

Economic Dispatch - Optimum allocation of generation among system generators.

Goal: Maximize system efficiency

Minimize system losses (can't bill customers)

Specifics:

Control voltage / vars

- Adjust generator exciter
- Reactors, caps (shunt)
- Tap-changing transformers

Control Power Flow

- Control P_{gen} at each generator
- Phase-shifting transformers
- Line switching

Frequency - (later)

- Prime mover control (droop controller)
- Load management.

$$P_{GEN} = P_{LOADS} + P_{TRANS/DIST \text{ LOSSES}}$$

$$P_G = \sum_{i=1}^n P_{Gi}$$

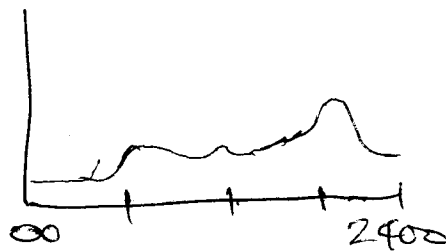
How should P_G be divided up among the n units?

Constraints: ~~them~~

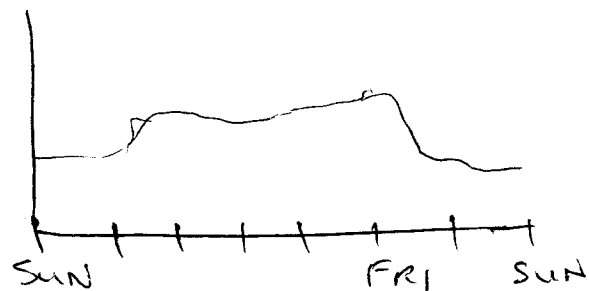
- UNIT COMMITMENT {
- On-line time regimnts - Coal 8hrs+
 - Some units down for maintenance
 - Should have rolling/spinning reserve in case units fail.
 - $P_{min}; < P_{Gi} < P_{max};$
 - ↑ Thermal constraints of turbine.
 - ↑ I²R of stator

Lingo - Load Characteristics:

