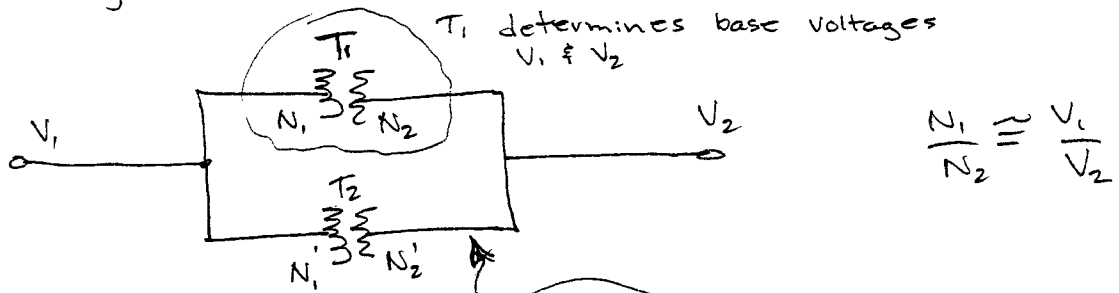
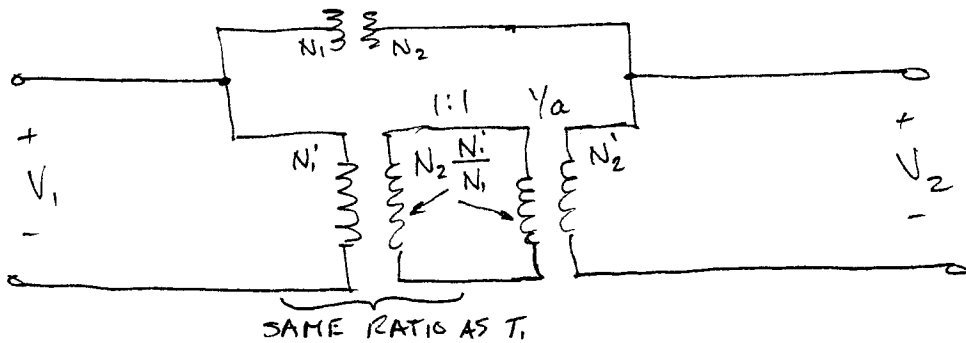


Study

Paralleling Transformers of Unlike Turns Ratio



What happens for $\frac{N_1'}{N_2'} \neq \frac{N_1}{N_2}$?



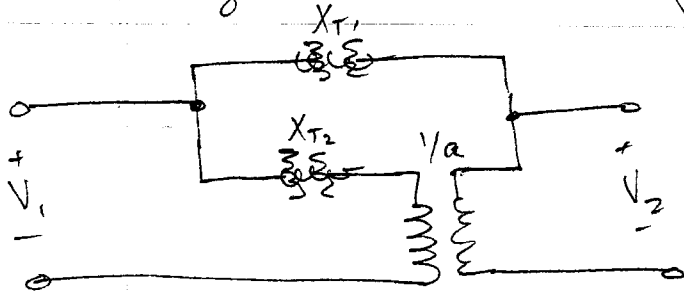
Replace T_2 with 2 XFMRs :- First is same ratio

as T_1 , $\frac{N_1}{N_2} = \frac{N_1'}{X}$

$X = N_2 \frac{N_1'}{N_1}$

Second XFMR has ratio of off-nominal turns

Per unit equivalent:



$$\frac{1}{a} = \frac{\left(N_2 \frac{N_1'}{N_1} \right)}{N_2'} = \frac{N_2 N_1'}{N_1 N_2'}$$

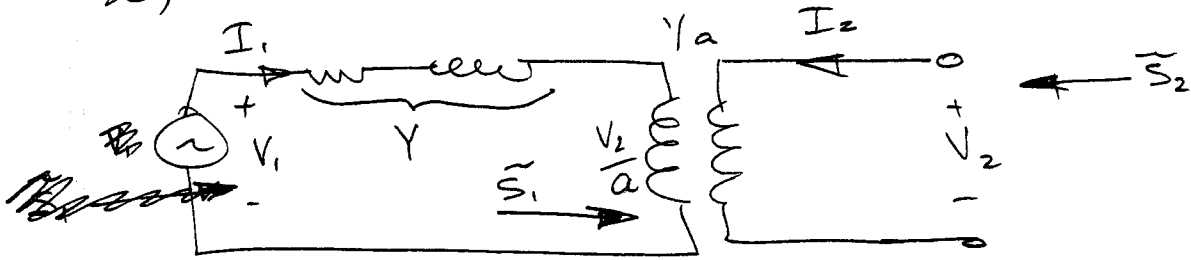
$$\therefore a = \frac{N_1 N_2'}{N_2 N_1'} = \text{p.u. turns ratio}$$

