12.6 - Defining the Transient Stability Problem

Assumptions:
- Ignore magnetic saturation
- Assume machine is lossless.
- Assume balanced 3ϕ operation.
- Assume P_m constant
- Assume $|E_f|$ constant

For salient pole machine:

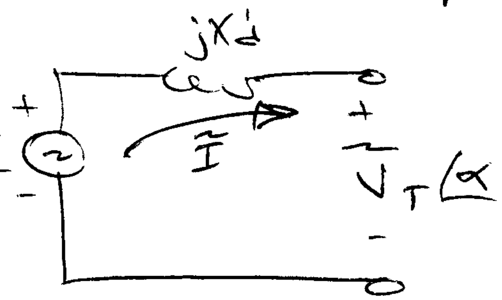
$$\tilde{E}_f = \tilde{V}_T + jX_q\tilde{I}_q + jX_d'\tilde{I}_d = \tilde{E}_q + j(X_d - X_q)\tilde{I}_d$$

Recall $\quad \tilde{I} = \tilde{I}_d + \tilde{I}_q \quad$ (see fig., p.515 in Gross)

For stability calculations, use X_d'.

Illustration on p. 515 shows that if $X_d' \approx X_q$ then machine can be simplified to

$\tilde{E}_q' = E_q' \angle \delta$



w.r.t. phase reference of system.

at $t = 0^-$, prior to disturbance.

$$P_e = \left.\frac{E_q' V}{X_d'}\sin\delta\right|_{X_d' = X_q}$$

If $X_d' \neq X_q$,

$$P_e = \frac{E_q' V}{X_d'}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} + \frac{1}{X_d'}\right)$$

$\sin 2\delta$

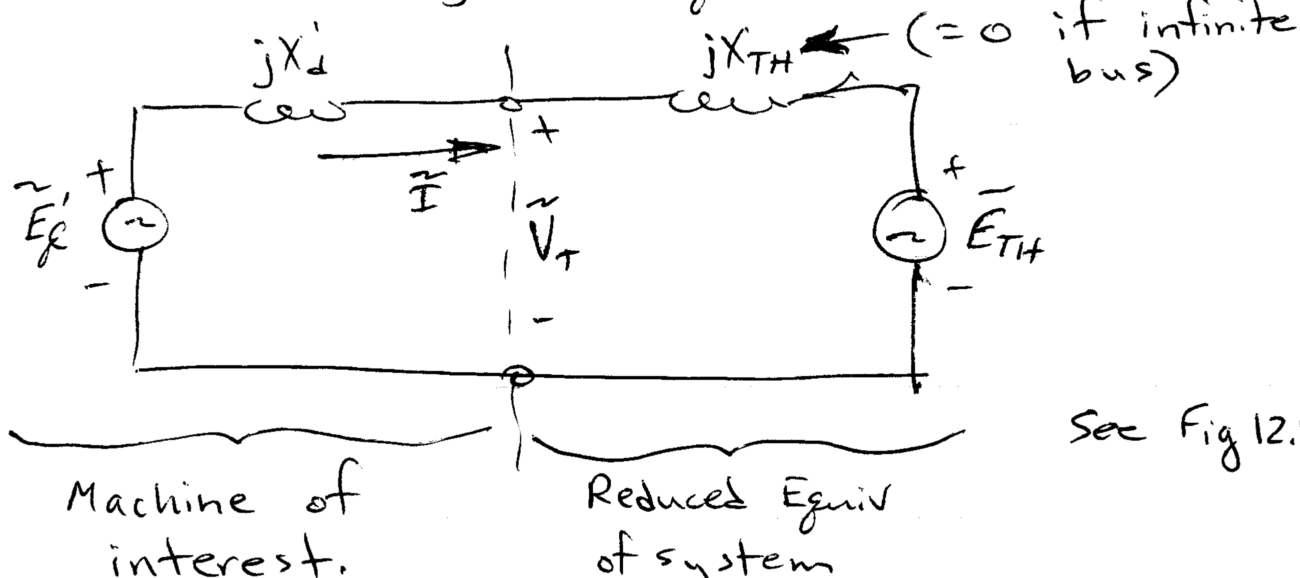
EE 486

Infinite Bus - Ideal voltage source of constant frequency

In terms of a generator, this means:

$\Rightarrow$
- Constant Internal voltage
- Zero impedance
- Infinite Inertia ($\omega_{ref} = \omega_s$)

When considering how a disturbance affects one particular machine, it is possible to approximate the remainder of the system as a reduced dynamic equivalent:



See Fig 12.17

If response does not involve governor or exciter, the relationship

$$P_m - P_e = \frac{H}{\pi f}\left(\frac{d^2\delta}{dt^2} + \underbrace{\frac{d\omega_{ref}}{dt}}_{0 \text{ if infinite bus}}\right)$$

can be used. This assumes that $\omega_{ref} = \omega_s$.

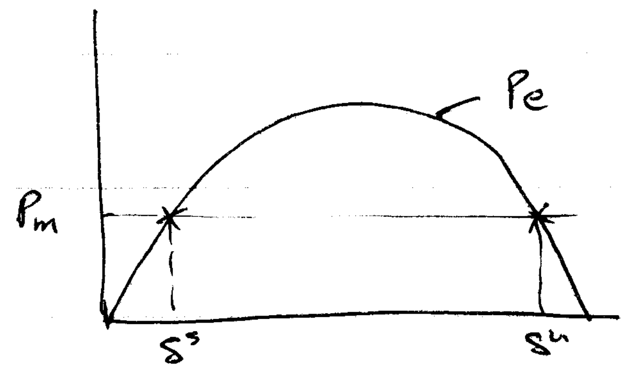
To evaluate,

P_m = constant $\qquad P_e = \dfrac{E_g' E_{TH}}{X_d' + X_{TH}} \sin\delta$.

Equilibria Points:

when $P_e = P_m$

$$P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}$$



Perturbations:

δ^s is stable: From δ^s, if δ increases, $\frac{d^2\delta}{dt} < 0$ $(P_m < P_e)$
$\Rightarrow$ δ decreases
if δ decreases, $\alpha > 0$ $(P_m > P_e)$
$\Rightarrow$ δ increases.

δ^u is unstable: From δ^u, if δ increases, $\alpha > 0$ $(P_m > P_e)$
$\Rightarrow$ δ increases further.
if $\underbrace{\delta \text{ decreases}}$, $\alpha < 0$ $(P_m < P_e)$

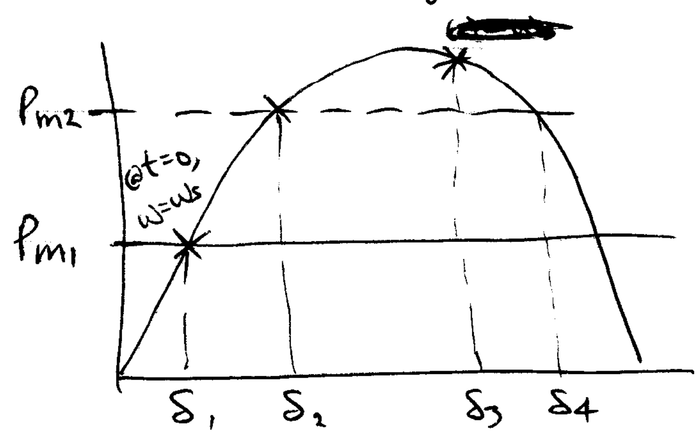
Pe proportional to $\sin\delta$

acceleration

$\therefore$ Better keep $\delta < 90°$ for stable equilibrium.

Conceptual example:

If P_m is initially P_{m1}, then suddenly increases to P_{m2}. What happens?



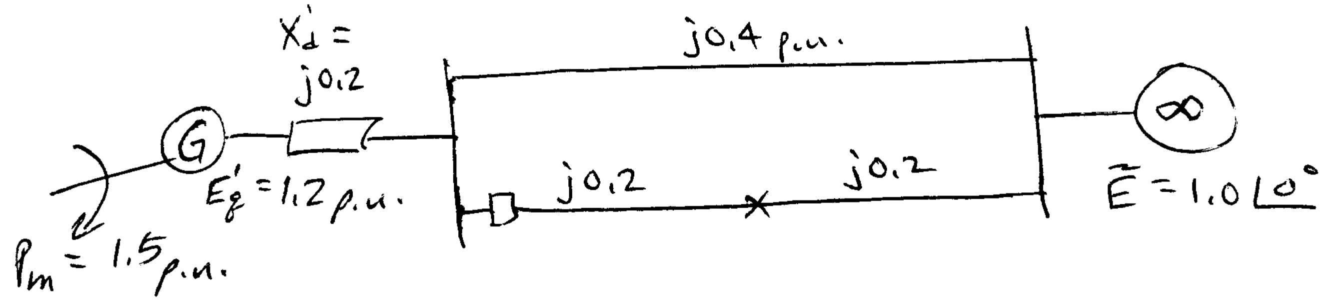
For $t \geq 0^+$, rotor accelerates, until δ reaches δ_2, where it begins to decelerate.

Oscillation in δ (about δ_2) continues, damping slowly.