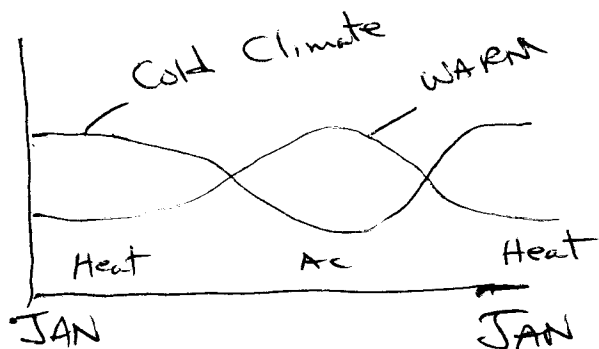
A) Daily -



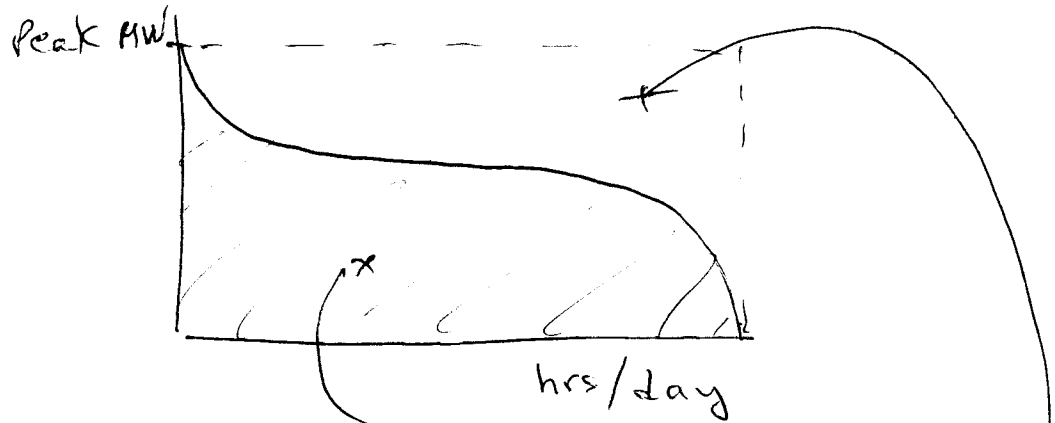
B) Weekly -



C) ~~Monthly~~ Annual



Load Duration Curve

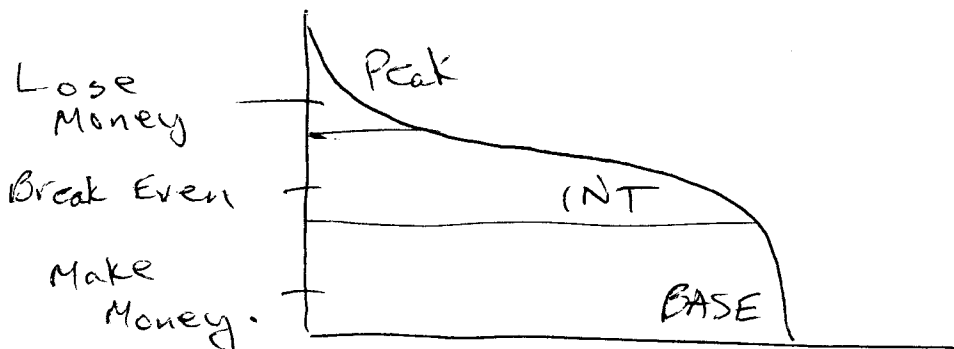
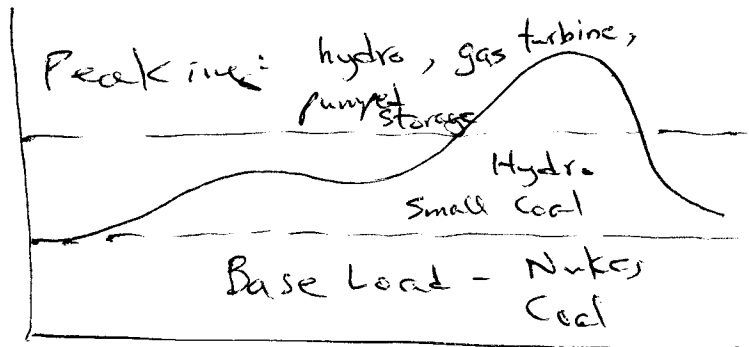


Load Factor: $LF = \frac{\text{Energy used}}{\text{Peak Power} \times \text{hrs}}$

0.4 - Bad

0.85 - Good

Strategy:

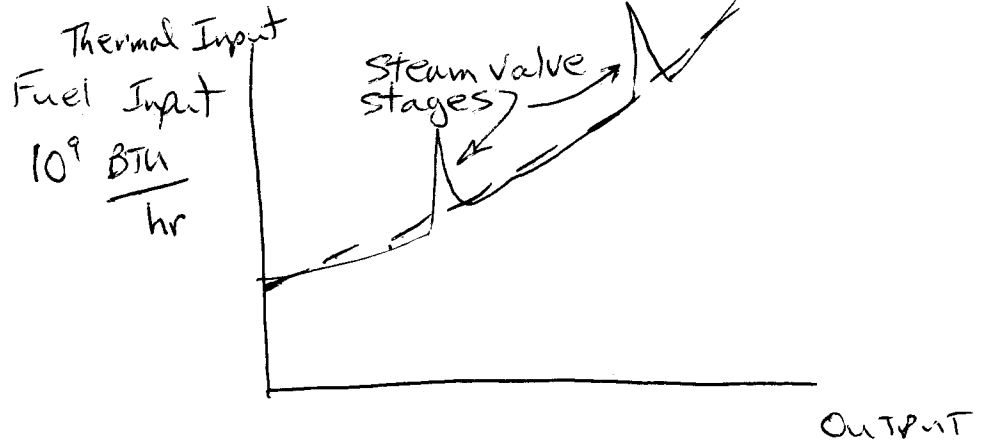


Ways a Utility can make money:

- Raise rates PUC / PSC must approve
- Sell more Base Load MWH
- Reduce Peak load (Load management)
 - Interruptible loads - water heaters, etc
 - Time of day rates
- *
 - Increase efficiency
 - Reduce Aux use in plant (10-15%)
 - Improve thermal efficiency (Net Heat Rate)
- * Economic Dispatch

