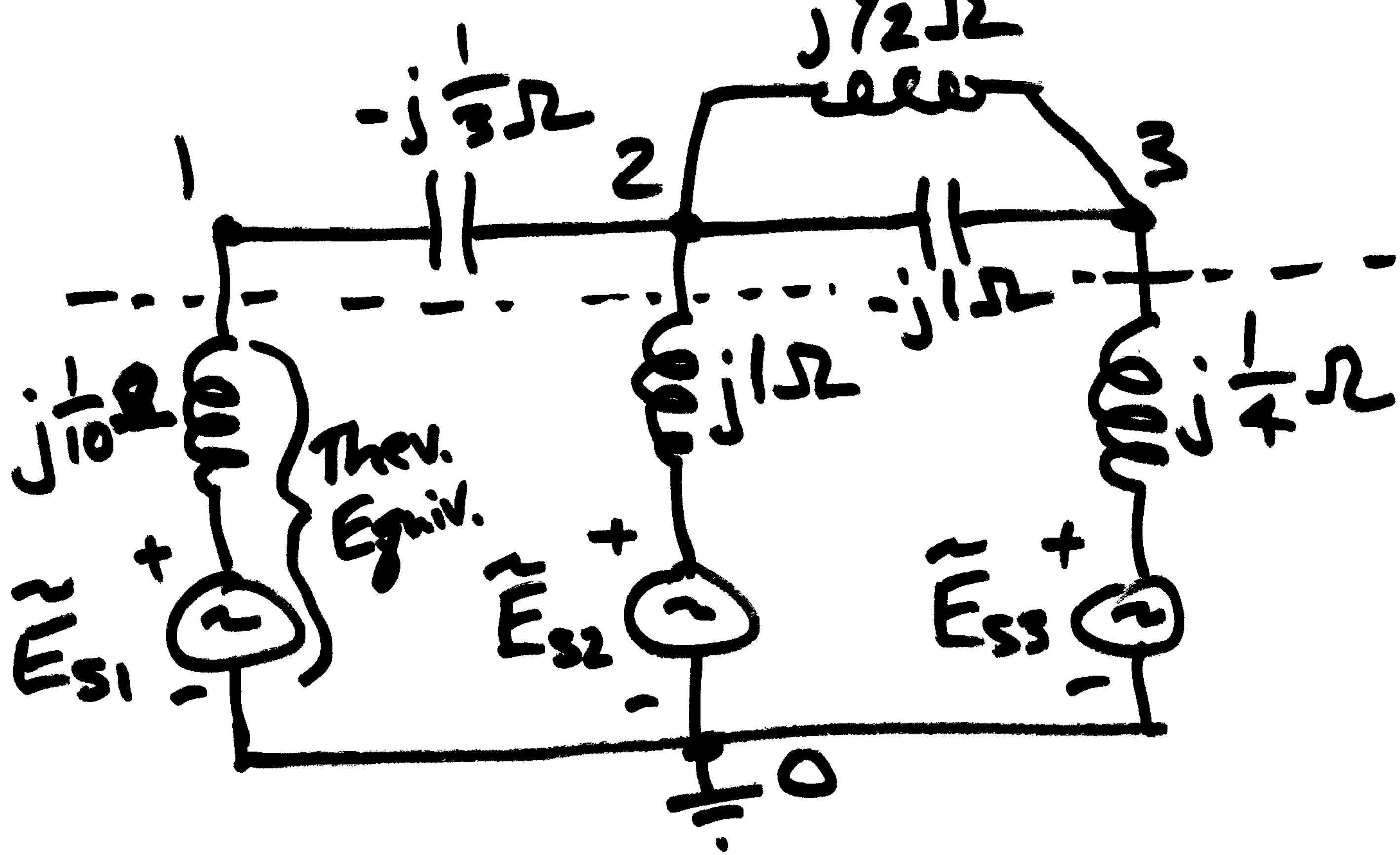
EE 5200 - Lecture 3 Tuesday Sep 2, 2003

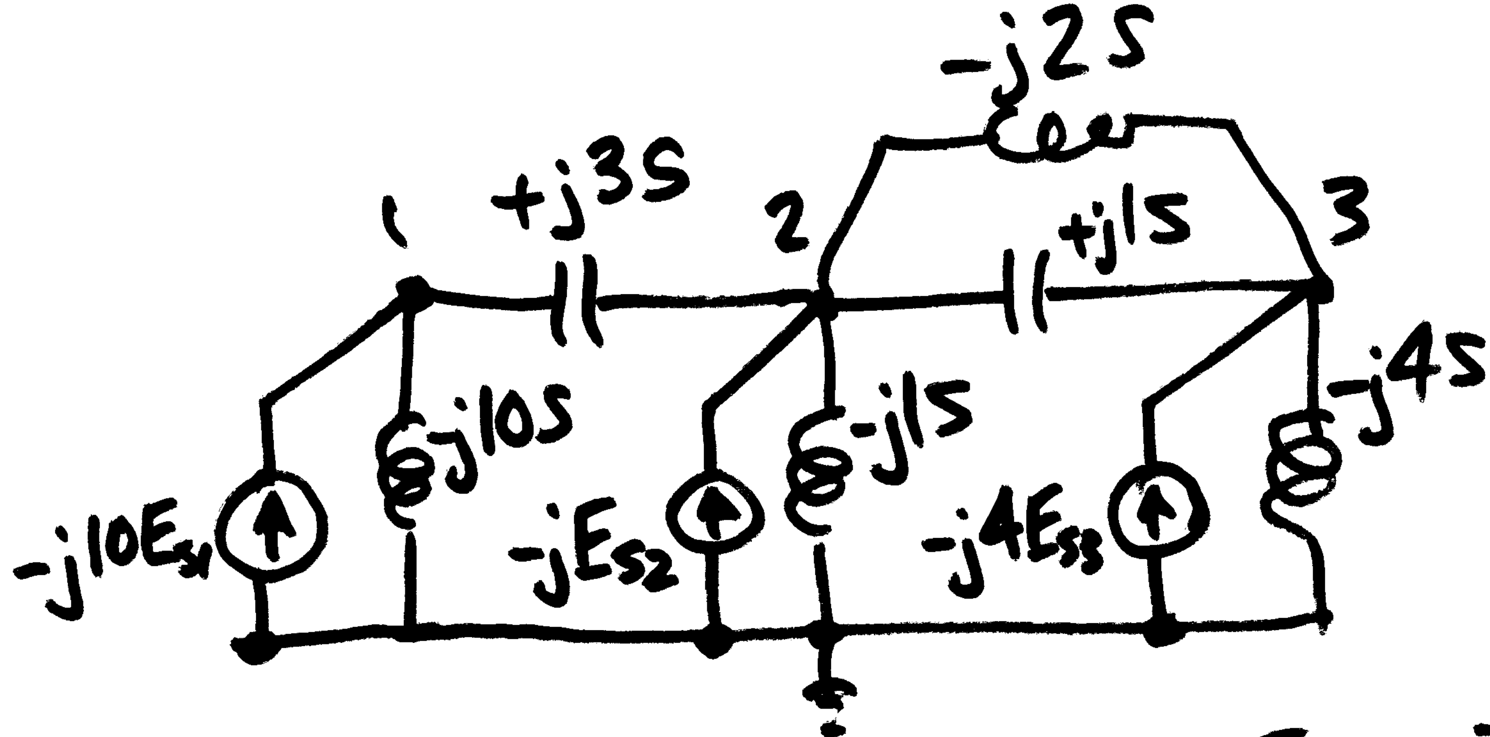
Topics for Today:

- Announcements
  - Text books should be in today, check w/book store.
  - Room B45 is open from 9am onward.
  - Office hrs: 1:30-2:30pm, Mon, Wed, Fri
  - Office: EERC 623. Phone: 906.487.2857
  - Ch.1 Solutions posted on web page. Finish by Sept. 5th.
  - Set of pre-req / review exercises given out this week.
- Continuing with Chapter 1 / Review:
  - Three-phase analysis, use of phasor diagrams
  - Per-phase (L-N phase-A) equivalent.
  - Per Unit system
  - Symmetrical Components
  - Sequence networks
- Next: Chapter 2 - Transformers and circuits w/transformers

Quick Review: Nodal Analysis



1) Convert Sources to Norton Equiv.
2) Convert all Z's into Y's
3) Construct $[Y]$ by inspection
4) Solve $[Y][V_{node}] = [I_{inj}]$