$\delta < \delta_2 \Rightarrow P_m > P_e \Rightarrow$ accel
$\delta > \delta_2 \Rightarrow P_m < P_e \Rightarrow$ decel
$\delta > \delta_4 \Rightarrow P_m > P_e \Rightarrow$ accel

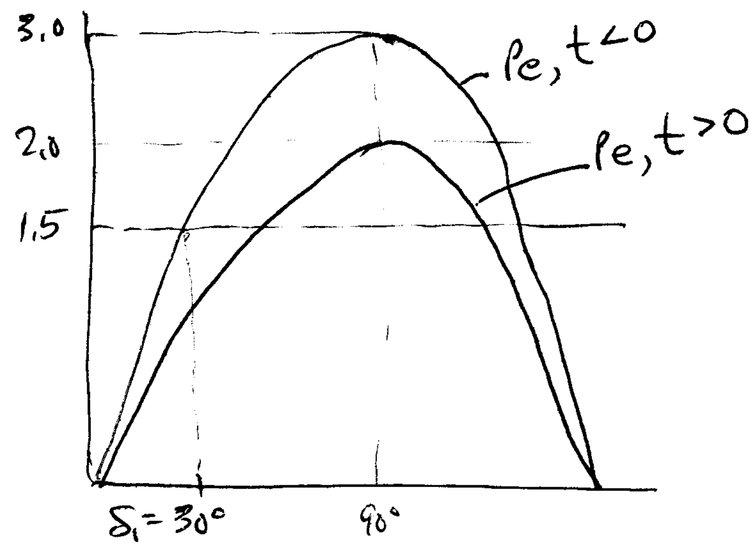
No chance to maintain stability if δ goes past δ_4.

Ex:



@ $t = 0^-$ $\quad X = j0.2$ p.u. (X refers to T-Line tie)

$$P_m = P_e = \underbrace{\frac{(1.2)(1.0)}{0.2 + 0.2}}_{= 3.0 = P_{e\,max}} \sin\delta = 1.5 \text{ p.u.} \Rightarrow \delta_1 = 30°$$



Breaker opens at $t = 0$,

$$P_{e\,max} = \frac{(1.2)(1.0)}{0.2 + 0.4} = 2.0 \text{ p.u.}$$

New equilibrium point is at

$$1.5 = P_{max} \sin\delta = 2.0 \sin\delta$$

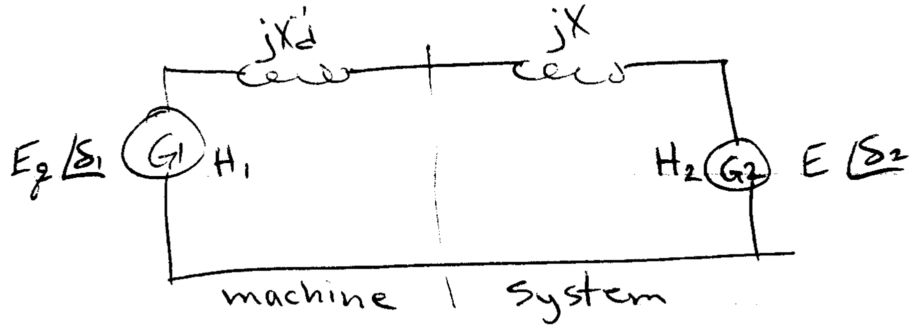
$$\sin\delta = 0.75 \Rightarrow \underline{\underline{\delta = 48.6°}} \text{ or } \cancel{131.4°}$$

unstable

Considerations / Questions

1) What if system equivalent cannot be assumed to be infinite bus (i.e. infinite inertia)?

This is a case where machines are <u>noncoherent</u>.



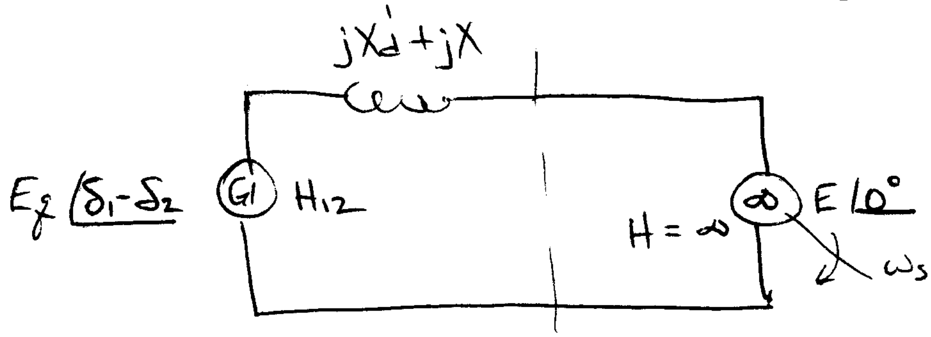
See pp. 705-706 in Grainger/Stevensen © 1994

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2} \Rightarrow \boxed{P_m - P_e = \frac{H_{12}}{\pi f} \frac{d^2 \delta_{12}}{dt^2}}$$

where $P_m = P_{m1} = -P_{m2}$

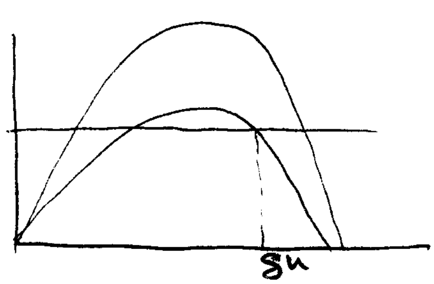
$P_e = P_{e1} = -P_{e2}$

$\delta_{12} = \delta_1 - \delta_2$



$\therefore$ Convert system equiv to ∞ bus.

2) How do we know if δ swings past δ^u in new P_{max} curve? All we know so far is how to find new s-s equilibrium point.



Focus on this for remainder of Ch. 12.

12.7 – EQUAL AREA CRITERION

Back to swing equation: $\boxed{P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}}$

Rearranging,
$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_m - P_e) = \frac{\pi f}{H}\left[P_m - \frac{E_g' V}{X}\sin\delta\right]$$

Because of $\sin\delta$ term in P_e, this is nonlinear differential equation. How to solve?

One "trick" that works:
$$\frac{d}{\cancel{dt}}\left(\frac{d\delta}{dt}\right)^2 = 2\frac{d\delta}{\cancel{dt}}\frac{d^2\delta}{dt^2}$$

We can substitute:
$$\frac{d^2\delta}{dt^2} = \frac{1}{2}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2\frac{dt}{d\delta}$$

$$\underbrace{\frac{1}{2}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2}\frac{dt}{d\delta} = \frac{\pi f}{H}(P_m - P_e)$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f}{H}(P_m - P_e)\frac{d\delta}{\cancel{dt}}$$

Integrating both sides,

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta = (\omega - \omega_s)^2$$

($\omega_{ref} = \omega_s$)

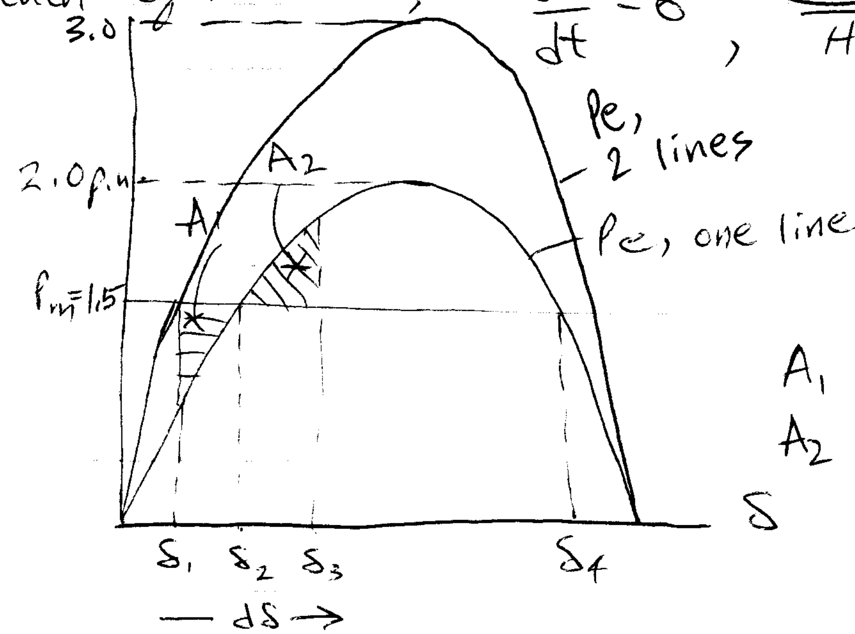
$\Rightarrow$ Area between P_m & P_e ($\Delta P \cdot d\delta$) is proportional to relative velocity squared. $(\omega - \omega_{ref})^2$

Recall that kinetic energy K.E. $\propto \omega^2$ also, so areas are proportional to changes in kinetic energy. If $P_m > P_e$ then area is positive and energy increases (ω increases). If $P_m < P_e$ then ω decreases and energy dcreases.

$\therefore$ When $P_m > P_e$, energy stored in rotor $\left(\frac{1}{2}J\omega^2\right)$,
when $P_e < P_m$, energy released from rotor.
Energy fluctuations are taken up by infinite bus.

To reach equilibrium, $\frac{d\delta}{dt} = 0$, $\frac{2\pi f}{H}\int_{\delta_0}^{\delta} (P_m - P_e)\, d\delta = 0$

which ω?



