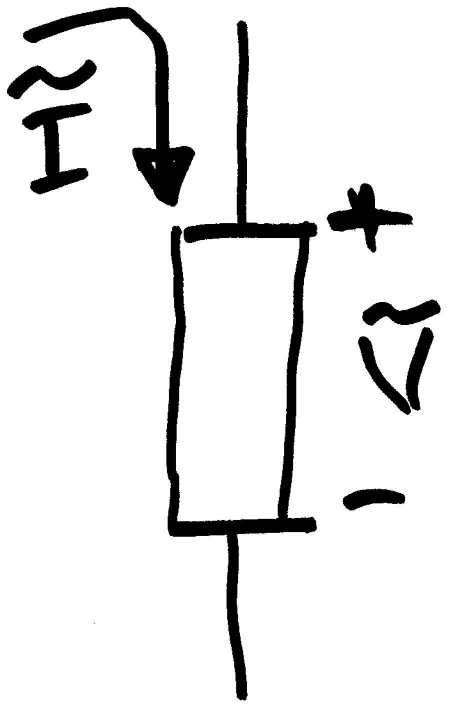
- Quick Review of circuit basics:
  - Active vs. passive sign convention
  - Dual-subscript notation, single-subscript (implied reference)
  - Closure of subscripts in mesh equation
  - Euler's Identity - basis for phasor analysis! See handout.
  - Drawing phasor diagrams, arrowheads
  - Three-phase, "open" vs. "closed" voltage phasor diagrams
  - Errata in text book - Figs. 1.16, 1.17.

- Study Chapters 1 and 2 for Thursday

Do Practices
Exercises at
end of Chapter!

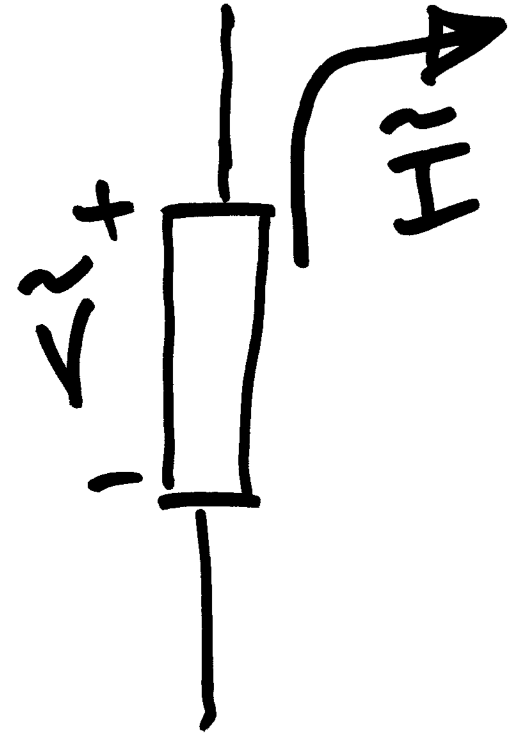
SIGN CONVENTION



PASSIVE

"Motor Convention"

"Load Convention"



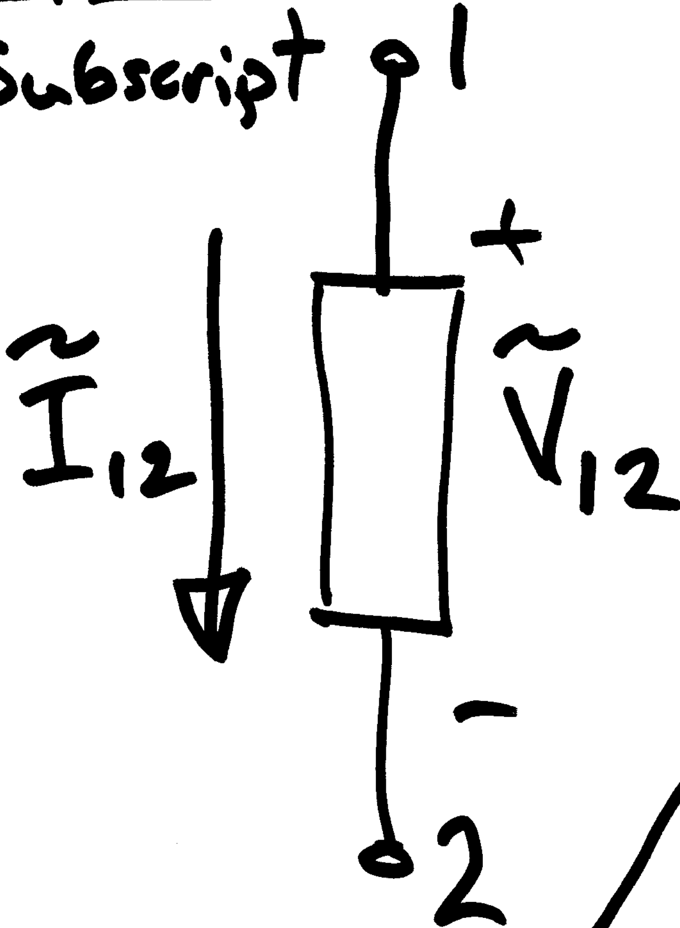
ACTIVE

"Generator Convention"