~~Back~~

For each unit:



$$HR = \frac{\text{Input Thermal power, BTU/hr}}{\text{Electrical output}}$$

Typical: $10.5 \times 10^6 \text{ BTU/MWH}$

Recognize form as $1/n$

But one $\text{BTU/hr} = 0.293 \text{ W}$

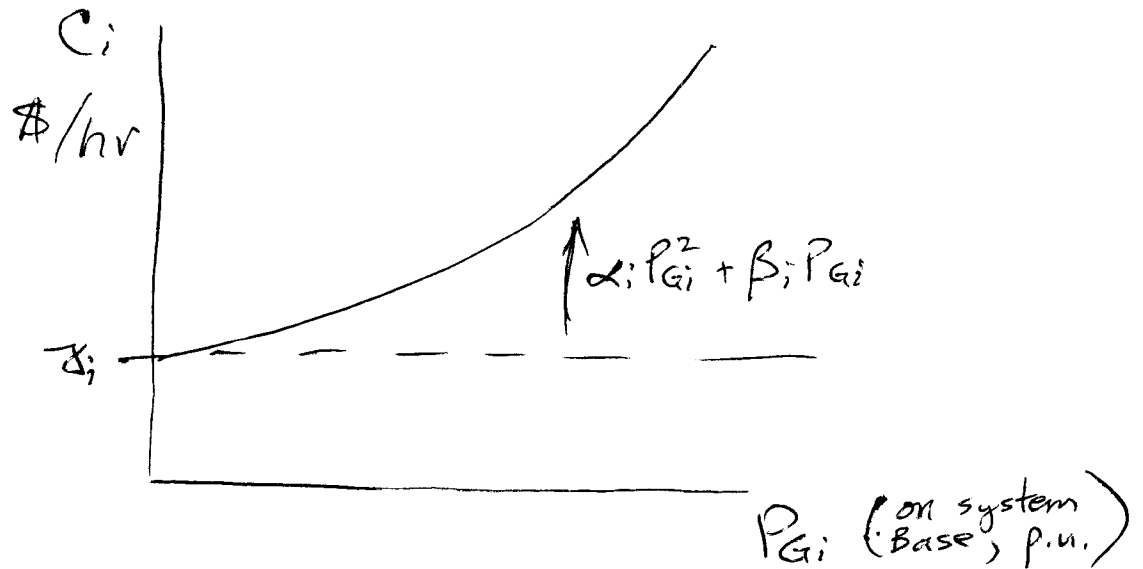
~~HR = BTU/hr~~ ~~2000~~

$$\eta = \frac{1}{\text{HR} \times 0.293 \times 10^{-6}} = \boxed{\frac{3.413 \times 10^6}{\text{HR}}}$$

Operating cost of unit i

$$C_i = F_i P_i$$

$\left\{ \begin{array}{l} \text{Input power in MBTU} \\ \text{Fuel cost in } \$/\text{MBTU} \text{ (order of Mag.) } \\ \text{Includes (+ labor, supplies, maint).} \end{array} \right.$



Empirically, $C_i = \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + F_i$

α, β, F in $\$/\text{hr}$

~~The curve is empirically described as~~
 ~~$C_i = \alpha_i P_i^2 + \beta_i P_i + \gamma_i$~~

Again,

$$P_G = P_L + P_{TL}$$

Problem: solve for n P_{Gi} 's subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

~~Example~~

Objective function:

$$\text{Min } F(x_1, x_2, x_3 \dots x_n)$$

$$\text{Constraints: } G_1(x_1, x_2 \dots x_n) = 0$$

⋮

$$G_m(x_1, x_2 \dots x_n) = 0$$

Usually (for our purposes) $m=1$

1) Form the Lagrangian:

$$\mathcal{L} = F(x_1, x_2 \dots x_n) - \lambda_1 G(x_1, x_2 \dots x_n)$$

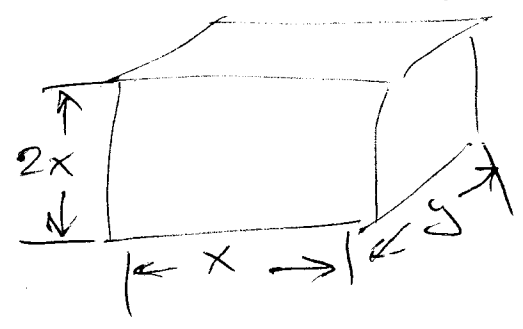
2) Find all partial derivatives of \mathcal{L} wrt $x_1, x_2 \dots x_n$ and set them = 0

3) solve for $(x_1, x_2, \dots, x_n), \lambda$
from partial derivatives & $G(x_1, x_2, \dots, x_n)$

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)

Min if pos definite.
Local Max if neg def, Saddle if indef.

