SESSION 3

Short Circuit Calculations, Unsymmetrical Faults

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Short Circuit Calculations

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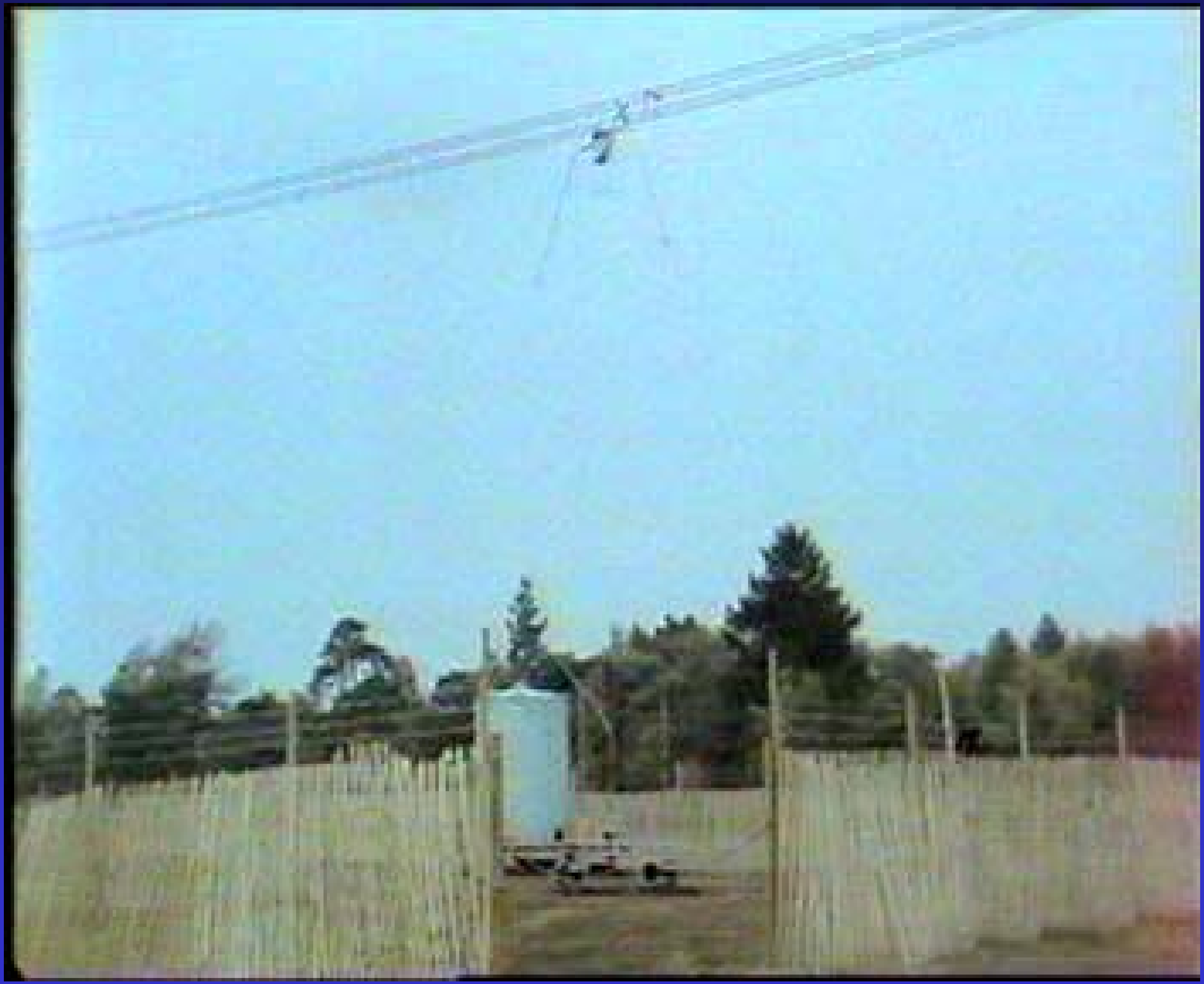
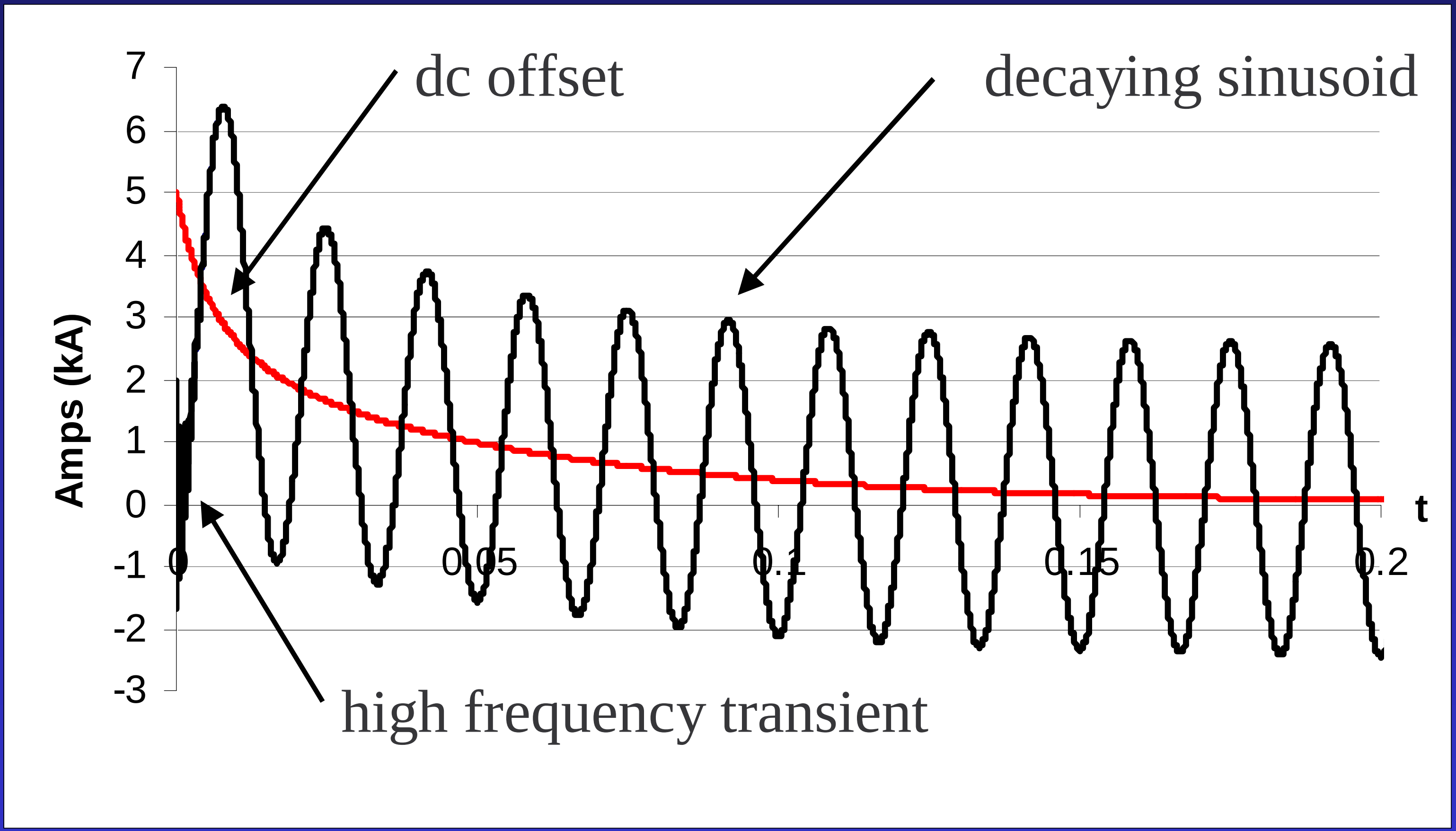
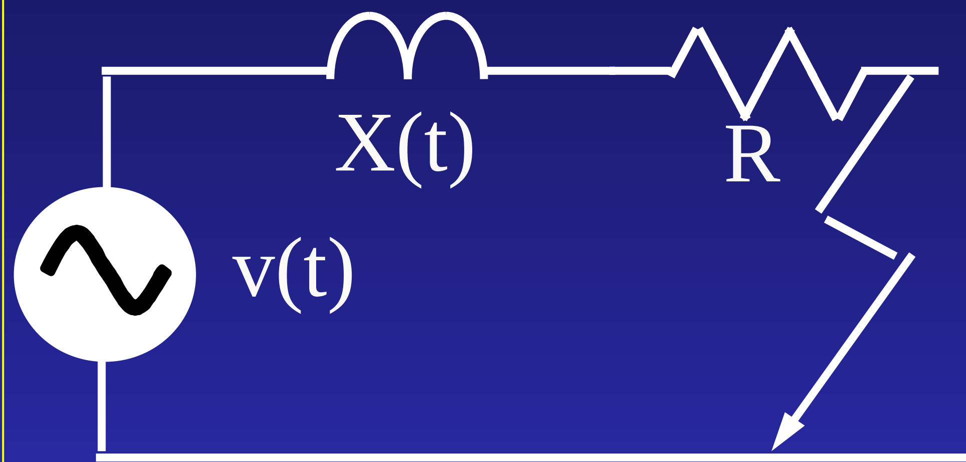
Line to Ground Short

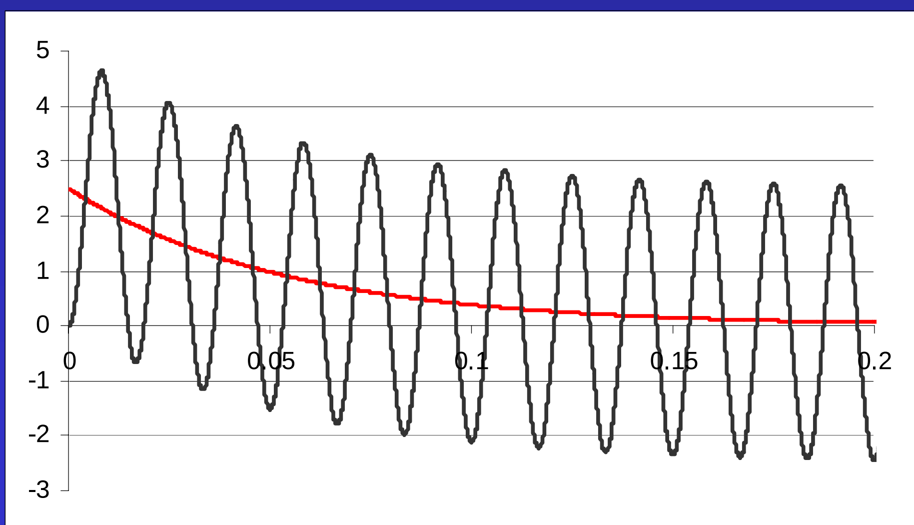


Short Circuit Fault Currents



Short Circuit Fault Currents (continued)