Three Methods to Analyze:

- 1) Admittance Method
- 2) Circuit Theory
- 3) Circulating Current Method (Approximate)

METHOD 1

So,



So we must find a way to model Y for Y_{12} as Y_a varies.



$$\tilde{S}_1 = -\tilde{S}_2$$

$$\hat{S}_1 = \frac{V_2}{a} I_1^*$$

$$S_2 = V_2 I_2^*$$

$$\frac{V_2}{a} I_1^* = -V_2 I_2^*$$

$$I_1^* = -a I_2^*$$

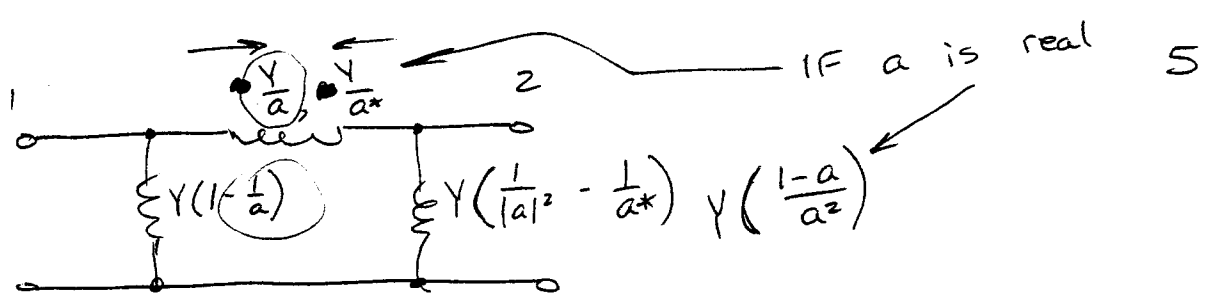
$$I_1 = -a^* I_2$$

$$I_1 = \left(V_1 - \frac{V_2}{a}\right) Y = Y V_1 - \frac{Y}{a} V_2 = -a^* I_2$$

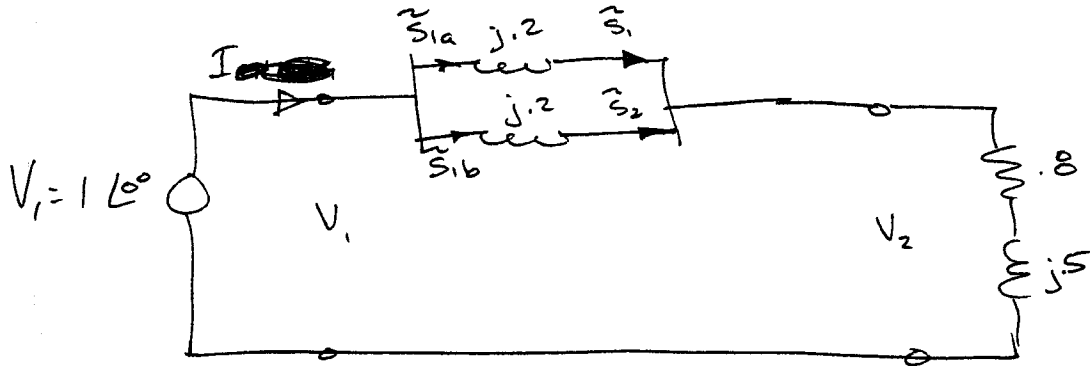
$$\therefore I_2 = -\frac{I_1}{a^*} = \frac{-Y V_1}{a^*} + \frac{Y}{a a^*} V_2$$

$$\therefore Y_{11} = Y \quad Y_{12} = -\frac{Y}{a}$$

$$Y_{21} = -\frac{Y}{a^*} \quad Y_{22} = \frac{Y}{a a^*} = \frac{Y}{|a|^2}$$



2) CIRCUIT THEORY APPROACH



$$V_2 = \frac{(.8 + j.5)(1 \angle 0^\circ)}{.8 + j.6} = .94 - j.08 = .9434 \angle -4.86^\circ$$

$$I_2 = \frac{.94 - j.08}{.8 + j.5} = 1 \angle -36.87^\circ$$

$$\tilde{S}_0 = VI^* = .9434 \angle -4.86^\circ (1 \angle 36.87^\circ) = .9434 \angle 32^\circ = .8 + j.5$$

$$\left. \begin{aligned} S_1 &= .4 + j.25 \\ S_2 &= .4 + j.25 \end{aligned} \right\} = \frac{S_{TOTAL}}{2}$$

