EE5200 - Stability Notes
Fall - 2003 - Bruce Mork

# SYSTEM STABILITY

Stability: Ability of machines in system to recover from system disturbances and still remain synchronized and ~~deliver load~~. return to a steady-state operating point.

Types:

**Steady-State Stability** - Use loadflow to:
Check for
1 - Phase angle across T-line $\ll 90°$
2 - $0.95 < V_{BUS} < 1.05$
3 - Gen's, T-Line's, XFMRs within P, Q, V, I limits.

Also - do incremental changes in operating points to check system sensitivities.

* **Transient Stability** - Check major disturbances
- Loss of generator
- Line switching
- Faults
- Load Switching

Track frequency changes ($f_s = 60$ Hz) and $\delta$ changes.
Objective: See if machines return to synch frequency with new s-s power angles.
Assumptions: ~~[illegible]~~

So-called "first swing" model, good for ~ 1.0 sec:
- $P_M$ constant, include inertial dynamics.
- $E_g$ doesn't change, use loadflow on electrical side.
- dc offsets & harmonics ignored
- Use symm components if unbalanced.

* This is what we'll focus on in this course.

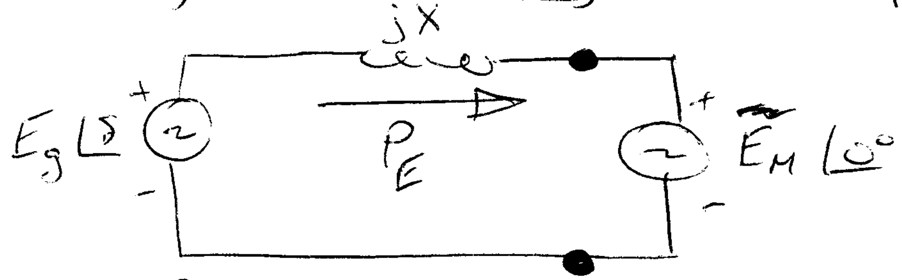
Transient Stability (cont'd):
If $t > 1.0$ sec is desired ("multi-swing") then can include governor & exciter.

DYNAMIC STABILITY – out to several minutes.
Effects Also included:
- Governors
- Exciters
- LTC XFMRS
- Dispatch / SCADA controls

Interactions can destabilize system even several minutes after disturbance occurs. (Even when transient stability is maintained.)

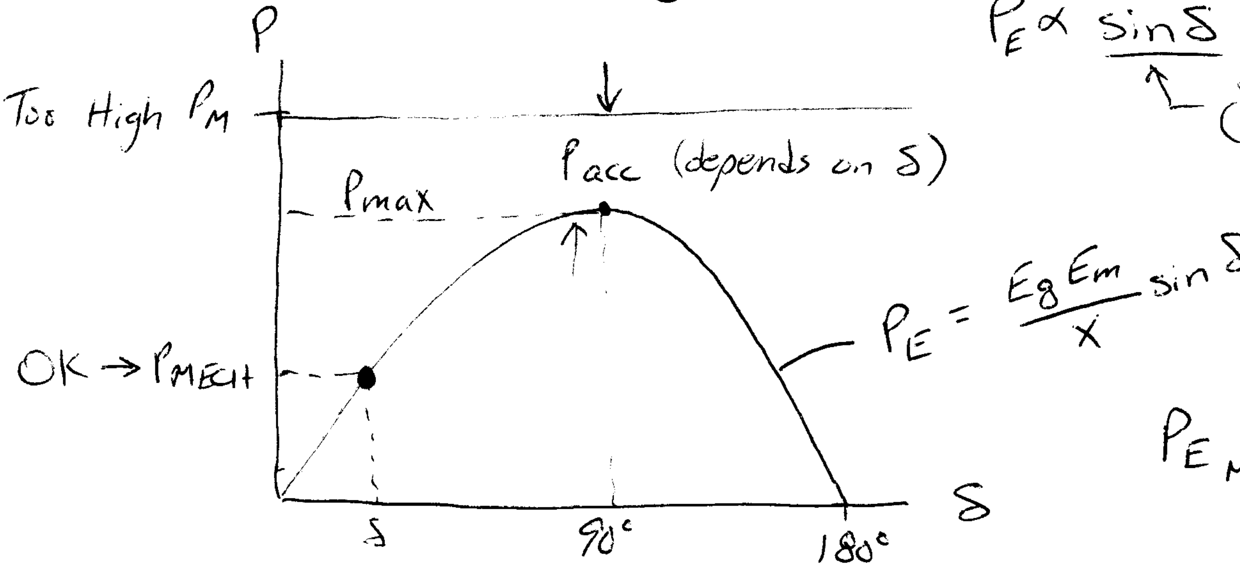
Ex:

## Steady-State Stability - Like power transfer




$$P = \frac{E_g E_M}{X} \sin\delta$$

$P_E \propto \sin\delta$ ← governor (steam throttle valve)

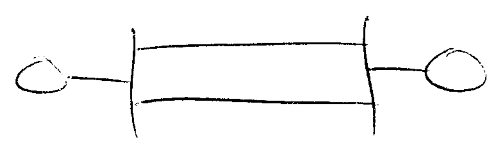



$$P_{E_{MAX}} = \frac{E_g E_m}{X}$$

If $P_M > P_{E_{MAX}}$, lose synch so must disconnect from system

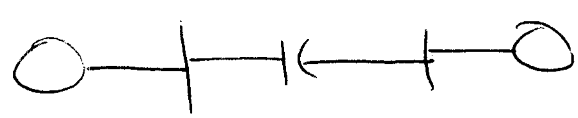
Keep $P_{MECH} \le P_{ELECT(MAX)}$ to assure stability.

To increase stability, we can:
- Increase $E_g$ or $E_m$
- Decrease X by
  a) Parallel lines:

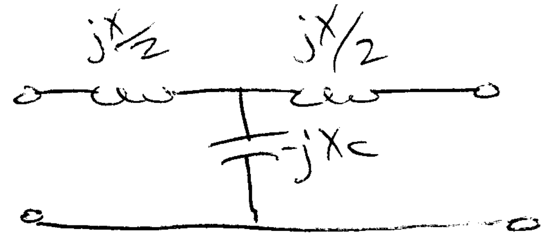


  b) Series Cap:

  $X = X_{LINE} - X_{CAP}$



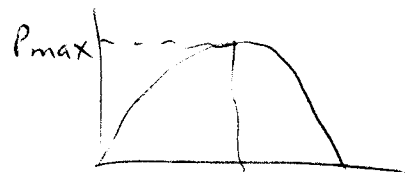
  c) Shunt Cap even helps:



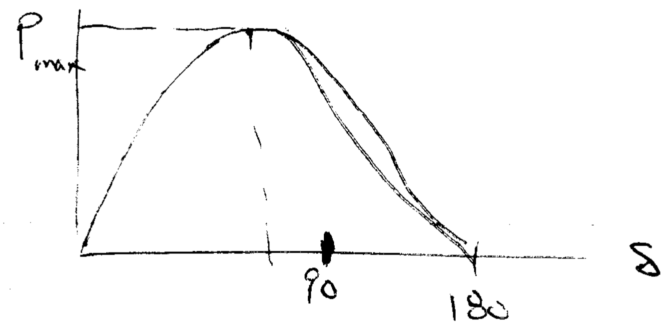

Transfer Impedance: $j\left(X - \frac{X^2}{4X_c}\right)$

For cylindrical rotor machines:

Keep $\delta < 90°$

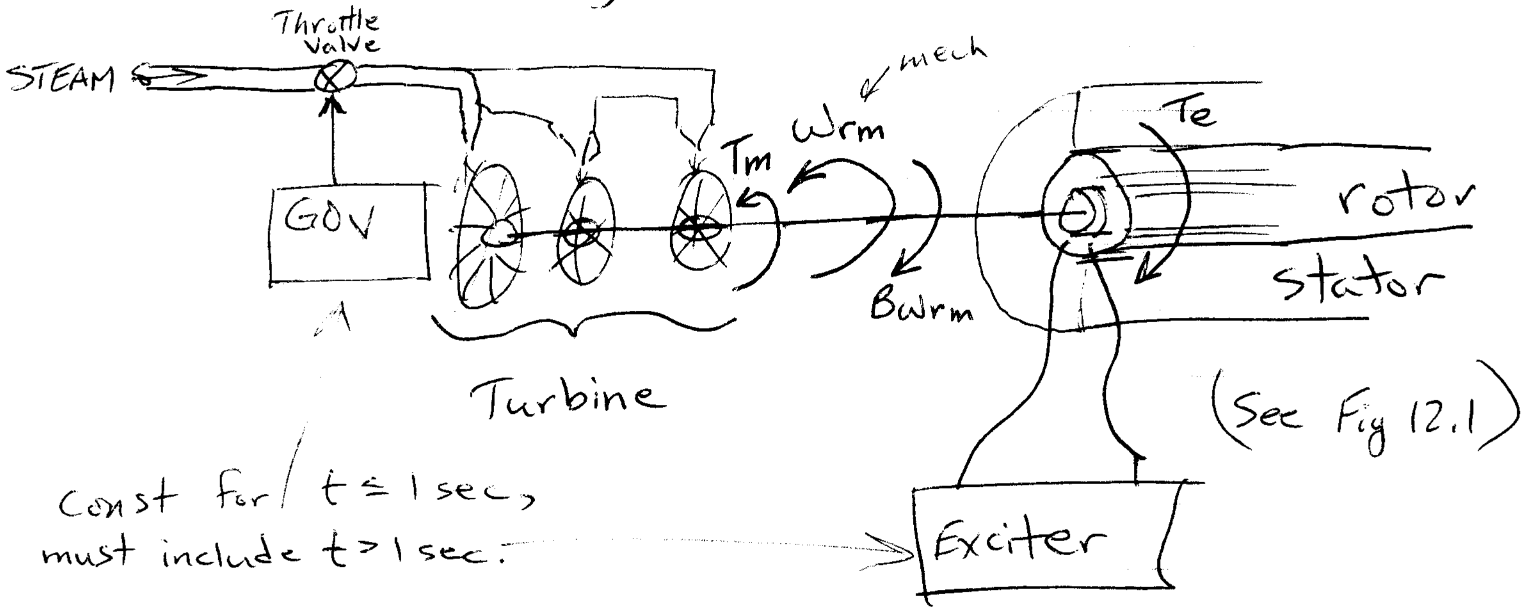



For salient machines, $\delta$ even less, since $P_{max}$ occurs at




In reality, dangerous to operate close to $P_{max}$, since small increase in $P_{mech}$ will put $\delta$ "over the top". Usually try to keep $\delta < 40°$.