A_1 inherently pos
A_2 inherently neg

Example: $\delta_0 = \delta_1 = 30°$ (release from δ_1 at $t=0$).
Clearly, equilibrium point will be $\delta = \delta_2$.
For $P_{e\,max} = 2.0$ & $P_m = 1.5$, $\delta_2 = 48.6° = 0.848$ rad

For equilibrium/stability,

$$\frac{2\pi f}{H}\int_{\delta_1}^{\delta_3} (P_m - P_e)\, d\delta = 0 \implies \underbrace{\int_{\delta_1}^{\delta_2} \overbrace{(P_m - P_e)}^{\text{pos}}\, \overset{\text{pos}}{d\delta}}_{A_1} = \underbrace{\int_{\delta_3}^{\delta_2} \overbrace{(P_m - P_e)}^{\text{neg}}\, \overset{\text{neg}}{d\delta}}_{A_2}$$

$$A_1 = \int_{0.524}^{0.848} (1.5 - 2\sin\delta)\,d\delta = 1.5\delta + 2\cos\delta \Big|_{0.524}^{0.848} = \underline{\underline{0.0773}}$$

Since equilibrium is (hopefully) about δ_2, A_2 will be equal to A1.

$$\therefore \int_{\delta_3}^{\delta_2} (P_m - P_e)\,d\delta = \int_{\delta_3}^{0.848} (1.5 - 2\sin\delta)\,d\delta = 0.0773$$

$$1.5\delta + 2\cos\delta \Big|_{\delta_3}^{0.848} = 0.0773$$

$$(1.5)(0.848) + 2\cos(0.848) - 1.5(\delta_3) - 2\cos(\delta_3) = 0.0773$$

$$2\cos\delta_3 + 1.5\delta_3 = 2.518$$

Solve? Use Newton Raphson, iteration, etc.

$$\delta_3 = 1.218 \text{ rad} \quad (69.8°)$$

Note: Max A_2 availabe to "counter" A_1 is

$$A_2 = \int_{\delta_4}^{\delta_2} (P_m - P_e)\,d\delta = \int_{2.293}^{0.848} (1.5 - 2\sin\delta)\,d\delta = \underline{\underline{0.478}}$$

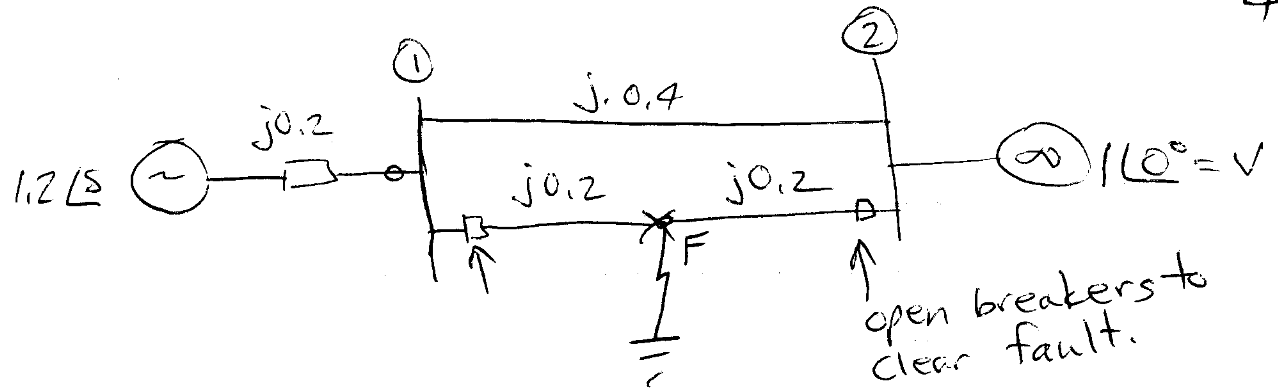
If δ_1 had been $0°$, $\quad A_1 = \int_{0}^{.524} (1.5 - 2\sin\delta)\,d\delta$

$$= 1.5(.524) + 2\cos .524 - 2$$

$$= \underline{0.5176}$$

∴ Stability would not have be possible!

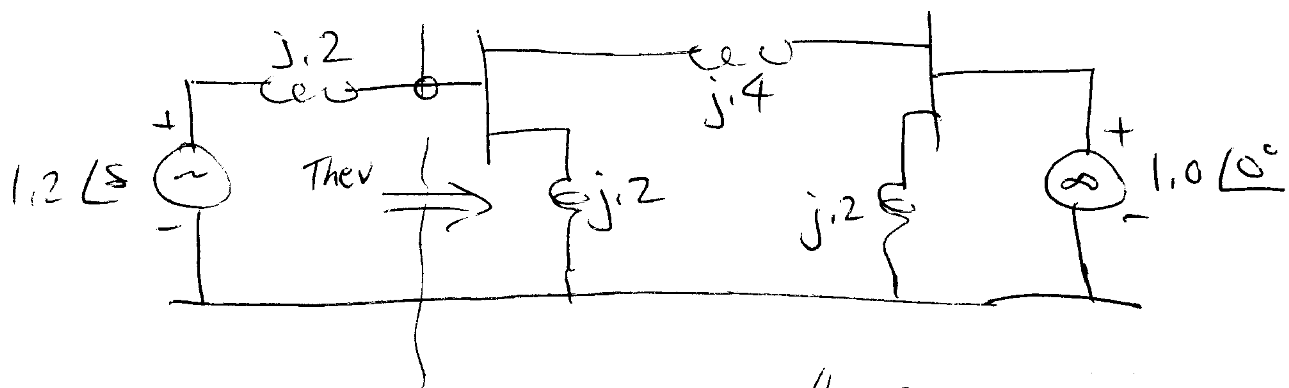
Ex:



What if fault occurs at $t=0$?

$$P_{e_{max}}(t=0^-) = \frac{(1.2)(1)}{j.2 + j.4 // j.4} = \frac{1.2}{.4} = 3.0 \text{ p.u.}$$

For fault, Thevenize at bus 1 look back to inf. bus.



$$Z_{TH} = j.4 // j.2 = j0.1333$$

$$V_{TH} = 1 \left(\frac{j.2}{j.6}\right) = 0.333$$

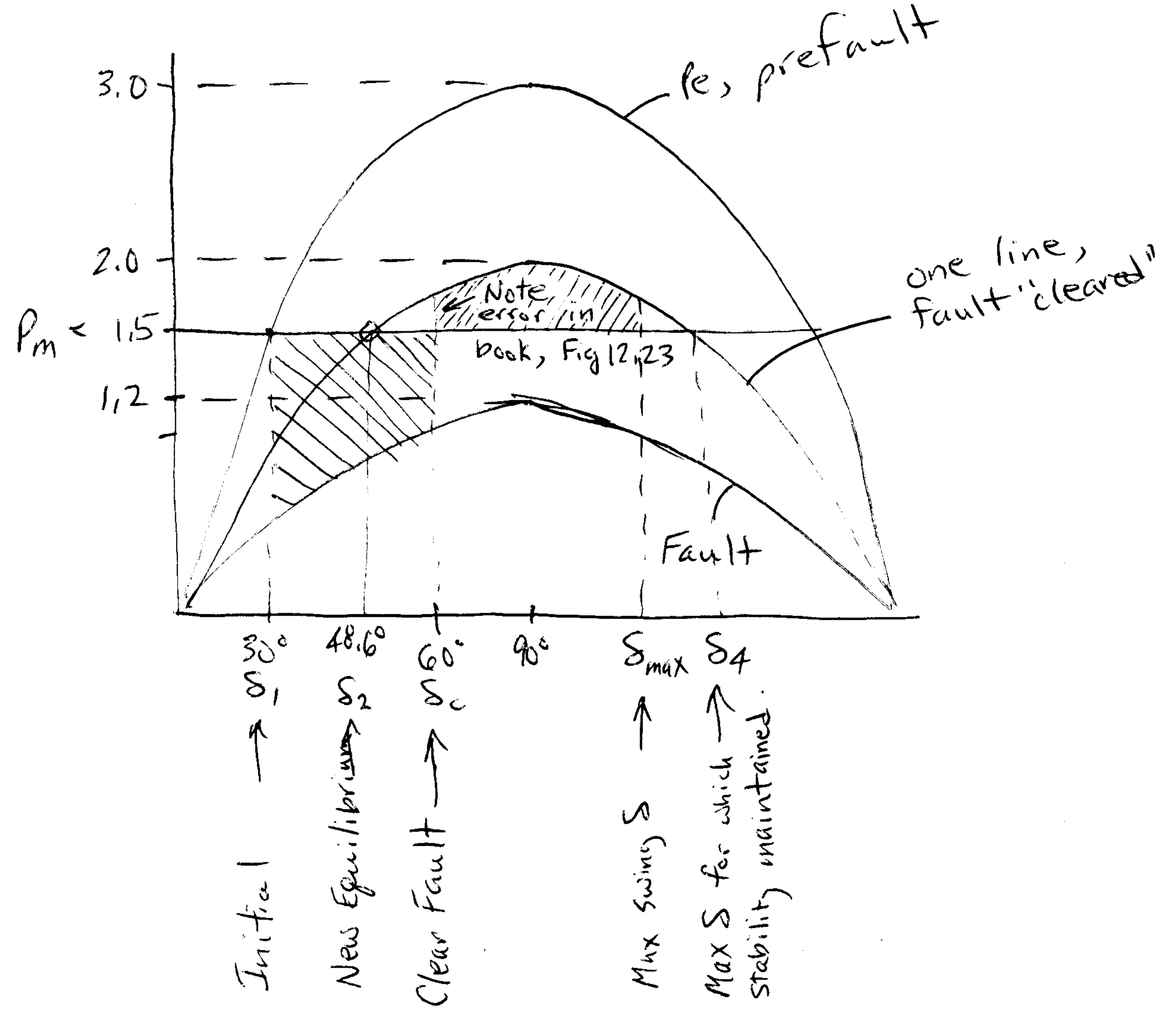
$$\therefore \quad P_{e_{Fault\,max}} = \frac{(1.2)(0.333)}{.2 + .1333} = 1.2$$

Note: $P_m = 1.5$ Conclusion?