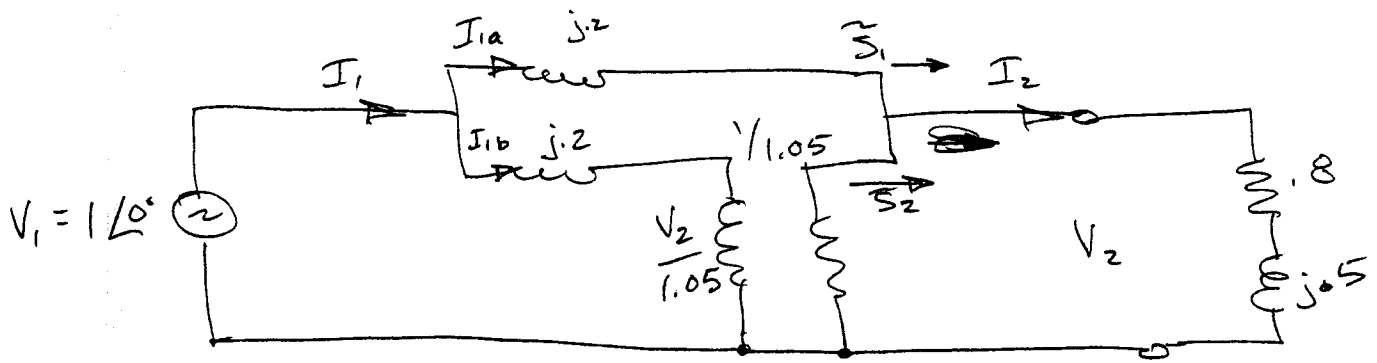
$$I_1 = \frac{1}{.8 + j.6} = 1(.8 - j.6) = .8 - j.6$$

$$S_{1a} = \frac{V_1 I_1^*}{2} = 4 + j.3$$

$$S_{1b} = \frac{V_1 I_1^*}{2} = 4 + j.3$$

Difference due to XFMR Inductance

Now add tap changer $a = 1.05$



$$\begin{cases} I_1 = I_{1a} + I_{1b} \\ I_1 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2} \\ I_2 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2} \left(\frac{1}{1.05} \right) \\ V_2 = I_2 (.8 + j.5) = \left[\frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2} \left(\frac{1}{1.05} \right) \right] (.8 + j.5) \end{cases}$$

$$V_2 = \left(1 - V_2 + \frac{1 - \frac{V_2}{1.05}}{1.05} \right) \left(\frac{.8 + j.5}{j.2} \right) = (1.952 - 1.907 V_2) \left(4.717 \angle -58^\circ \right)$$

$$V_2 = -1.907 V_2 (4.717 \angle -58^\circ) + (1.952) (4.717 \angle -58^\circ)$$

$$= -8.995 \angle -58^\circ V_2 + 9.207 \angle -58^\circ$$

$$V_2 = \frac{9.207 \angle -58^\circ}{1 + 8.995 \angle -58^\circ} = \frac{9.207 \angle -58^\circ}{9.563 \angle -52.9^\circ} = 963 \angle -5.1^\circ$$

$$I_{1a} = \frac{1 - V_2}{j.2} = 4730 \angle -25.54^\circ$$

$$I_{1b} = \frac{1 - \frac{V_2}{1.05}}{j.2} = 59 \angle -46.75^\circ$$

$$S_{1a} = V_1 I_{1a}^* = .427 + j.204$$

$$S_{1b} = V_1 I_{1b}^* = .4067 + j.4324$$

$$\tilde{S}_1 = V_2 I_{1a}^* = (.963 \angle -5.1^\circ) (.4732 \angle 25.536^\circ)$$

$$= .427 + j.16$$

7

~~Complex~~

$$\tilde{S}_2 = V_2 \frac{I_{1b}^*}{a} = .963 \angle -5.1^\circ \left(\frac{.59 \angle -46.75^\circ}{1.05} \right)^* = .406 + j.3616$$

$$\tilde{S} = V_2 I_2^* = (.963 \angle 5.1^\circ) \left(\frac{V_2}{2} \right)^* = .983 \angle 32^\circ = \tilde{S}_1 + \tilde{S}_2$$