"Source Convention"

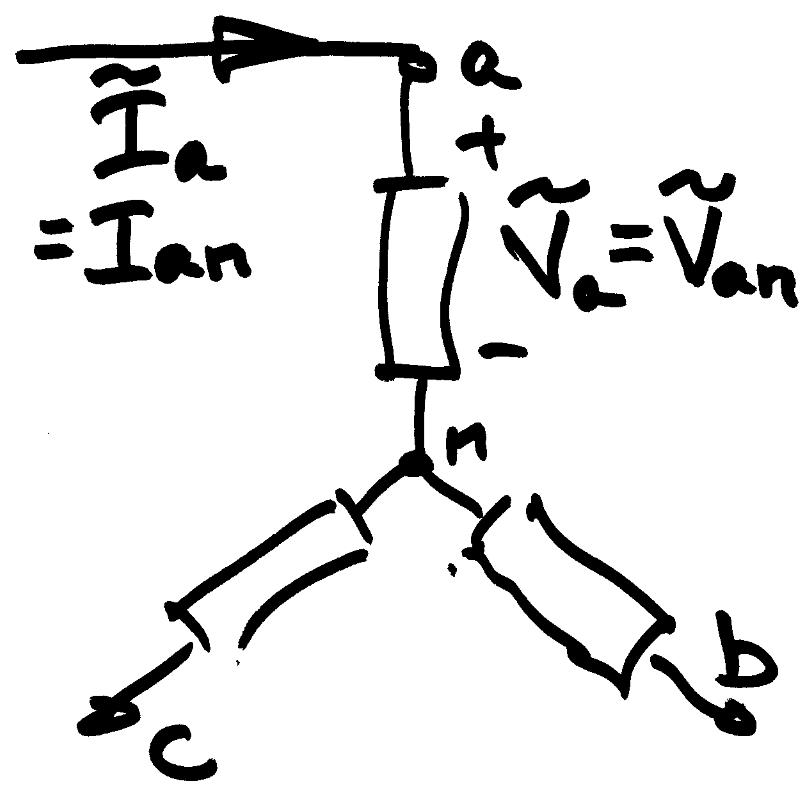
Subscripts

Double-Subscript

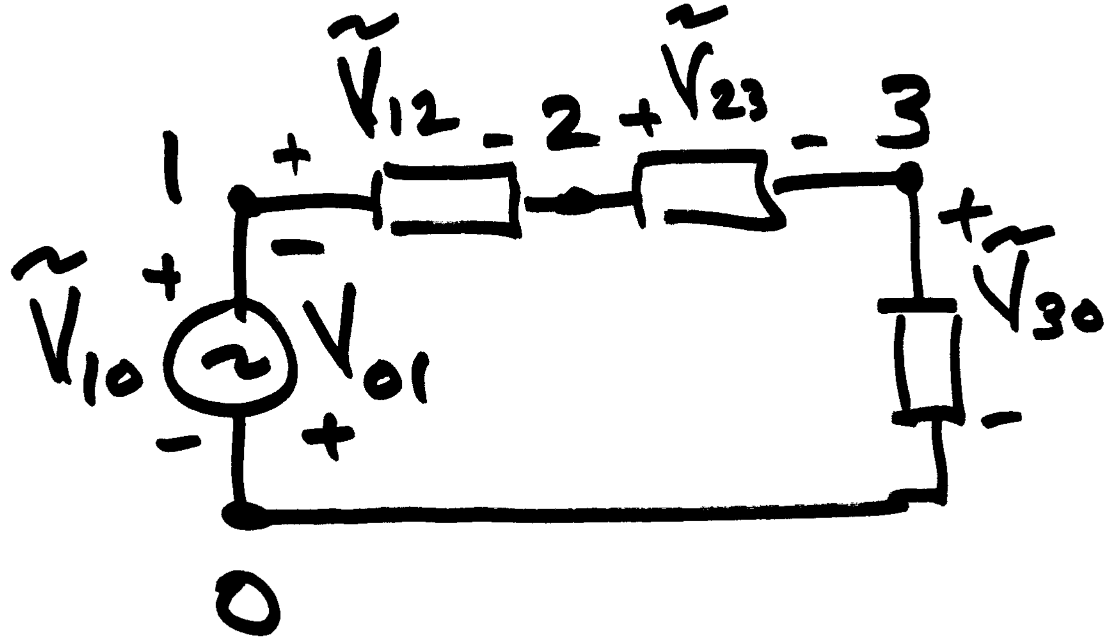


Always use Voltage drops! (Voltage rises seen in old texts).