# Transient Stability: The Problem




Tm - mechanical torque, N-m
Te - electromagnetic counter torque, N-m
J - mass polar moment of inertia
B - Damping torque coefficient
- Bearing Friction
- Windage
- Magnetic ~~leak~~ Losses (leakage)
- Any Drag Torques in General
- Damper Winding Torque
- Exciter Torque

$\omega_{rm}$ - mechanical rotor speed, rad/sec

Steady-State: $P_m = P_e$, $T_m = T_e$. But:
* Loss of Line, fault, load, etc. reduces $P_E$
  $P_M$ will stay constant (gradually reduce, $t > 1$ sec)

$P_A = P_M - P_E$ = Accellerating power, speeds up rotor
* Loss of generator in system makes $P_E > P_M$ and decelerates other generators in system.

Oscillations of machine $\delta$'s w.r.t. each other is called SWING.

# Adding Detail to Model

1) Governor - Measures $f$ and
(each machine) a) increases $P_m$ if $f < 60$ Hz
b) decreases $P_m$ if $f > 60$ Hz

2) Add excitation -
Measure $V_T$ (bus voltage)
a) Reduce $I_f$ if $V_T > 1.0$ p.u.
b) Increase $I_f$ if $V_T < 1.0$ p.u.

3) Fast valving
4) Power System Stabilizer
5) Single Pole Reclosing - Japan
6) Fast Reclosing
7) Load Shedding
8) Switched Capacitors
9) Braking Resistors

So computer simulation is a must.

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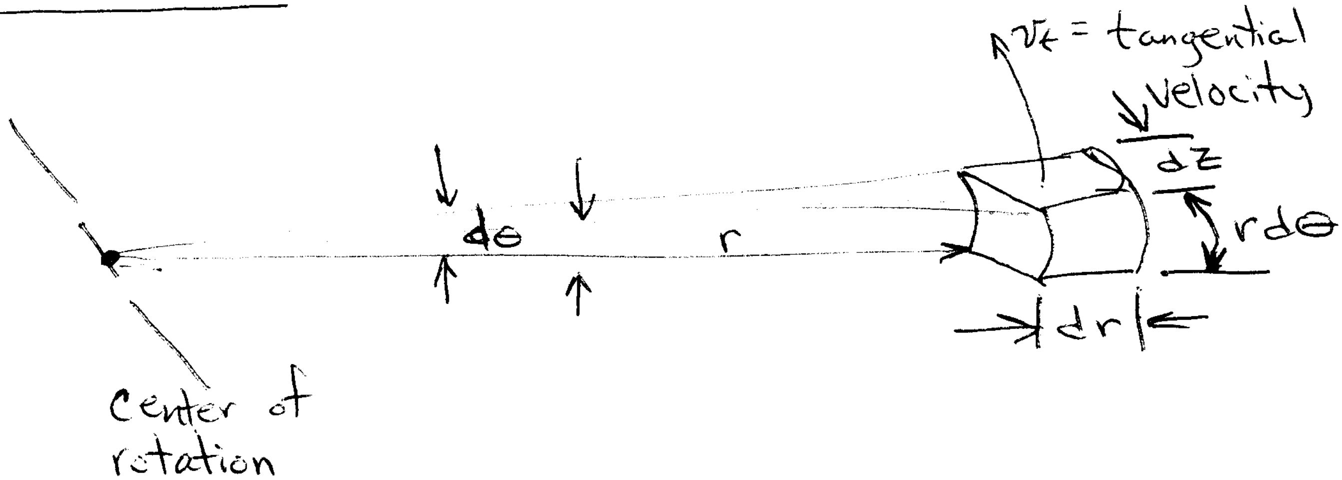
To improve stability
1) Make $H$ Large
2) Reduce $P_m$ during fault
a) Fast Valving
b) Gov - slow down during fault but too slow to react

## 2.2 - Inertia

Newton's second Law: $F = ma$




Tangential Forced on diff element: $\rightarrow$ $F_t = (dm)\, a_t = dm \dfrac{dv_t}{dt}$

$$= r(dm) \frac{d\omega_{rm}}{dt} \Bigg|_{v_t = r\omega_{rm}}$$

The accelerating Torque

$$T_a = rF_t = r^2(dm) \frac{d\omega_{rm}}{dt} = J\alpha \quad N\text{-}m$$

$\therefore$

$dJ = r^2 dm$ = mass polar moment of inertia of element. $(Kg\text{-}m^2)$

If material has density $\rho$, $dm = \rho \underbrace{dv}_{\text{volume}}$

$$dv = (rd\theta)(dr)(dz)$$

$$\therefore \quad J = \int_0^{2\pi} \int_0^{Z} \int_0^{R} \rho\, r^3\, dr\, dz\, d\theta$$

For actual rotor, field slots and use of different materials can make this complex problem.

i.e. $R = f(\theta, z)$
$Z = f(?)$

If rotor is homogeneous,

$$J = \rho \frac{R^4 Z 2\pi}{4} = \frac{\pi \rho R^4 Z}{2}$$

Total Cylinder Mass is

$$M = \rho \int_0^{2\pi} \int_0^{Z} \int_0^{R} r\,dr\,dz\,d\theta$$

$$= \rho \frac{R^2 Z 2\pi}{2} = \boxed{\pi \rho R^2 Z} \text{ kg}$$

Substituting, $\boxed{J = \frac{MR^2}{2}}$ Kg-m$^2$

If rotor not homogeneous, use numerical integration.

General Case:

$$T_a = J \frac{d\omega_{rm}}{dt}$$

Sometimes, radius of gyration is given:

$K$ = radius from axis of rotation that a concentrated mass $M$ could be placed to give identical $M$.

For diff mass or point mass, $J = MK^2$

$$K = \sqrt{\frac{J}{M}}$$

$\therefore$ If $K$ & $M$ are given, $J$ can be calculated.

~~Sometimes M is~~

Possible confusion:

| | Mass | Weight | g accel. of grav |
|---|---|---|---|
| SI: | Kg | N | 9.8 m/sec² |
| Eng: | slugs | lb | 32.2 ~~m~~ ft/sec² |

$W = Mg$ lbs

also lbm vs. lbf

$M' = Mg$ lbm

$W = Mg$ lbf

Common Practice in US: (unfortunately):

J given as $WK^2$

$W$ = mass in lbm
$K$ = rad gyration in feet

$$\therefore \quad J = WK^2 \ \text{lbm-ft}^2 \left(\frac{0.3048\ \text{m}}{\text{ft}}\right)^2 \left(\frac{0.4536\ \text{Kg}}{\text{lbm}}\right)$$

$$= \_\_\_\_\_\_ \ \text{Kg-m}^2$$

"If" several masses are on same shaft

Then $J_{TOT} = J_1 + J_2 + J_3 + \cdots J_n$

Then: <u>Convert</u>

$$H = \frac{\frac{1}{2} J_{TOT} \left(2\omega_s/N_P\right)^2}{S_{3\phi,\ BASE}}$$

Conversion of H to different system base,

Ex: Gen base is 500 MVA, System base is 100 MVA

$$\boxed{H_{\text{new base}} = H_{\text{old base}} \frac{S_{3\phi,\ \text{OLD BASE}}}{S_{3\phi,\ \text{NEW BASE}}}}$$

$$H_{new} = 5 \cdot H_{old}$$

Typical H values:

| | | |
|---|---|---|
| 2-pole thermal : | 2.5 - 6.0 | ON MACHINE BASE |
| 4-pole thermal : | 4.0 - 10.0 | |
| Hydro : | 2.0 - 4.0 | |

**Ex:** Nuclear Generator,

$$WR^2 = 5.82\times10^6 \text{ lbm-ft}^2$$
$$MVA = 1333 \text{ MVA}$$
$$N_s = 1800$$

$$J = WR^2 \times \text{conversion factors}$$

$$= \left(5.82\times10^6 \text{ lbm-ft}^2\right)\left(\frac{\text{kg}}{2.2\text{ lbm}}\right)\left(\frac{\text{m}}{3.28\text{ ft}}\right)^2$$

$$= \cancel{8.065\times10^5 \text{ kg-m}^2} = \underline{2.459\times10^5 \text{ kg-m}^2}$$

$$H = \frac{\frac{1}{2}\left(2.459\times10^5\right)\left[2\left(\frac{2\pi\times1800}{60}\right)/2\right]^2}{1333\times10^6} = \underline{3.27 \text{ sec}}$$

On 100 MVA system base,

$$H = 3.27\left(\frac{1333}{100}\right) = \boxed{43.56 \text{ sec}}$$

**Shortcut:**

$$\boxed{H = \frac{2.31\times10^{-10}\, WR^2 \times RPM^2}{\underbrace{S_{\text{machine base}}}_{\text{in MVA}}}}$$

$WR^2$ in lbm-ft$^2$

<u>Coherent Machines</u>: 2 or more machines operating in parallel, whose rotors swing together ($\delta_1 = \delta_2$).
(Only possible if sharing equal p.u. loads, and running on same droop characteristic)

<u>Ex:</u>

Unit 1: 500 MVA 0.85 PF, 20 kV 3600 RPM } $H_1 = 4.8$

Unit 2: 1333 MVA 0.9 PF, 22 kV 1800 RPM } $H_2 = 3.27$

~~$H_{?} = 4.8$~~ SYSTEM BASE = 100 MVA

To combine, must convert to same base.

$$H_{TOT} = 4.8\,\frac{500}{100} + 3.27\,\frac{1333}{100} = \boxed{67.59 \text{ sec}}$$

Alternate Method: $H = \dfrac{\text{MJ stored}}{\text{MVA}}$

$$H = \frac{(4.8\ \text{MJ/MVA})(500\ \text{MVA}) + (3.27\ \text{MJ/MVA})(1333\ \text{MVA})}{100\ \text{MVA}}$$

$\delta$ & $\omega$ <u>must</u> be in electrical units -- since one machine is 4-pole, other is 2-pole.