if

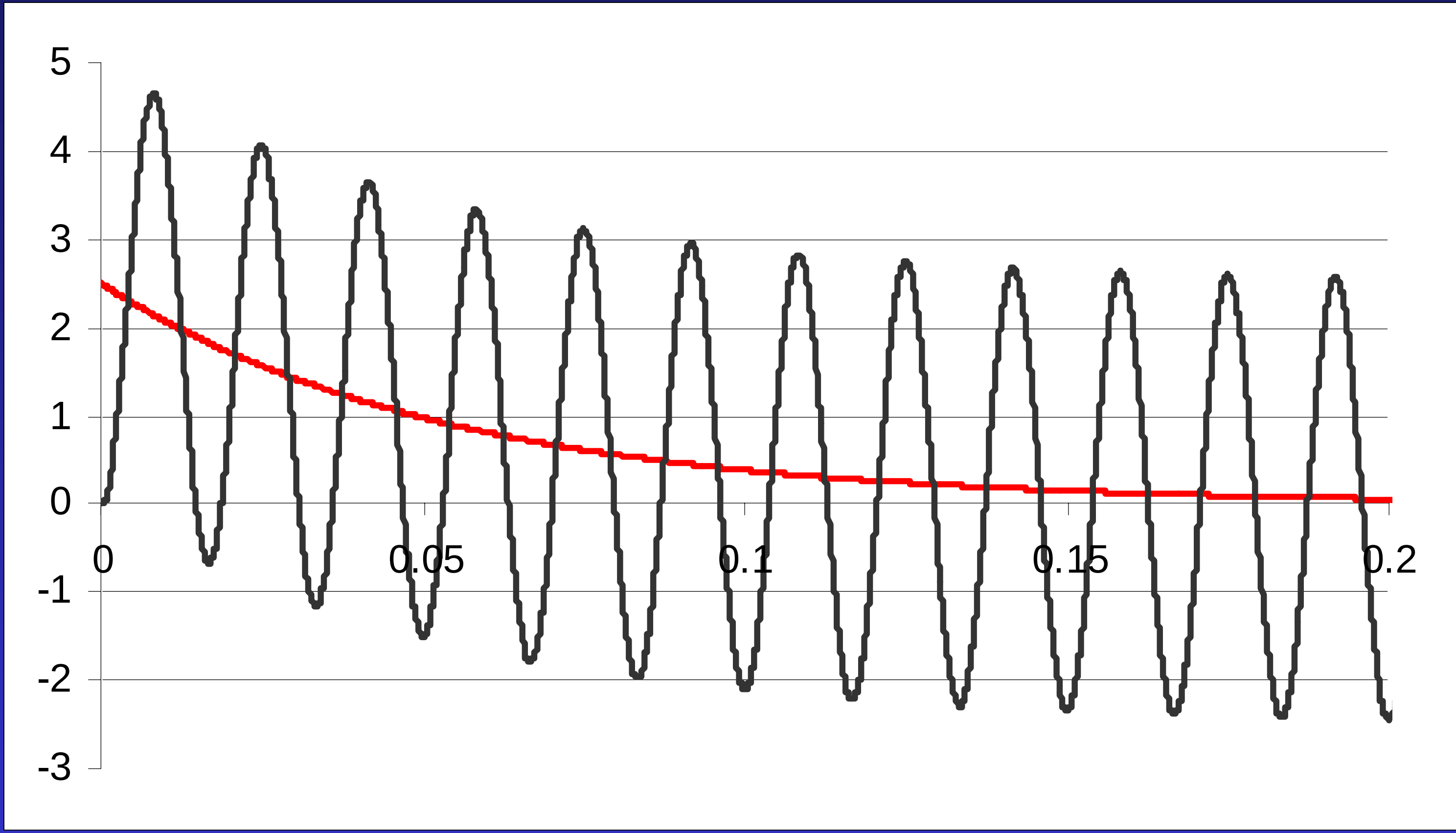
$$v(t) = V_m \cos(\omega t + \alpha)$$

$$Z(t)\angle\theta = R + jX(t)$$

then

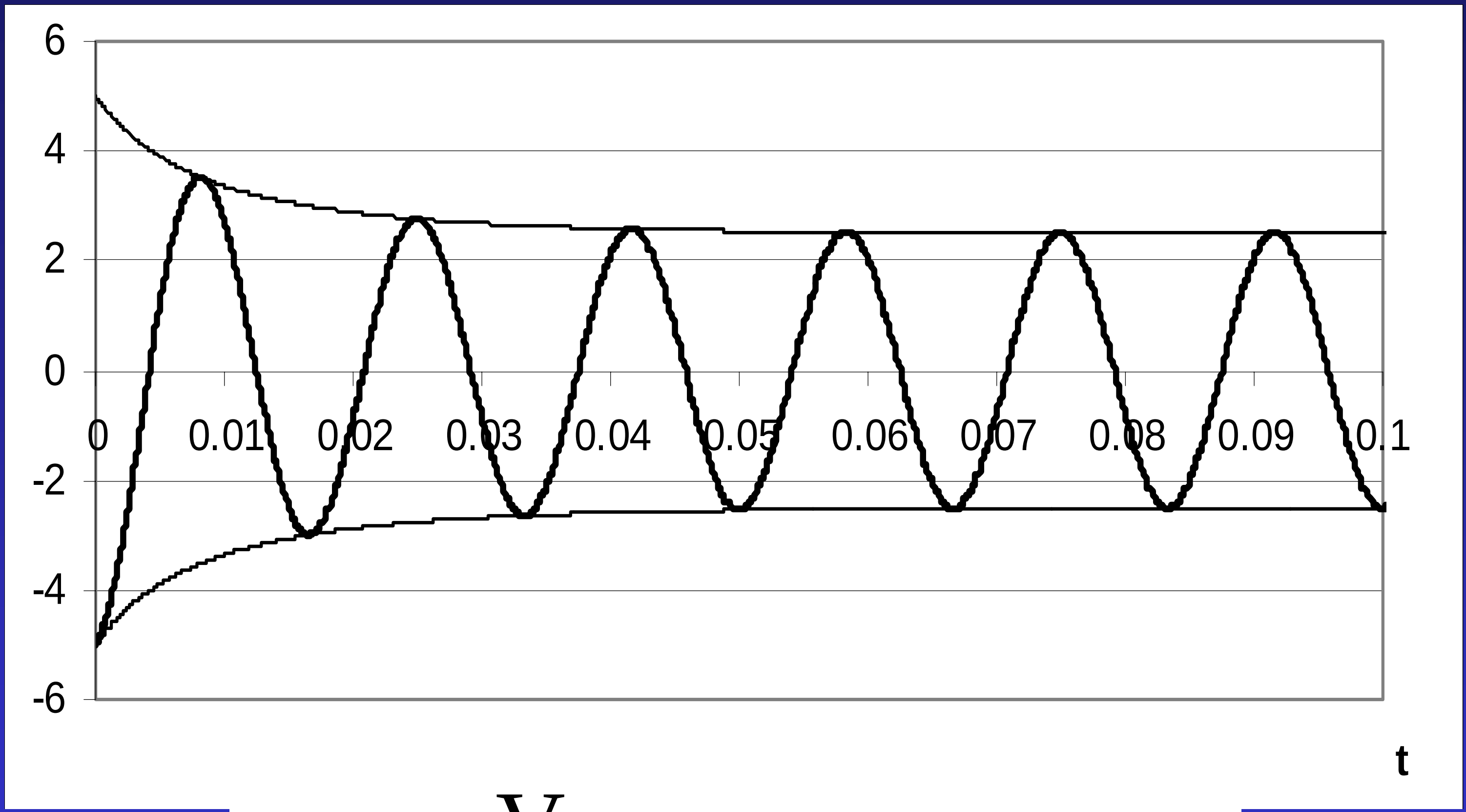
$$i(t) = \frac{V_m}{Z(t)}\left[\cos(\omega t + \alpha - \theta) - \cos(\omega t_0 + \alpha - \theta)e^{-R\omega t / X(t)}\right]$$

dc offset



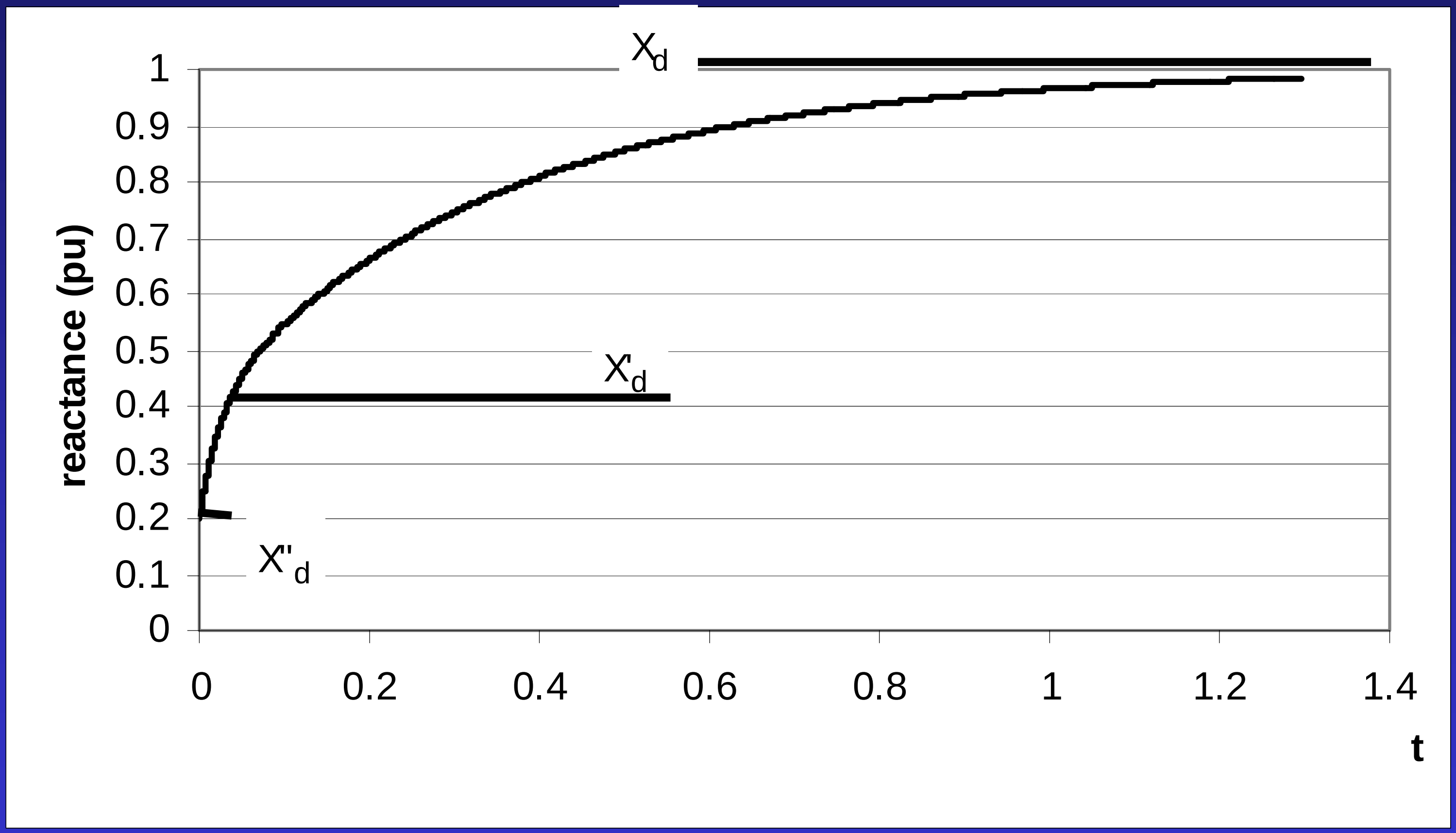
$$i(t) = \frac{V_m}{Z}\left[\cos(\omega t + \alpha - \theta) - \cos(\omega t_0 + \alpha - \theta)e^{-R\omega t/X}\right]$$

Variable magnitude

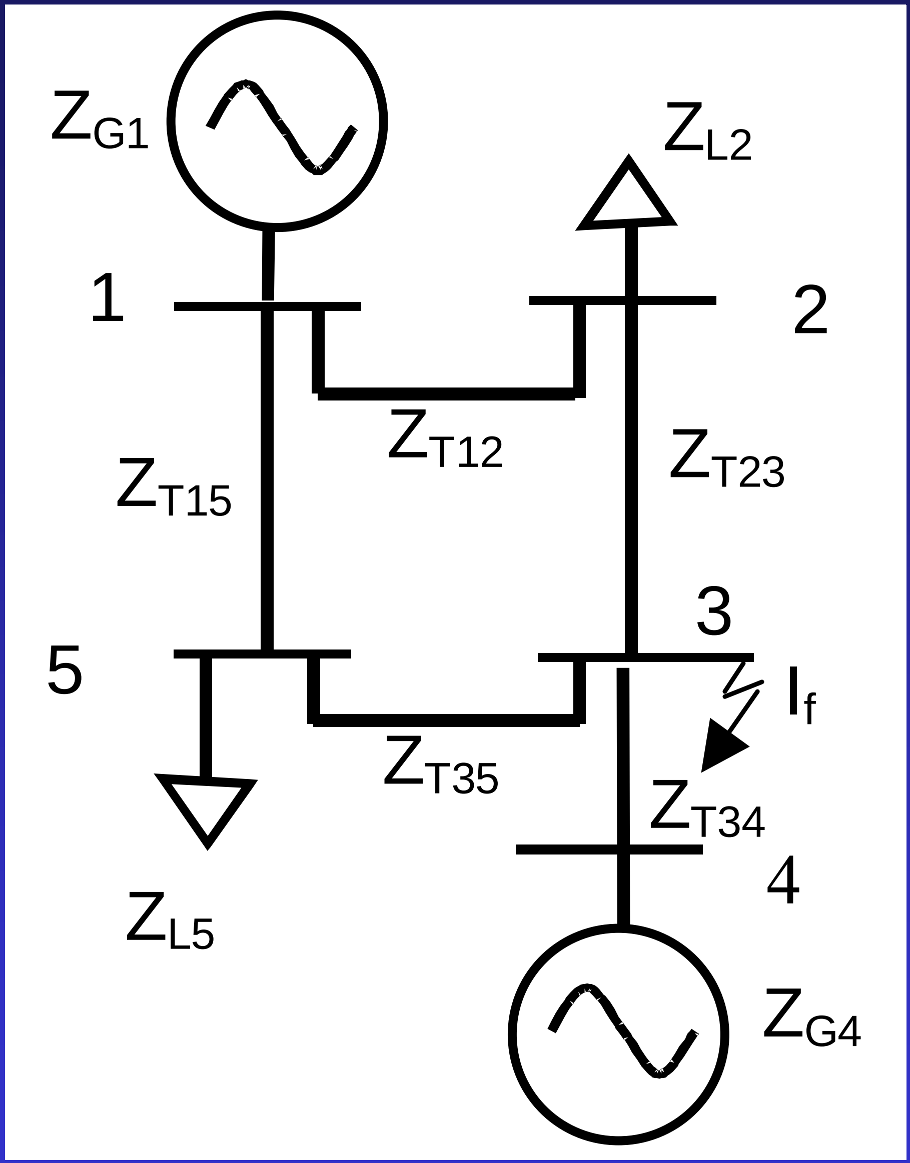


$$i(t) = \frac{V_m}{Z(t)} \cos(\omega t + \alpha - \theta)$$

Reactance of Synchronous Machine

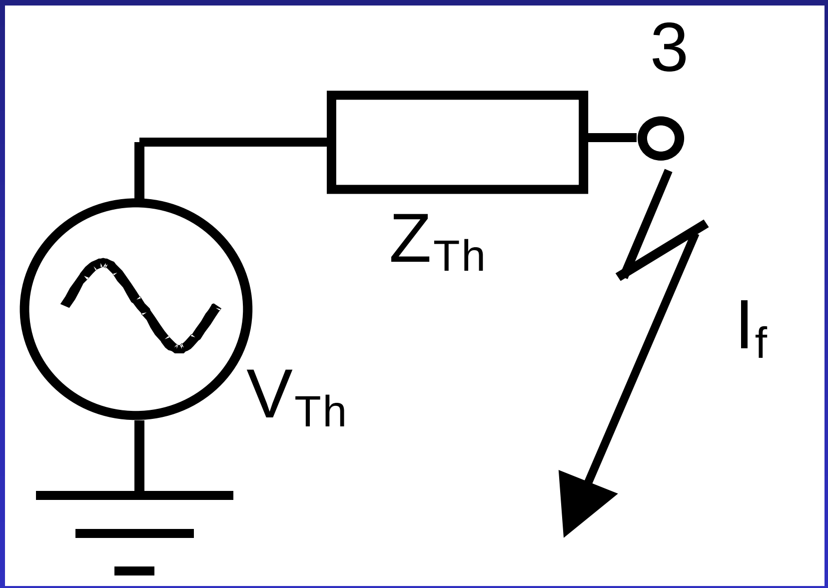


- Use sub-transient reactance to calculate fault currents
  - worst case for when CB is opening
- Do not calculate dc offset directly
  - use X/R to approximate offset (asymmetry ratio) at opening time
  - X/R is the time constant of the decay in cycles



How do you calculate the fault current (I_f) at bus 3?

