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# APPENDIX B

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## SALIENT-POLE THEORY OF SYNCHRONOUS MACHINES

The equivalent circuit for a synchronous generator derived in Chap. 8 is in fact valid only for machines built with cylindrical rotors, and not for machines built with salient-pole rotors. Likewise, the expression for the relationship between the torque angle  $\delta$  and the power supplied by the generator [Eq. (8-20)] is valid only for cylindrical rotors. In Chap. 8, we ignored any effects due to the saliency of rotors and assumed that the simple cylindrical theory applied. This assumption is in fact not too bad for steady-state work, but it is quite poor for examining the transient behavior of generators and motors.

The problem with the simple equivalent circuit of induction motors is that it ignores the effect of the *reluctance torque* on the generator. To understand the idea of reluctance torque, refer to Fig. B-1. This figure shows a salient-pole rotor with no windings inside a three-phase stator. If a stator magnetic field is produced as shown in the figure, it will induce a magnetic field in the rotor. Since it is *much* easier to produce a flux along the axis of the rotor than it is to produce a flux across the axis, the flux induced in the rotor will line up with the axis of the rotor. Since there is an angle between the stator magnetic field and the rotor magnetic field, a torque will be induced in the rotor which will tend to line up the rotor with the stator field. The magnitude of this torque is proportional to the sine of twice the angle between the two magnetic fields ( $\sin 2\delta$ ).

Since the cylindrical rotor theory of synchronous machines ignores the fact that it is easier to establish a magnetic field in some directions than in others (i.e., ignores the effect of reluctance torques), it is inaccurate when salient-pole rotors are involved.

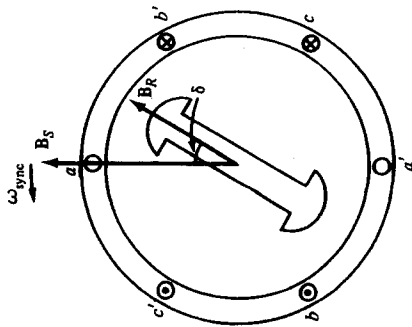


FIGURE B-1

A salient-pole rotor, illustrating the idea of reluctance torque. A magnetic field is induced in the rotor by the stator magnetic field, and a torque is produced on the rotor which is proportional to the sine of twice the angle between the two fields.

### B-1 DEVELOPMENT OF THE EQUIVALENT CIRCUIT OF A SALIENT-POLE SYNCHRONOUS GENERATOR

As was the case for the cylindrical rotor theory, there are four elements in the equivalent circuit of a synchronous generator:

1. The internal generated voltage of the generator  $E_A$
2. The armature reaction of the synchronous generator
3. The stator winding's self-inductance
4. The stator winding's resistance

The first, third, and fourth elements are unchanged in the salient-pole theory of synchronous generators, but the armature-reaction effect must be modified to explain the fact that it is easier to establish a flux in some directions than in others.

This modification of the armature-reaction effects is accomplished as explained below. Figure B-2 shows a two-pole salient-pole rotor rotating counterclockwise within a two-pole stator. The rotor flux of this generator is called  $B_R$ , and it points upward. By the equation for the induced voltage on a moving conductor in the presence of a magnetic field,

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \quad (1-45)$$

the voltage in the conductors in the upper part of the stator will be positive out of the page, and the voltage in the conductors in the lower part of the stator will be into the page. The plane of maximum induced voltage will lie directly under the rotor pole at any given time.

If a lagging load is now connected to the terminals of this generator, then a

current will flow whose peak is delayed behind the peak voltage. This current is shown in Fig. B-2b.

The stator current flow produces a magnetomotive force that lags  $90^\circ$  behind the plane of peak stator current, as shown in Fig. B-2c. In the cylindrical theory, this magnetomotive force then produces a stator magnetic field  $B_S$  that lines up with the stator magnetomotive force. However, it is actually easier to produce a magnetic field in the direction of the rotor than it is to produce one in the direction perpendicular to the rotor. Therefore, we will break down the stator magnetomotive force into components parallel to and perpendicular to the rotor's axis. Each of these magnetomotive forces produces a magnetic field, but more flux is produced per ampere-turn along the axis than is produced perpendicular (in *quadrature*) to the axis.

The resulting stator magnetic field is shown in Fig. B-2d, compared to the field predicted by the cylindrical rotor theory.