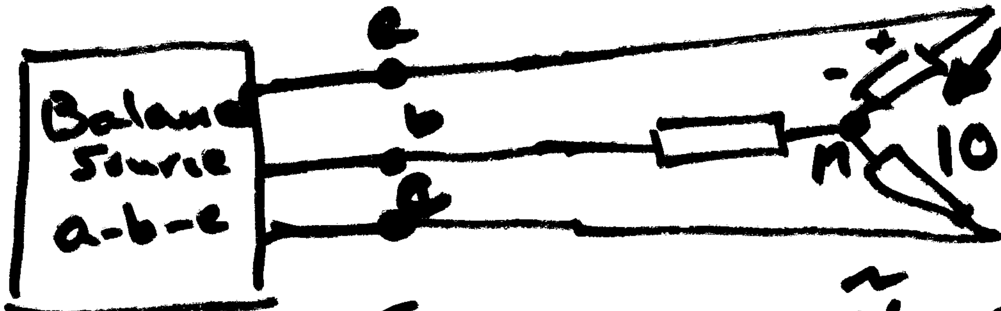
$$[Y]\quad[V_{node}] = [I_{inj}]$$

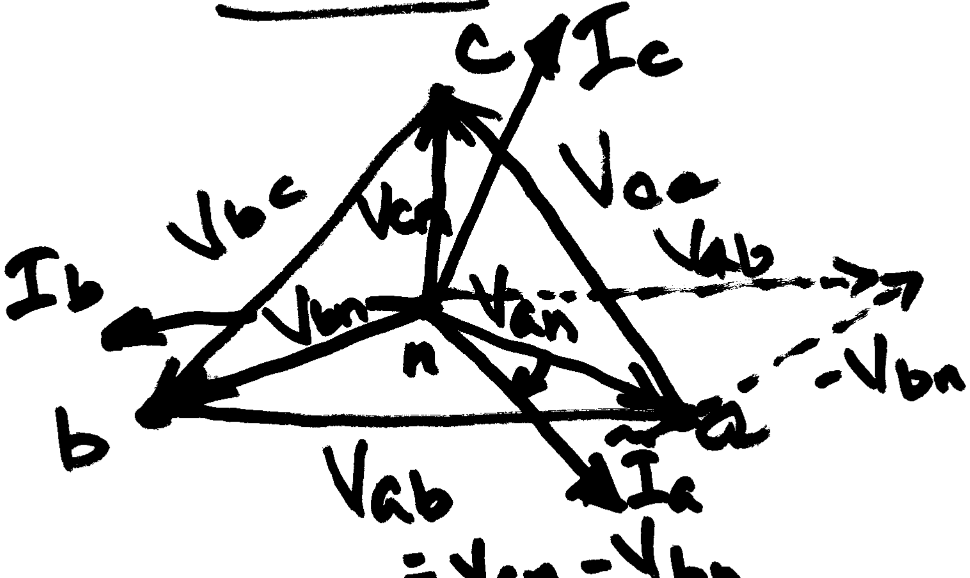
$$j\begin{bmatrix}-7 & -3 & 0\\ -3 & 1 & 1\\ 0 & 1 & -5\end{bmatrix}\begin{bmatrix}\tilde{V}_1\\ \tilde{V}_2\\ \tilde{V}_3\end{bmatrix} = \begin{bmatrix}-j10E_{s1}\\ -jE_{s2}\\ -j4E_{s3}\end{bmatrix}$$

Ex: 1.2 3ϕ ckt

- 3ϕ Source, balanced a-b-c
- Y-conn load $Z_L = 10\angle 20^\circ\,\Omega$
- $\tilde{V}_{ab} = 173.2\angle 0^\circ\ V_{RMS}$

$\tilde{I}_a = \dfrac{\tilde{V}_{an}}{Z_{an}} = \dfrac{100\angle -30^\circ}{10\angle 20^\circ} = 10\angle -50^\circ$





$\tilde{V}_{an} = 100\angle -30^\circ\ V_{RMS}$

$V_{bn} = 100\angle -150^\circ\ V$

$V_{cn} = 100\angle +90^\circ\ V$

$\tilde{V}_{ab} = 173.2\angle 0^\circ\ V$

$\tilde{V}_{bc} = 173.2\angle -120^\circ\ V$

$\tilde{V}_{ca} = 173.2\angle +120^\circ\ V$

Z_Δ's, Table 1.2 shows that

$$Z_\Delta = \frac{\text{sum of pairwise products of } Z_Y\text{'s}}{\text{the opposite } Z_Y} \tag{1.32}$$

Similar statements apply to the admittance transformations.

Example 1.3. The terminal voltage of a Y-connected load consisting of three equal impedances of $20\angle 30°\,\Omega$ is 4.4 kV line to line. The impedance of each of the three lines connecting the load to a bus at a substation is $Z_L = 1.4\angle 75°\,\Omega$. Find the line-to-line voltage at the substation bus.