Use Thevenin equivalent



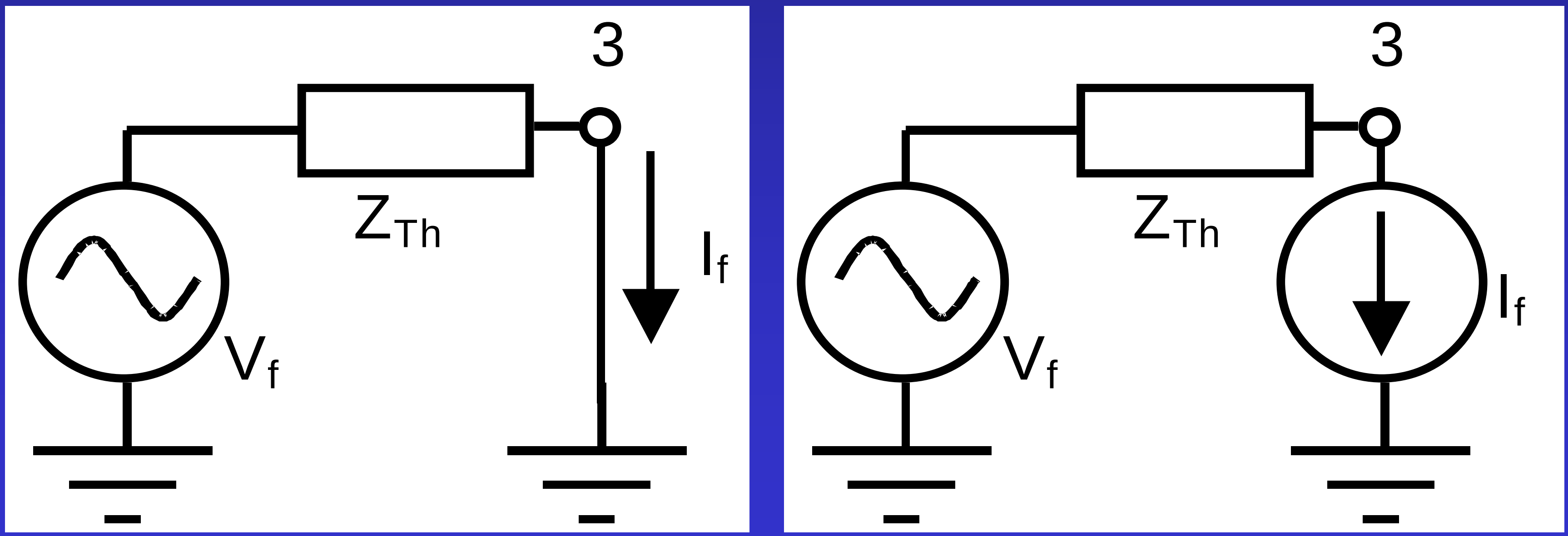
- V_{Th}: open circuit voltage
  - also called the pre-fault voltage, V_f
  - often use $1\angle 0°$

Use for:

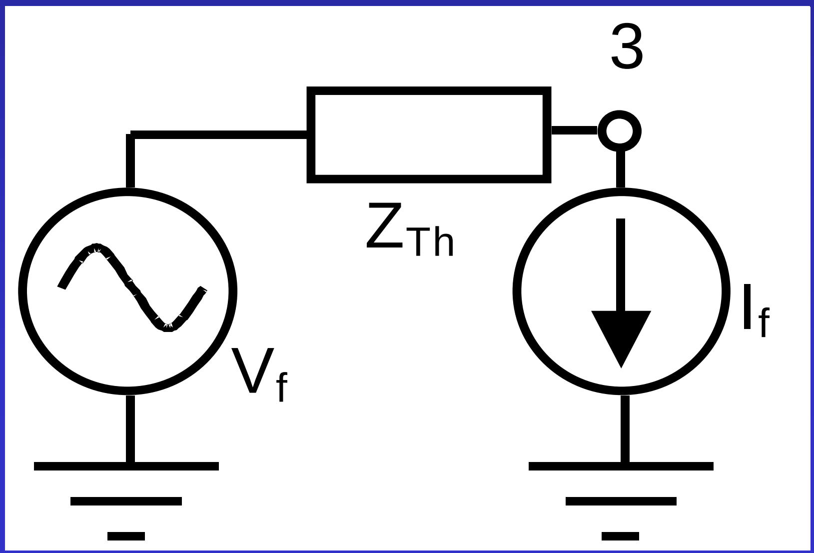
- symmetrical faults
  - single phase networks
  - three phase faults in three phase networks (per phase equivalent)
- concept for solving all faults

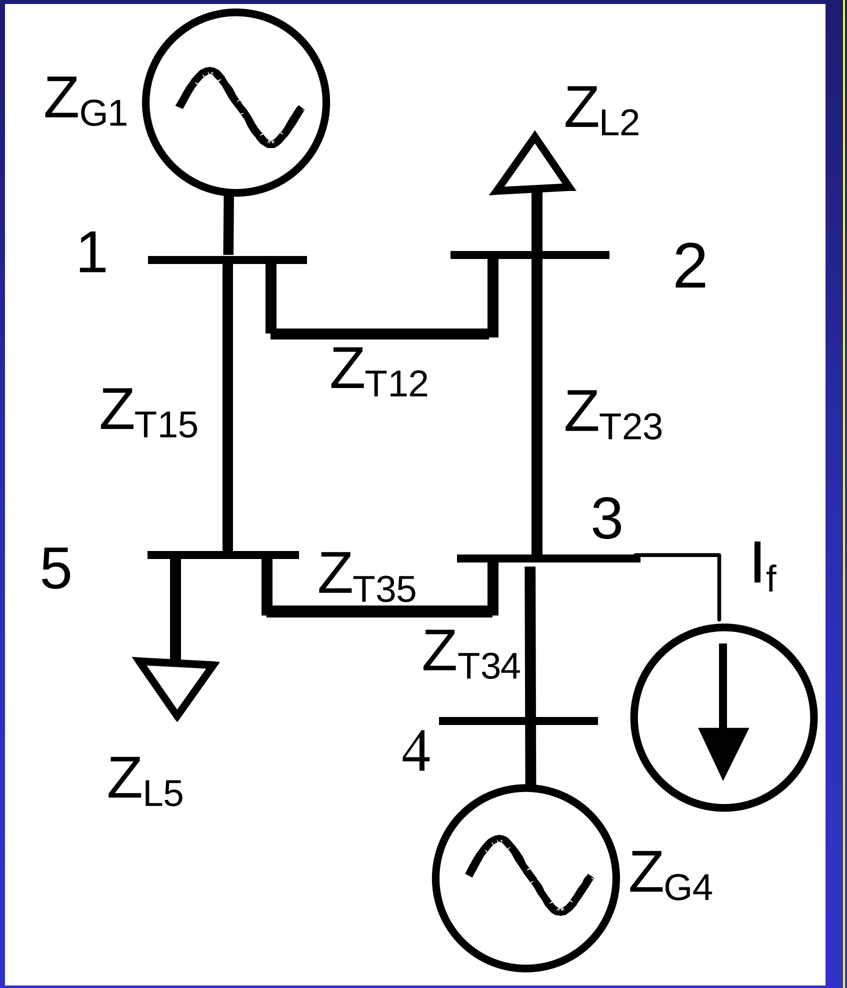
- This method calculates the current going into the fault
- Often want the current elsewhere in the network (through a circuit breaker, in bus bars, in other lines, etc.)

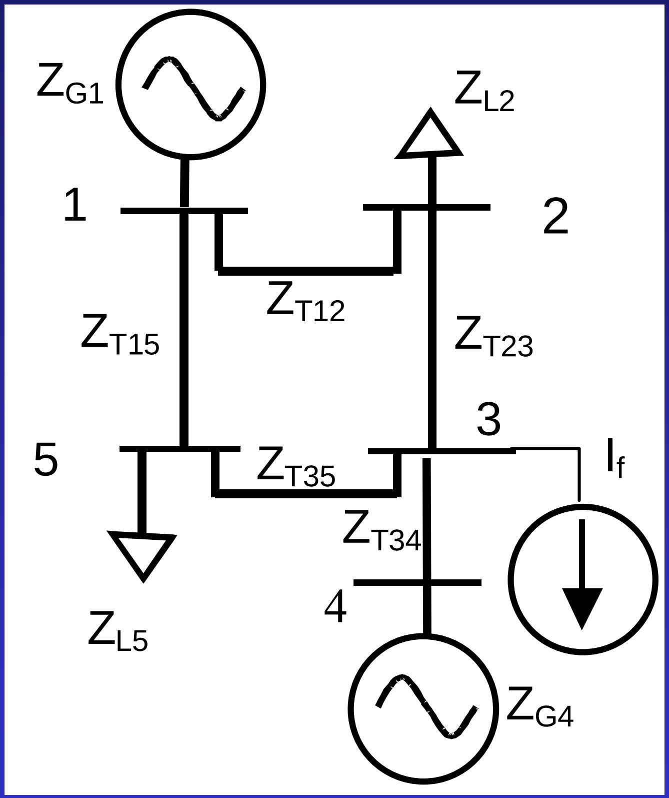
A current source, with the magnitude of the fault current, placed at the fault, has the same effect on the rest of the circuit as the short circuit.



Adding a current source to the original circuit has the same electrical effect as adding it to the equivalent circuit







All the voltages and all the currents can be found using the new circuit.

Use superposition:

– Use the original sources (or assume currents are 0 and voltages are $1\angle 0°$)