## 12.1 The Swing Equation:

$$T_A = T_{acc} = T_m - T_e = J\alpha = J\frac{d\omega_{rm}}{dt}$$

$(T = J\alpha \Leftrightarrow F = ma)$

In terms of Power,

$$P_m - P_e = \omega_{rm} J \frac{d\omega_{rm}}{dt}$$

$P_m = \omega_{rm} T_m$ $\quad$ $P_e = \omega_{rm} T_e$

Review: $\delta' =$ electrical torque angle $= \angle \vec{B}_{r_e} - \angle \vec{B}_{s_e} = \angle \tilde{E}_f - \angle \tilde{V}_T$

Here,
$\delta$ taken as angle between $\omega_{ref}$ and rotor

$$\delta = \angle \vec{B}_r - \angle \omega_{ref}$$ see fig 12.2

$\omega$ ref can be synch, or set to match some other machine in system. Usually, it is both, i.e. matches $\omega$ of ~~[illegible]~~ infinite bus. If there is no infinite bus, then probably set to $\omega$ of machine having largest "H".

For an N-Pole machine,

$$\omega_{re} = \omega_{rm}\left(\frac{N_p}{2}\right)$$

~~The~~ The electrical angular velocity:

$$\omega_{re} = \omega_{ref} + \frac{d\delta}{dt}$$

$$\omega_{rm} = \frac{\omega_{re}}{N_p/2}$$

$$P_m - P_e = \omega_{rm} J \frac{d\omega_{rm}}{dt}$$

Substituting, $$P_m - P_e = \left(\frac{\omega_{re}}{N_p/2}\right)(J)\frac{d}{dt}\left(\frac{\omega_{re}}{N_p/2}\right)$$

$$= \frac{\omega_{re}}{N_p^2/4} J \frac{d}{dt}\left(\omega_{ref} + \frac{d\delta}{dt}\right)$$

$$P_m - P_e = \frac{\omega_{re} J}{N_p^2/4}\left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2}\right)$$

Converting to per unit,

$$P_{m,pu} - P_{e,pu} = \frac{\omega_{re} J}{N_p^2/4 \cdot S_{3\phi\,BASE}}\left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2}\right)$$

At this point we define

$$H = \frac{\text{Kinetic Energy of all rotating parts at } \omega_s}{S_{3\phi \text{ rated}}} = \frac{\frac{1}{2} J \omega_{sm}^2}{S_{3\phi\,\text{rated}}}$$

mechanical speed

$$\text{units} = \frac{\text{Joules}}{\text{watt}} = \frac{\text{Watt}\cdot\text{s}}{\text{watt}} = \text{s}$$

$$H = \frac{\frac{1}{2} J \left( \underbrace{\frac{\omega_{se}}{N_p/2}}_{\omega_{s,mech}} \right)^2}{S_{3\phi\,BASE}}$$

Substituting, $J = \dfrac{2H\, S_{3\phi\,BASE}\, N_p^2}{4\,\omega_{se}^2}$

$$J = \frac{H\, N_p^2\, S_{3\phi\,BASE}}{2\,\omega_{se}^2}$$

if $\overset{\text{synch}}{\omega_{se}} = \overset{\text{actual}}{\omega_{re}}$ (not too far wrong if stability maintained)

Substituting, $P_{m,pu} - P_{e,pu} = \left[ \dfrac{\omega_{re}}{\cancel{N_p^2/4}\, \cancel{S_{3\phi\,BASE}}} \right] \left[ \dfrac{H\, \cancel{N_p^2}\, \cancel{S_{3\phi\,BASE}}}{\cancel{2}\, \omega_{re}^{\cancel{2}}/2} \right] \left( \dfrac{d\,\omega_{ref}}{dt} + \dfrac{d^2\delta}{dt^2} \right)$

$$\boxed{P_m - P_e = \frac{H}{\pi f} \left( \underbrace{\frac{d\,\omega_{ref}}{dt}} + \frac{d^2\delta}{dt^2} \right)}$$

Term = 0 if infinite bus $\omega_{ref} = \omega_s$ is used.

Again,

$P_m$ = turbine input power (mech) } Per Unit
$P_e$ = generator output } Per Unit
$H$ = Inertia Constant in seconds.
$\delta$ = $\angle \vec{B}_r - \angle \omega_{ref}$, radians
$\omega_{ref}$ = ang velocity of reference machine, rad/sec
$t$ = time in seconds

# 12.6 - Defining the Transient Stability Problem

Assumptions:
- Ignore magnetic saturation
- Assume machine is lossless.
- Assume balanced $3\phi$ operation.
- Assume $P_m$ constant
- Assume $|E_f|$ constant

For salient pole machine:

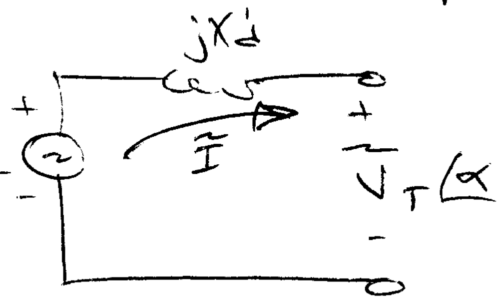
$$\tilde{E}_f = \tilde{V}_T + jX_q\tilde{I}_q + jX_d'\tilde{I}_d = \tilde{E}_q + j(X_d - X_q)\tilde{I}_d$$

Recall $\quad \tilde{I} = \tilde{I}_d + \tilde{I}_q$ $\quad$ (see fig, p.515 in Gross)

For stability calculations, use $X_d'$.

Illustration on p. 515 shows that if $X_d' \approx X_q$ then machine can be simplified to

$\tilde{E}_q' = E_q' \angle \delta$




w.r.t. phase reference of system.

$$P_e = \left.\frac{E_q' V}{X_d'} \sin\delta \right|_{X_d' = X_q}$$

If $X_d' \neq X_q$,

$$P_e = \frac{E_q' V}{X_d'} \sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} + \frac{1}{X_d'}\right)$$

$\sin 2\delta$

at $t = 0^-$, prior to disturbance.

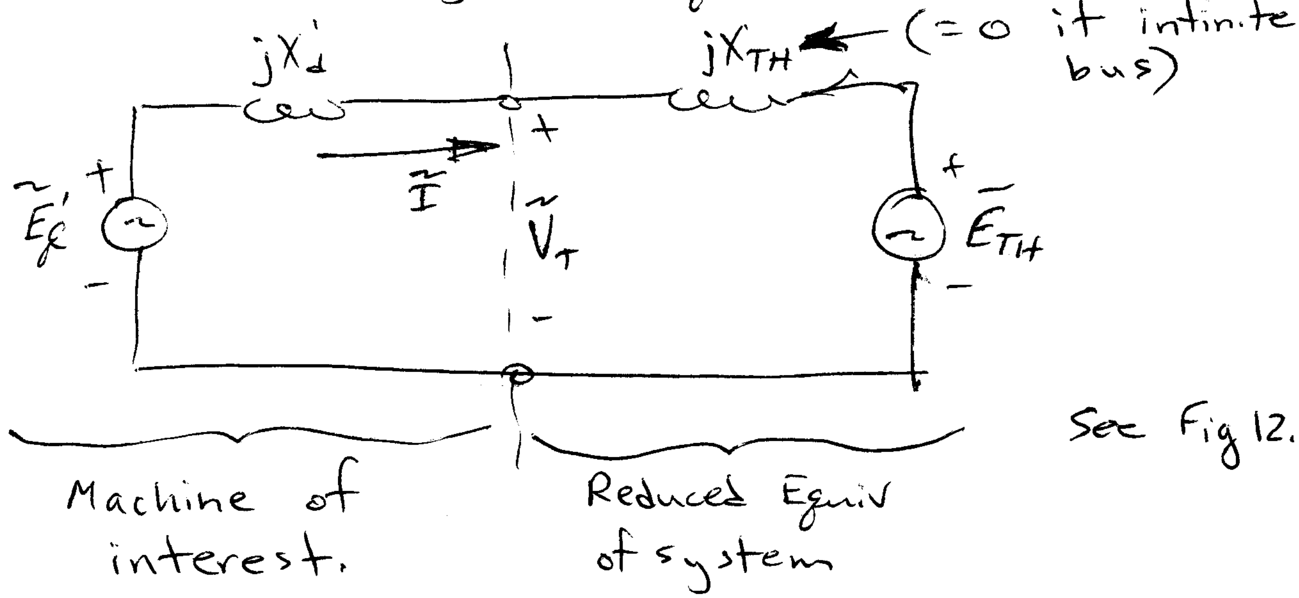
EE486

## Infinite Bus - Ideal voltage source of constant frequency

In terms of a generator, this means:

$\Rightarrow$
- Constant Internal Voltage
- Zero impedance
- Infinite Inertia ($\omega_{ref} = \omega_s$)

When considering how a disturbance affects one particular machine, it is possible to approximate the remainder of the system as a reduced dynamic equivalent:




See Fig 12.17

If response does not involve governor or exciter, the relationship

$$P_m - P_e = \frac{H}{\pi f}\left(\frac{d^2\delta}{dt^2} + \underbrace{\frac{d\omega_{ref}}{dt}}_{0 \text{ if infinite bus}}\right)$$

can be used. This assumes that $\omega_{ref} = \omega_s$.

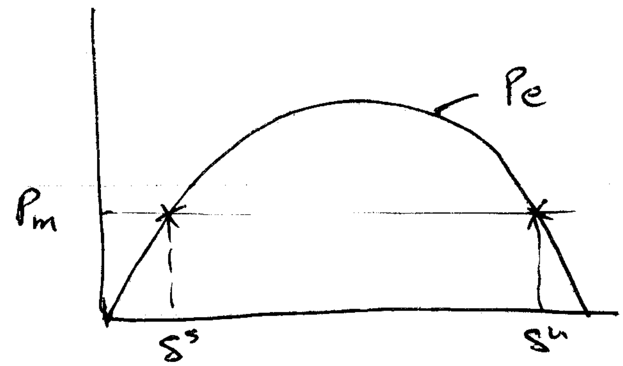
To evaluate,

$P_m =$ constant $\qquad P_e = \dfrac{E_g' E_{TH}}{X_d' + X_{TH}} \sin\delta$.