Single subscript notation: Second subscript is assumed.



Closure in subscripts:



KVL: $\Sigma V_s = 0$ (Sum of V drops!)

starting at node 0:

$$-\tilde{V}_{10} + \tilde{V}_{12} + \tilde{V}_{23} + V_{30} = 0$$

$$\tilde{V}_{10} = \tilde{V}_{12} + \tilde{V}_{23} + \tilde{V}_{30}$$

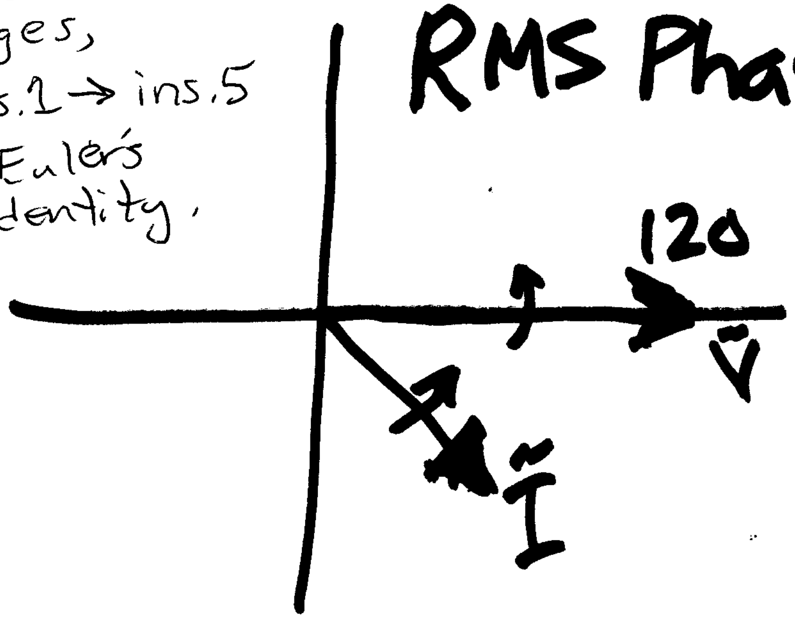
or:

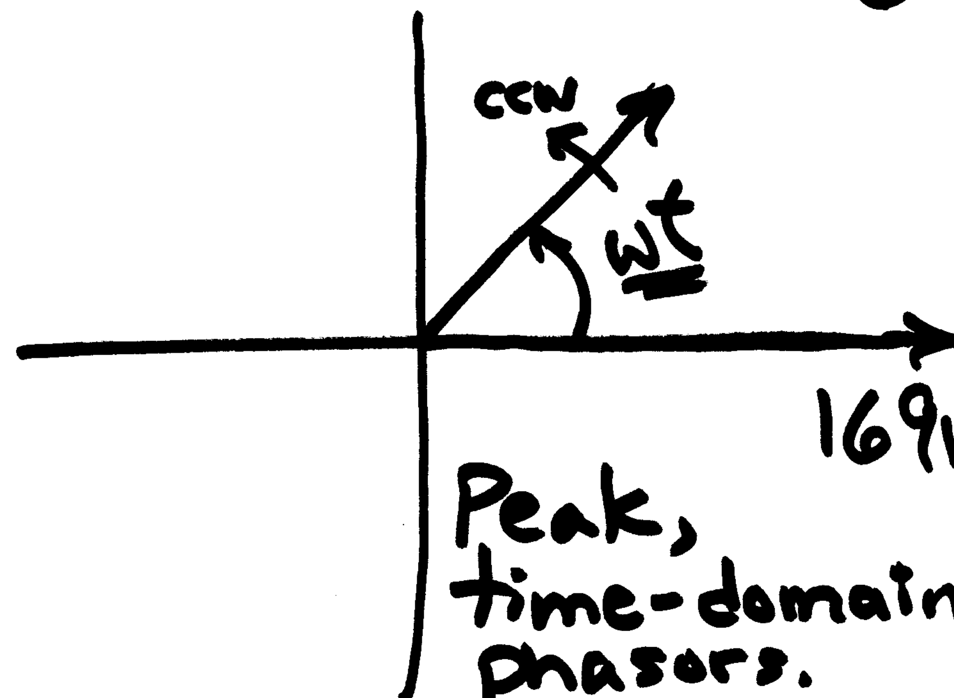
$$0 = V_{01} + V_{12} + V_{23} + V_{30}$$

Phasor Analysis (use "cos reference")

see next 5 pages, ins.1 → ins.5 for Euler's Identity.

RMS Phasors





Peak, time-domain phasors.

$\tilde{V} = \cancel{120 \cos(\omega t)}$?