ex: ~~8.1~~ Box of dimensions x, y, z



Maximize volume for $S = 432 \text{ cm}^2$

Objective function: $V = xy z$
 $= 2x^2 y$

Constraint: $2(xy + 2x^2 + 2xy) - 432 = 0$

$\mathcal{L} = 2x^2 y - \lambda (4x^2 + 6xy - 432)$

$\frac{\partial \mathcal{L}}{\partial x} = 4x - 8\lambda x + 6y = 0$

$\frac{\partial \mathcal{L}}{\partial y} = 2x^2 - 6\lambda x = 0$

Constraint: $2(xy + 2x^2 + 2xy) - 432 = 0$

solve simultaneously.
 $\lambda = 2$
 $x = 6 \text{ cm}$
 $y = 8 \text{ cm}$
 $V = 576 \text{ cm}^3$

Applying to Economic Dispatch:

$$\text{Objective: } C = \sum_{i=1}^n C_i = \sum_{i=1}^n \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i$$

$$\text{Constraints: } G = P_G - P_L = 0 = \sum_{i=1}^n P_{Gi} - \cancel{P_L} P_L$$

(ignore line losses for now.)

\uparrow gen \uparrow Total Loads.

$$\textcircled{1} \quad \mathcal{L} = C - \lambda \left(\sum_{i=1}^n P_{Gi} - P_L \right)$$

$\textcircled{2}$ Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial C}{\partial P_{Gi}} - \lambda (1) = \frac{\partial C}{\partial P_{Gi}} - \lambda$$

Since λ is the same in every term, one way to satisfy conditions is:

$$\frac{\partial C}{\partial P_{G1}} - \lambda = 0, \quad \frac{\partial C}{\partial P_{G2}} - \lambda = 0 \quad \dots \quad \frac{\partial C}{\partial P_{Gn}} - \lambda = 0$$

Therefore, each plant must be at same incremental ~~cost~~ cost, λ_i ($\lambda_1 = \lambda_2 = \dots = \lambda_n = \lambda$)

For each unit,

$$\lambda_i = \frac{\partial C_i}{\partial P_{Gi}} = 2\alpha_i P_{Gi} + \beta_i$$

Example 8.2

Unit 1:

$$25 \text{ MW} < P_{G1} < 150 \text{ MW}$$

$$C_1 = 0.01 P_{G1}^2 + 2 P_{G1} + 100$$

α_1 (100) $\frac{\$/\text{hr}}{\text{MW}^2}$ β_1 (2) $\frac{\$/\text{hr}}{\text{MW}}$ γ_1 (100) $\frac{\$/\text{hr}}{\text{MW}}$

Unit 2:

$$30 \text{ MW} < P_{G2} < 200 \text{ MW}$$

$$C_2 = 0.004 P_{G2}^2 + 2.6 P_{G2} + 80$$

α_2 (40) $\frac{\$/\text{hr}}{\text{MW}^2}$ β_2 (2.6) $\frac{\$/\text{hr}}{\text{MW}}$ γ_2 (80) $\frac{\$/\text{hr}}{\text{MW}}$

How to divide P_{G1} & P_{G2} within range
 $55 \text{ MW} \leq P_L \leq 350 \text{ MW}$?

For ex, $P_L = 282 \text{ MW}$

First, select $S_{\text{BASE}} = 100 \text{ MVA}$ & convert data to p.u.