Now, each component of the stator magnetic field produces a voltage of its own in the stator winding by armature reaction. These armature-reaction voltages are shown in Fig. B-2e.

The total voltage in the stator is thus

$$V_\phi = E_A + E_d + E_q \quad (\text{B-1})$$

where  $E_d$  is the direct-axis component of the armature-reaction voltage and  $E_q$  is the quadrature-axis component of armature reaction voltage (see Fig. B-3). As in the case of the cylindrical rotor theory, each armature-reaction voltage is *directly proportional to its stator current* and *delayed  $90^\circ$  behind the stator current*. Therefore, each armature-reaction voltage can be modeled by

$$E_d = -jx_d I_d \quad (\text{B-2})$$

$$E_q = -jx_q I_q \quad (\text{B-3})$$

and the total stator voltage becomes

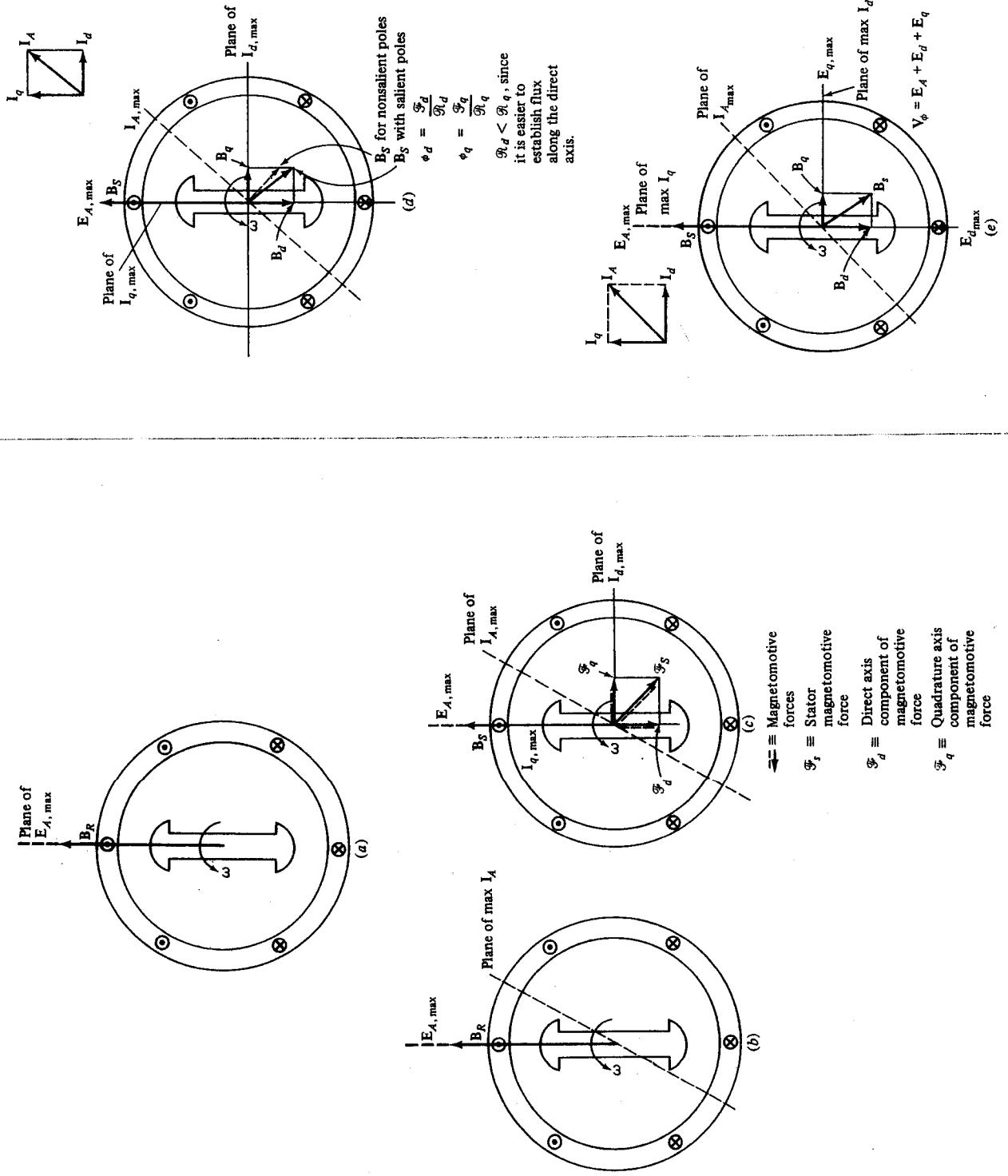
$$V_\phi = E_A - jx_d I_d - jx_q I_q \quad (\text{B-4})$$