Solution. The magnitude of the voltage to neutral at the load is $4400/\sqrt{3} = 2540$ V. If V_{an}, the voltage across the load, is chosen as reference,

$$V_{an} = 2540\angle 0°\ \text{V} \qquad \text{and} \qquad I_{an} = \frac{2540\angle 0°}{20\angle 30°} = 127.0\angle -30°\ \text{A}$$

The line-to-neutral voltage at the substation is

$$\begin{aligned} V_{an} + I_{an}Z_L &= 2540\angle 0° + 127\angle -30° \times 1.4\angle 75° \\ &= 2540\angle 0° + 177.8\angle 45° \\ &= 2666 + j125.7 = 2670\angle 2.70°\ \text{V} \end{aligned}$$

$\tilde{V}_{an,\text{source}}$

and the magnitude of the voltage at the substation bus is

$$\sqrt{3} \times 2.67 = 4.62\,\text{kV}$$

Figure 1.22 shows the per-phase equivalent circuit and quantities involved.

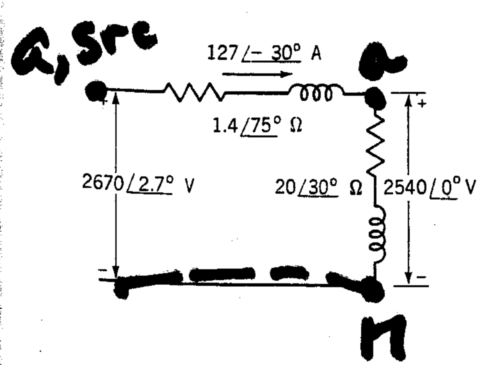


FIGURE 1.22
Per-phase equivalent circuit for Example 1.3.

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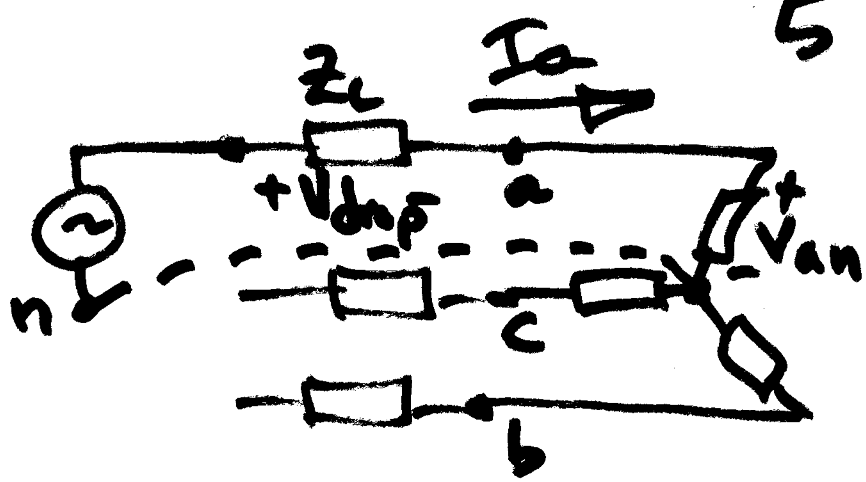

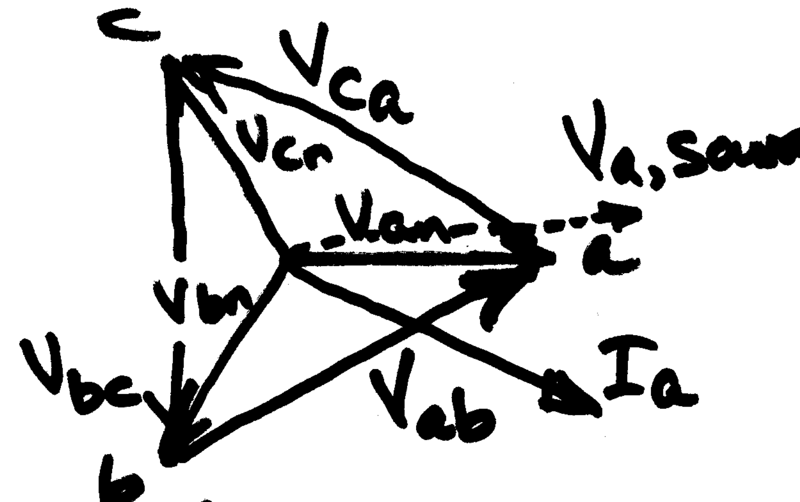

$$\tilde{I}_a = \frac{\tilde{V}_{an}}{Z_L} = \frac{2540\angle 0°\,\text{V}}{20\angle 30°\,\Omega} = 127\angle -30°\,\text{A}$$