~~$(0.01 \frac{\$/\text{hr}}{\text{MW}^2})$~~

$$\alpha_1 = (100)^2 (0.01) = 100$$

$$\alpha_2 = 40$$

$$\beta_1 = (100)(2) = 200$$

$$\beta_2 = 260$$

$$\gamma_1 = 100$$

$$\gamma_2 = 80$$

$$0.25 \leq P_{G1} \leq 1.50 \text{ p.u.}$$

$$0.30 \leq P_{G2} \leq 2.00 \text{ p.u.}$$

$$0.55 \leq P_L \leq 3.50 \text{ p.u.}$$

$$\lambda_1 = \frac{\partial C_1}{\partial P_{G1}} = 200 P_{G1} + 200$$

$$\lambda_2 = \frac{\partial C_2}{\partial P_{G2}} = 80 P_{G2} + 260$$

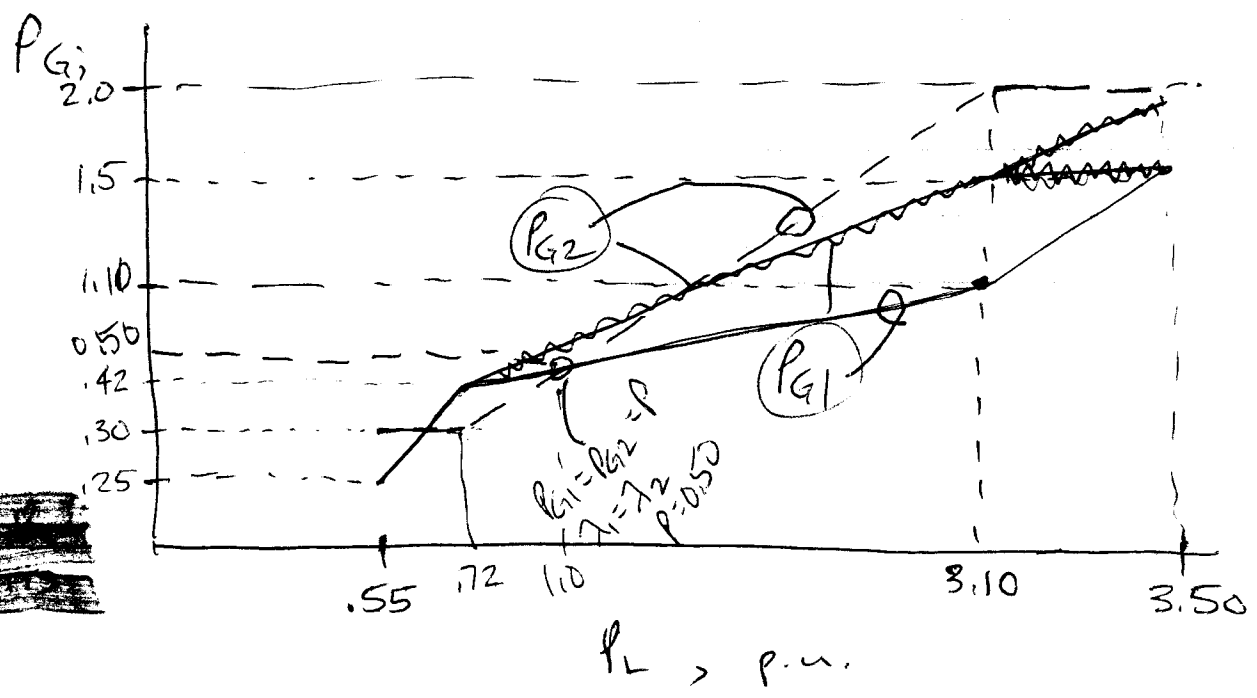
$$P_{G1} + P_{G2} = 282 \text{ p.u.}$$

Solving, setting $\lambda_1 = \lambda_2 = \lambda$

$P_{G1} = 1.02$ p.u. (102 MW)

$P_{G2} = 1.80$ p.u. (180 MW)

Looking at complete range, $55 \text{ MW} \leq P_L \leq 350 \text{ MW}$



@ 0.55 p.u. $P_{G1} = 0.25$, $P_{G2} = 0.30$
 $\lambda_1 = 250$, $\lambda_2 = 284$

→ ∴ must increase unit 1 first, until $\lambda_1 = 284$. This happens at $P_{G1} = \frac{284 - 200}{200} = 0.42 \text{ p.u.}$

→ Then λ_1 & λ_2 can be equal until one unit hits P_{max} . @ 3.5 p.u., ~~PG1 = 1.5, PG2 = 1.1~~
 $\lambda_1 = 500$ @ $P_{G1} = 1.5 \text{ p.u.}$ $\lambda_2 = 420$ @ $P_{G2} = 2.0 \text{ p.u.}$
 ∴ P_{G2} limits out first. $P_{G1} = \frac{420 - 200}{200} = 1.1 \text{ p.u.}$

→ From $P_L = 3.10$ and up, only P_{G1} increases.

- Gross

Solution

We first select $S_{3\phi_{base}} = 100$ MVA; then convert all data into per-unit.