The armature resistance and self-reactance must now be included. Since the armature self-reactance  $X_A$  is independent of the rotor angle, it is normally added to the direct and quadrature armature-reaction reactances to produce the *direct synchronous reactance* and the *quadrature synchronous reactance* of the generator:

$$X_d = x_d + X_A \quad (\text{B-5})$$

$$X_q = x_q + X_A \quad (\text{B-6})$$

The armature resistance voltage drop is just the armature resistance times the armature current  $I_A$ .



**FIGURE B-2**  
The effects of armature reaction in a salient-pole synchronous generator. (a) The rotor magnetic field induces a voltage in the stator which peaks in the wires directly under the pole faces. (b) If a lagging load is connected to the generator, a stator current will flow that peaks at an angle behind  $E_A$ . (c) This stator current  $I_A$  produces a stator magnetomotive force in the machine. (d) The stator magnetomotive force produces a stator flux  $B_S$ . However, the direct-axis component of magnetomotive force produces more flux per ampere-turn than the quadrature-axis component does, since the reluctance of the direct-axis flux path is lower than the reluctance of the quadrature-axis flux path. (e) The direct- and quadrature-axis stator fluxes produce armature reaction voltages in the stator of the machine.

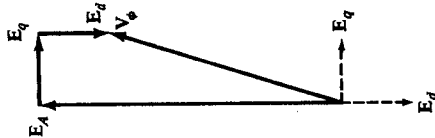


FIGURE B-3

The phase voltage of the generator is just the sum of its internal generated voltage and its armature reaction voltages.

Therefore, the final expression for the phase voltage of a salient-pole synchronous motor is

$$V_\phi = E_A - jX_d I_d - jX_q I_q - R_A I_A \tag{B-7}$$

and the resulting phasor diagram is shown in Fig. B-4.

Note that this phasor diagram requires that the armature current be resolved into components in parallel with  $E_A$  and in quadrature with  $E_A$ . However, the angle between  $E_A$  and  $I_A$  is  $\delta + \theta$ , which is *not usually known* before the diagram is constructed. Normally, only the power-factor angle  $\theta$  is known in advance.

It is possible to construct the phasor diagram without advance knowledge of the angle  $\delta$ , as shown in Fig. B-5. The solid lines in Fig. B-5 are the same as the lines shown in Fig. B-4, while the dotted lines present the phasor diagram as though the machine had a cylindrical rotor with synchronous reactance  $X_d$ .

The angle  $\delta$  of  $E_A$  can be found by using information known at the terminals of the generator. Notice that the phasor  $E_A''$ , which is given by

$$E_A'' = V_\phi + R_A I_A + jX_q I_A \tag{B-8}$$

is collinear with the internal generated voltage  $E_A$ . Since  $E_A''$  is determined by the current at the terminals of the generator, the angle  $\delta$  can be determined with a

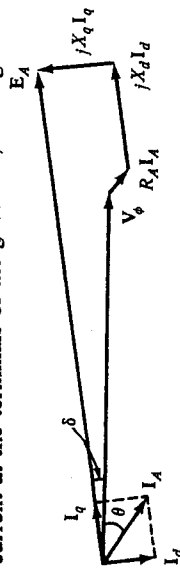


FIGURE B-4

The phasor diagram of a salient-pole synchronous generator.