$v(t) = \underbrace{120\sqrt{2}}_{V_{peak}} \cos \omega t$

$\underbrace{120}_{V_{RMS}} \angle 0^\circ \; V_{RMS}$

Phasors

$\boxed{\begin{array}{l} \longrightarrow V \\ \longrightarrow I \end{array}}$

$\longrightarrow P, Q, S$

$\longrightarrow P$

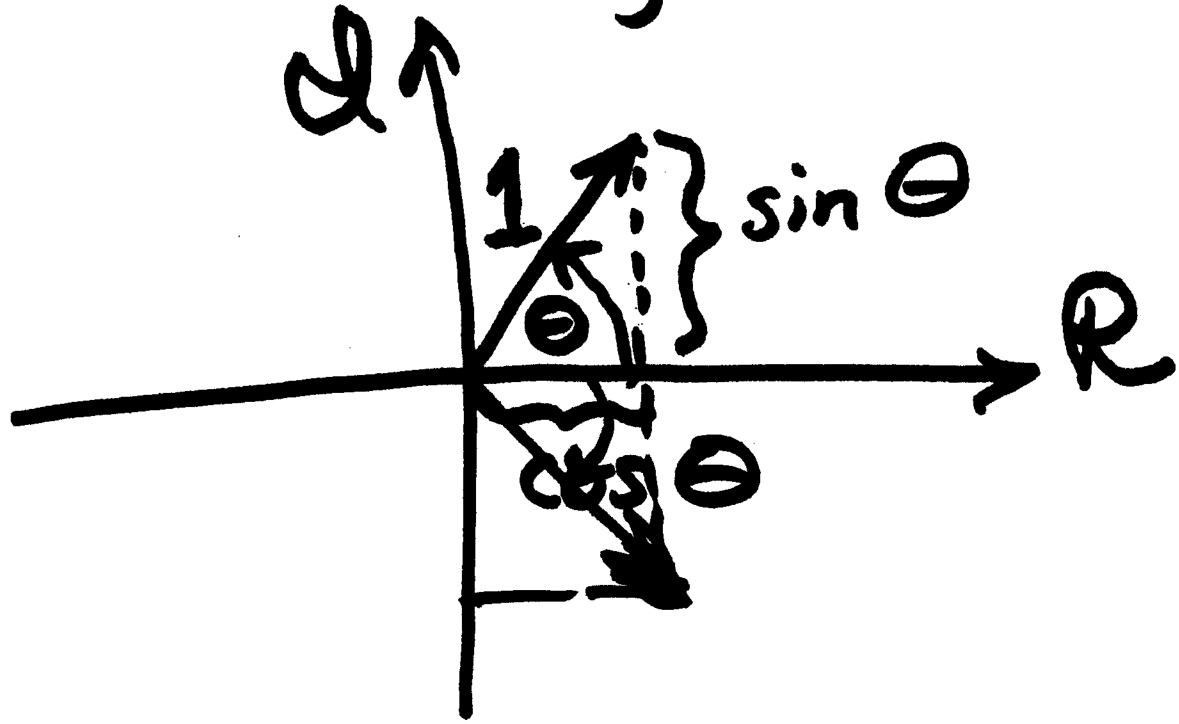
$\longrightarrow Q$

Power Tri.

Euler's Identity

ins. 1

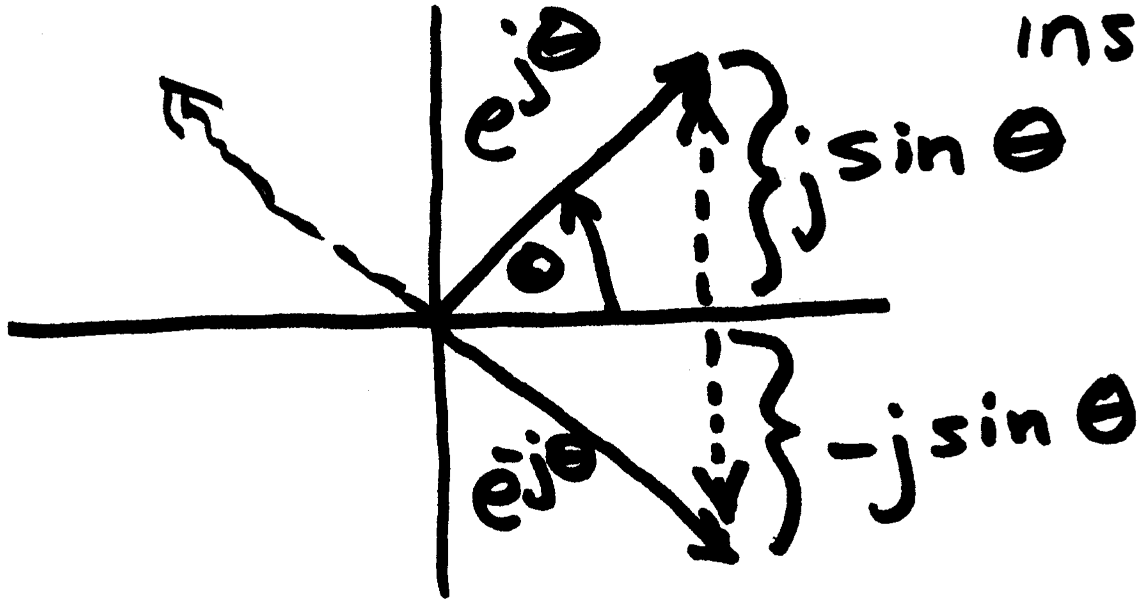
$e^{j\theta}$ = Unit vector at an angle θ.