$$\alpha_1 = (S_{3\phi_{base}})^2 0.01 = 100 \quad \alpha_2 = 40$$

$$\beta_1 = (S_{3\phi_{base}}) 2.00 = 200 \quad \beta_2 = 260$$

$$\gamma_1 = 100 \quad \gamma_2 = 80$$

$$0.25 \leq P_{G1} \leq 1.50 \text{ pu}$$

$$0.30 \leq P_{G2} \leq 2.00 \text{ pu}$$

$$0.55 \leq P_L \leq 3.50 \text{ pu}$$

$$\lambda_1 = \frac{\partial C_1}{\partial P_{G1}} = 200P_{G1} + 200$$

$$\lambda_2 = \frac{\partial C_2}{\partial P_{G2}} = 80P_{G2} + 260$$

Let us tabulate and plot results as we develop them, as shown in Figure 8.3. We start by calculating λ_1 and λ_2 for minimum-generation conditions (point 1). Observe that $\lambda_2 > \lambda_1$. Since we wish to make the λ 's equal, the strategy is to load unit 1 first. We do this until $\lambda_1 = 284$, which occurs at

$$P_{G1} = \frac{284 - 200}{200} = 0.42 \text{ (point 2)}$$

Now, calculate λ_1 and λ_2 at the maximum-generation condition (point 3). Observe

2.82 = 284

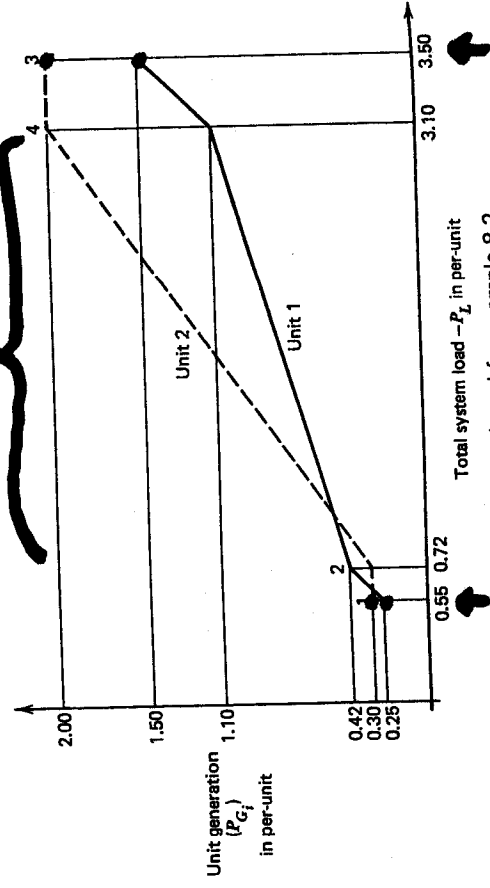


Figure 8.3. Results plotted for example 8.2.

now that $\lambda_1 > \lambda_2$, suggesting that we unload unit 1 first until we bring λ_1 down to $\lambda_2 = 420$. This happens at

$$P_{G1} = \frac{420 - 200}{200} = 1.10 \text{ (point 4)}$$

Observe that for $0.72 \leq P_L \leq 3.10$, it is possible to maintain equal λ 's. The equations are

$$\lambda_1 = \lambda_2$$

$$200P_{G1} + 200 = 80P_{G2} + 260$$

and

$$P_{G1} + P_{G2} = P_L$$

These linear relations are plotted in Figure 8.3. For $P_L = 282$ MW = 2.82 pu,

$$P_{G2} = 2.82 - P_{G1}$$

$$P_{G1} = 0.4P_{G2} + 0.3$$

$$= 1.128 - 0.40P_{G1} + 0.3$$

$$1.4P_{G1} = 1.428; P_{G1} = 1.02 \quad (102 \text{ MW})$$

$$P_{G2} = 2.82 - 1.02 = 1.80 \quad (180 \text{ MW})$$

Results are presented in Table 8.1.

Table 8.1. Results for Example 8.2.

Point	P_{G1}	P_{G2}	P_L	λ_1	λ_2
1	0.25	0.30	0.55	250	284
2	0.42	0.30	0.72	284	284
3	1.50	2.00	3.50	500	420
4	1.10	2.00	3.10	420	420

In the case where the incremental cost functions (λ_i) are linearized, a simple straightforward general solution is possible

$$2\alpha_i P_{G_i} - \lambda = -\beta_i \quad i = 1, 2, \dots, n \quad (8.13a)$$

$$P_{G1} + P_{G2} + \dots + P_{Gn} = P_L \quad (8.13b)$$

$$(8.13c)$$

$$\begin{bmatrix} 2\alpha_1 & 0 & 0 & \dots & 1 \\ 0 & 2\alpha_2 & 0 & \dots & 1 \\ 0 & 0 & 2\alpha_3 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ P_{G3} \\ \vdots \\ P_L \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ \vdots \\ -\lambda \end{bmatrix}$$

Solve the linear set for the P_{G_i} 's and λ .