Impossible to maintain stability, if fault persists.

If fault can be cleared,

$$P_{e\,max} = \frac{(1.2)(1)}{.2+.4} = 2.0 \quad \text{(from before, one line)}$$



The accelerating area, A_1 is

$$\int_{\delta_1}^{\delta_c} (P_m - P_e)\,d\delta = \int_{.524}^{1.047} (1.5 - 1.2\sin\delta)\,d\delta$$

$$= 1.5\,\delta + 1.2\cos\delta \Big|_{.524}^{1.047} = 0.346$$

How far will δ increase after fault is cleared?

$$|A_2| = |A_1|: \quad \int_{\delta_{max}}^{1.047} (1.5 - 2.0 \sin\delta)\, d\delta = 0.346$$

$$1.5\delta + 2.0\cos\delta \Big|_{\delta_{max}}^{1.047} = 0.346$$

$$1.5\,\delta_{max} + 2\cos\delta_{max} = 0.346$$

$$\Rightarrow \delta_{max} = 1.848 \text{ rad} = 105.9^\circ$$

Max it could "swing" is out to $131.4^\circ = \delta_4$

<u>Question:</u> What is critical clearing angle? This is largest angle that fault could be cleared and stability still maintained. Call this δ_{cc}.

<u>Ex 12.5</u>

$$\int_{0.524}^{\delta_{cc}} \overbrace{(1.5\cancel{\delta} - 1.2 \sin\delta)}^{\text{faulted}}\, d\delta = \int_{2.293}^{\delta_{cc}} (1.5 - 2\sin\delta)\, d\delta$$

Solving,
$$1.5\delta + 1.2\cos\delta \Big|_{.524}^{\delta_{cc}} = 1.5\delta + 2\cos\delta \Big|_{2.293}^{\delta_{cc}}$$

$$\underline{\underline{\delta_{cc} = 1.196 \text{ rad} = 68.6^\circ}}$$

How long do we have to clear?

Not so simple, since the accelerating torque varies with δ. Some key relationships:

$$P_{acc} = P_m - P_e = P_m - 1.2 \sin\delta$$

P_{acc} gradually decreases as δ increases to δ_{cc}

$$T_{acc} = \left(P_m - 1.2 \sin\delta\right) \omega_{mech}(\delta)$$

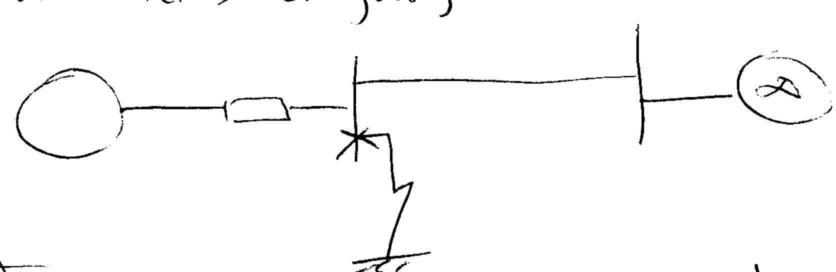
$$\omega = \omega_0 + \int_0^{t_{cc}} \alpha(\delta)\, dt \quad \leftarrow \quad \omega(t) = \omega_0 + \alpha t$$

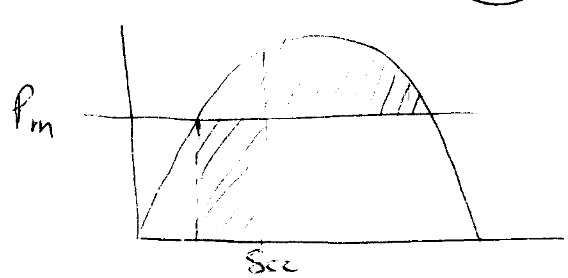
From swing equation,

$$\omega_{cc}^2 - \omega_1^2 = \frac{2\pi f}{H} \int_{\delta_1}^{\delta_{cc}} (P_m - P_e)\, d\delta \ ?$$

Should be able to get closed-form expression for $t_{CRITICAL}$. Usually this is done trial & error, in available texts anyway.

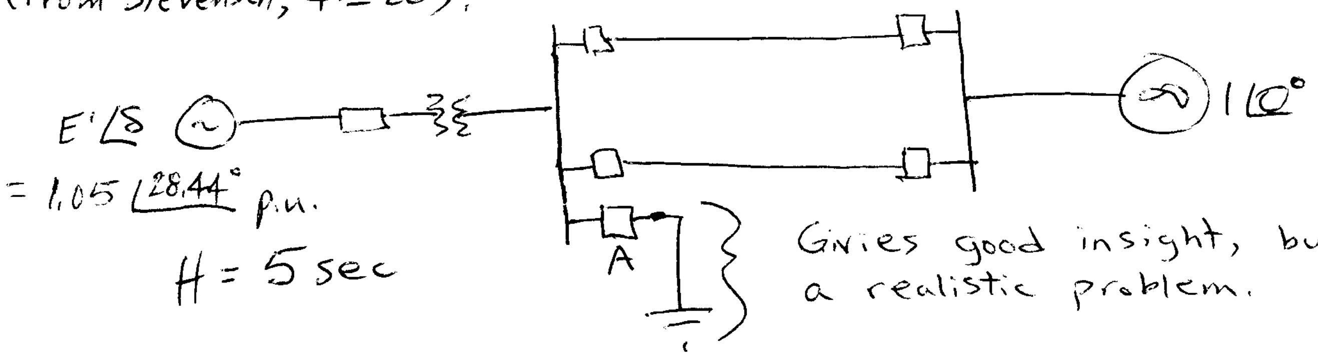
Easier case:





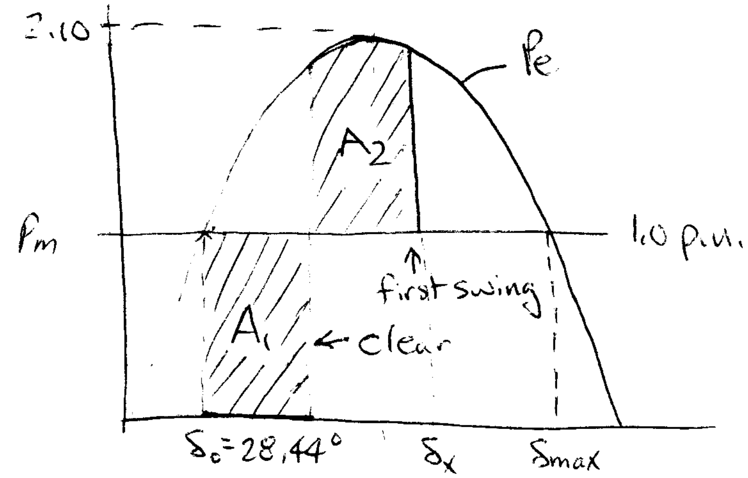
$P_e = 0$ during fault!

More on Equal Area Criterion: <u>Fault on Gen Bus</u>
(From Stevenson, 4th Ed).



Gives good insight, but not a realistic problem.

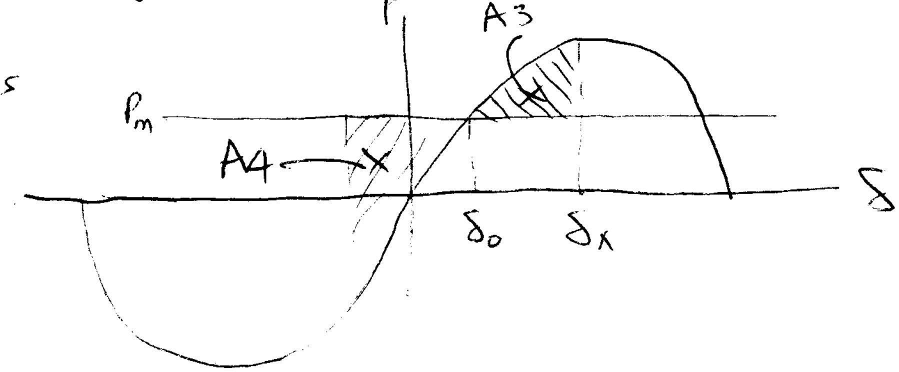
Assume a bus fault occurs (Breaker A is closed).
Prefault P_{max} is 2.10 p.u.