$$\therefore e^{j\theta} = \cos\theta + j\sin\theta \quad (1)$$

$$e^{-j\theta} = \cos\theta - j\sin\theta \quad (2)$$

ins. 2



If we add the 2 eqns

(1) + (2)

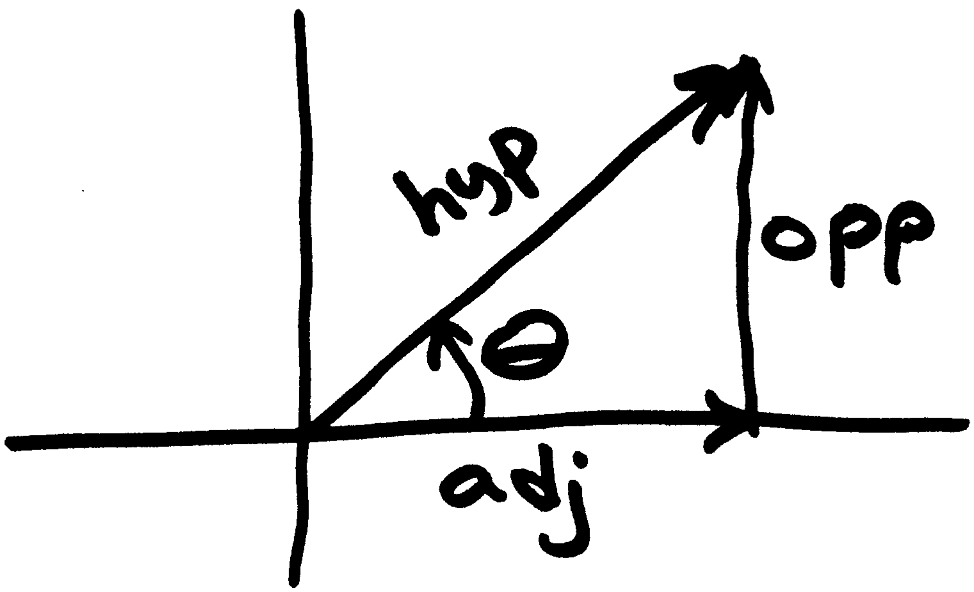
$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\Rightarrow \boxed{\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

Subtracting,

$$\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{j2}}$$

Referring back to figure ins. 3 on preceding page, this matches with the basic trig you learned:



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

Hyperbolic Functions are similar exponential functions, where the exponent of e is, in general, a complex number, $z = a + jb$

$$\sinh(z) = \frac{e^{z} - e^{-z}}{2}$$

$$= \frac{e^{a}e^{jb} - e^{-a}e^{-jb}}{2}$$

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2}$$

$$= \frac{e^{a}e^{jb} + e^{-a}e^{-jb}}{2}$$

Engineering Mathematics Handbook. ins. 5
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6.09 HYPERBOLIC FUNCTIONS

Examples for real values of z:

$z = a + j\cancel{b}$ 0

$z = a + x$

(1) Definitions

A hyperbolic function is a combination of e^x and e^{-x} and is introduced as follows:

Hyperbolic sine of $x = \sinh x = \dfrac{e^x - e^{-x}}{2}$

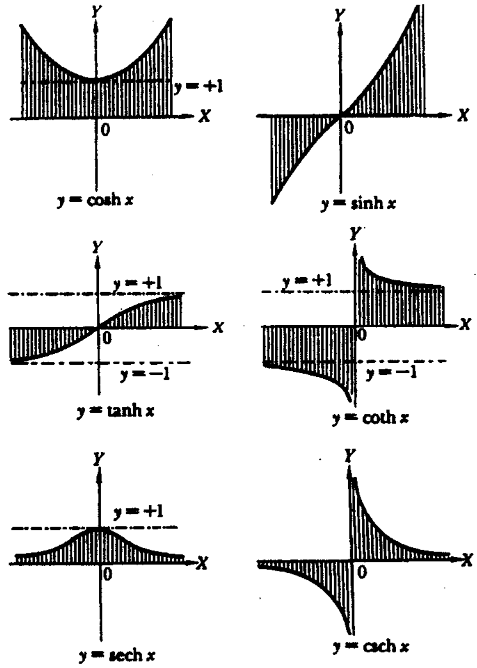
Hyperbolic cosine of $x = \cosh x = \dfrac{e^x + e^{-x}}{2}$