	<u>Before Tc</u>		<u>After T.C.</u>	
			<u>Circ. Approx.</u>	<u>Ckt. Analysis</u>
V_1	1		1	1
V_2		.943 $\angle -4.86^\circ$.963 $\angle -4.87^\circ$.963 $\angle -5.1^\circ$
at V_2 {	P.	.4	.398	4.27
	r_2	.4	.418	.406
	Q_1	.25	.135	.159
	Q_2	.25	.375	.362

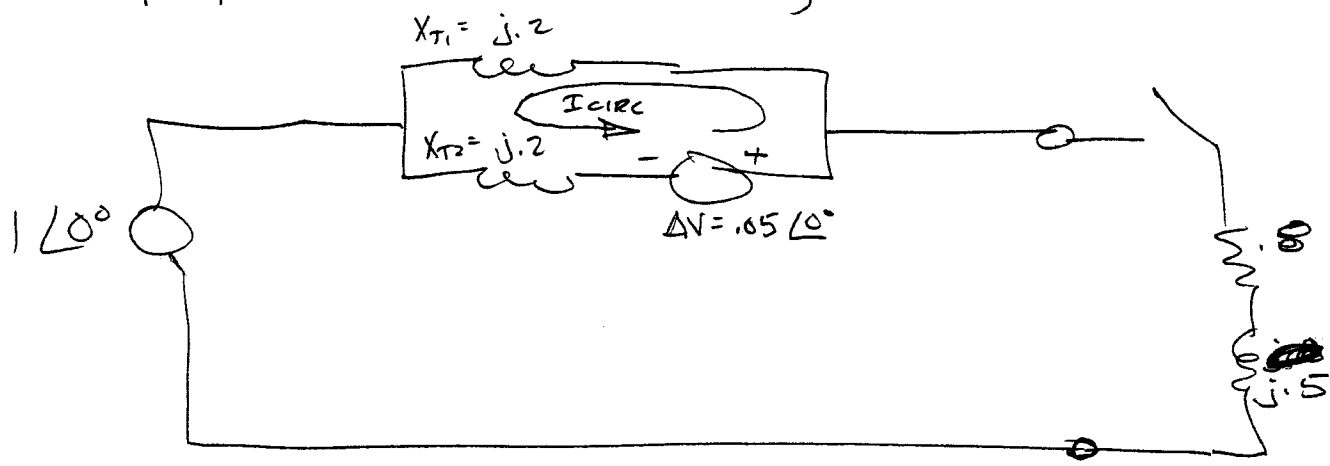
~~Q~~ ~~to~~ ~~XFMR~~ ~~with~~ ~~top~~ ~~tap~~ ~~to~~ ~~higher~~ ~~than~~ ~~rated~~ ~~secondary~~ ~~voltage.~~ ~~higher~~ ~~tap~~ ~~setting.~~

So:

Q is shifted to XFMR of higher tap setting.
 P is still divided almost evenly.

Approximate solution:

Superposition of circulating current



First, look at XFMR before load applied.

$$I_{CIRC} = \frac{.05 \angle 0^\circ}{j.4} = .125 \angle -90^\circ = -j.125$$

From first example,

$$I_{1a} = .4 - j.3 + j.125 = .4 - j.175$$

$$I_{1b} = .4 - j.3 - j.125 = .4 - j.425$$

~~$V_2 = V_1 - I_{1a} X_{T1}$~~

~~$V_2 = V_1 - (I_{1a} X_{T1})$~~

~~$= 1 - (.4 - j.175)(j.2) = .98 \angle -4.45^\circ$~~

~~$= 1 - (j.08 + .025) = .98 \angle -4.45^\circ$~~

By Superposition,

$$V_2 = V_2 \text{ BEFORE TAP CHANGE} + \frac{\Delta V (jX_{T1})}{jX_{T1} + jX_{T2}} \left[\frac{Z_{LOAD}}{Z_{TOTAL}} \right]$$

Voltage at Load due to ΔV

$$\Delta V_{oc} = \frac{\Delta V (jX_{T1})}{(jX_{T1} + jX_{T2})}$$

$$Z_{TH} = jX_{T1} \parallel jX_{T2}$$

$$= .943 \angle -4.86^\circ + \frac{.05 (j.2)}{j.4} \left[\frac{.8 + j.5}{.8 + j.6} \right]$$