$$V_{drop} = \tilde{I}_a(1.4\angle 75°\,\Omega) = 177.8\angle 45°\,\text{V}$$

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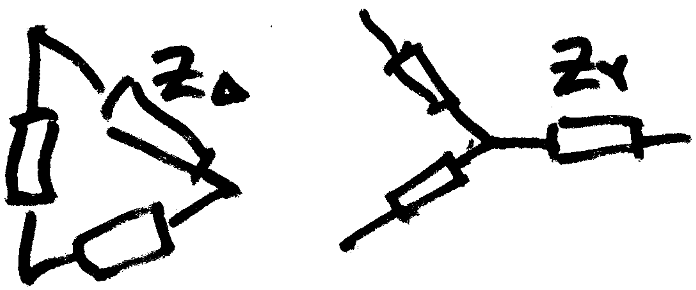
Only need for unbalanced Ys or Δs.

For balanced:

$$\boxed{Z_\Delta = 3Z_Y}$$



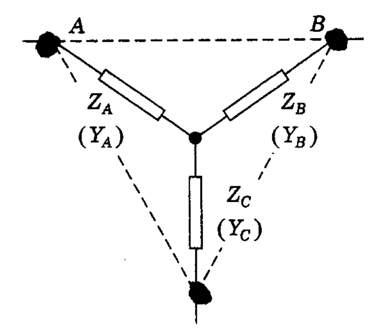
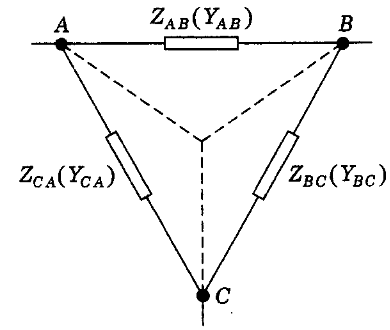
$S_\Delta = S_Y$

$P_\Delta = P_Y$

$Q_\Delta = Q_Y$

$P_\Delta = 3\frac{V_{LL}^2}{R_\Delta} = 3\frac{V_{LN}^2}{R_Y} = P_Y$

TABLE 1.2
Y-Δ and Δ-Y transformations†

	
Δ → Y $Z_A = \frac{Z_{AB}Z_{CA}}{Z_{AB}+Z_{BC}+Z_{CA}}$ $Z_B = \frac{Z_{BC}Z_{AB}}{Z_{AB}+Z_{BC}+Z_{CA}}$ $Z_C = \frac{Z_{CA}Z_{BC}}{Z_{AB}+Z_{BC}+Z_{CA}}$	Y → Δ $Z_{AB} = \frac{Z_AZ_B+Z_BZ_C+Z_CZ_A}{Z_C}$ $Z_{BC} = \frac{Z_AZ_B+Z_BZ_C+Z_CZ_A}{Z_A}$ $Z_{CA} = \frac{Z_AZ_B+Z_BZ_C+Z_CZ_A}{Z_B}$
Δ → Y $Y_A = \frac{Y_{AB}Y_{CA}+Y_{BC}Y_{AB}+Y_{CA}Y_{BC}}{Y_{BC}}$ $Y_B = \frac{Y_{AB}Y_{CA}+Y_{BC}Y_{AB}+Y_{CA}Y_{BC}}{Y_{CA}}$ $Y_C = \frac{Y_{AB}Y_{CA}+Y_{BC}Y_{AB}+Y_{CA}Y_{BC}}{Y_{AB}}$	Y → Δ $Y_{AB} = \frac{Y_AY_B}{Y_A+Y_B+Y_C}$ $Y_{BC} = \frac{Y_BY_C}{Y_A+Y_B+Y_C}$ $Y_{CA} = \frac{Y_CY_A}{Y_A+Y_B+Y_C}$