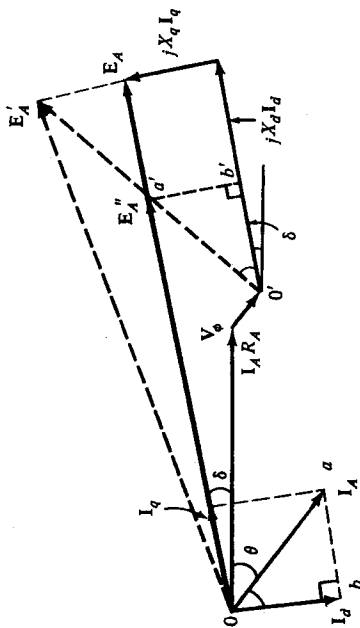


FIGURE B-5

Constructing the phasor diagram with no a priori knowledge of  $\delta$ ,  $E_A''$  lies at the same angle as  $E_A$ , and  $E_A''$  may be determined exclusively from information at the terminals of the generator. Therefore, the angle  $\delta$  may be found, and the current can be divided into  $d$  and  $q$  components.

knowledge of the armature current. Once the angle  $\delta$  is known, the armature current can be broken down into direct and quadrature components, and the internal generated voltage can be determined.

**Example B-1.** A 480-V 60-Hz  $\Delta$ -connected four-pole synchronous generator has a direct-axis reactance of  $0.1 \Omega$  and a quadrature-axis reactance of  $0.075 \Omega$ . Its armature resistance may be neglected. At full load, this generator supplies 1200 A at a power factor of 0.8 lagging.

- (a) Find the internal generated voltage  $E_A$  of this generator at full load, assuming that it has a cylindrical rotor of reactance  $X_d$ .
- (b) Find the internal generated voltage  $E_A$  of this generator at full load, assuming it has a salient-pole rotor.

**Solution**

- (a) Since this generator is  $\Delta$ -connected, the armature current at full load is

$$I_A = \frac{1200 \text{ A}}{\sqrt{3}} = 693 \text{ A}$$

The power factor of the current is 0.8 lagging, so the impedance angle  $\theta$  of the load is

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

Therefore, the internal generated voltage is

$$\begin{aligned} E_A &= V_\phi + jX_d I_A \\ &= 480 \angle 0^\circ \text{ V} + j(0.1 \Omega)(693 \angle -36.87^\circ \text{ A}) \\ &= 480 \angle 0^\circ + 69.3 \angle 53.13^\circ \\ &= 480 + 41.58 + j55.44 \\ &= 521.6 + j55.4 = 524.5 \angle 6.1^\circ \text{ V} \end{aligned}$$

Notice that the torque angle  $\delta$  is  $6.1^\circ$ .

(b) Assume that the rotor is salient. To break down the current into direct- and quadrature-axis components, it is necessary to know the *direction* of  $E_A$ . This direction may be determined from Eq. (B-8):

$$\begin{aligned} E_A'' &= V_\phi + R_A I_A + jX_d I_d + jX_q I_q & (\text{B-8}) \\ &= 480 \angle 0^\circ \text{ V} + 0 \text{ V} + j(0.075 \Omega)(693 \angle -36.87^\circ \text{ A}) \\ &= 480 \angle 0^\circ \text{ V} + 52 \angle 53.13^\circ \text{ V} \\ &= 480 + 31.2 + j41.6 \\ &= 511.2 + j41.6 = 513 \angle 4.65^\circ \text{ V} \end{aligned}$$

The direction of  $E_A$  is  $\delta = 4.65^\circ$ . The magnitude of the direct-axis component of current is thus

$$\begin{aligned} I_d &= I_A \sin(\theta + \delta) \\ &= (693 \text{ A}) \sin(36.87^\circ + 4.65^\circ) \\ &= 459 \text{ A} \end{aligned}$$

and the magnitude of the quadrature-axis component of current is

$$\begin{aligned} I_q &= I_A \cos(\theta + \delta) \\ &= (693 \text{ A}) \cos(36.87^\circ + 4.65^\circ) \\ &= 519 \text{ A} \end{aligned}$$

Combining magnitudes and angles yields

$$\begin{aligned} I_d &= 459 \angle -85.35^\circ \text{ A} \\ I_q &= 519 \angle 4.65^\circ \text{ A} \end{aligned}$$

The resulting internal generated voltage is

$$\begin{aligned} E_A &= V_\phi + R_A I_A + jX_d I_d + jX_q I_q \\ &= 480 \angle 0^\circ \text{ V} + 0 \text{ V} + j(0.1 \Omega)(459 \angle -85.35^\circ) + j(0.075 \Omega)(519 \angle 4.65^\circ) \\ &= 480 \angle 0^\circ + 45.9 \angle 4.65^\circ + 38.9 \angle 94.65^\circ \\ &= 522.6 + j42.49 = 524.3 \angle 4.65^\circ \text{ V} \end{aligned}$$

Notice that the *magnitude* of  $E_A$  is not much affected by the salient poles, but the *angle* of  $E_A$  is considerably different with salient poles than it is without salient poles.