Equilibria Points:

when $P_e = P_m$

$$P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}$$




Perturbations:

$\delta^s$ is stable: From $\delta^s$, if $\delta$ increases, $\frac{d^2\delta}{dt} < 0$ $(P_m < P_e)$
$\Longrightarrow$ $\delta$ decreases
if $\delta$ decreases, $\alpha > 0$ $(P_m > P_e)$
$\Longrightarrow$ $\delta$ increases.

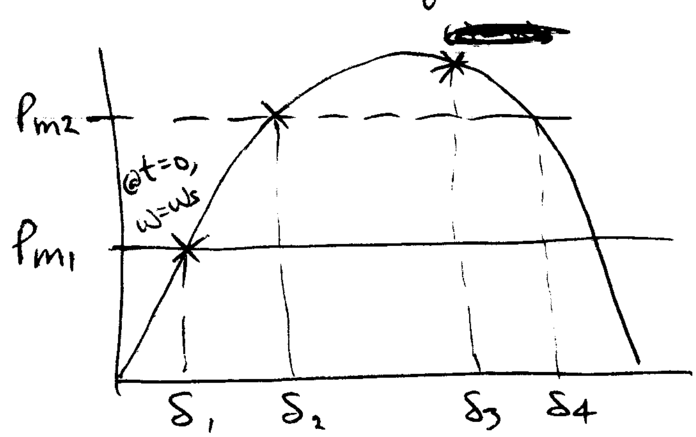
$\delta^u$ is unstable: From $\delta^u$, if $\delta$ increases, $\alpha > 0$ $(P_m > P_e)$
$\Longrightarrow$ $\delta$ increases further.
if $\underbrace{\delta \text{ decreases}}_{P_e \text{ proportional to } \sin\delta}$, $\alpha < 0$ $(P_m < P_e)$

acceleration

$\therefore$ Better keep $\delta < 90°$ for stable equilibrium.

Conceptual example:

If $P_m$ is initially $P_{m1}$, then suddenly increases to $P_{m2}$. What happens?

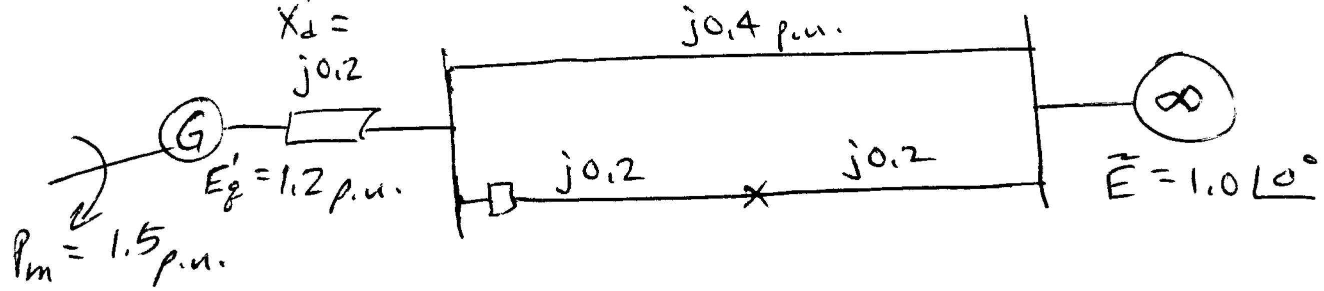



For $t \geq 0^+$, rotor accelerates, until $\delta$ reaches $\delta_2$, where it begins to decelerate.
Oscillation in $\delta$ (about $\delta_2$) continues, damping slowly.

$\delta < \delta_2 \Rightarrow P_m > P_e \Rightarrow$ accel
$\delta > \delta_2 \Rightarrow P_m < P_e \Rightarrow$ decel
$\delta > \delta_4 \Rightarrow P_m > P_e \Rightarrow$ accel

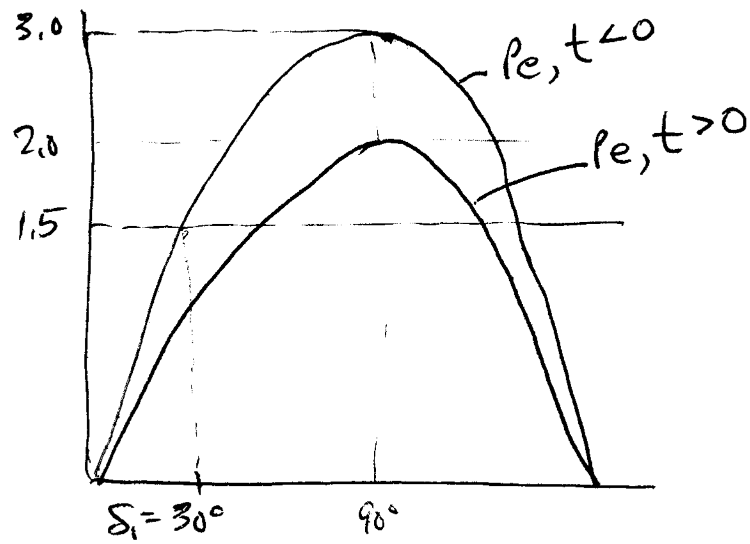
No chance to maintain stability if $\delta$ goes past $\delta_4$.

Ex:




@ $t = 0^-$ $\quad X = j0.2$ p.u. (X refers to T-Line tie)

$$P_m = P_e = \underbrace{\frac{(1.2)(1.0)}{0.2 + 0.2}}_{= 3.0 = P_{e\,max}} \sin\delta = 1.5 \text{ p.u.} \Rightarrow \delta_1 = 30°$$




Breaker opens at $t = 0$,

$$P_{e\,max} = \frac{(1.2)(1.0)}{0.2 + 0.4} = 2.0 \text{ p.u.}$$

New equilibrium point is at

$$1.5 = P_{max} \sin\delta = 2.0 \sin\delta$$

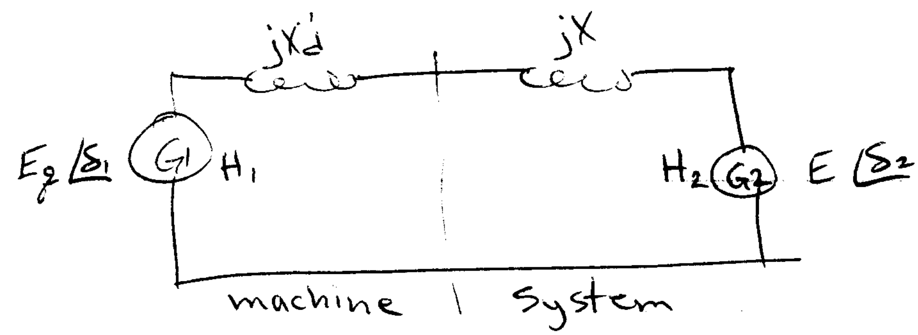
$$\sin\delta = 0.75 \Rightarrow \underline{\underline{\delta = 48.6°}} \text{ or } \cancel{131.4°}$$

unstable

## Considerations / Questions

1) What if system equivalent cannot be assumed to be infinite bus (i.e. infinite inertia)?

This is a case where machines are <u>noncoherent</u>.



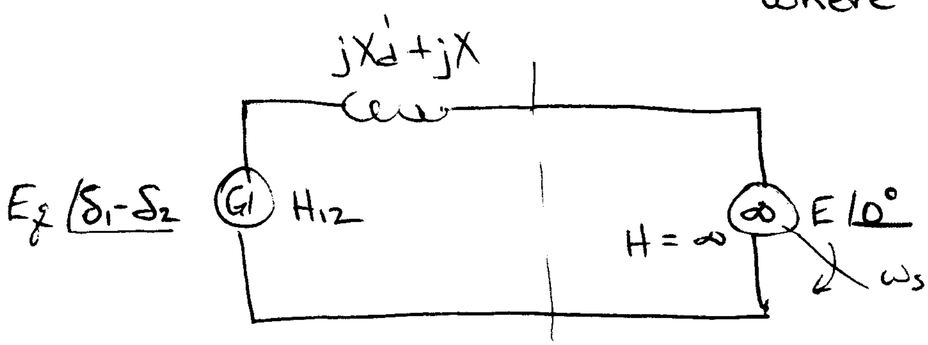

See pp. 705-706 in Grainger/Stevensen © 1994

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2} \Rightarrow \boxed{P_m - P_e = \frac{H_{12}}{\pi f} \frac{d^2 \delta_{12}}{dt^2}}$$

where $P_m = P_{m1} = -P_{m2}$

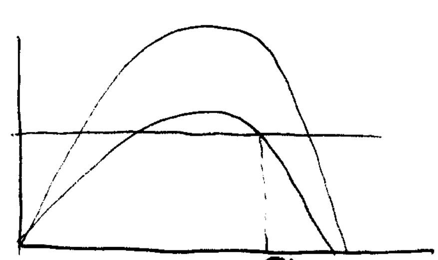
$P_e = P_{e1} = -P_{e2}$

$\delta_{12} = \delta_1 - \delta_2$




$\therefore$ Convert system equiv to $\infty$ bus.

2) How do we know if $\delta$ swings past $\delta^u$ in new $P_{max}$ curve? All we know so far is how to find new s-s equilibrium point.




Focus on this for remainder of Ch. 12.

## 12.7 – EQUAL AREA CRITERION

Back to swing equation: $\boxed{P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}}$

Rearranging, $\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_m - P_e) = \frac{\pi f}{H}\left[P_m - \frac{E_g' V}{X}\sin\delta\right]$

Because of $\sin\delta$ term in $P_e$, this is nonlinear differential equation. How to solve?