– Use just the current source, gives changes due to the fault

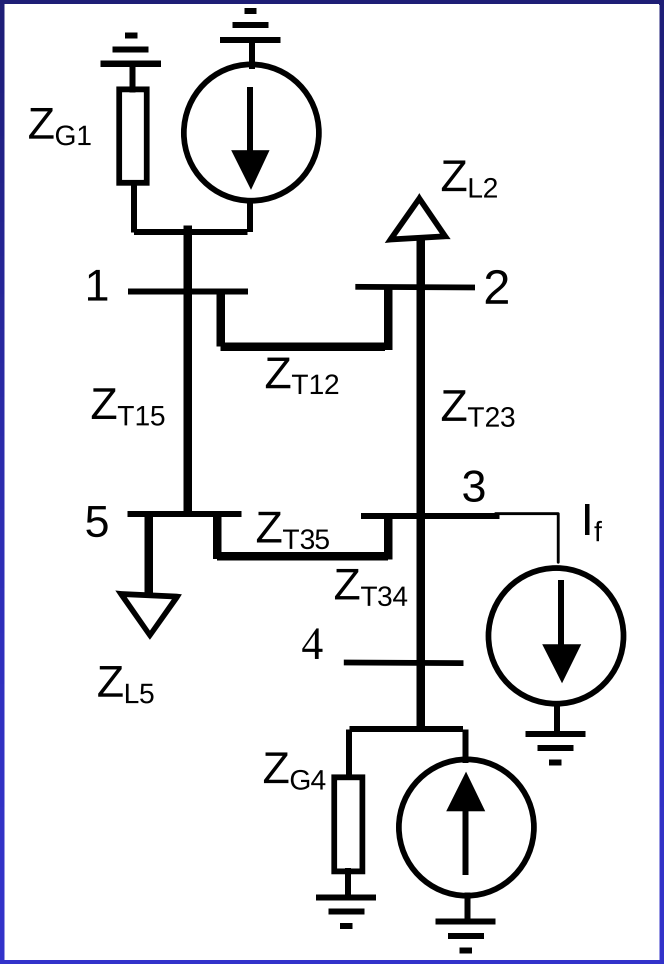
Calculation details

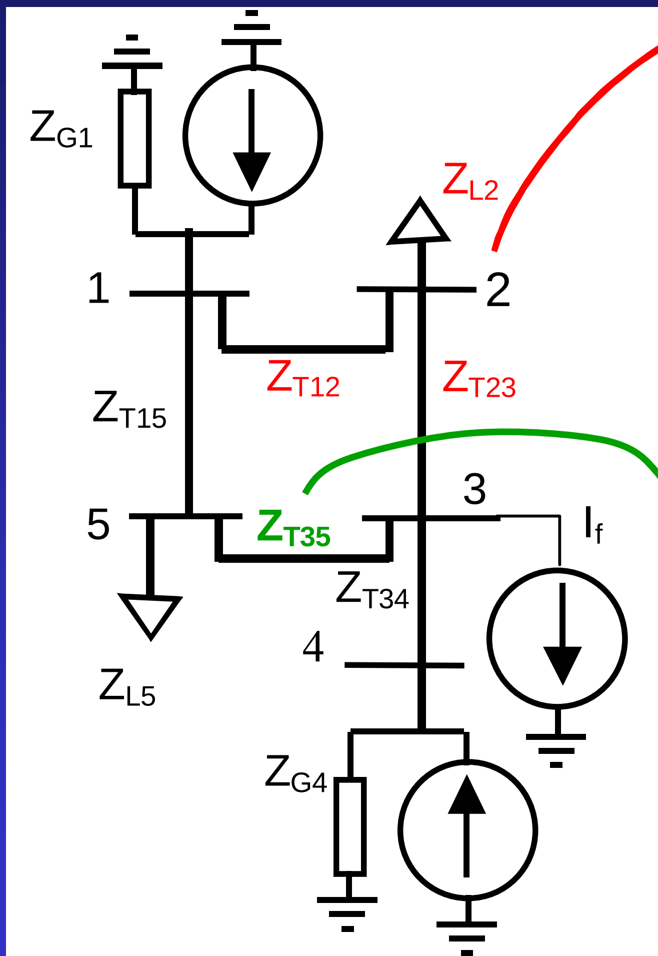
Bus Admittance Matrix: Y_{bus}

$$I = Y_{bus}\, V$$

vector of current injections

vector of bus voltages





Diagonals:

Σ of Y connected at node

$Y_{22} = 1/Z_{T12} + 1/Z_{L2} + 1/Z_{T23}$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & 0 & 0 \\ 0 & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ 0 & 0 & Y_{43} & Y_{44} & 0 \\ Y_{51} & 0 & Y_{53} & 0 & Y_{55} \end{bmatrix}$$

Off diagonals:

- of Y connected between nodes

$Y_{53} = -1/Z_{T35}$

Y_{bus} is sparse (lots of zero)

- small size
- easy to build
- quick algorithms to manipulate

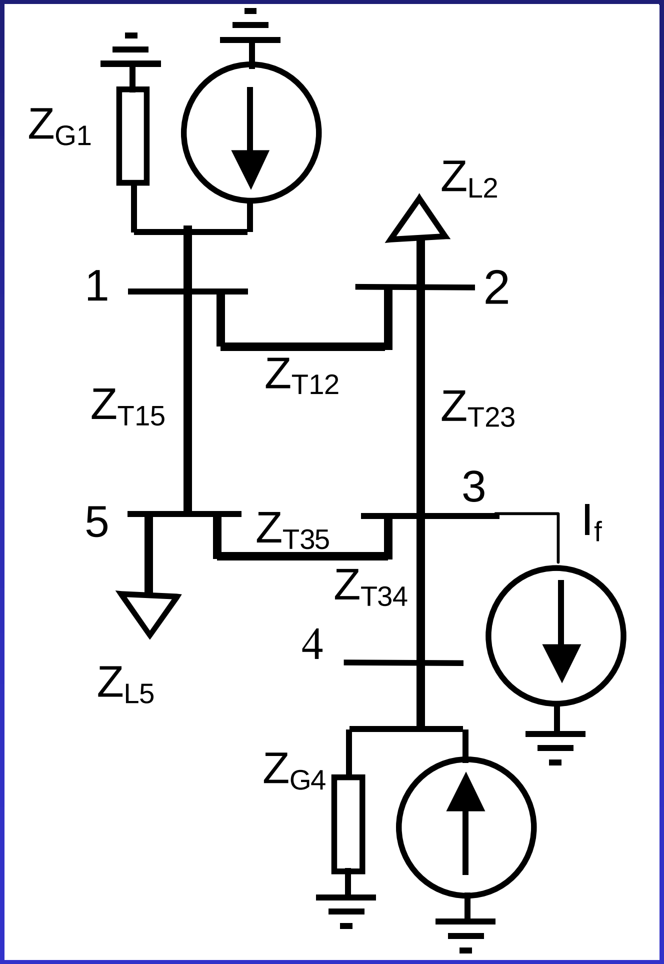
Bus Impedance Matrix: Z_{bus}

$$V = Z_{bus}\ I$$

vector of bus voltages

vector of current injections

$$Z_{bus} = Y_{bus}^{-1}$$



$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{bmatrix}$$

Z_{bus} is a full matrix

- slow to invert
- slow to manipulate
- avoid using if possible

Diagonal element is the Thevenin Impedance at that bus

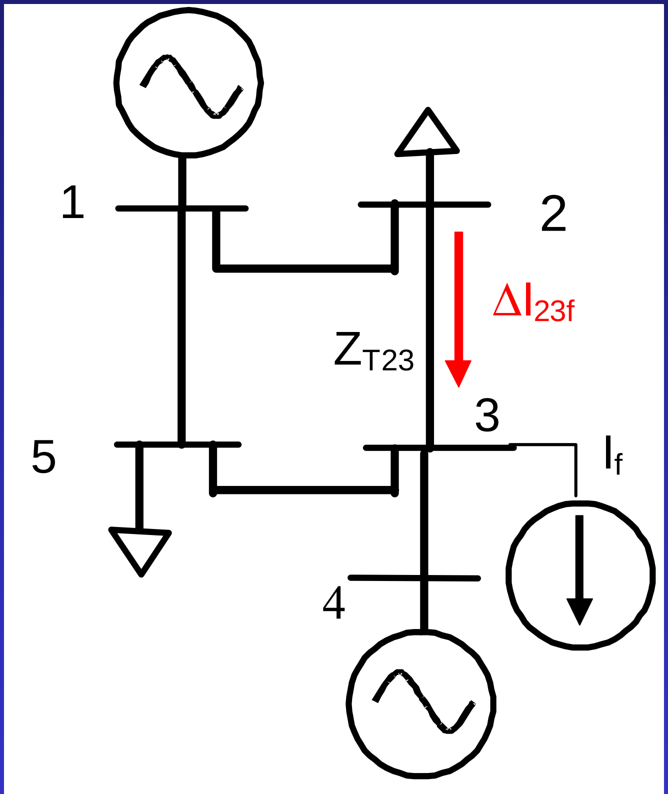
- Z_{33} is the Thevenin Impedance at bus 3

To find the change in voltages due to the fault current at bus 3:

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \\ \Delta V_5 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -I_f \\ 0 \\ 0 \end{bmatrix}$$

To calculate current in line 2-3:

$$\Delta I_{23f} = \frac{\Delta V_2 - \Delta V_3}{Z_{T23}}$$



Asymmetry ratio: S

- Method calculates the RMS worst case of the 60 Hz component
  - this is needed to set protective relays
- For circuit breaker selection, need to know the total RMS current (60 Hz and dc offset)
  - Circuit breakers are rated on symmetrical fault current, assuming an X/R ratio of 15

Asymmetry ratio

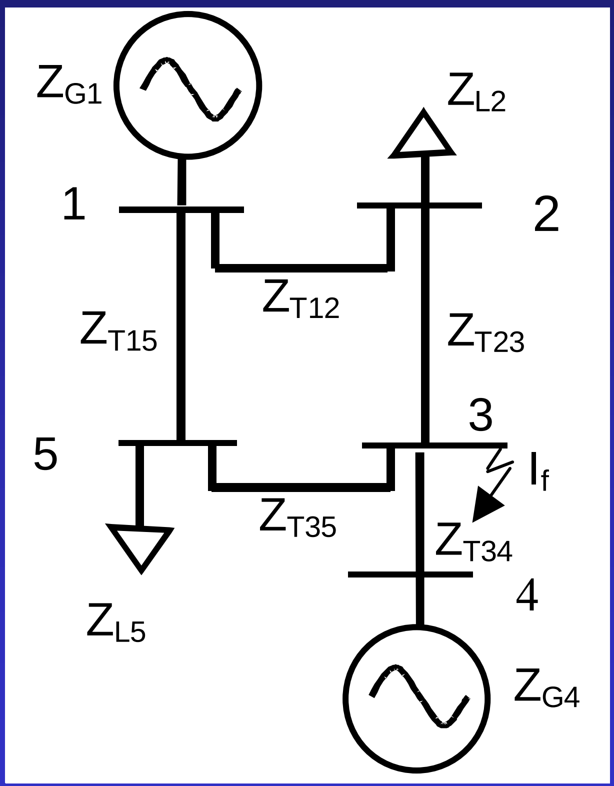
$$I_{RMS} = \sqrt{I_{ac}^2 + I_{dc}^2}$$

$$I_{RMS} = \sqrt{I_{ac}^2 + \left(\sqrt{2} I_{ac} e^{-\omega t / X/R}\right)^2}$$

$$I_{RMS} = I_{ac}\sqrt{1 + 2 \cdot e^{-2\omega t / X/R}}$$

$$\frac{I_{RMS}}{I_{ac}} = S(t) \text{ where } S(t) \text{ is } \sqrt{1 + 2 \cdot e^{-2\omega t / X/R}}$$

Calculating X/R



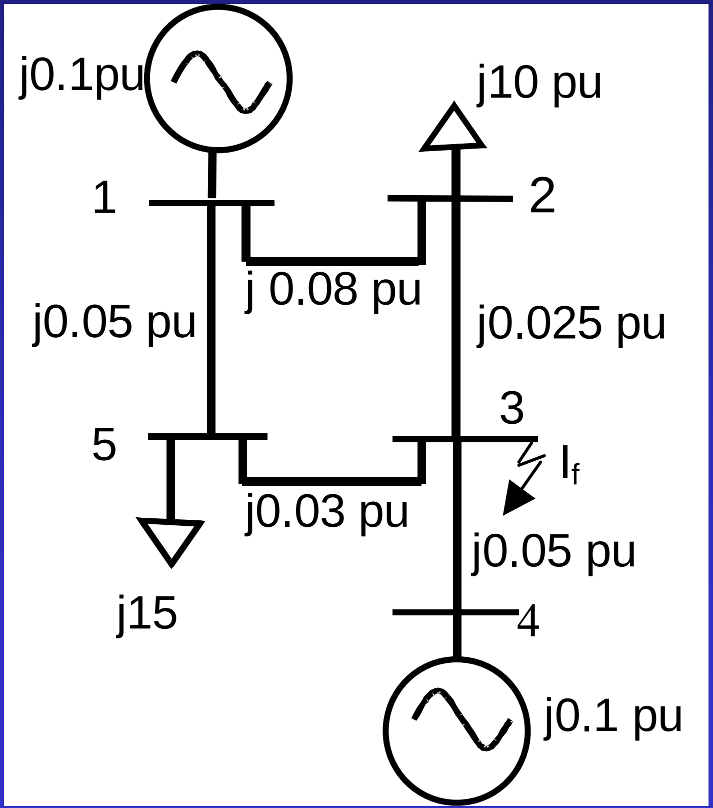
- Equations presented are for a single order system
- For higher order systems,
  - ignore R and find equivalent X,
  - ignore X, and find equivalent R
- Actual decay is typically quicker than this approximation

Algorithm

- Calculate Thevenin Impedance
- Calculate current into fault
- Find the needed elements of Z_{bus} (not all of them)
- Find the needed changes in bus voltages
- Find the line currents of of interest
- Find the dc offset

Example

- The oneline drawing of a system, along with Z_{bus} are shown below.



i) Calculate the fault current for a three phase solid fault at bus 3.

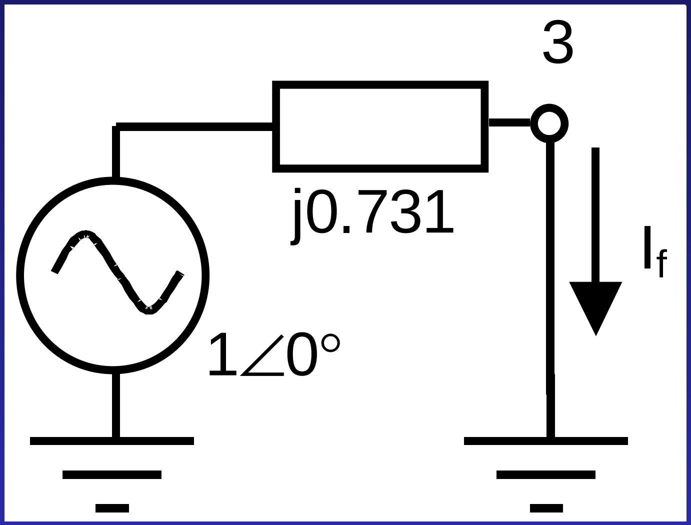
ii) Calculate the voltages at bus 1, 2, and 3 during this fault.

iii) Calculate the current flowing from bus 2 to bus 3.

$$Z_{bus} = j\begin{bmatrix} 0.0656 & 0.0538 & 0.0502 & 0.0334 & 0.0559 \\ 0.0538 & 0.0831 & 0.0675 & 0.0450 & 0.0623 \\ 0.0502 & 0.0675 & 0.0731 & 0.0487 & 0.0644 \\ 0.0334 & 0.0450 & 0.0487 & 0.0658 & 0.0429 \\ 0.0559 & 0.0623 & 0.0644 & 0.0429 & 0.0799 \end{bmatrix} pu$$

Part (i): Calculate the fault current for a three phase solid fault at bus 3.