$j.5 + (jX_{T1} \parallel jX_{T2})$

$$V_2 = .963 \angle -4.87^\circ$$

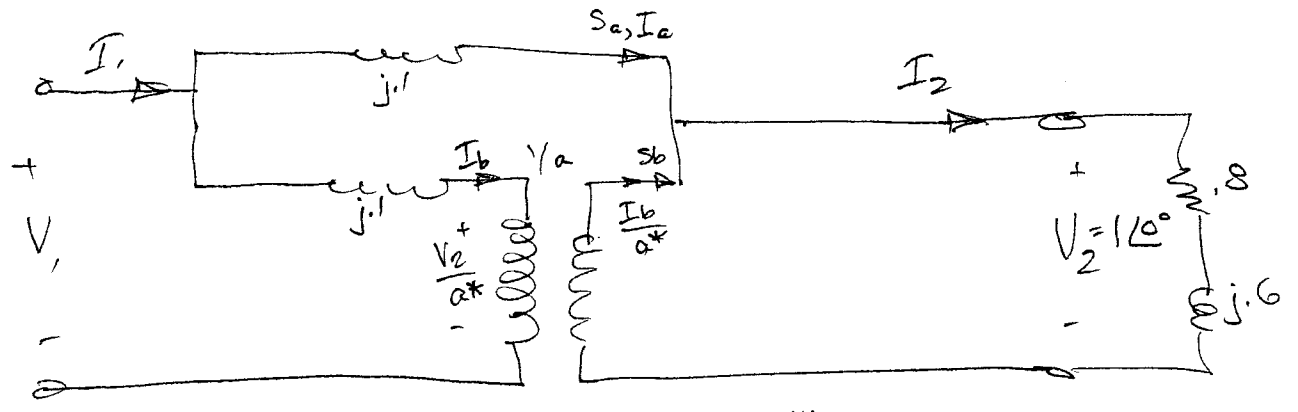
$$S_{1a} = V_2 I_{1a}^* = (.963 \angle -4.87^\circ) (.437 \angle 23.63^\circ)^* = .421 \angle 18.76^\circ$$

$$= .398 + j.135$$

$$S_{2a} = V_2 I_{2a}^* = (.963 \angle -4.87^\circ) (.584 \angle 46.73^\circ)^* = .562 \angle 41.86^\circ$$

$$= .418 + j.375$$

Phase Shifting XFMR



$$a = 1 \angle 30^\circ = e^{j\pi/6}$$

Without phase shifter,

$$P_{LOAD} = \frac{V_2^2}{Z^*} = 1 \angle 0^\circ / (.8 + j.6)^* = .8 + j.6$$

$$S_a = .4 + j.3$$

$$S_b = .4 + j.3$$

$$I_a = .4 - j.3$$

$$I_b = .4 - j.3$$

$$V_1 = 1 \angle 0^\circ + (.4 + j.3)(j.1)$$

$$= 1.6307 \angle 2.2^\circ$$

With Phase Shifter,

$$\frac{V_1 - 1}{j.1} + \frac{V_1 - \frac{1}{1 \angle 30^\circ}}{j.1} \left(\frac{1}{1 \angle 30^\circ} \right) = \frac{1}{.8 + j.6} = I_2$$

$$V_1 \left(1 + \frac{1}{1 \angle 30^\circ} \right) - 1 - \left(\frac{1}{1 \angle 30^\circ} \right)^2 = j.1 (.8 + j.6)$$

$$V_1 (1.999 \angle -1.5^\circ) - 1 - (1.999 \angle -1.5^\circ) = .1 \angle 53.13^\circ + 1.999 \angle -1.5^\circ$$

$$V_1 = 1.031 \angle 1.73^\circ = 1.031 + j.013$$

$$I_2 = \frac{1.028 + j.016}{j.1} = 1.028 \angle -60.3^\circ = .159 - j.28$$

$$I_a = \frac{1.031 + j.013 - 1}{j.1} = .131 - j.311$$

$$\frac{I_b}{a^*} = I_2 - I_a = .8 - j.6 - .131 + j.311 = .669 - j.289$$

$$S_a = .131 + j.311$$

$$S_b = .669 + j.289$$



NOTE SHIFT IN POWER FLOW THRU XFMR

BEFORE

AFTER

I_a	$.4 + j.3$	$.131 - j.311$
I_b/a^*	$.4 - j.3$	$.669 - j.289$
P_a	.4	.131
P_b	.4	.669
Q_a	+1.3	.311
Q_b	+1.3	.289
V_1	$1.031 \angle 2.2^\circ$	$1.031 \angle 7.3^\circ$
V_2	$1.0 \angle 0^\circ$	$1.0 \angle 0^\circ$

if α is at positive angle, power flow through phase shifting XFMR increases