One "trick" that works: $\frac{d}{\cancel{dt}}\left(\frac{d\delta}{dt}\right)^2 = 2\frac{d\delta}{\cancel{dt}}\left(\frac{d^2\delta}{dt^2}\right)$

We can substitute. $\frac{d^2\delta}{dt^2} = \frac{1}{2}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2\frac{dt}{d\delta}$

$$\frac{1}{2}\underbrace{\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2}\frac{dt}{d\delta} = \frac{\pi f}{H}(P_m - P_e)$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f}{H}(P_m - P_e)\frac{d\delta}{dt}$$

Integrating both sides,

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta = (\omega - \omega_s)^2$$

$(\omega_{ref} = \omega_s)$

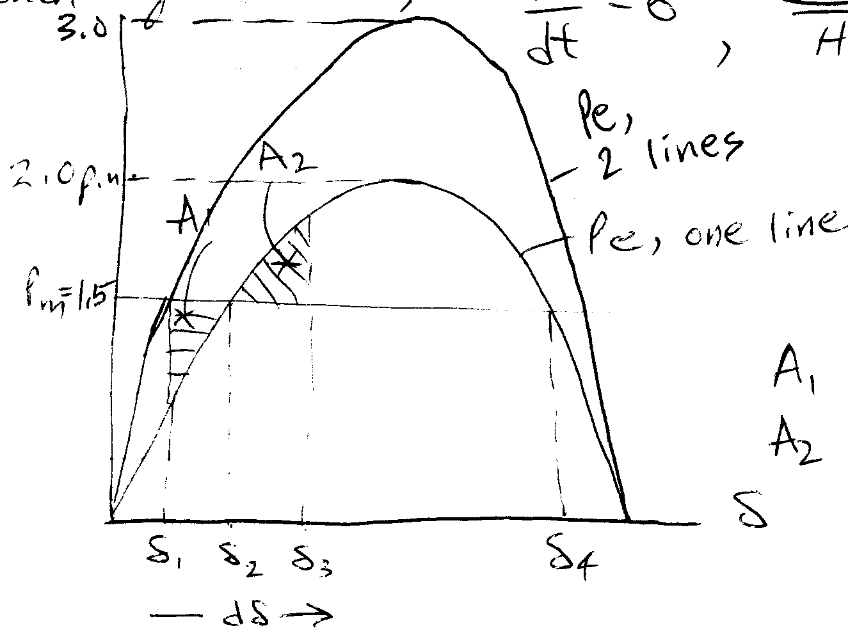
$\Rightarrow$ Area between $P_m$ & $P_e$ $(\Delta P \cdot d\delta)$ is proportional to relative velocity squared. $(\omega - \omega_{ref})^2$

Recall that kinetic energy K.E. $\propto \omega^2$ also, so areas are proportional to changes in kinetic energy. If $P_m > P_e$ then area is positive and energy increases ($\omega$ increases). If $P_m < P_e$ then $\omega$ decreases and energy dereases.

$\therefore$ When $P_m > P_e$, energy stored in rotor $\left(\frac{1}{2}J\omega^2\right)$,
when $P_e < P_m$, energy released from rotor.
Energy fluctuations are taken up by infinite bus.

which $\omega$?

To reach equilibrium, $\frac{d\delta}{dt} = 0$, $\frac{2\pi f}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta = 0$




$A_1$ inherently pos
$A_2$ inherently neg

Example: $\delta_0 = \delta_1 = 30°$ (release from $\delta_1$ at $t=0$).
Clearly, equilibrium point will be $\delta = \delta_2$.
For $P_{e\,max} = 2.0$ & $P_m = 1.5$, $\delta_2 = 48.6° = 0.848$ rad

For equilibrium/stability,

$$\frac{2\pi f}{H}\int_{\delta_1}^{\delta_3}(P_m - P_e)\,d\delta = 0 \Rightarrow \underbrace{\int_{\delta_1}^{\delta_2}\overbrace{(P_m - P_e)}^{pos}\overset{pos}{d\delta}}_{A_1} = \underbrace{\int_{\delta_3}^{\delta_2}\overbrace{(P_m - P_e)}^{neg}\overset{neg}{d\delta}}_{A_2}$$

$$A_1 = \int_{0.524}^{0.848} (1.5 - 2\sin\delta)\,d\delta = 1.5\delta + 2\cos\delta \Big|_{0.524}^{0.848} = \underline{\underline{0.0773}}$$

Since equilibrium is (hopefully) about $\delta_2$, $A_2$ will be equal to A1.

$$\therefore \int_{\delta_3}^{\delta_2} (P_m - P_e)\,d\delta = \int_{\delta_3}^{0.848} (1.5 - 2\sin\delta)\,d\delta = 0.0773$$

$$1.5\delta + 2\cos\delta \Big|_{\delta_3}^{0.848} = 0.0773$$

$$(1.5)(0.848) + 2\cos(0.848) - 1.5(\delta_3) - 2\cos(\delta_3) = 0.0773$$

$$2\cos\delta_3 + 1.5\delta_3 = 2.518$$

solve? Use Newton Raphson, iteration, etc.

$$\delta_3 = 1.218 \text{ rad} \quad (69.8°)$$

Note: Max $A_2$ availabe to "counter" $A_1$ is

$$A_2 = \int_{\delta_4}^{\delta_2} (P_m - P_e)\,d\delta = \int_{2.293}^{0.848} (1.5 - 2\sin\delta)\,d\delta = \underline{\underline{0.478}}$$

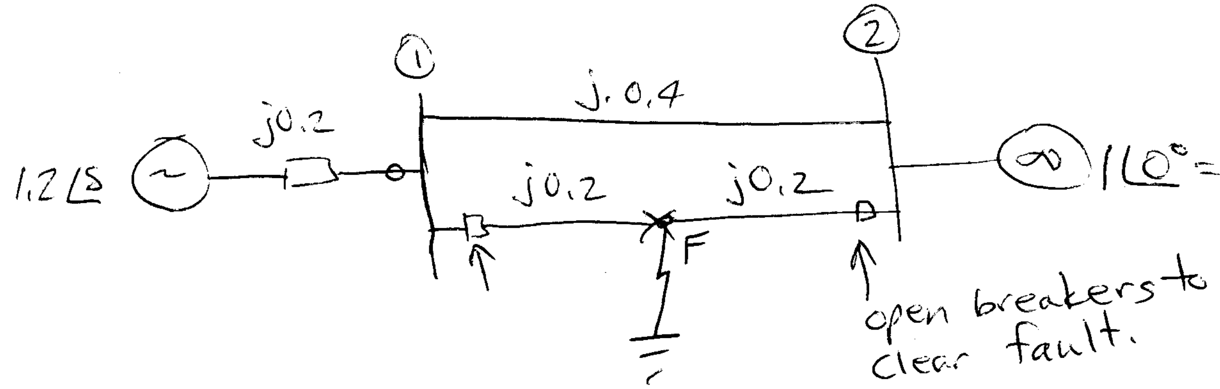
If $\delta_1$ had been $0°$, $\quad A_1 = \int_0^{.524} (1.5 - 2\sin\delta)\,d\delta$

$$= 1.5(.524) + 2\cos .524 - 2$$

$$= \underline{\underline{0.5176}}$$

$\therefore$ Stability would not have be possible!

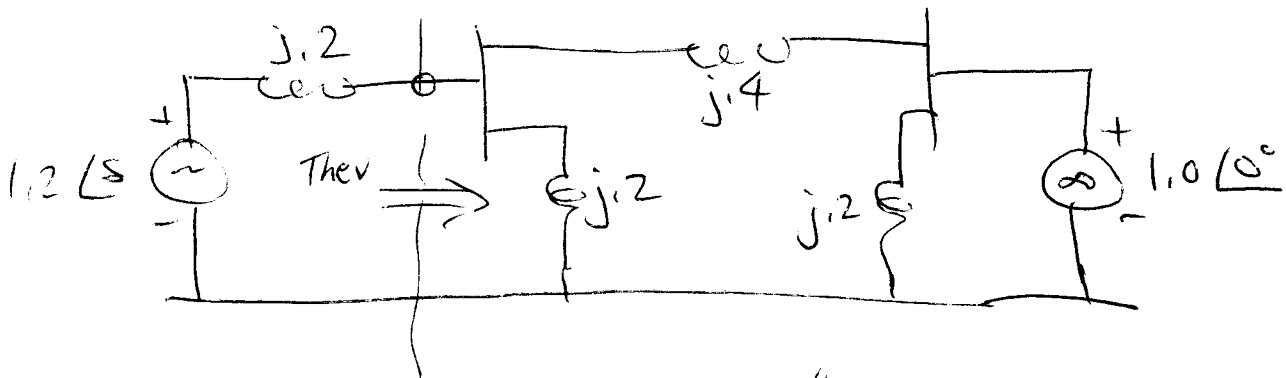
Ex:




What if fault occurs at t=0 ?

$$P_{e_{max}}(t=0^-) = \frac{(1.2)(1)}{j.2 + j.4 // j.4} = \frac{1.2}{.4} = 3.0 \text{ p.u.}$$

For fault, Thevenize at bus 1 look back to inf. bus.