- Assume all initial bus voltages are $1\angle 0°$
  - therefore all initial currents are 0
- Use the 3,3 element of Z_{bus} for the Thevenin impedance.



$$I_f = \frac{1}{j0.0731}$$

$$I_f = -j13.68\,pu$$

Part (ii): Calculate the voltages at bus 2, 3, and 4 during this fault.

Multiply the negative of the fault current by elements in the 3rd column to get the change in voltages

$$\begin{bmatrix} -0.687 \\ -0.923 \\ -1 \\ \Delta V_4 \\ \Delta V_5 \end{bmatrix} = j \begin{bmatrix} 0.0656 & 0.0538 & 0.0502 & 0.0334 & 0.0559 \\ 0.0538 & 0.0831 & 0.0675 & 0.0450 & 0.0623 \\ 0.0502 & 0.0675 & 0.0731 & 0.0487 & 0.0644 \\ 0.0334 & 0.0450 & 0.0487 & 0.0658 & 0.0429 \\ 0.0559 & 0.0623 & 0.0644 & 0.0429 & 0.0799 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ --j13.68 \\ 0 \\ 0 \end{bmatrix}$$

Part (ii): Calculate the voltages at bus 2, 3, and 4 during this fault. (continued)

The voltages during the fault are the initial voltages minus the changes in the voltages

$$V_{1f} = V_{1i} - \Delta V_1 = 1\angle 0° - 0.687 = 0.313 pu$$

$$V_{2f} = V_{2i} - \Delta V_2 = 1\angle 0° - 0.923 = 0.0766 pu$$

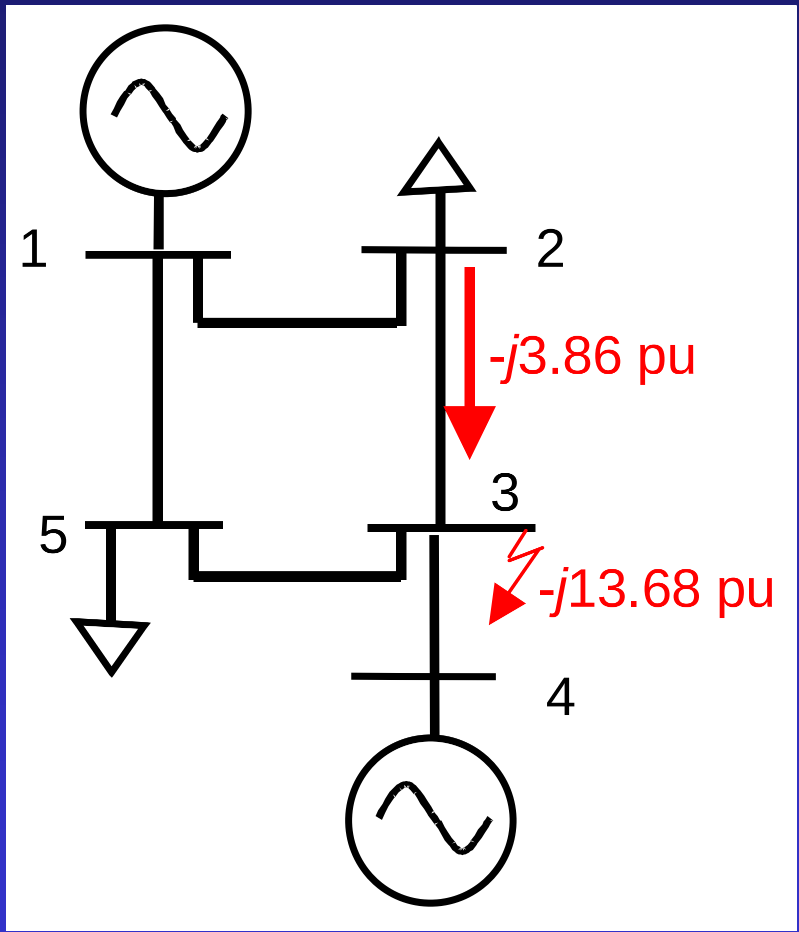
$$V_{3f} = V_{3i} - \Delta V_3 = 1\angle 0° - 1 = 0 pu$$

Part (iii): Calculate the current flowing from bus 2 to bus 3.

The current is the difference between the voltages divided by the impedance

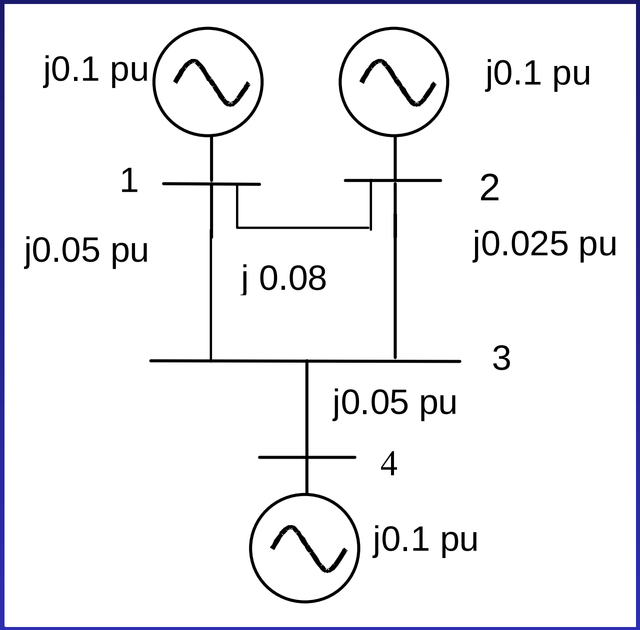
$$I_{23f} = \frac{V_{2f} - V_{3f}}{Z_{T23}}$$

$$I_{23f} = \frac{0.0766 - 0}{j0.025} = -j3.86pu$$



Problem

A small power system is sketched below along with Z_{bus}.



$$Z_{bus} = j\begin{bmatrix} 4.783 & 3.044 & 3.261 & 2.174 \\ 3.044 & 4.534 & 3.634 & 2.422 \\ 3.261 & 3.634 & 4.658 & 3.106 \\ 2.174 & 2.422 & 3.106 & 5.404 \end{bmatrix} \times 10^{-2}\ pu$$

a) What is the fault current for a three-phase fault to ground at bus 3?

b) What are the voltages at busses 2, 3, and 4?

c) What is the current due to the fault from bus 4 to 3 and from 2 to 3?

Unsymmetrical Faults

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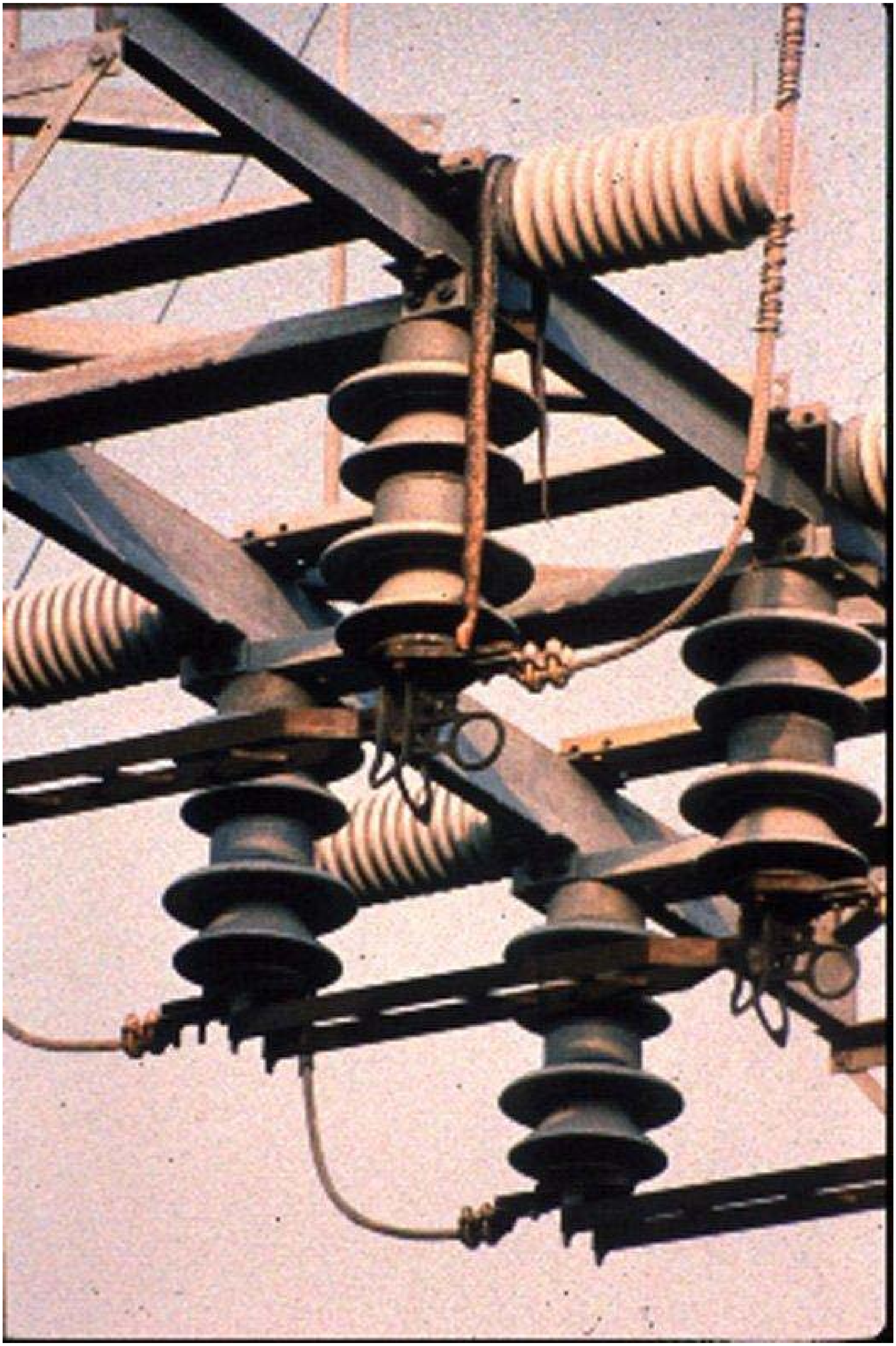