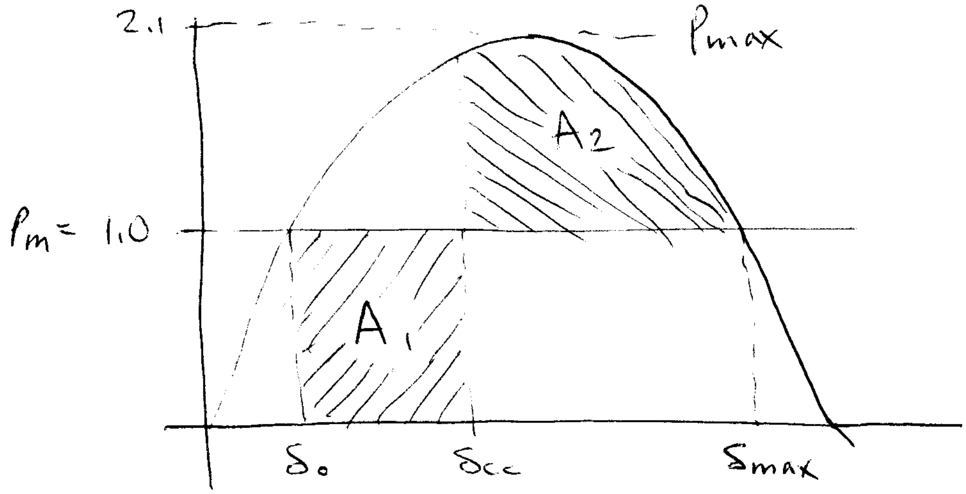
When fault occurs in this case, $P_e = 0$. $(P_m - P_e) = P_m$
P_e remains at zero until fault is cleared (open A).
δ will continue to increase until $A_2 = A_1$.

After clearing, equilibrium is still at δ_o

Now, δ swings about δ_o, with $A_3 = A4$



What is maximum δ_x for which stability can be maintained?



$$\underbrace{\int_{\delta_o}^{\delta_{cc}} (P_m - \cancel{P_e})\, d\delta}_{A_1} = \underbrace{\int_{\delta_{max}}^{\delta_{cc}} (P_m - P_e)\, d\delta}_{A_2}$$

$$P_m \delta \Big|_{\delta_o}^{\delta_{cc}} = P_m \delta + P_{max} \cos\delta \Big|_{\delta_{max}}^{\delta_{cc}}$$

$$P_m \delta_{cc} - P_m \delta_o = \cancel{P_m \delta_{cc}} - P_m \delta_{max} + P_{max} \cos\delta_{cc} - P_{max} \cos\delta_{max}$$

$$P_m (\delta_{max} - \delta_o) = P_{max} (\cos\delta_{cc} - \cos\delta_{max})$$

$$\cos\delta_{cc} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_o) + \cos\delta_{max}$$

but $\delta_{max} = \pi - \delta_o$, $P_m = P_{max} \sin\delta_o$

$$\boxed{\delta_{cc} = \cos^{-1}\left[(\pi - 2\delta_o) \sin\delta_o - \cos\delta_o \right]}$$ only for bus fault at gen!

For this case, it is possible to directly calculate t_{cc}, the time before which the fault must be removed.

$P_A = P_m$, since P_e is zero in this case. Therefore, the accelerating Torque and the acceleration are roughly constant.

From swing equation, $\alpha = \frac{d^2\delta}{dt} = \frac{\pi f}{H}(P_m - \underset{0}{P_e})$

$$\therefore \quad \alpha = \frac{\pi f}{H} P_m \ \frac{\text{rad}}{\text{sec}^2}$$

$$\omega(t) = \underset{0}{\omega_0} + \alpha t = \frac{\pi f}{H} P t$$

$(\omega\ \text{ref} = \omega_s)$

$$\delta(t) = \delta_0 + \int_0^t \omega(t)\,dt = \delta_0 + \frac{\pi f}{2H} t^2$$

$$\therefore \quad \delta_{cc} = \delta_0 + \frac{\pi f}{2H} t_{cc}^2$$

$$\Rightarrow \quad \boxed{t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\pi f P_m}} \quad \text{only for bus fault at gen! } (P_e = 0)}$$

For previous problem, $\delta_0 = 28.44° = 0.496$ rad

$\cancel{\delta_{max} = \pi - \delta_0}$

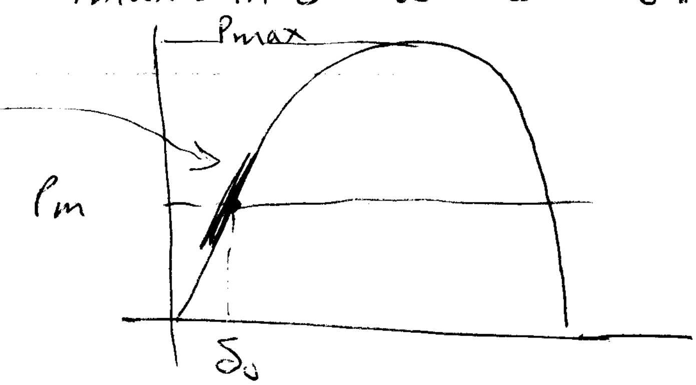
$$\delta_{cc} = \cos^{-1}\left[(\pi - 2\times 0.496)\sin 0.496 - \cos 0.496\right] = 1.426 = \underline{\underline{81.70°}}$$

$$t_{cc} = \sqrt{\frac{2(5)(1.426 - 0.496)}{(60)(\pi)(1.0)}} = \underline{\underline{0.222\ \text{sec}}}$$

Natural Frequency of Oscillation -

$$\omega_n = \sqrt{\frac{\omega_s P_{max}\cos\delta}{2H}}$$ (For small disturbances.)

$S_p = P_{max}\cos\delta$ is the "synchronizing power coefficient". It is the slope of $P_e = P_{max}\sin\delta$ at $\delta = \delta_o$.



$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\omega_s P_{max}\cos\delta}{2H}}$$

Ex: $\delta_o = 28.44°$, $H = 5$ sec, $P_{max} = 2.10$ p.u.

$$\omega_n = \sqrt{\frac{(377)(2.10\cos 28.44°)}{2(5)}} = 8.343 \ \frac{\text{elec rad}}{\text{sec}}$$

$$f_n = \underline{1.33 \text{ Hz}}$$

∴ Small perturbations in load (normal occurrence throughout day) will cause low-freq machine oscillations. Usually on order of ~1 Hz. They are quickly damped by prime mover, generator and system load. Note that R of transmission system has virtually no damping effect.

12.8

Solution of Swing Equation:

For practical cases, critical clearing time cannot be calculated with equal area criterion. Engineers must know how much time they have to clear the fault – the fault must be detected, a trip signal sent to the CB, and then breaker must operate. t_{cc} must be known before the CB & protective relaying can be speced and before coordinating the relaying.

If we can solve for $\delta(t)$, then we can determine t_{cc}. The approach is typically to use trial & error, clearing fault at different times.

An analog computer implementation is given in book.

$\underbrace{\frac{\pi f}{H}}_{\text{gain}}(P_m - P_e)$ is integrated. (Note inverting integrator).

basically:

$$\boxed{\begin{aligned} \alpha(t) &= \frac{d^2\delta(t)}{dt} = \frac{\pi f}{H}\Big(P_m - \underbrace{P_{max}\sin(\delta(t))}_{P_e}\Big) \\ \omega(t) &= \omega_0 + \int \alpha(t)\,dt \\ \delta(t) &= \delta_0 + \int \omega(t)\,dt \end{aligned}}$$

Note that P_{max} changes with fault, clear, reclose, line switching.

The book sets up a problem using Runge-Kutta numerical integration. This is more typical of the modern solution approach on computer.

Results for the previous example, where $\delta_{cc} = 68.6°$, are given in Figs 12.25 & 12.26.

→ In these programs, t_{cc} is specified and simulation is run to see if stability is maintained. Best guess/judgement is

** that if ~~first pos/neg fl~~ first negative fluctuation in δ is $\leq$ to first pos fluctuation then system will be stable.
i.e. if $(\delta_{max} - \delta_o) \geq (\delta_o - \delta_{min}) \Rightarrow$ stable.

For this simple system (two machines), δ_{cc} could be found by equal-area. t_{cc} could be found simply by running simulation and noting the value of t when $\delta = \delta_{cc}$.

What if there are more than 2 machines?