$$Z_{TH} = j.4 // j.2 = \cancel{j.1333} \; j0.1333$$

$$V_{TH} = 1 \left(\frac{j.2}{j.6}\right) = 0.333$$

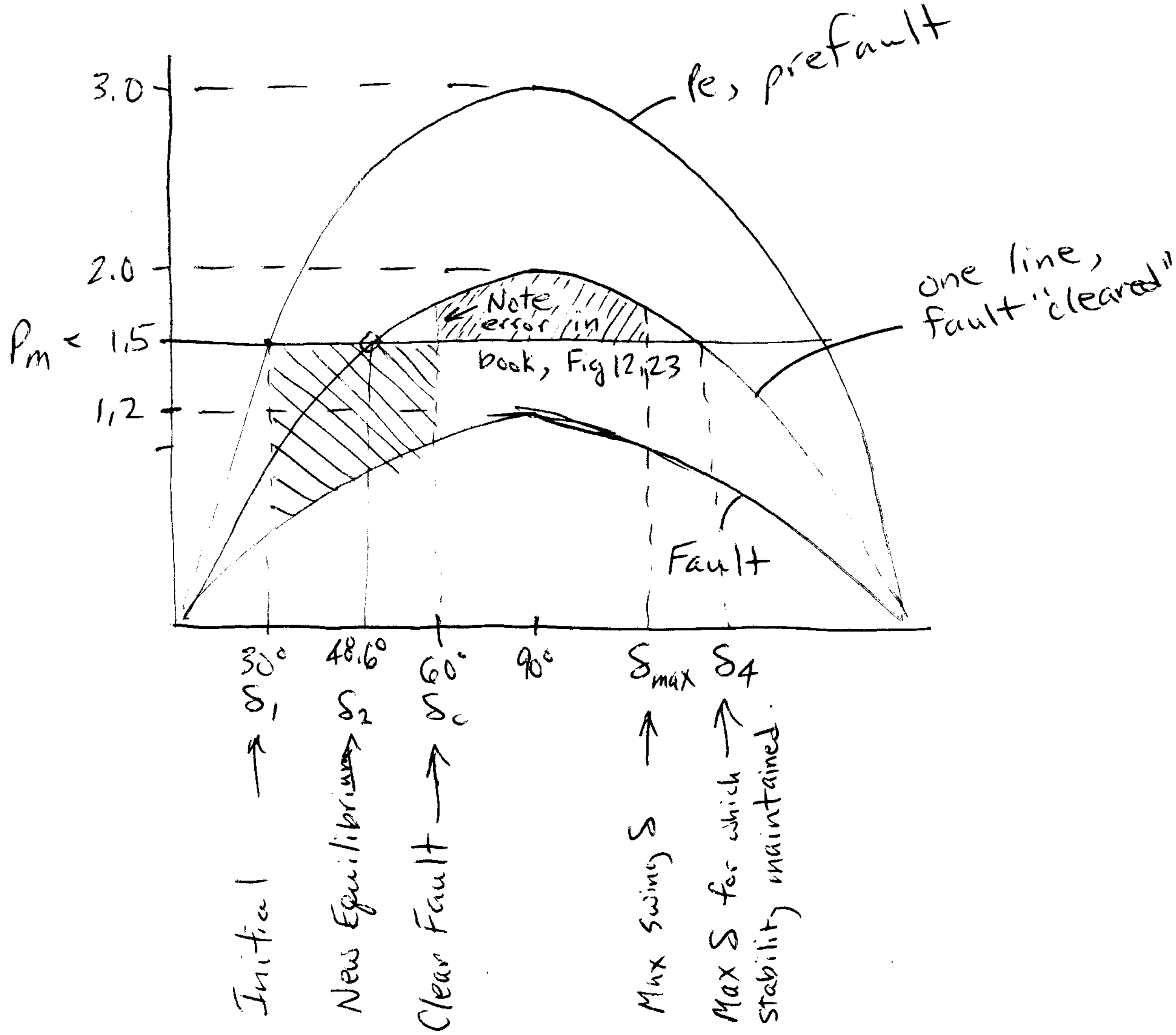
$$\therefore \quad P_{e_{Fault\,max}} = \frac{(1.2)(0.333)}{.2 + .1333} = 1.2$$

Note: $P_m = 1.5$ Conclusion?

Impossible to maintain stability, if fault persists.

If fault can be cleared,

$$P_{e_{max}} = \frac{(1.2)(1)}{.2+.4} = 2.0 \quad \text{(from before, one line)}$$




The accelerating area, $A_1$ is

$$\int_{\delta_1}^{\delta_c} (P_m - P_e)\, d\delta = \int_{.524}^{1.047} (1.5 - 1.2 \sin\delta)\, d\delta$$

$$= 1.5\,\delta + 1.2 \cos\delta \Big|_{.524}^{1.047} = 0.346$$

How far will $\delta$ increase after fault is cleared?

$|A_2| = |A_1|$:

$$\int_{\delta_{max}}^{1.047} (1.5 - 2.0 \sin\delta)\, d\delta = 0.346$$

$$1.5\delta + 2.0\cos\delta \Big|_{\delta_{max}}^{1.047} = 0.346$$

$$1.5\,\delta_{max} + 2\cos\delta_{max} = 0.346$$

$$\Rightarrow \delta_{max} = 1.848 \text{ rad}$$
$$= 105.9^\circ$$

Max it could "swing" is out to $131.4^\circ = \delta_4$

<u>Question:</u> What is critical clearing angle? This is largest angle that fault could be cleared and stability still maintained. Call this $\underline{\underline{\delta_{cc}}}$.

<u>Ex 12.5</u>

$$\int_{0.524}^{\delta_{cc}} (\overbrace{1.5 - 1.2 \sin\delta}^{\text{faulted}})\, d\delta = \int_{2.293}^{\delta_{cc}} (1.5 - 2\sin\delta)\, d\delta$$

Solving,

$$1.5\delta + 1.2\cos\delta \Big|_{.524}^{\delta_{cc}} = 1.5\delta + 2\cos\delta \Big|_{2.293}^{\delta_{cc}}$$

$$\underline{\underline{\delta_{cc} = 1.196 \text{ rad} = 68.6^\circ}}$$

How long do we have to clear?

Not so simple, since the accelerating torque varies with $\delta$. Some key relationships:

$$P_{acc} = P_m - P_e = P_m - 1.2 \sin\delta$$

$P_{acc}$ gradually decreases as $\delta$ increases to $\delta_{cc}$.

$$T_{acc} = (P_m - 1.2 \sin\delta)\, \omega_{mech}(\delta)$$

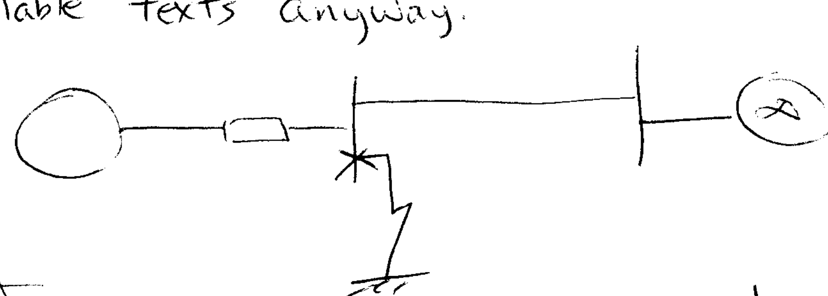
$$\omega = \omega_0 + \int_0^{t_{cc}} \alpha(\delta)\, dt \quad \leftarrow \quad \omega(t) = \omega_0 + \alpha t$$

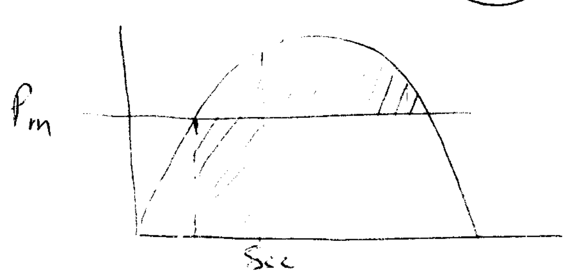
From swing equation,

$$\omega_{cc}^2 - \omega_1^2 = \frac{2\pi f}{H} \int_{\delta_1}^{\delta_{cc}} (P_m - P_e)\, d\delta \;?$$

Should be able to get closed-form expression for $t_{CRITICAL}$. Usually this is done trial & error, in available texts anyway.

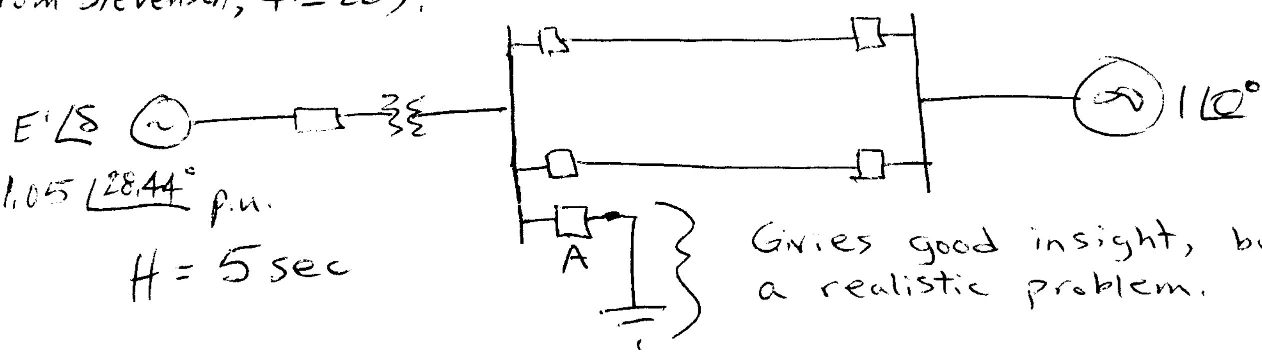
Easier case:





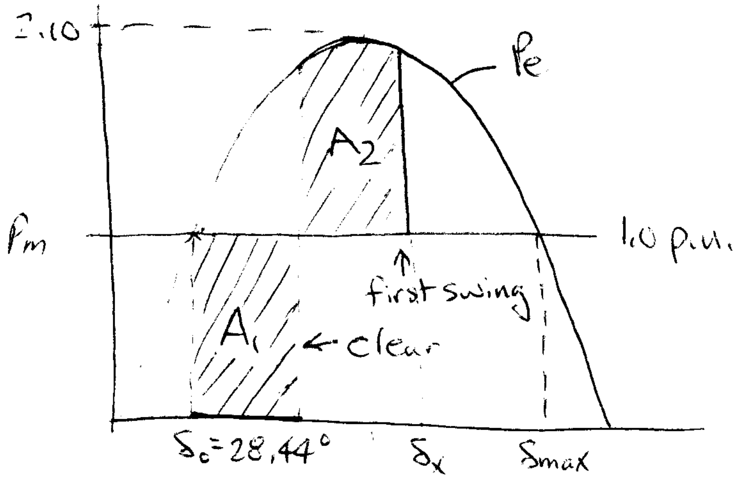

$P_e = 0$ during fault!

More on Equal Area Criterion: <u>Fault on Gen Bus</u>
(From Stevenson, 4th Ed).




Gives good insight, but not a realistic problem.