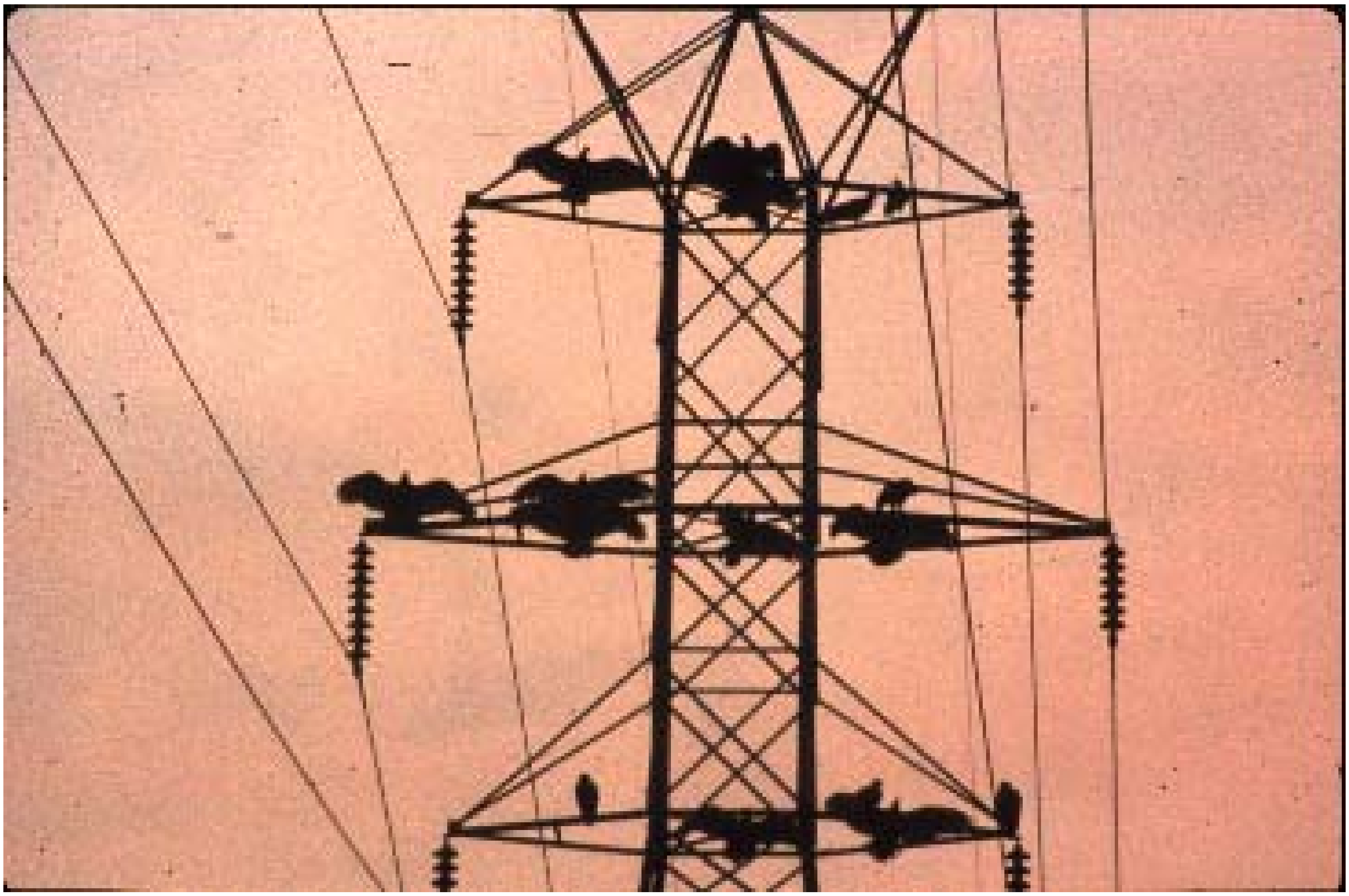


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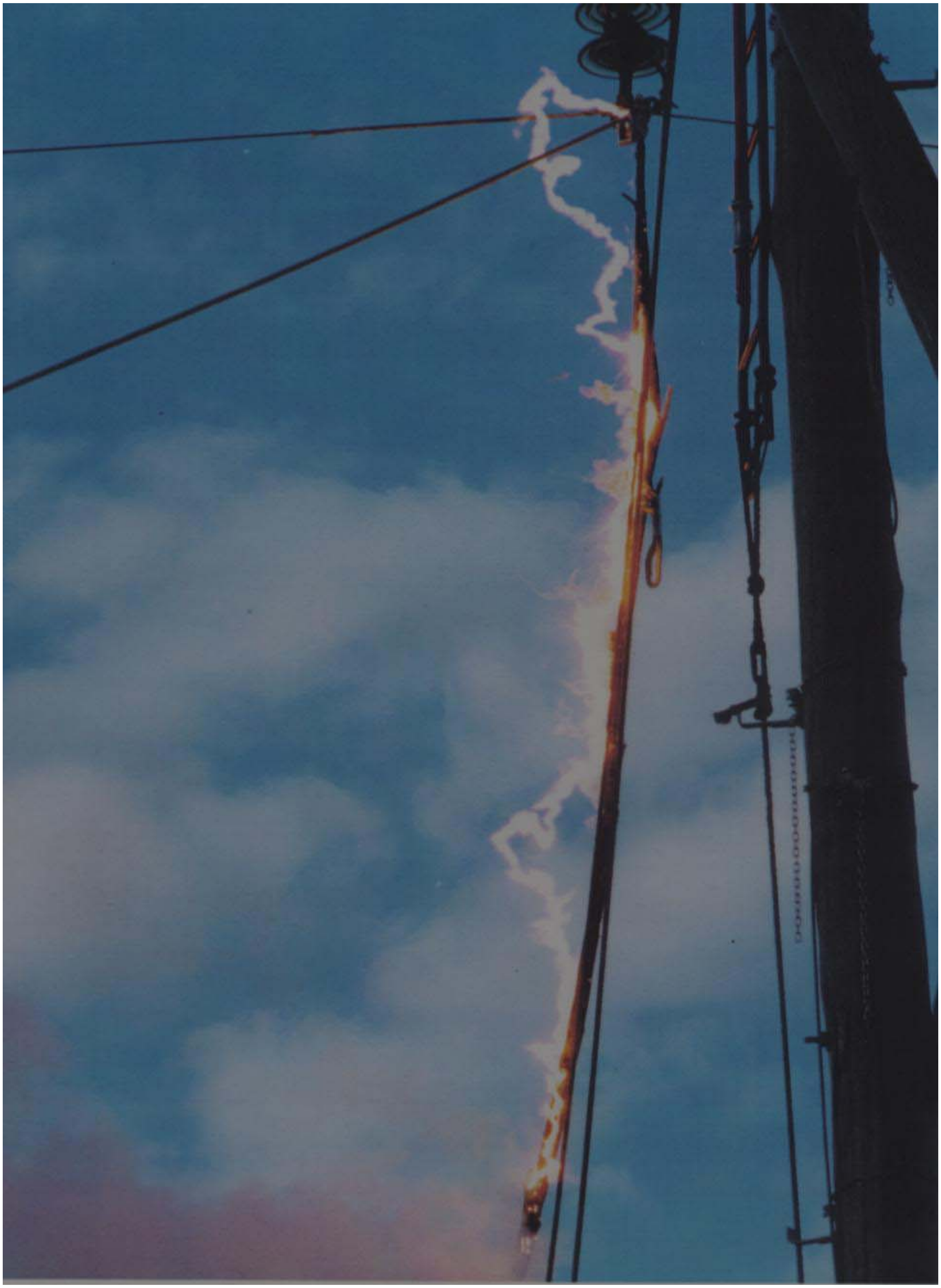


SINGLE LINE TO GROUND FAULT

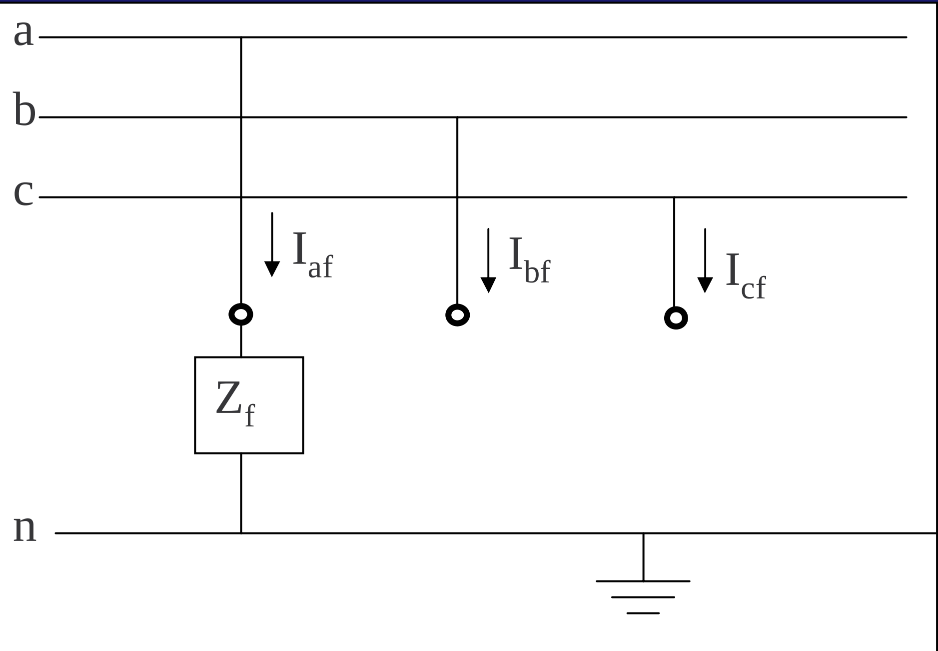


Fig.1. Single line to ground fault on phase A

Boundary Conditions at fault location

$$I_{bf} = I_{cf} = 0$$

$$V_f = Z_f * I_{af}$$

From the current equations,

$$I_0 + a^2 I_1 + aI_2 = I_0 + aI_1 + a^2 I_2 = 0$$

$$(a^2 - a)I_1 = (a^2 - a)I_2$$

$$I_1 = I_2$$

Thus $I_0 + (a^2 + a) I_1 = 0$

$$I_0 = I_1 = I_2$$

Now $V_a = Z_f * I_{af}$

Or, $V_0 + V_1 + V_2 = Z_f * 3I_0$

SINGLE LINE TO GROUND FAULT (CONTD.)

To satisfy the voltage and current boundary conditions, the sequence networks should b connected in series as shown in Fig. 2.

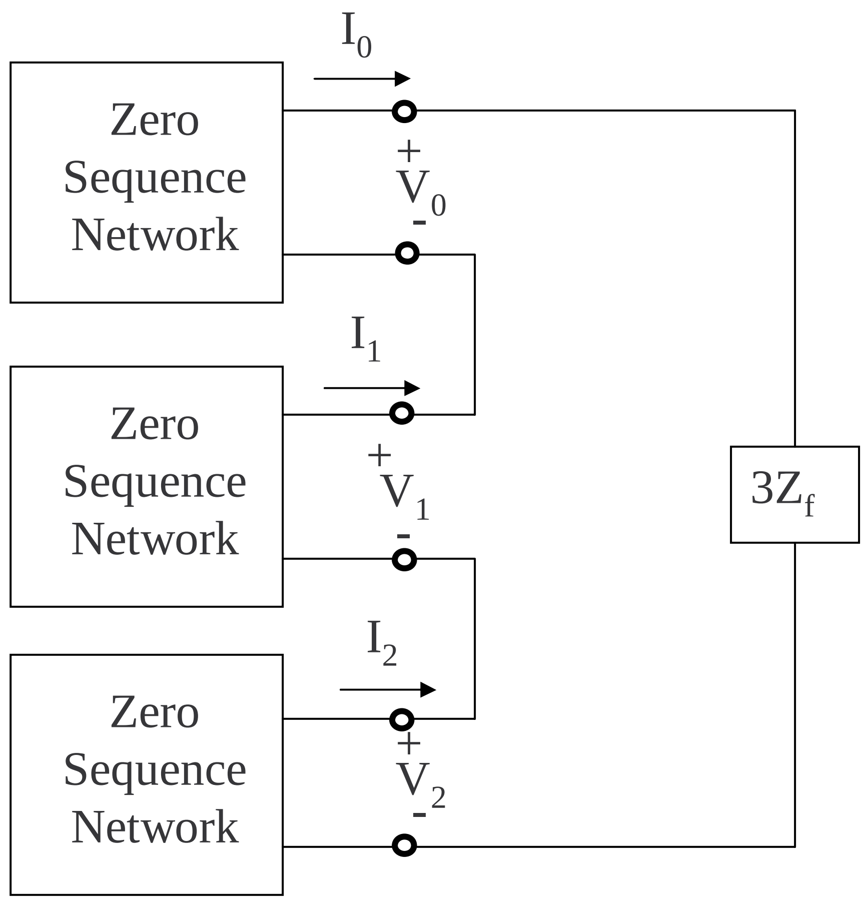


Fig.2. Sequence connection for A-G fault

LINE TO LINE FAULT

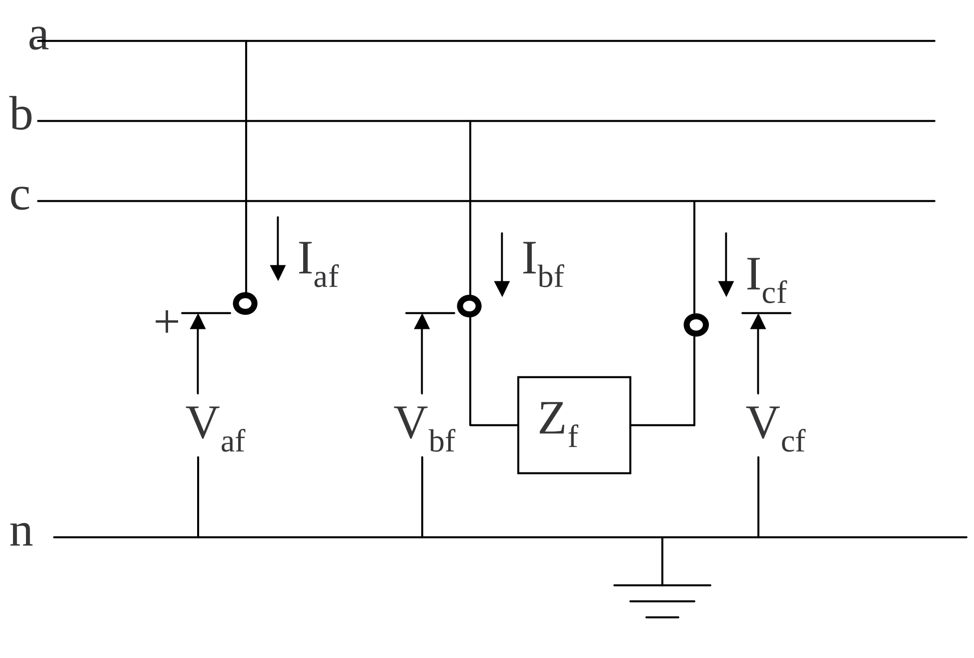


Fig.3. Line to line fault

Current boundary Conditions

$$I_{af} = 0$$

$$I_{bf} = - I_{cf}$$

Thus,

$$I_0 = 0$$

$$a^2 I_1 + aI_2 + aI_1 + a^2 I_2 = 0$$

$$(a^2 + a)I_1 = -(a^2 + a)I_2$$

$$I_1 = - I_2$$

Voltage boundary conditions

$$V_{bf} = V_{cf} + Z_f \; I_{bf}$$

$$V_0 + a^2 V_1 + aV_2 = Z_f (I_0 + a^2 I_1 + aI_2)$$

$$+ V_0 + aV_1 + a^2 V_2$$

This leads to

$$(a^2 - a)V_1 = (a^2 - a)I_1 \; Z_f + (a^2 - a)V_2$$

$$V_1 = V_2 + Z_f \; I_1$$

LINE TO LINE FAULT (CONTD.)