## B-2 TORQUE AND POWER EQUATIONS OF SALIENT-POLE MACHINES

The power output of a synchronous generator with a cylindrical rotor as a function of the torque angle was given in Chap. 8 as

$$P = \frac{3V_\phi E_A}{X_S} \sin \delta \quad (\text{8-20})$$

This equation assumed that the armature resistance was negligible. Making the same assumption, what is the output power of a salient-pole generator as a function of torque angle? To find out, refer to Fig. B-6. The power out of a synchronous generator is the sum of the power due to the direct-axis current and the power due to the quadrature-axis current:

$$\begin{aligned} P &= P_d + P_q & (\text{B-9}) \\ &= 3V_\phi I_d \cos(90^\circ - \delta) + 3V_\phi I_q \cos \delta \\ &= 3V_\phi I_d \sin \delta + 3V_\phi I_q \cos \delta \end{aligned}$$

From Fig. B-6, the direct-axis current is given by

$$I_d = \frac{E_A - V_\phi \cos \delta}{X_d} \quad (\text{B-10})$$

and the quadrature-axis current is given by

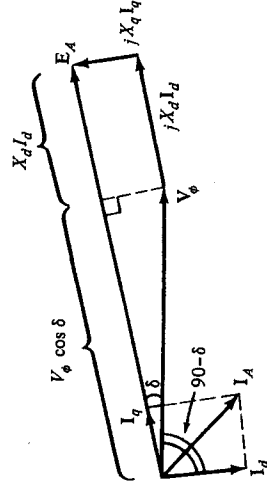
$$I_q = \frac{V_\phi \sin \delta}{X_q} \quad (\text{B-11})$$

Substituting Eqs. (B-10) and (B-11) into Eq. (B-9) yields

$$\begin{aligned} P &= 3V_\phi \left( \frac{E_A - V_\phi \cos \delta}{X_d} \right) \sin \delta + 3V_\phi \left( \frac{V_\phi \sin \delta}{X_q} \right) \cos \delta \\ &= \frac{3V_\phi E_A}{X_d} \sin \delta + 3V_\phi^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin \delta \cos \delta \end{aligned}$$

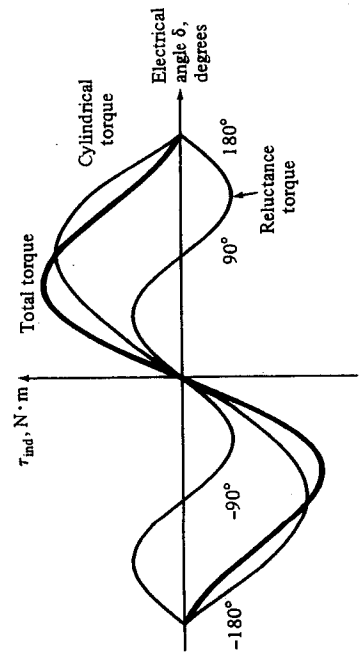
Since  $\sin \delta \cos \delta = \frac{1}{2} \sin 2\delta$ , this expression reduces to

$$P = \frac{3V_\phi E_A}{X_d} \sin \delta + \frac{3V_\phi^2}{2} \left( \frac{X_d - X_q}{X_d X_q} \right) \sin 2\delta \quad (\text{B-12})$$



$$P = 3V_\phi I_d \cos(90^\circ - \delta) + 3V_\phi I_q \cos \delta$$

FIGURE B-6 Determining the power output of a salient-pole synchronous generator. Both  $I_d$  and  $I_q$  contribute to the output power, as shown in the figure.



**FIGURE B-7** Plot of torque versus rotor angle for a salient-pole synchronous generator. Note the component of torque due to rotor reluctance.

The first term of this expression is the same as the power in a cylindrical rotor machine, and the second term is the additional power due to the reluctance torque in the machine.

Since the induced torque in the generator is given by  $\tau_{ind} = P_{conv}/\omega_m$ , the induced torque in the motor can be expressed as

$$\tau_{ind} = \frac{3V_{\phi}E_A}{\omega_m X_d} \sin \delta + \frac{3V_{\phi}^2}{2\omega_m} \left( \frac{X_d - X_q}{X_d X_q} \right) \sin 2\delta \tag{B-13}$$

The induced torque out of a salient-pole generator as a function of the torque angle  $\delta$  is plotted in Fig. B-7.