Hyperbolic tangent of $x = \tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Hyperbolic cotangent of $x = \coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Hyperbolic secant of $x = \operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$

Hyperbolic cosecant of $x = \operatorname{csch} x = \dfrac{2}{e^x - e^{-x}}$



(2) Relationships

$\cosh^2 x - \sinh^2 x = 1$ $\qquad \tanh x = \dfrac{\sinh x}{\cosh x}$ $\qquad \operatorname{sech} x \cosh x = 1$

$\tanh^2 x + \operatorname{sech}^2 x = 1$ $\qquad \coth x = \dfrac{\cosh x}{\sinh x}$ $\qquad \operatorname{csch} x \sinh x = 1$

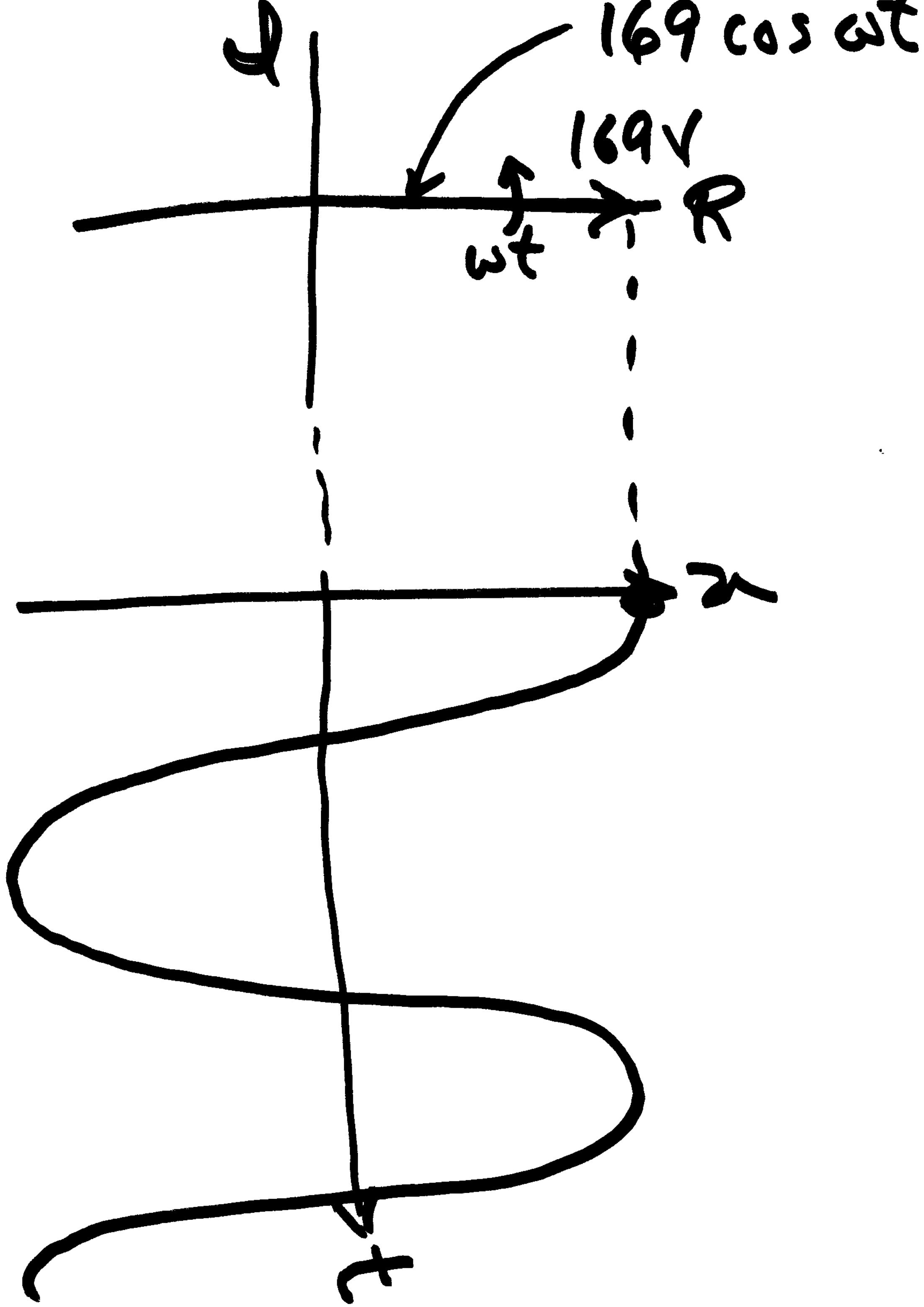
$\coth^2 x - \operatorname{csch}^2 x = 1$ $\qquad \tanh x \coth x = 1$