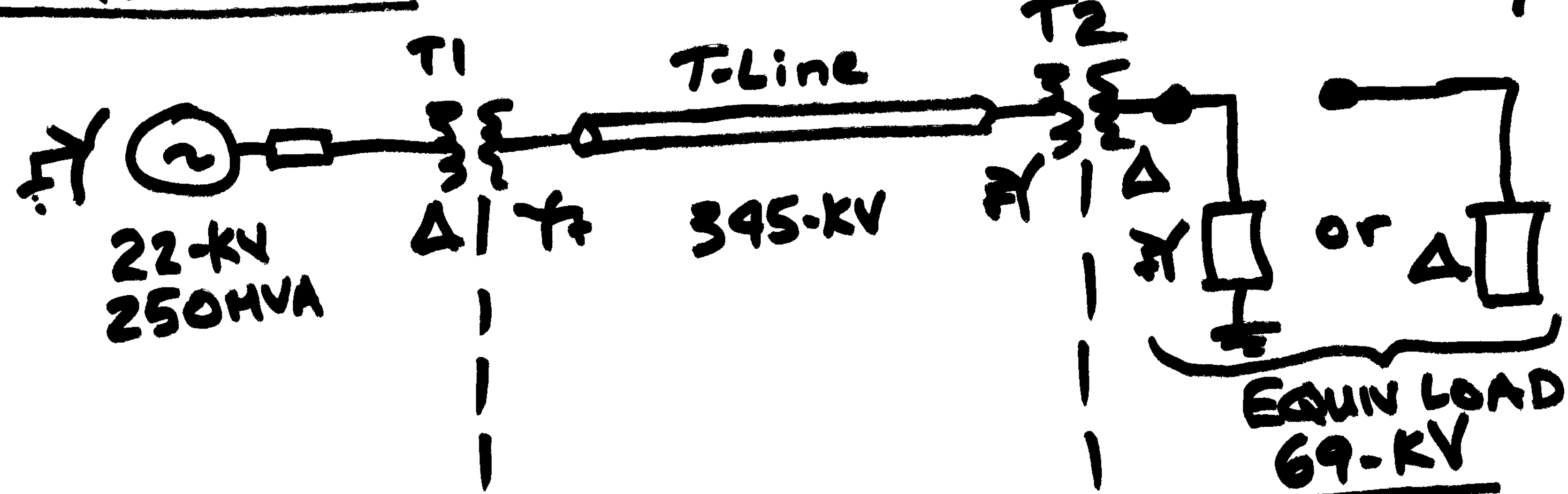
† Admittances and impedances with the same subscripts are reciprocals of one another.

Z_Y in terms of the delta impedances Z_Δ's is

$$Z_Y = \frac{\text{product of adjacent } Z_\Delta\text{'s}}{\text{sum of } Z_\Delta\text{'s}} \tag{1.31}$$

So, when all the impedances in the Δ are equal (that is, balanced Z_Δ's), the impedance Z_Y of each phase of the equivalent Y is one-third the impedance of each phase of the Δ which it replaces. Likewise, in transforming from Z_Y's to

Per Unit Quantities



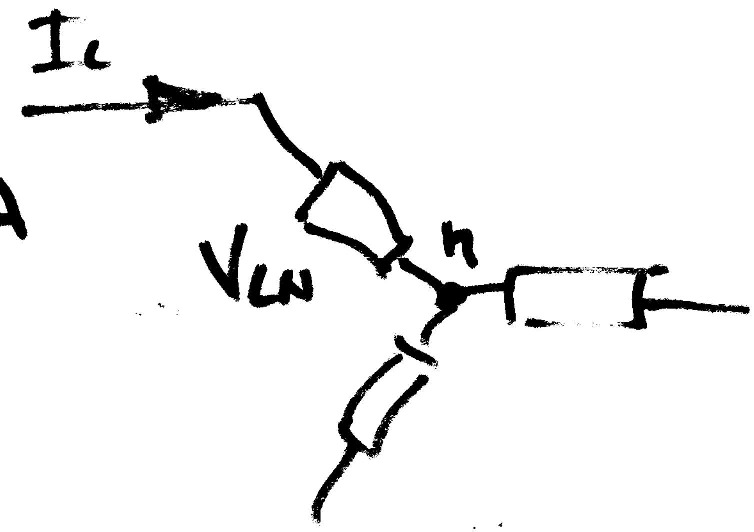
BASE QTY			
S_{BASE}	100 MVA	100 MVA	100 MVA
V_{L-L}	22 KV	345 KV	69 KV
V_{L-N}	12.7 KV	199.2 KV	39.8 KV
I_{LINE}			
$I_{\phi,\Delta}$			
Z_Y			
Z_Δ			

S_{BASE} : Assumed/defined, typ: 100 MVA

V_{LL} : Rated V_{LL} of system zone

V_{LN} : $V_{LL}/\sqrt{3}$

I_{LINE} : $\dfrac{S_{BASE}/3 \text{ VA}}{\frac{V_{LL}}{\sqrt{3}} \text{ V}}$



Continue from here on Thursday.