**PROBLEMS**

- B-1.** A 2300-V 1000-kVA 0.8-PF-lagging 60-Hz four-pole Y-connected synchronous generator has a direct-axis reactance of 1.1  $\Omega$ , a quadrature-axis reactance of 0.8  $\Omega$ , and an armature resistance of 0.15  $\Omega$ . Friction, windage, and stray losses may be assumed negligible. The generator's open-circuit characteristic is given by Fig. P8-1.
  - (a) How much field current is required to make  $V_T$  equal to 2300 V when the generator is running at no load?
  - (b) What is the internal generated voltage of this machine when it is operating at rated conditions? How does this value of  $E_A$  compare to that of Prob. 8-2b?
  - (c) What fraction of this generator's full-load power is due to the reluctance torque of the rotor?
- B-2.** A 14-pole Y-connected three-phase water-turbine-driven generator is rated at 120 MVA, 13.2 kV, 0.8 PF lagging, and 60 Hz. Its direct-axis reactance is 0.62  $\Omega$ , and its quadrature-axis reactance is 0.40  $\Omega$ . All rotational losses may be neglected.
  - (a) What internal generated voltage would be required for this generator to operate at the rated conditions?

- (b) What is the voltage regulation of this generator at the rated conditions?
  - (c) Sketch the power-versus-torque-angle curve for this generator. At what angle  $\delta$  is the power of the generator maximum?
  - (d) How does the maximum power out of this generator compare to the maximum power available if it were of cylindrical rotor construction?
- B-3.** Suppose that a salient-pole machine is to be used as a motor.
- (a) Sketch the phasor diagram of a salient-pole synchronous machine used as a motor.
  - (b) Write the equations describing the voltages and currents in this motor.
  - (c) Prove that the torque angle  $\delta$  between  $E_A$  and  $V_{\phi}$  on this motor is given by

$$\delta = \tan^{-1} \frac{I_A X_q \cos \theta - I_A R_A \sin \theta}{V_{\phi} + I_A X_q \sin \theta + I_A R_A \cos \theta}$$

**B-4.** If the machine in Prob. B-1 is running as a motor at the rated conditions, what is the maximum torque that can be drawn from its shaft without it slipping poles when the field current is zero?

**TABLE A.2**  
**Typical reactances of three-phase synchronous machines†**

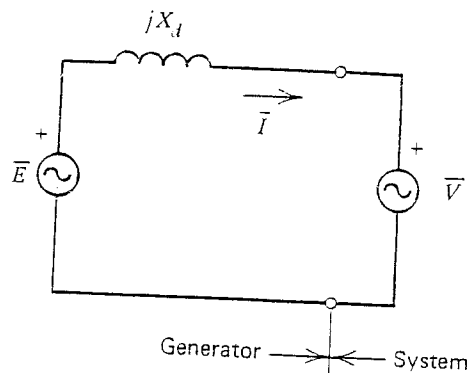
Values are per unit. For each reactance a range of values is listed below the typical value‡

	Turbine-generators					
	2-pole		4-pole		Salient-pole generators	
	Conventional cooled	Conductor cooled	Conventional cooled	Conductor cooled	With dampers	Without dampers
$X_d$	1.76 1.7-1.82	1.95 1.72-2.17	1.38 1.21-1.55	1.87 1.6-2.13	1 0.6-1.5	1 0.6-1.5
$X_q$	1.66 1.63-1.69	1.93 1.71-2.14	1.35 1.17-1.52	1.82 1.56-2.07	0.6 0.4-0.8	0.6 0.4-0.8
$X'_d$	0.21 0.18-0.23	0.33 0.264-0.387	0.26 0.25-0.27	0.41 0.35-0.467	0.32 0.25-0.5	0.32 0.25-0.5
$X''_d$	0.13 0.11-0.14	0.28 0.23-0.323	0.19 0.184-0.197	0.29 0.269-0.32	0.2 0.13-0.32	0.30 0.2-0.5
$X_2$	$= X''_d$	$= X''_d$	$= X''_d$	$= X''_d$	0.2 0.13-0.32	0.40 0.30-0.45
$X_0$ §						

†Data furnished by ABB Power T & D Company, Inc.