To satisfy the voltage and current boundary conditions, the sequence networks should be connected as shown in Fig. 4.

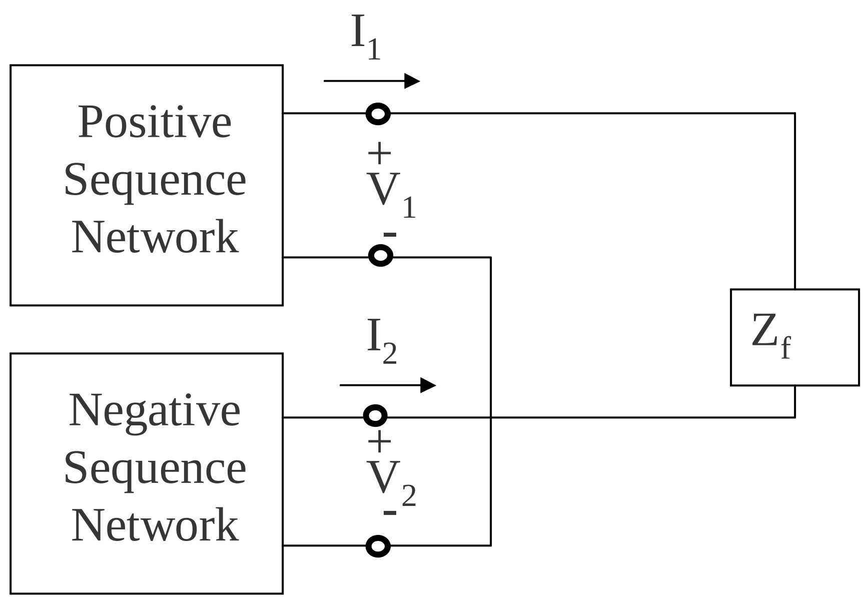


Fig.4. Connection of sequence network for BC fault

DOUBLE LINE TO GROUND FAULT

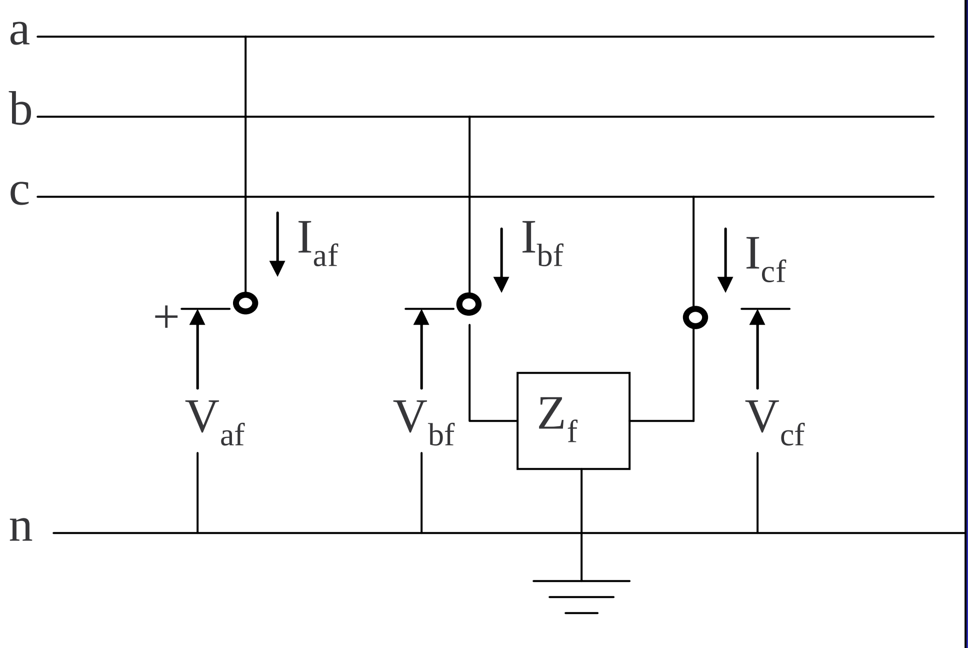


Fig.5. Double line to ground fault

Current boundary Conditions at the fault location,

$I_{af} = 0$

Thus,

$I_0 + I_1 + I_2 = 0$

Voltage boundary conditions

$V_{bf} = (I_{bf} + I_{cf})Z_f$

and,

$V_{bf} = V_{cf}$

Thus,

$V_0 + a^2V_1 + aV_2 = V_0 + aV_1 + a^2V_2$

Thus $V_1 = V_2$

Also,

$V_0 + a^2V_1 + aV_2 = (2I_0 + a^2I_1 + aI_2 + aI_1 + a^2I_2)\ Z_f$

Thus $V_0 - V_1 = 3Z_f I_0$

DOUBLE LINE TO GROUND FAULT (CONTD.)

Considering the voltage and current boundary conditions, the sequence networks are connected as shown in Fig. 6.

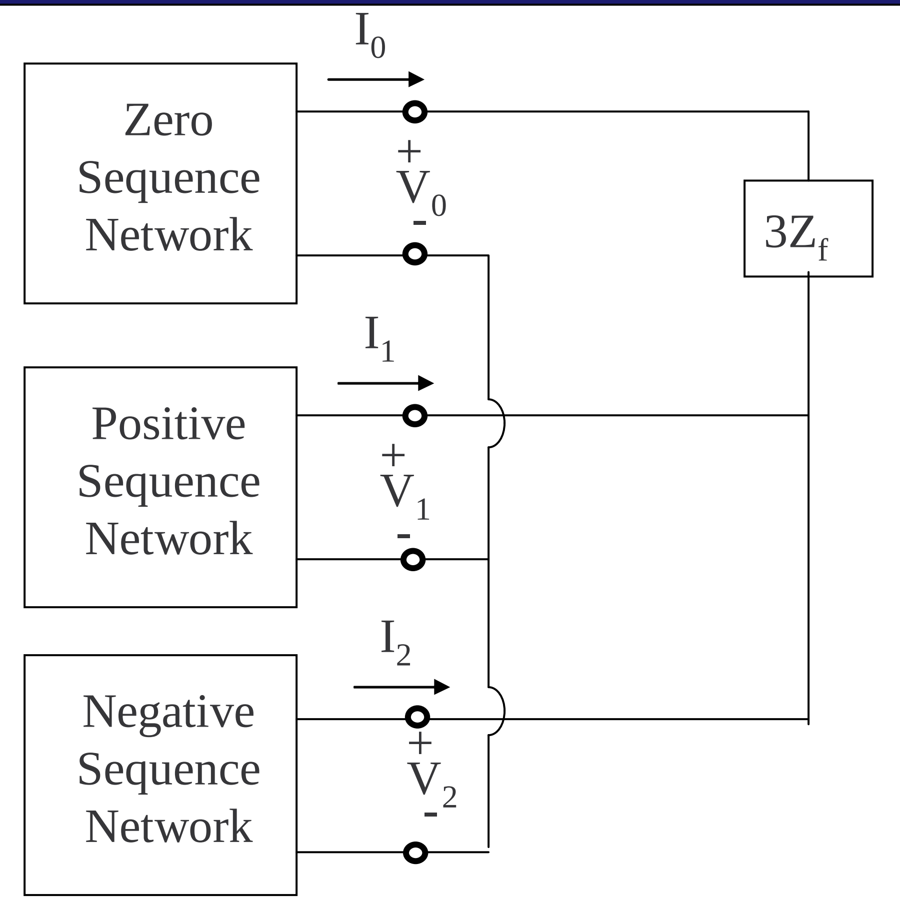


Fig.6. Sequence network connection for double line to ground fault

Example 1

Consider the power system shown in Fig. 7.

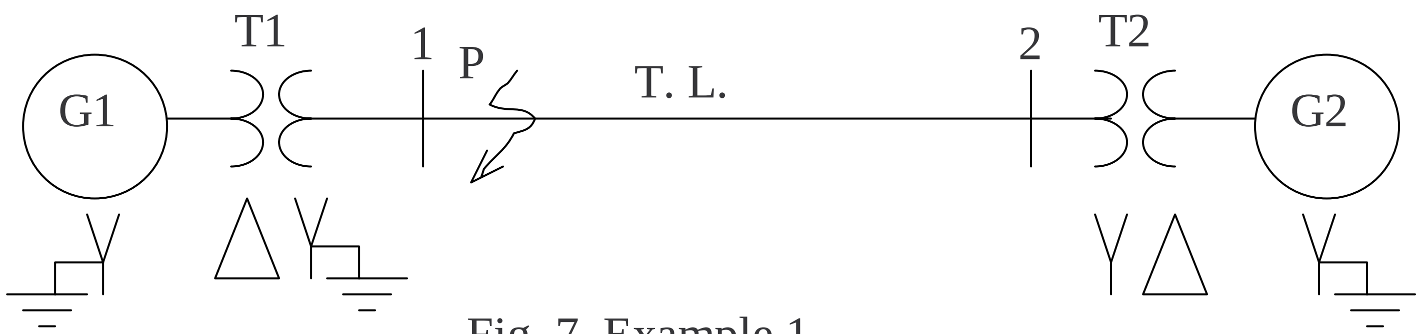


Fig. 7. Example 1

Ratings

G1 and G2: 100 MVA, 20 kV

$X_1 = X_2 = 0.3$ pu

$X_0 = 0.04$ pu

T_1 and T_2: 100 MVA, 345/20 kV

$X_{T1} = X_{T2} = 0.08$ pu

T. L. $X_1 = X_2 = 0.15$ pu

$X_0 = 0.5$ pu

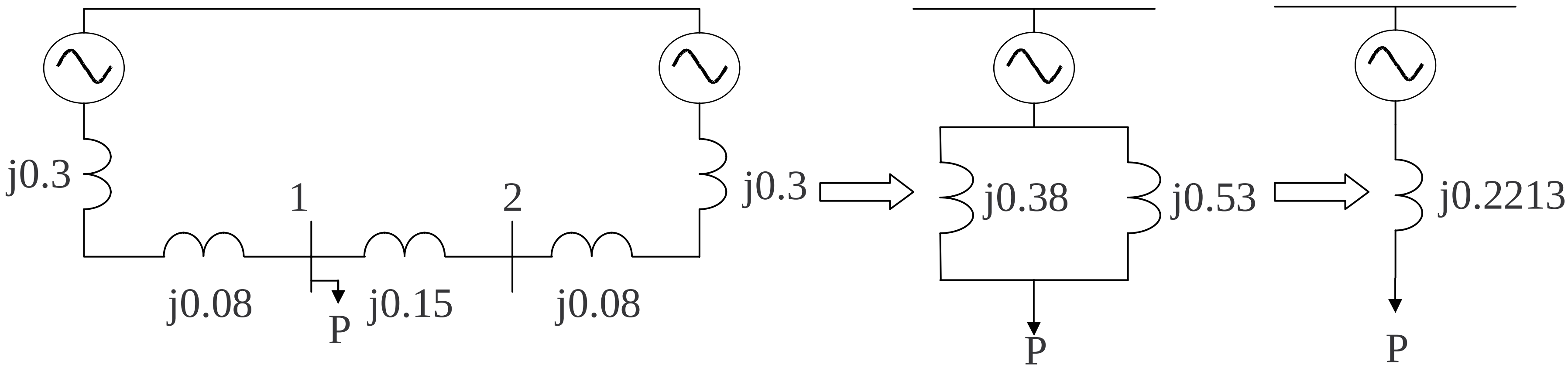
Find the voltages at Bus 1 and the currents from T_1 to P due to

(i) Single line to ground fault at P

(ii) Line to line fault at P

(iii) Double line to ground fault at P

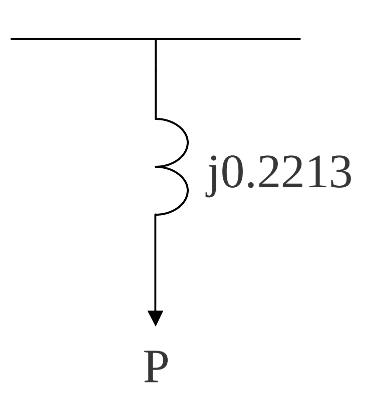
Positive sequence network



If $E_1 = E_2 = V = 1\angle 0°$

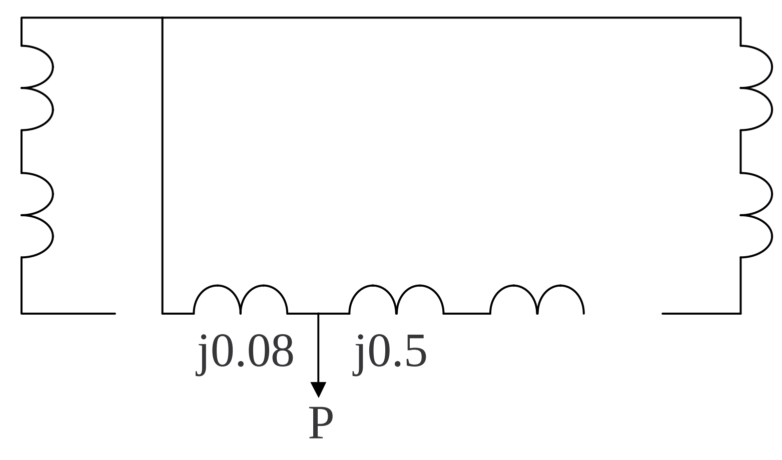
$Z_{th} = (0.3 + j0.8) \| (j0.3 + j0.08 + j0.15) = j0.2213$