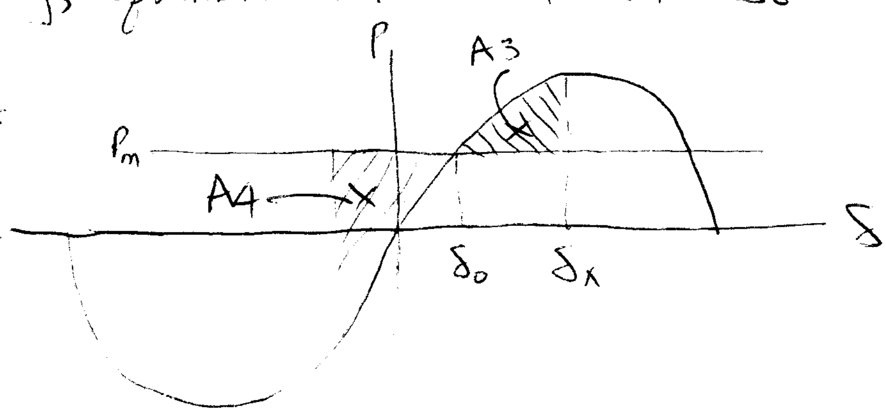
Assume a bus fault occurs (Breaker A is closed).
Prefault $P_{max}$ is 2.10 p.u.



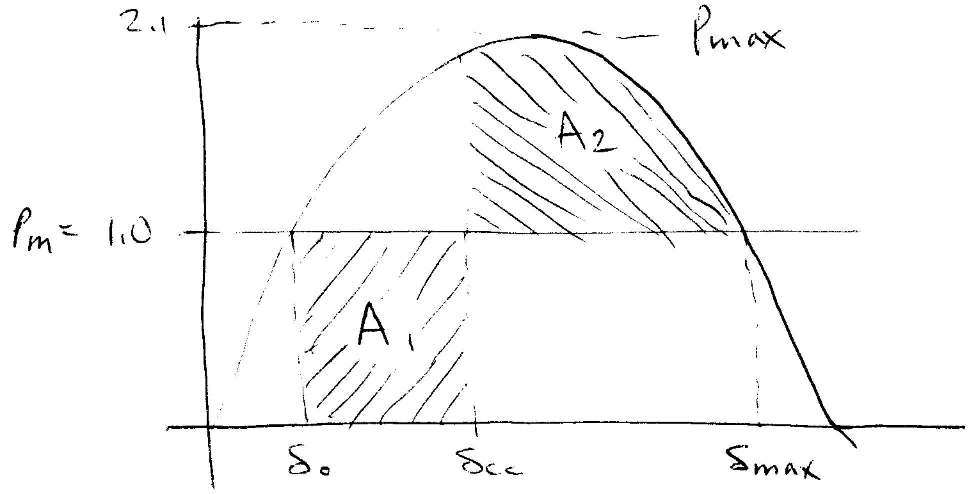

When fault occurs in this case, $P_e = 0$. $(P_m - P_e) = P_m$
$P_e$ remains at zero until fault is cleared (open A).
$\delta$ will continue to increase until $A_2 = A_1$.

After clearing, equilibrium is still at $\delta_o$

Now, $\delta$ swings about $\delta_o$, with $A_3 = A4$



What is maximum $\delta_x$ for which stability can be maintained?




$$\underbrace{\int_{\delta_o}^{\delta_{cc}} (P_m - \cancel{P_e})\, d\delta}_{A_1} = \underbrace{\int_{\delta_{max}}^{\delta_{cc}} (P_m - P_e)\, d\delta}_{A_2}$$

$$P_m \delta \Big|_{\delta_o}^{\delta_{cc}} = P_m \delta + P_{max} \cos\delta \Big|_{\delta_{max}}^{\delta_{cc}}$$

$$P_m \delta_{cc} - P_m \delta_o = \cancel{P_m \delta_{cc}} - P_m \delta_{max} + P_{max} \cos\delta_{cc} - P_{max} \cos\delta_{max}$$

$$P_m (\delta_{max} - \delta_o) = P_{max} (\cos\delta_{cc} - \cos\delta_{max})$$

$$\cos\delta_{cc} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_o) + \cos\delta_{max}$$

but $\delta_{max} = \pi - \delta_o$, $\quad P_m = P_{max} \sin\delta_o$

$$\boxed{\delta_{cc} = \cos^{-1}\left[ (\pi - 2\delta_o) \sin\delta_o - \cos\delta_o \right]}$$ only for bus fault at gen!

For this case, it is possible to directly calculate $t_{cc}$, the time before which the fault must be removed.

$P_A = P_m$, since $P_e$ is zero in this case. Therefore, the accelerating Torque and the acceleration are roughly constant.

From swing equation, $\alpha = \frac{d^2\delta}{dt} = \frac{\pi f}{H}(P_m - P_e)$, $P_e \to 0$

$$\therefore \quad \alpha = \frac{\pi f}{H} P_m \ \frac{rad}{sec^2}$$

$$\omega(t) = \omega_0 + \alpha t = \frac{\pi f}{H} P t$$

$\omega_0 \to 0$ ($\omega$ ref $= \omega_s$)

$$\delta(t) = \delta_0 + \int_0^t \omega(t)\,dt = \delta_0 + \frac{\pi f}{2H} t^2$$

$$\delta_{cc} = \delta_0 + \frac{\pi f}{2H} t_{cc}^2$$

$$\Rightarrow \quad \boxed{t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\pi f P_m}}}$$

only for bus fault at gen! ($P_e = 0$)

For previous problem, $\delta_0 = 28.44° = 0.496$ rad

~~$\delta_{max} = \pi - \delta_0$~~

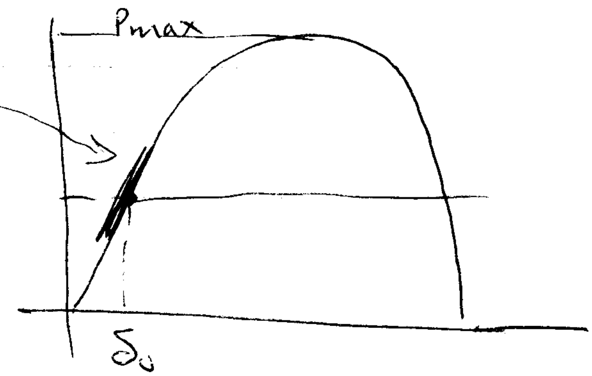
$$\delta_{cc} = \cos^{-1}\left[(\pi - 2\times 0.496)\sin 0.496 - \cos 0.496\right] = 1.426 = \underline{\underline{81.70°}}$$

$$t_{cc} = \sqrt{\frac{2(5)(1.426 - 0.496)}{(60)(\pi)(1.0)}} = \underline{\underline{0.222 \text{ sec}}}$$

## Natural Frequency of Oscillation -

$$\omega_n = \sqrt{\frac{\omega_s P_{max} \cos\delta}{2H}}$$ (For small disturbances.)

$S_p = P_{max} \cos\delta$ is the "synchronizing power coefficient". It is the slope of $P_e = P_{max} \sin\delta$ at $\delta = \delta_o$.




$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\omega_s P_{max} \cos\delta}{2H}}$$

Ex: $\delta_o = 28.44°$, $H = 5$ sec, $P_{max} = 2.10$ p.u.

$$\omega_n = \sqrt{\frac{(377)(2.10 \cos 28.44°)}{2(5)}} = 8.343 \ \frac{\text{elec rad}}{\text{sec}}$$

$$f_n = \underline{\underline{1.33 \text{ Hz}}}$$

∴ Small perturbations in load (normal occurrence throughout day) will cause low-freq machine oscillations. Usually on order of ~1 Hz. They are quickly damped by prime mover, generator and system load. Note that R of transmission system has virtually no damping effect.

12.8

## Solution of Swing Equation:

For practical cases, critical clearing time cannot be calculated with equal area criterion. Engineers must know how much time they have to clear the fault – the fault must be detected, a trip signal sent to the CB, and then breaker must operate. $t_{cc}$ must be known before the CB & protective relaying can be speced and before coordinating the relaying.

If we can solve for $\delta(t)$, then we can determine $t_{cc}$. The approach is typically to use trial & error, clearing fault at different times.

An analog computer implementation is given in book.

$\underbrace{\frac{\pi f}{H}}_{\text{gain}}(P_m - P_e)$ is integrated. (Note inverting integrator).

basically:

$$\boxed{\begin{aligned} \alpha(t) &= \frac{d^2\delta(t)}{dt} = \frac{\pi f}{H}\Big(P_m - \underbrace{P_{max}\sin(\delta(t))}_{P_e}\Big) \\ \omega(t) &= \cancel{\omega_0}^{\,0} + \int \alpha(t)\,dt \\ \delta(t) &= \delta_0 + \int \omega(t)\,dt \end{aligned}}$$

Note that $P_{max}$ changes with fault, clear, reclose, ~~etc~~ line switching.

The book sets up a problem using Runge-Kutta numerical integration. This is more typical of the modern solution approach on computer.

Results for the previous example, where $\delta_{cc} = 68.6°$, are given in Figs 12.25 & 12.26.

→ In these programs, $t_{cc}$ is specified and simulation is run to see if stability is maintained. Best guess/judgement is

** that if ~~first pos/neg fl~~ first negative fluctuation in $\delta$ is $\leq$ to first pos fluctuation then system will be stable.
i.e. if $(\delta_{max} - \delta_0) \geq (\delta_0 - \delta_{min}) \Rightarrow$ stable.

For this simple system (two machines), $\delta_{cc}$ could be found by equal-area. $t_{cc}$ could be found simply by running simulation and noting the value of $t$ when $\delta = \delta_{cc}$.

What if there are more than 2 machines?