‡Reactances of older machines will generally be close to minimum values.

§ $X_0$  varies so critically with armature winding pitch that an average value can hardly be given. Variation is from 0.1 to 0.7 of  $X''_d$ .



FROM:

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Figure 6.11. Simplified positive-sequence network.

so that equation (6.22a) becomes

$$\bar{S} = \frac{VE_f}{X_d} \angle 90^\circ - \delta - j \frac{V^2}{X_d} \quad (6.22c)$$

$$= \frac{VE_f}{X_d} \sin \delta + j \left[ \frac{VE_f}{X_d} \cos \delta - \frac{V^2}{X_d} \right] \quad (6.22d)$$

Therefore,

$$P = \text{Re}[\bar{S}] \quad (6.23a)$$

$$= \frac{VE_f}{X_d} \sin \delta \quad (6.23b)$$

and

$$Q = \text{Im}[\bar{S}] \quad (6.24a)$$

$$= \frac{VE_f}{X_d} \cos \delta - \frac{V^2}{X_d} \quad (6.24b)$$

characteristics

## 6.5 The Salient-Pole Synchronous Machine

The simple circuit in Figure 6.11 is applicable to only the nonsalient-pole, or round-rotor, synchronous machine. The reason is that the  $IX_d$  voltage is produced by a rotating magnetic field that is caused by  $I$  (actually, the combined effect of all three stator currents). It happens that if the phasor diagram representing stator quantities is referenced to the physical rotor axis in a certain way, the *phase* position of  $I$  corresponds to the *spatial* position of the revolving magnetomotive force (mmf) caused by  $I$ . This was unimportant in the nonsalient rotor case, because the machine's air gap (the space between the rotor and the inner stator surface) was uniform in all directions, and thus the coefficient of  $I$  was the same for all phase orientations of  $I$ .

The rotor  $d$  axis is the magnetic axis of the rotor field and centered on the air gap at its narrowest point. The positive-axis orientation is out of the north pole. The  $q$



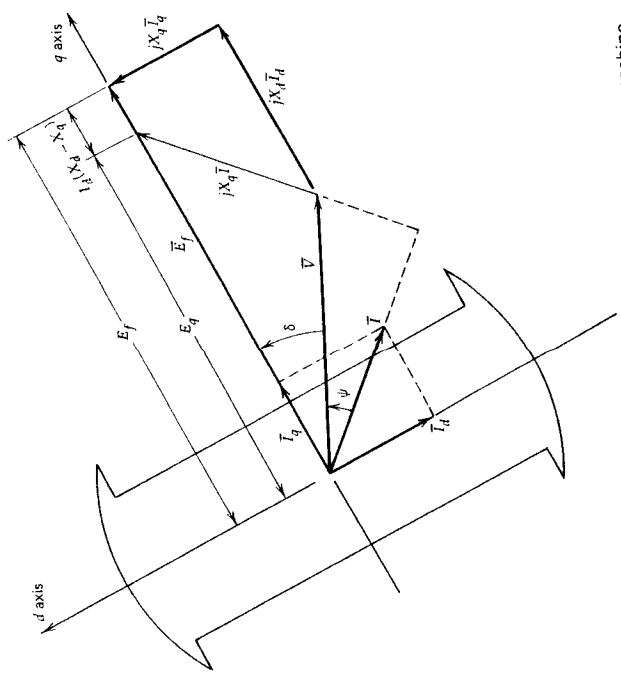


Figure 6.17. Phasor diagram for a salient pole synchronous machine.

axis lags the  $d$  axis by 90 electrical deg (i.e., sweeps past a reference point on the stator one-quarter voltage cycle after the  $d$  axis), as illustrated in Figure 6.17. The stator phasor diagram is placed on the rotor  $d, q$  axes such that  $E_f$  falls along the positive  $q$  axis. Thus, the revolving stator field can be broken into two components  $d$  and  $q$ . The relative magnitudes of the  $d, q$  components are proportional to  $I_d$  and  $I_q$ . The corresponding induced voltages are calculated as  $I_d X_d$  and  $I_q X_q$ ; their correct phase orientation are shown in Figure 6.17. The parameters  $X_d$  and  $X_q$  are routinely available for synchronous machines. Neglecting stator resistance the terminal voltage, current, and  $E_f$  are interrelated as follows:

$$E_f = jX_d I_d + jX_q I_q + \bar{V} \tag{6.34}$$

where

- $\bar{V} = V/\sqrt{2}$  = terminal voltage.
- $I = I/\sqrt{2}$  = terminal current.
- $E_f = E_f/\sqrt{2}$  = internal stator voltage.
- $I_d = I_d/\sqrt{2} - 90^\circ$  = direct-axis component of  $I$ .
- $I_q = I_q/\sqrt{2}$  = quadrature-axis component of  $I$ .