$\sinh(-x) = -\sinh x$ $\qquad \tanh(-x) = -\tanh x$ $\qquad \cosh(-x) = \cosh x$

$\operatorname{sech}(-x) = \operatorname{sech} x$ $\qquad \coth(-x) = -\coth x$ $\qquad \operatorname{csch}(-x) = -\operatorname{csch} x$

(3) Limit Values

x	$\sinh x$	$\cosh x$	$\tanh x$	$\coth x$	$\operatorname{sech} x$	$\operatorname{csch} x$
$-\infty$	$-\infty$	$+\infty$	-1	-1	0	0
-1	-1.1752	$+1.5431$	-0.7616	-1.3130	$+0.6480$	-0.8509
0	0	$+1$	0	$\mp\infty$	$+1$	$\mp\infty$
$+1$	$+1.1752$	$+1.5431$	$+0.7616$	$+1.3130$	$+0.6480$	$+0.8509$
$+\infty$	$+\infty$	$+\infty$	$+1$	$+1$	0	0



169 cos ωt
169V
R
ωt
I
R
t

Three Phase Phasor Analysis 8

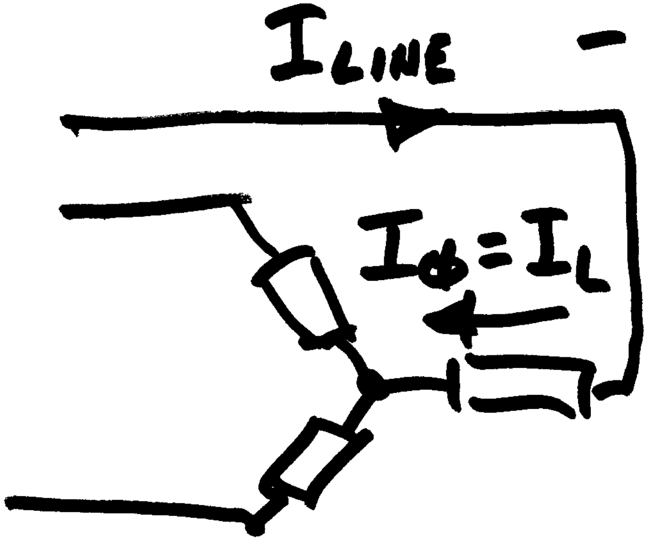
- Must deal with all V's & I's

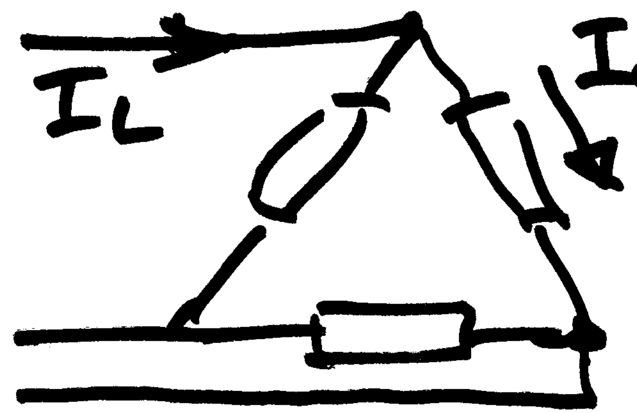
Don't!

- Say "Line Voltage"
- Say "phase voltage" unless you also say whether it is $\underline{\Delta \text{ or } Y}$.

ALWAYS!

- Say V_{LN} or V_{LL}
- say I_{LINE} ($= I_\phi$ in Y)

