

5.11 Six conductors of ACSR *Drake* constitute a 60-Hz double-circuit three-phase line arranged as shown in Fig. 5.11. The vertical spacing, however, is 14 ft; the longer horizontal distance is 32 ft; and the shorter horizontal distances are 25 ft. Find,

(*a*) the inductance per phase (in H/mi) and the inductive reactance in Ω/mi.

(*b*) the capacitive reactance to neutral in ohm-miles and the charging current A/mi per phase and per conductor at 138 kV.

Solution:



(*a*)

$$\text{GMR} = 0.0373\ \textit{Drake}$$

In original positions in the transposition cycle,

$$\begin{aligned}
\text{distance } a\text{-}b &= \sqrt{14^2 + 3.5^2} = 14.43 \text{ ft} \\
\text{distance } a\text{-}b' &= \sqrt{14^2 + 28.5^2} = 31.75 \text{ ft} \\
\text{distance } a\text{-}a' &= \sqrt{25^2 + 28^2} = 37.54 \text{ ft} \\
D^p_{ab} &= D^p_{bc} = \sqrt[4]{14.43^2 \times 31.75^2} = 21.04 \text{ ft} \\
D_{ac} &= \sqrt[4]{25^2 \times 28^2} = 26.46 \text{ ft} \\
D_{eq} &= \sqrt[3]{21.04^2 \times 26.46} = 22.71 \text{ ft} \\
D_s &= \left[\left(\sqrt{0.0373 \times 37.54}\right)^2 \sqrt{0.0373 \times 32}\right]^{\frac{1}{3}} = 1.152 \text{ ft} \\
L &= 2 \times 10^{-7} \ln \frac{22.71}{1.152} = 5.693 \times 10^{-7} \text{ H/m} \\
&= 5.963 \times 10^{-7} \times 10^3 \times 1609 = 0.959 \text{ mH/mi} \\
X_L &= 377 \times 0.959 \times 10^{-3} = 0.362 \ \Omega/\text{mi/phase}
\end{aligned}$$

(*b*) $r = \frac{1.108}{2 \times 12} = 0.0462$ ft as in part (*a*) above, except that r is substituted for D_s:

$$D_{sC} = \left[\left(\sqrt{0.0462 \times 37.54}\right)^2 \sqrt{0.0462 \times 32}\right]^{\frac{1}{3}} = 1.282 \text{ ft}$$

From part (*a*) above, $D_{eq} = 22.71$ ft and

$$\begin{aligned}
X_C &= 2.965 \times 10^{-4} \ln \frac{22.71}{1.282} = 85,225 \ \Omega \cdot \text{mi/phase to neutral} \\
I_{chg} &= \frac{138,000/\sqrt{3}}{85,225} = 0.935 \text{ A/mi/phase} = 0.467 \text{ A/mi/conductor}
\end{aligned}$$

Chapter 6 Problem Solutions

6.1 An 18-km 60-Hz single circuit three-phase line is composed of *Partridge* conductors equilaterally spaced with 1.6 m between centers. The line delivers 2500 kW at 11 kV to a balanced load. Assume a wire temperature of 50°C.

(*a*) Determine the per-phase series impedance of the line.

(*b*) What must be the sending-end voltage when the power factor is (*i*) 80% lagging (*ii*) unity (*iii*) 90% leading?

(*c*) Determine the percent regulation of the line at the above power factors.

(*d*) Draw phasor diagrams depicting the operation of the line in each case.

Solution:

(*a*)

$$\begin{aligned}
R &= 0.3792 \times \frac{18}{1.609} = 4.242 \ \Omega \\
\text{From Table A.3, } X_a &= 0.465 \ \Omega/\text{mi}
\end{aligned}$$