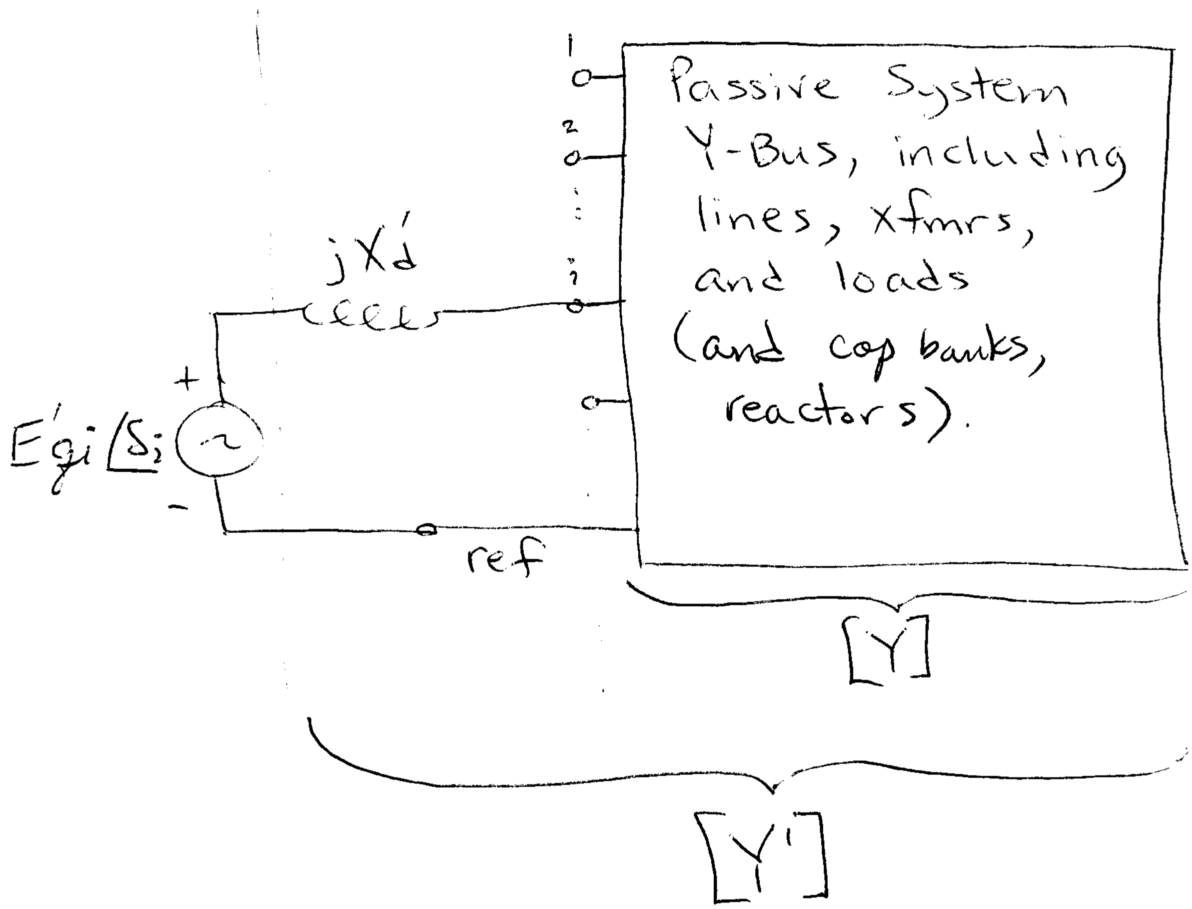
Before going on,

- Do you understand swing equation?
- Do you understand [Y] system description?
- Do you have equal area method figured out?

Prob 12.15

12.9 - Multimachine Systems

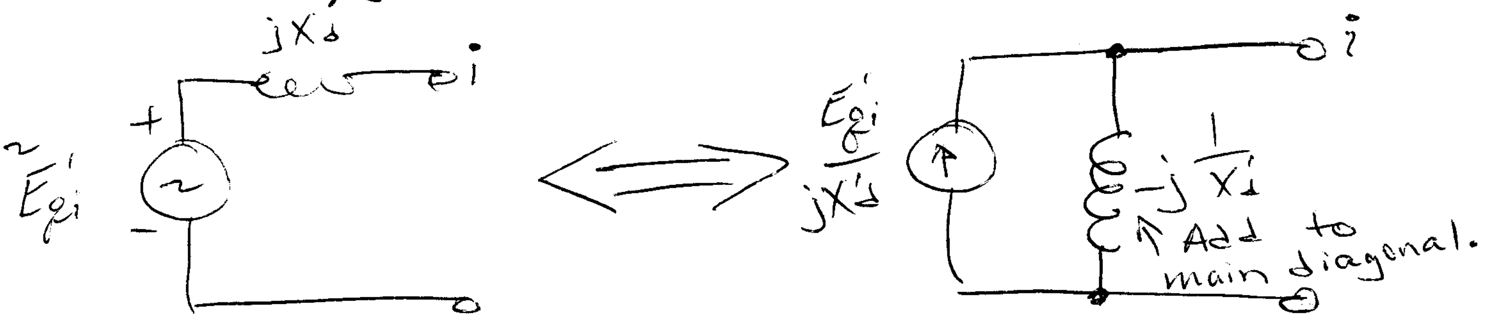
A network approach must be used where more than 2 machines are to be simulated.



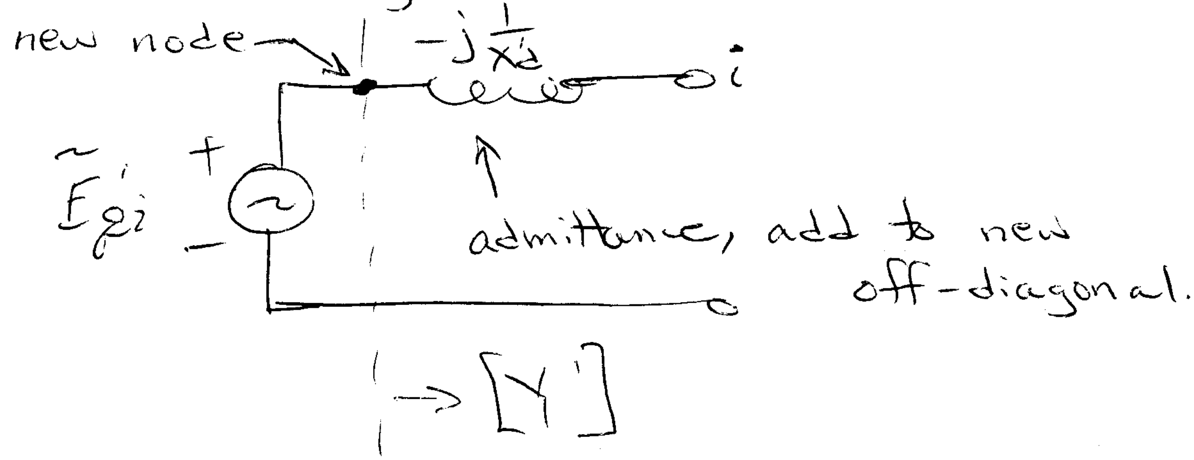
- Pre-~~dist~~ transient bus voltages typically obtained from load flow program.
- All buses not connected to a generator ~~are~~ have no current injected.

$$\begin{array}{c} \\ \\ \\ \\ \text{All zero}\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \end{array} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_{n+1} \\ \vdots \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} Y' \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \\ V_{n+1} \\ \vdots \\ V_{n+m} \end{bmatrix} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} n \text{ generators} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \begin{array}{l} m \text{ buses w/o generators,} \\ \text{Voltage due to } I_1 \rightarrow I_n \end{array} \end{array}$$

[Y'] is formed by adding Norton admittance of generators to main diagonal of [Y]. (No extra nodes added to network).



In this book, author adds an extra node at each generator bus:



Therefore, end up with $2n+m$ nodes in system. We will follow author's method, even though not the best in terms of computer application.

Nodes not connected to a generator are typically eliminated, using matrix partitioning.

$[Y']$ is partitioned:

$$\begin{array}{c|c|c} & 1 \to n & n+1 \to n+m \\ \hline 1 \downarrow n & [Y_{AA}] & [Y_{AB}] \\ \hline n+1 \downarrow n+m & [Y_{BA}] & [Y_{BB}] \end{array}$$

$$[Y_{red}] = [Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}] = n \times n$$

See derivation, p. 533

Ex: Go thru Ex. 12.7

Effect of loads: Loads are added to main diagonal term of $[Y]$

Ex: See example 12.8

Calculating $P_{e\,max}$ for various ~~[illegible]~~ conditions is done using $[Y]$ for each of the operating states.

Ex: If $[Y] = \begin{bmatrix} -j2.5 & j2.5 \\ j2.5 & -j2.5 \end{bmatrix}$ Prefault

$E_1 = E'_{g1} = 1.2$ p.u. $\quad \delta_1 = \delta \qquad y_1 \angle \gamma_1 = 2.5 \angle -90^\circ$

$E_2 = 1.0$ p.u. $\quad \delta_2 = 0$ (infinite bus) $\quad y_2 \angle \gamma_2 = 2.5 \angle +90^\circ$

$$P_{e1} = \sum_{j=1}^{2} E_1 E_j y_{1j} \cos(\delta_1 - \delta_j - \gamma_{1j})$$

$$= \cancel{(1.2)^2(2.5)\cos \overbrace{90^\circ}^{\delta_1-\delta_1-(-90)}} + (1.2)(1.0)(2.5)\cos(\delta_1 - 0 - 90)$$

$$= 3.0 \sin\delta \Rightarrow P_{e\,max} = 3.0$$

Using same method,

$Pe_{max} = 1.2$ during fault.
$Pe_{max} = 2.0$ ~~[illegible]~~ when cleared.

Each machine's swing equation is solved separately, although δ depends on other machines.

$$P_{mi} - P_{ei} = \frac{H_i}{\pi f}\left(\frac{d^2\delta_i}{dt^2} + \frac{d\omega_{ref}}{dt}\right) \quad i = 1, 2 \ldots n$$

Caution: use ~~same base~~ system base for all H and P values.