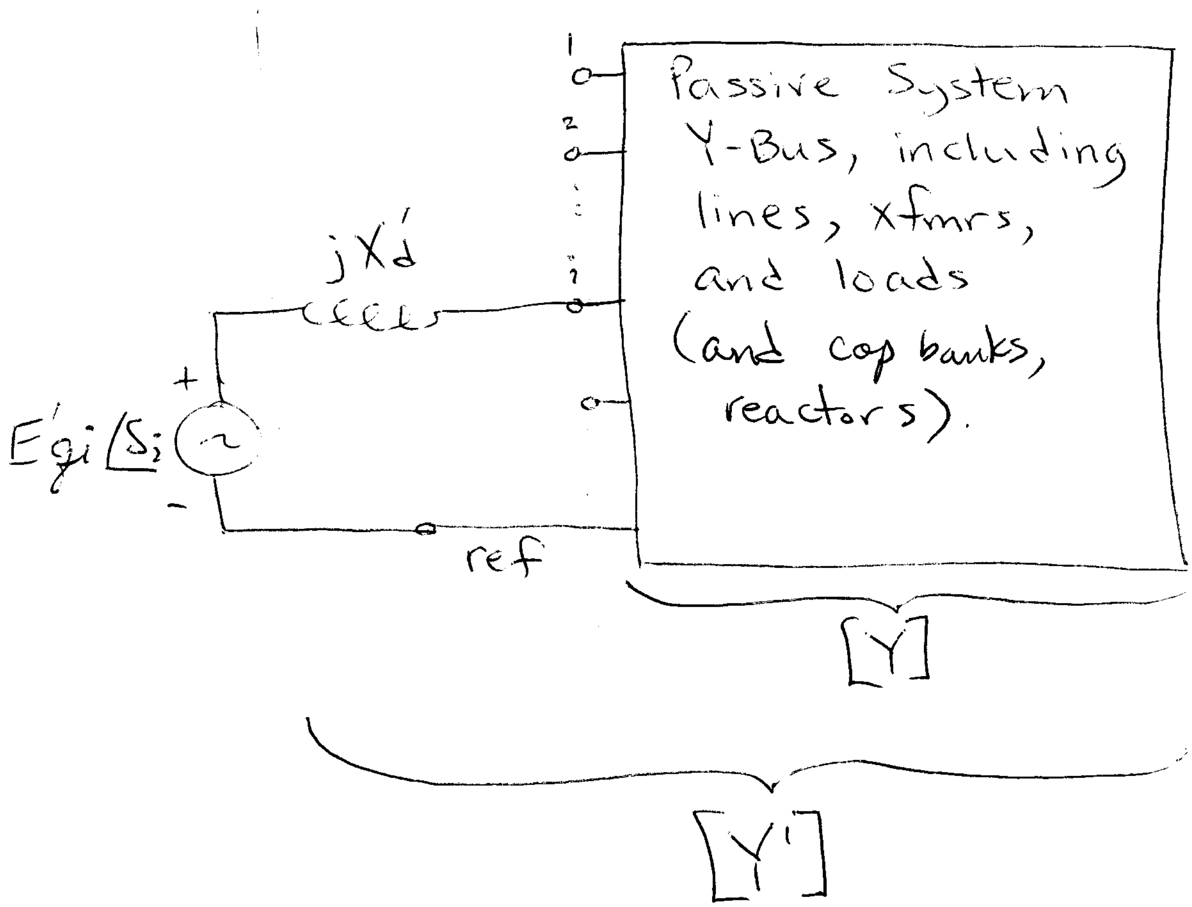
Before going on,

- Do you understand swing equation?
- Do you understand [Y] system description?
- Do you have equal area method figured out?

Prob 12.15 

## 12.9 – Multimachine Systems

A network approach must be used where more than 2 machines are to be simulated.




- Pre-~~dist~~transient bus voltages typically obtained from load flow program.
- All buses not connected to a generator ~~are~~ have no current injected.

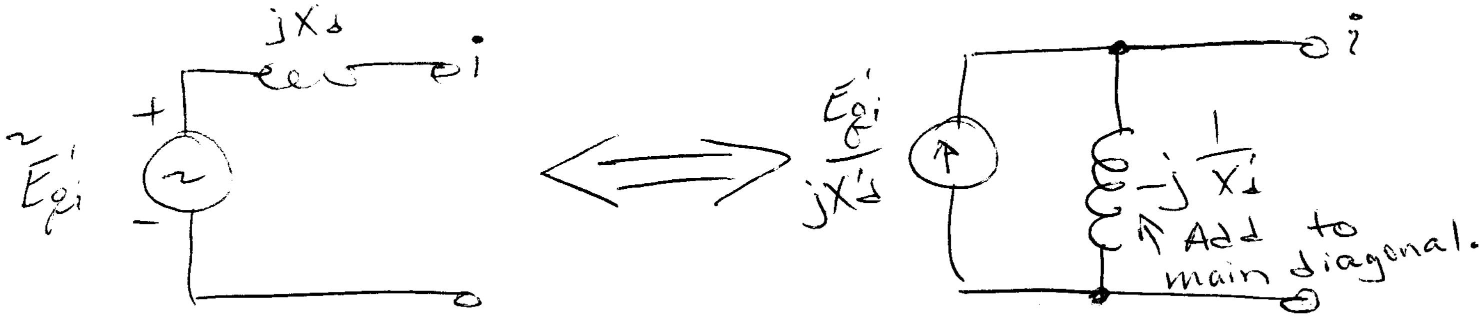
$$\begin{bmatrix} I_1\\ I_2\\ \vdots\\ I_n\\ I_{n+1}\\ \vdots\\ I_{n+m}\end{bmatrix} = \begin{bmatrix} Y' \end{bmatrix} \begin{bmatrix} E_1\\ E_2\\ \vdots\\ E_n\\ V_{n+1}\\ \vdots\\ V_{n+m}\end{bmatrix}$$

All zero: $I_{n+1} \ldots I_{n+m}$

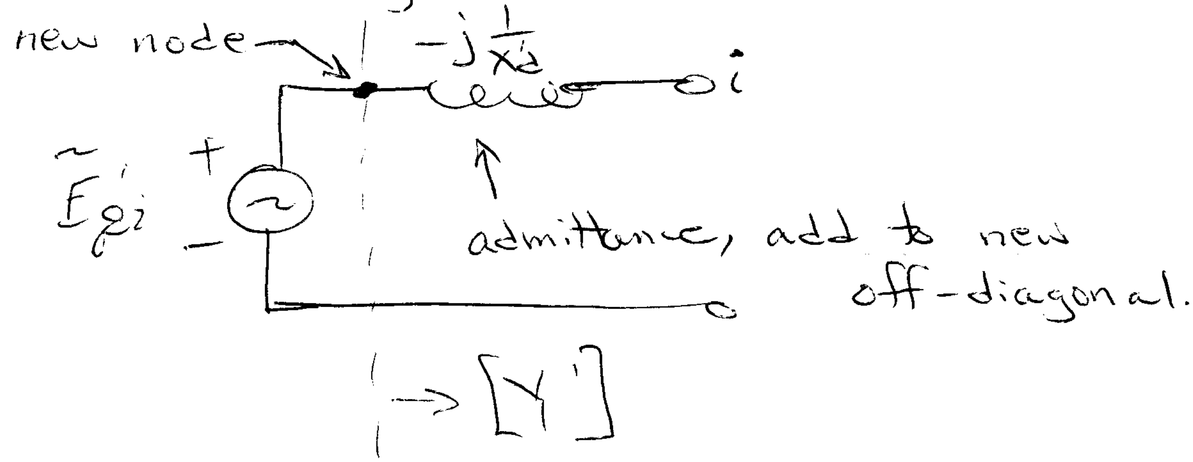
$E_1 \ldots E_n$: n generators

$V_{n+1} \ldots V_{n+m}$: m buses w/o generators, Voltage due to $I_1 \rightarrow I_n$

$[Y']$ is formed by adding Norton admittance of generators to main diagonal of $[Y]$. (No extra nodes added to network).




In this book, author adds an extra node at each generator bus:




Therefore, end up with $2n+m$ nodes in system. We will follow author's method, even though not the best in terms of computer application.

Nodes not connected to a generator are typically eliminated, using matrix partitioning.

$[Y']$ is partitioned:

$$\begin{array}{c|cc} & 1 \to n & n+1 \to n+m \\ \hline 1 \downarrow n & [Y_{AA}] & [Y_{AB}] \\ n+1 \downarrow n+m & [Y_{BA}] & [Y_{BB}] \end{array}$$

$$[Y_{red}] = [Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}] = n \times n$$

See derivation, p. 533

<u>Ex:</u> Go thru Ex. 12.7

<u>Effect of loads:</u> Loads are added to main diagonal term of $[Y]$

Ex: See example 12.8

Calculating $P_{emax}$ for various ~~[illegible]~~ conditions is done using $[Y]$ for each of the operating states.

<u>Ex:</u> If $[Y] = \begin{bmatrix} -j2.5 & j2.5 \\ j2.5 & -j2.5 \end{bmatrix}$ <u>Prefault</u>

$E_1 = E'_{g1} = 1.2$ p.u. $\quad \delta_1 = \delta \quad\quad y_1 \angle \gamma_1 = 2.5 \angle -90^\circ$

$E_2 = 1.0$ p.u. $\quad \delta_2 = 0$ (infinite bus) $\quad y_2 \angle \gamma_2 = 2.5 \angle +90^\circ$

$$P_{e1} = \sum_{j=1}^{2} E_1 E_j y_{1j} \cos(\delta_1 - \delta_j - \gamma_{1j})$$

$$= \cancel{(1.2)^2(2.5)\cos \overset{\delta_1 - \delta_1 - (-90)}{90^\circ}} + (1.2)(1.0)(2.5)\cos(\delta_1 - 0 - 90)$$

$$= 3.0 \sin\delta \Rightarrow P_{emax} = 3.0$$

Using same method,

$Pe_{max} = 1.2$ during fault.
$Pe_{max} = 2.0$ ~~due~~ when cleared.

Each machine's swing equation is solved separately, although $\delta$ depends on other machines.

$$P_{mi} - P_{ei} = \frac{H_i}{\pi f}\left(\frac{d^2\delta_i}{dt^2} + \frac{d\,\omega_{ref}}{dt}\right) \qquad i = 1, 2 \ldots n$$

Caution: use ~~same base~~ system base for all H and P values.

Usually pick infinite bus machine for $\omega_{ref}$

$$P_{mi} - P_{ei} = \frac{H_i}{\pi f}\left(\frac{d^2\delta_i}{dt^2}\right) \qquad i = 1, 2, \ldots n$$

Reformulate as

$$\frac{d\delta_i}{dt} = \omega_i$$

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i}\Big(P_{mi} - \underbrace{\sum_{j=1}^{n} E_i E_j y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})}_{P_{ei}}\Big)$$

Initial conditions: $\omega_i = 0$ for all machines $\underline{\omega_i(0)}$
$\delta_{0i} = \delta_i(0)$