Negative sequence network



$Z_{1th} = j0.2213$

Zero sequence network



$Z_{0th} = j0.08$

(i) Single line to ground fault

$$I_1 = I_2 = I_0 = \frac{E}{(Z_{0th} + Z_{1th} + Z_{2th})} \frac{1\angle 0^\circ}{(j0.2213 + j0.2213 + j0.08)} = -j1.9134$$

The following are the sequence current components from T1 --> P

$$I_1 = I_1 * \frac{0.53}{(0.53 + 0.38)} = -j1.114$$

$$I_2 = -j1.114$$

$$I_0 = -j1.9134$$

The sequence voltage components at the fault location are:

$$V_1 = E_1 - I_1 Z_1 = 1.0 - (-j1.114)(j0.38) = 0.57668$$

$$V_2 = -I_2 Z_2 = -(j1.114)(j0.38) = -0.42332$$

$$V_0 = -I_0 Z_0 = -j1.913(j0.8) = -0.15304$$

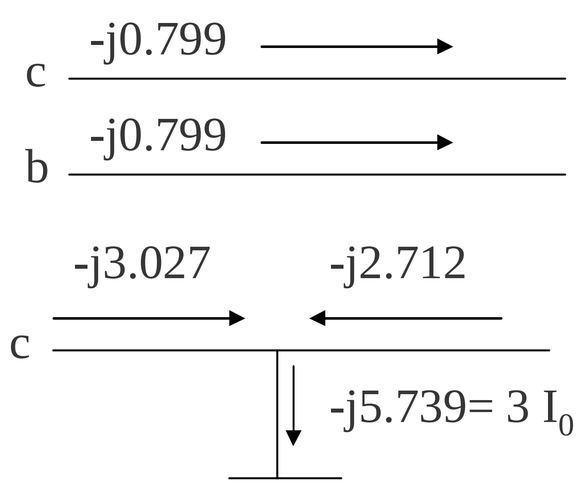
The current phase components from T1 --> P

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} * \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_a = -j1.114 - j1.114 - j1.913 = -j3.027$$

$$I_b = a^2(-j1.114) + a(-j1.114) - j1.913 = -j0.799$$

$$I_c = a(-j1.114) + a^2(-j1.114) - j1.913 = -j0.799$$



The phase voltages at the fault location are:

$$V_a = V_0 + V_1 + V_2 = -0.15304 + 0.57668 - 0.42332 = 0$$

$$V_b = V_0 + a^2V_1 + aV_2 = -0.15304 + (1\angle 240°)(0.57668) + (1\angle 120°)(-0.42332)$$
$$= -0.2297 - j0.866$$

$$V_c = V_0 + aV_1 + a^2V_2 = -0.15304 + (1\angle 120°)(0.57668) + (1\angle 240°)(-0.42332)$$
$$= -0.2297 + j0.866$$

(ii) Line to line fault

Currents:

$$I_1 = -I_2 = \frac{E}{(Z_{1th} + Z_{2th})} = \frac{1\angle 0°}{(j0.2213 + j0.2213)} = -j2.26$$

$$I_1(T1 --> P) = -j2.26 \frac{0.53}{(0.38 + 0.53)} = -j1.316$$

$$I_2(T1 --> P) = j1.316$$

$$I_a = 0$$

$$I_b = a^2(-j1.316) + a(j1.316) = 2.279$$

$$I_c = a\,(-j1.316) + a^2(j1.316) = 2.279$$

Voltages:

$$V_{a1} = E - I_{a1}Z_1 = 1.0 - (-j1.316)(j0.38) = 0.5$$

$$V_{a2} = -I_{a2}Z_2 = 0.5$$

$$V_a = 1\angle 0°$$

$$V_b = -0.5\angle 0°$$

$$V_c = 0.5\angle 0°$$

SEQUENCE PHASE SHIFT THROUGH WYE/DELTA TRANSFORMER BANK

- Positive and negative sequence impedance is independent of connection
- These networks ignore phase shift
- When current and voltages are transformed from one side to the other phase shift must be considered.
- Standard ANSI connection has been shown in figs. 8 (a) and 8 (b).

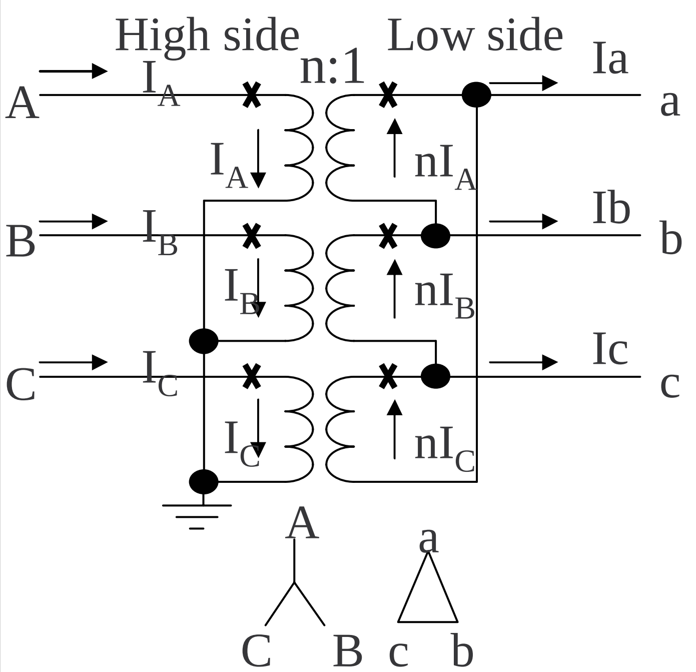


Fig. 8 (a) Wye/Delta

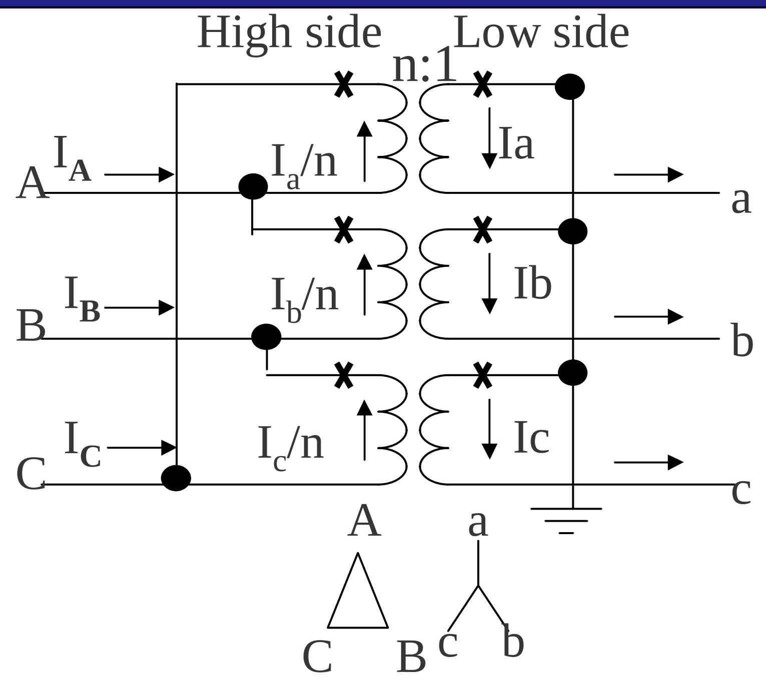


Fig. 8 (b) Delta/Wye

Refer to fig. 8(a)

Positive sequence current :

$$I_{a1} = n(I_{A1} - I_{C1}) = n(I_{A1} - aI_{A1}) = nI_{A1}(1 - a) = n\sqrt{3}\, I_{A1}\angle -30^\circ$$

In per unit,

$$I_{A1} = I_{a1}\angle 30^\circ$$

Positive sequence volatge :

$$V_{A1} = n(V_{a1} - a^2V_{a1}) = nV_a(1 - a^2) = n\sqrt{3}\, V_{a1}\angle 30^\circ$$

or in per unit,

$$V_{A1} = V_{a1}\angle 30^\circ$$

Negative sequence current :

$$I_{a2} = n(I_{A2} - I_{C2}) = n(I_{A2} - a^2I_{A2}) = nI_{A2}(1 - a^2) = n\sqrt{3}\, I_{A2}\angle 30^\circ$$

or In per unit

$$I_{A2} = I_{a2}\angle -30^\circ$$

Negative sequence voltage :

$$V_{A2} = n(V_{a2} - V_{b2}) = n(V_{a2} - aV_{a2}) = nV_{a2}(1 - a) = n\sqrt{3}\, V_{a2}\angle -30^\circ$$

or In per unit

$$V_{A2} = V_{a2}\angle -30^\circ$$

Refer to fig. 8 (b)

Postitive sequence current :

$$I_{A1} = \frac{1}{n}(I_{a1} - I_{b1}) = \frac{1}{n}(I_{a1} - a^2 I_{a1}) = \frac{\sqrt{3}}{n} I_{a1}\angle 30°$$

or, in per unit,

$$I_{A1} = I_{a1}\angle 30°$$

Postitive sequence voltage :

$$V_{a1} = \frac{1}{n}(V_{A1} - aV_{A1}) = \frac{\sqrt{3}}{n} V_{A1}\angle -30°$$

or, in per unit,

$$V_{A1} = V_{a1}\angle 30°$$

Negative sequence current :

$$I_{A2} = \frac{1}{n}(I_{a2} - aI_{a2}) = \frac{\sqrt{3}}{n} I_{a2}\angle -30°$$

or In per unit

$$I_{A2} = I_{a2}\angle -30°$$

Negative sequence voltage

$$V_{A2} = V_{a2}\angle -30°$$

In general, in Y - Δ or Δ - Y connections:

(i) Positive sequence High voltage = Positive sequence Low voltage $\angle +30°$

(ii) Negative sequence High voltage = Negative sequence Low voltage $\angle -30°$

Example 2

Find the currents and voltages at G1 of Example 1, due to single line to ground fault at P.

(i) Currents:

$$I_{ag1} = -j1.114\angle -30° = -j1.114\angle -120°$$

$$I_{ag2} = -j1.114\angle +30° = j1.114\angle -60°$$

$$I_{ag0} = 0$$

$$I_{ag} = I_{ag1} + I_{ag2} = -j1.114\angle -120° + j1.114\angle -60° = -j1.9295$$

$$I_{bg} = a^2 I_{ag1} + aI_{ag2} = 1\angle 240°(1.114\angle -120°) + 1\angle 120°(1.114\angle -60°)$$

$$= (1.114\angle 120°) + (1.114\angle 60°) = j1.9295$$

$$I_{cg} = 0$$

Example 2 (contd.)

(ii) Voltages :

$$V_{ag1} = E - I_{a1} Z_{g1} = 1.0 - (-j1.114)(j0.3)$$

$$= 0.666 \text{ (from sequence Network neglecting phase shift)}$$

$$V_{ag2} = - I_{a2} Z_{g2}$$

$$= -0.3342 \text{ (from sequence Network neglecting phase shift)}$$

$$V_{ag1} = 0.666 \angle -30°$$

$$V_{ag2} = -0.334 \angle 30°$$

$$V_{ag} = V_{ag1} + V_{ag2} = 0.666 \angle -30° - 0.334 \angle 30° = 0.577 \angle -60°$$

$$V_{bg} = a^2 V_{ag1} + aV_{ag2} = 0.666 \angle 210° - 0.334 \angle 150°$$

$$= 0.577 \angle 240.12°$$

$$V_{cg} = aV_{ag1} + a^2 V_{ag2} = 1.0 \angle 90°$$

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} * \begin{bmatrix} V_{ag0} \\ V_{ag1} \\ V_{ag2} \end{bmatrix} = \begin{bmatrix} 0.577 \angle -60° \\ 0.577 \angle 240.12° \\ 1 \angle 90° \end{bmatrix}$$

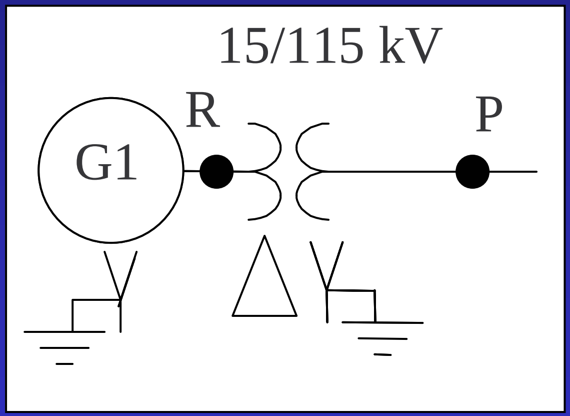
PROBLEMS

PROBLEM #1

For the system shown below, find the currents in the transformer and generator windings in per unit and amperes for a single-line-to-ground fault at

(i) P

(ii) R



G: 100 MVA, 15 kV

$X_1=X_2= 0.1$,

$X_0 = 0.05$

Transformer:

100 MVA, 15/115 kV

$X = 0.1$

PROBLEMS

PROBLEM #2

Solve examples 1 and 2 for a double-line-to-ground fault at P.

PROBLEMS

PROBLEM #3

Solve examples 1 and 2 for a line-line fault at P.