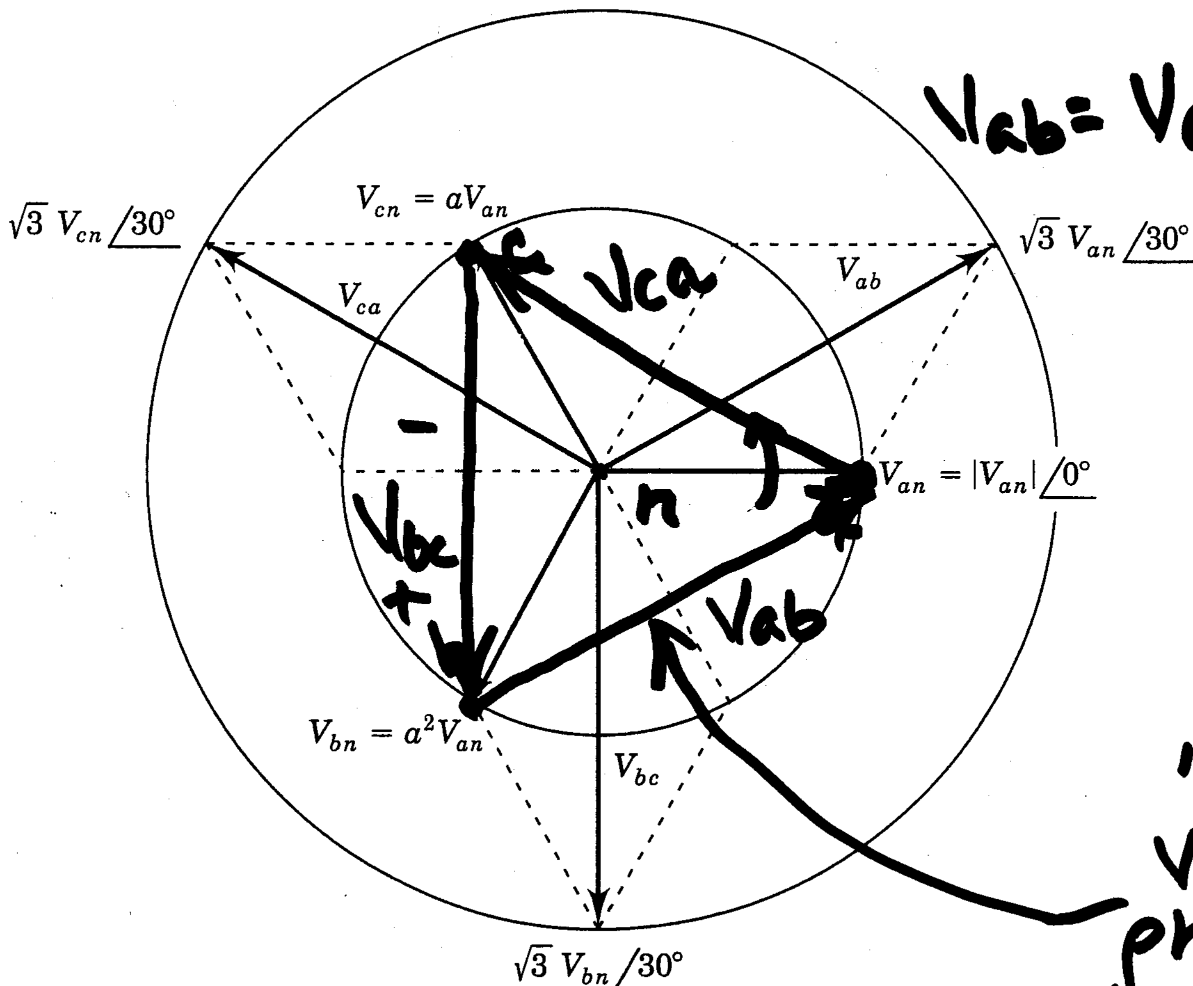




$|I_\phi| = \frac{|I_L|}{\sqrt{3}}$

9

$V_{ab} = V_{an} - V_{bn}$

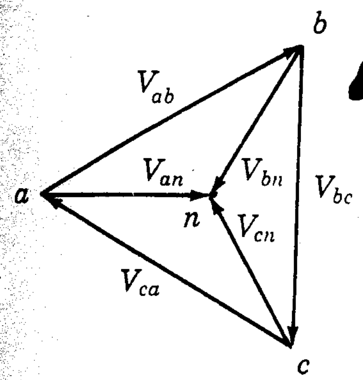


"closed voltage phasor diagram"

FIGURE 1.15
Phasor diagram of line-to-line voltages in relation to line-to-neutral voltages in a balanced three-phase circuit.

("open" voltage phasor diagram)

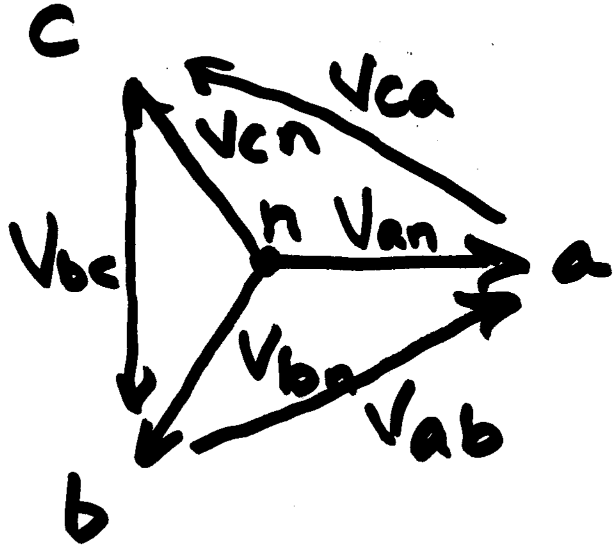
10



Construction is mathematical correct, but not common practice!

FIGURE 1.16
Alternative method of drawing the phasors of Fig. 1.15.

also Fig 1.17



"Correct" common practice.