Usually pick infinite bus machine for ω_{ref}

$$P_{mi} - P_{ei} = \frac{H_i}{\pi f}\left(\frac{d^2\delta_i}{dt^2}\right) \quad i = 1, 2, \ldots n$$

Reformulate as

$$\frac{d\delta_i}{dt} = \omega_i$$

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i}\Big(P_{mi} - \underbrace{\sum_{j=1}^{n} E_i E_j y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})}_{P_{ei}}\Big)$$

Initial conditions: $\omega_i = 0$ for all machines $\underline{\omega_i(0)}$
$\delta_{0i} = \delta_i(0)$

$$\frac{d\delta(t)}{dt} = f = \omega(t)$$

$$\frac{d\omega(t)}{dt} = g = \alpha(t) = \frac{\pi f}{H}(P_m - P_e)$$

$$\delta(0) = \delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max}}\right)$$ INITIATE

$$\omega(0) = 0 \qquad \alpha(0) = $$ CALCULATE

$f(\delta,t)$: omeg $= \omega(t)$

$g(\delta,t)$ alpha $= \frac{\pi f}{H}\left[P_m - P_{max}\sin\delta(t)\right]$

~~K1 = OMEG*dt~~
~~K1 = ALPHA*dt~~
~~K2 =~~
~~L2 = $\alpha(\delta + \frac{K1}{2})$*dt~~
~~K3~~
~~L3 = $\alpha(\delta + \frac{L2}{2})$*dt~~
~~K4~~
~~L4 =~~

K1 = OMEG*dt = dδ
L1 = ALPHA(δ)*dt
K2 = OMEG*dt
L2 = ALPHA(δ + K1/2)*dt
K3 = OMEG*dt
L3 = ALPHA(δ + K2/2)*dt
K4 = OMEG*dt
L4 = ALPHA(δ + K3)*dt

~~DX~~ DOMEG = OMEG*DT
DDELT = (L1 + 2*L2 + 2*L3 + L4)/6.0

OMEG = OMEG + DOMEG
DELTA = DELTA + DDELT

~~FUNCTION OMEGA (X, t~~
~~FUNCTION ALPHA (DELTA, PM, PMAX)~~

OMEGA (0:1000)
DELTA (0:1000)
ALPHA (0:1000)

LECTURE 22 & 23

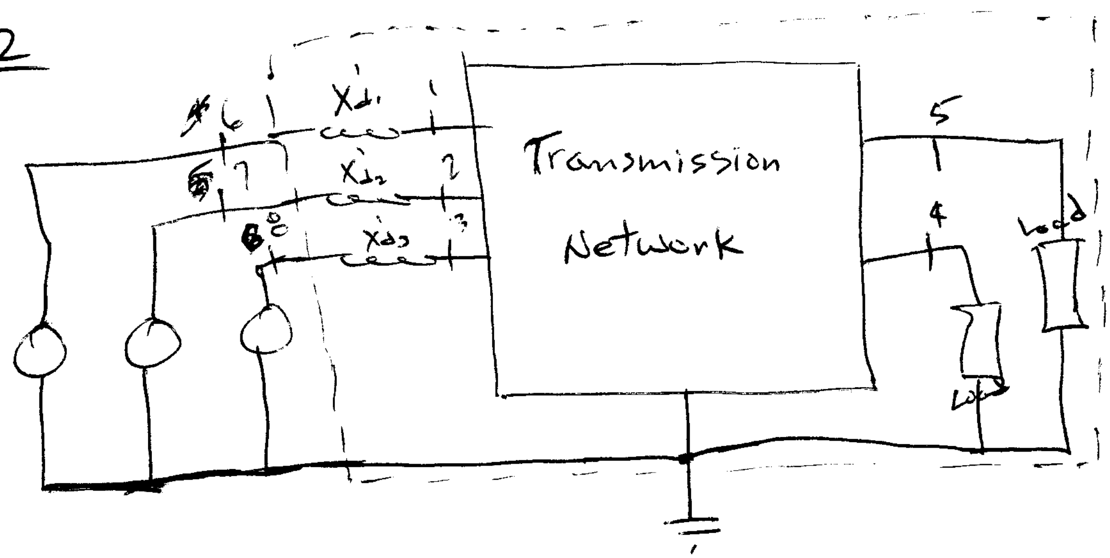
MULTI-MACHINE STUDIES:
(Can't use equal-area if > 2 machines)

- Use **load** flow to establish initial conditions.

- Convert loads to Admittances
(Can be constant admittance, or voltage dependent.)

ex:
$$Y_{L5} = \frac{P_L - jQ_L}{|V_L|^2}$$ then add to Y_{55}

Fig 14.2



Two ways to absorb X_d'
a) Make new bus
b) Use Norton Equiv

Electrical power:

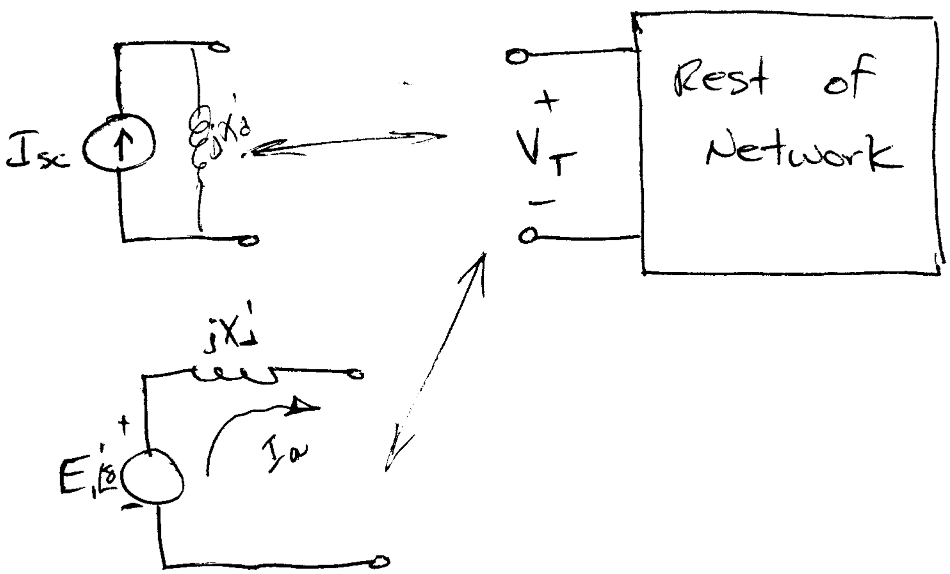
Add buses:

$$P_{e_1} = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}| \cos(\delta_{12} - \theta_{12})$$
$$+ |E_1'||E_3'||Y_{13}| \cos(\delta_{12} - \theta_{12})$$

So do Summation

OR: use Norton Equiv

For each Mach:



~~By you know~~

1) Use V_T, and power flow to find $|E'|$ = constant.
(From load flow) and δ_0 & I_0

2) Switch system (Fault, etc)

$$P_e = E' I_a^* = E' \left[\frac{(E' - V_T)}{jX_d'} \right]^*$$

Modified Euler Method — Adapted From Stagg & El-Abiad

1) Use V_T and power flow from Load flow to find $|E'|_o$, δ_o, I_{ao}, P_m

$$P_m = Re\{E'_o \, I^*_{ao}\} = \text{constant}$$

2) Perform system switching (Fault) etc (Modify Y-Bus)

3) Use $I_{sc(0)} = \dfrac{E'}{jX'_d}$ and solve $V_{(0^+)} = Y^{-1} I_{sc(0^+)}$ for all machines.

4) Calculate new machine currents based on

$$\frac{E'_o - V}{jX'_d} = I_{a(0^+)}$$

5) Calculate machine electrical Power

$$P_e = Re\{E'_o \, I^*_a(0^+)\}$$

6)
$$\omega(t+\Delta t) = \omega(t) + \left[\frac{d}{dt}(\omega(0))\right]\Delta t$$

from swing eqn.

$$= \boxed{\omega(t) + \left[\frac{\pi f}{H}(P_m - P_e)\right]\Delta t} \quad \text{est.}$$

$$\delta(t+\Delta t) = \delta(t) + \frac{d\delta(t)}{dt}\Delta t$$

$$= \boxed{\delta(t) + [\omega(t) - \omega_s]\Delta t} \quad \text{est.}$$

7) Calculate est Voltages behind jX'_d $\Longrightarrow$ $|E'| \angle \delta(t+\Delta t)$

8) Use $|E'| \angle \delta(t+\Delta t)$ (est) to determine $V(t+\Delta t) = Y^{-1} I_{sc(t+\Delta t)}$

9) Calculate $P_e = Re\{E'_{(t+\Delta t)} \, I^*_{a(t+\Delta t)}\}$

10) $$\boxed{\omega(t+\Delta t) = \omega(t) + \left[\frac{\frac{d\omega(t)}{dt} + \frac{d\omega(t+\Delta t)}{dt}}{2}\right]\Delta t}$$ final

$$\boxed{\delta(t+\Delta t) = \delta(t) + \left[\frac{\frac{d\delta(t)}{dt} + \frac{d\delta(t+\Delta t)}{dt}}{2}\right]\Delta t}$$ final)

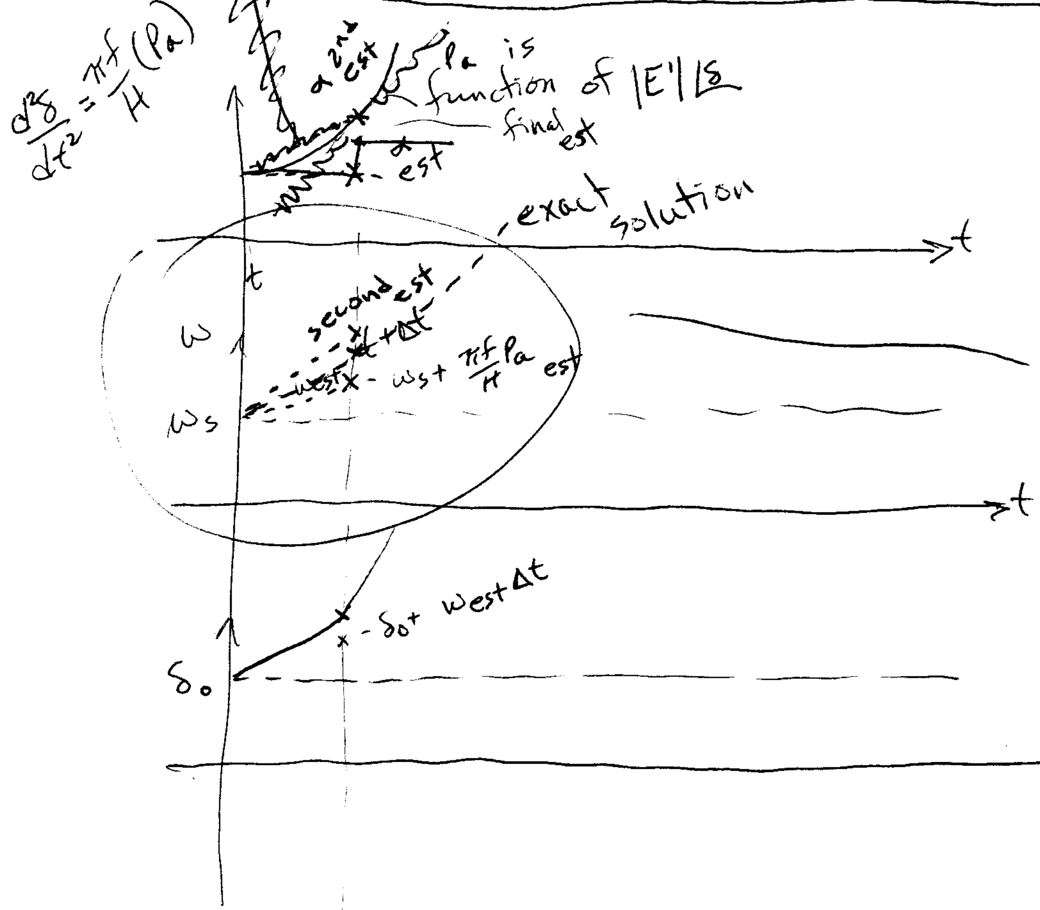
11) Calculate final est. of voltages behind jX_d' $\Rightarrow$ $|E'|\angle\delta(t+\Delta t)$

$t = t + \Delta t$

IF no switching @ t GOTO 4

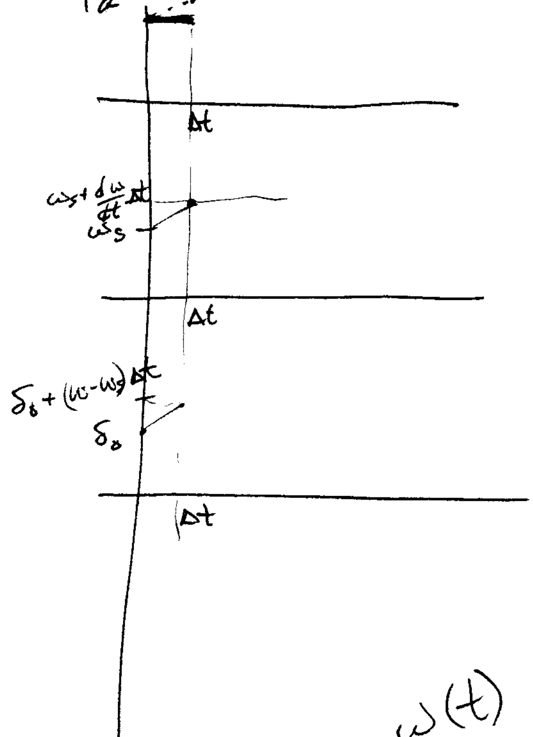
IF switching @ t GOTO 2

IF $t = t_f$ - quit.

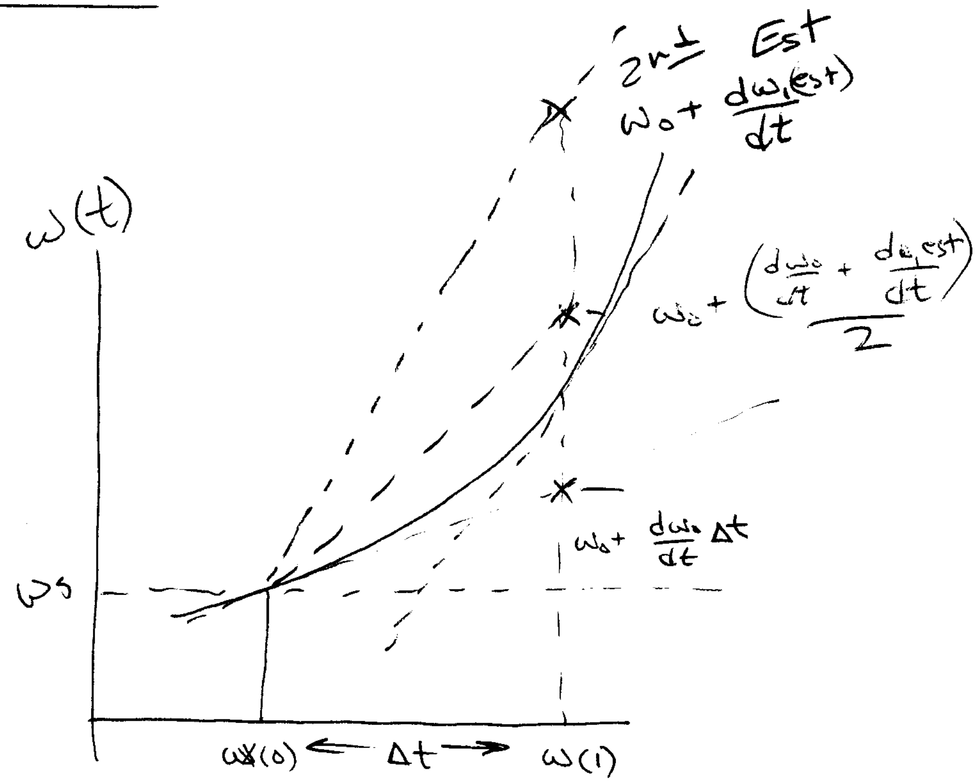


see next page

$$P_a = \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

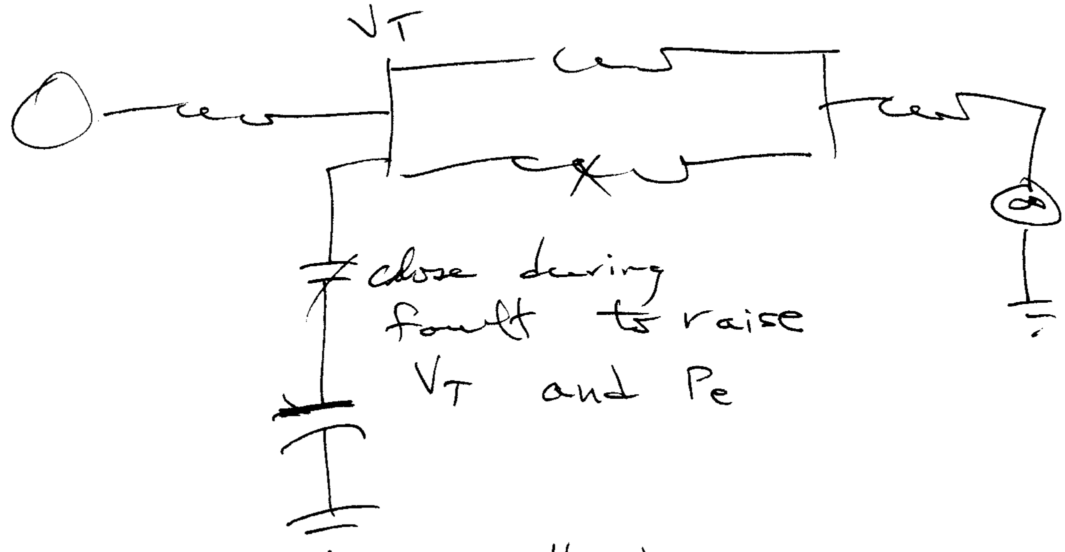


MODIFIED
EULER



Factors Affecting Stability

- Gov
- Exciter
- PSS (Power System Stabilizer)
  works on Δf from f_s changes V_{INT}
  faster than Exciter ΔV controls
- STATIC VAR COMP



Hydro Quebec

Fast Value

Braking Resist