Now, consider the problem of finding  $E_f$ , given the generator terminal voltage and load condition ( $S$  and  $\text{pf}$ ) for a machine of known parameters  $X_d$  and  $X_q$ . Referring to Figure 6.17, locate  $\bar{V} = V/\sqrt{2}$  (no generality is lost by assigning the phase of  $\bar{V}$  to be  $0^\circ$ ). Furthermore,

$$I = \frac{S}{\bar{V}}$$

$$\psi = \pm \cos^{-1}(\text{pf})$$

$$\bar{I} = I/\sqrt{2} \angle -\psi$$

There is an apparent dilemma. We need to break  $\bar{I}$  into  $\bar{I}_d, \bar{I}_q$  components, but in order to do this we must locate the  $d, q$  axes (i.e.,  $\delta$  must be found). How can we efficiently determine  $\delta$ ? Recall

$$\bar{I}_q = \bar{I} - \bar{I}_d \tag{6.35}$$

$$\bar{E}_f = j(X_d - X_q)\bar{I}_d + jX_q\bar{I} + \bar{V} \tag{6.36a}$$

$$= j(X_d - X_q)\bar{I}_d + \bar{E}_q \tag{6.36b}$$

where

$$\bar{E}_q = jX_q\bar{I} + \bar{V} \tag{6.36c}$$

The voltage  $\bar{E}_q$  is easily calculated, since we know all right-hand terms. Now, observe that

$$j(X_d - X_q)\bar{I}_d = (X_d - X_q)I_d/90^\circ + \delta - 90^\circ \tag{6.37a}$$

$$= (X_d - X_q)I_d/\underline{\delta} \tag{6.37b}$$

That is,  $j(X_d - X_q)\bar{I}_d$  lies along the  $q$  axis. But  $\bar{E}_f = E_f/\underline{\delta}$  also lies on the  $q$  axis! Therefore,  $\bar{E}_q$  lies on the  $q$  axis or

$$\delta = \text{Arg}[\bar{E}_q] \tag{6.38}$$

Now that  $\delta$  is known

$$I_d = I \sin(\delta + \psi) \tag{6.39}$$

$$E_f = E_q + (X_d - X_q)I_d \tag{6.40a}$$

and

$$E_f = E_f/\underline{\delta} \tag{6.40b}$$

Now, consider power relationships

$$\bar{S} = \bar{V}\bar{I}^* \tag{6.41a}$$

$$= V[I_q(\delta - jI_d\delta)]^* \tag{6.41b}$$

$$= V[-\delta[I_q + jI_d]] \tag{6.41c}$$

$$= (V \cos \delta - jV \sin \delta) \left( \frac{V \sin \delta}{X_q} + j \frac{E_f - V \cos \delta}{X_d} \right) \tag{6.41d}$$

$$= \frac{E_f V}{X_d} \sin \delta + \frac{V^2 \sin 2\delta}{2X_d X_q} (X_d - X_q) + j \left[ \frac{E_f V}{X_d} \cos \delta - \frac{V^2}{2X_d X_q} (X_d + X_q) + \frac{V^2 \cos 2\delta}{2X_d X_q} (X_d - X_q) \right] \tag{6.41e}$$

If we define

$$S_1 = \frac{E_f V}{X_d} \tag{6.42a}$$

$$S_2 = \frac{V^2}{2X_d X_q} (X_d - X_q) \tag{6.42b}$$

$$Q_0 = \frac{V^2}{2X_d X_q} (X_d + X_q) \tag{6.42c}$$

Then,

$$P = S_1 \sin \delta + S_2 \sin 2\delta \tag{6.43a}$$

$$Q = S_1 \cos \delta + S_2 \cos 2\delta - Q_0 \tag{6.43b}$$

The  $2\delta$  terms are sometimes called reluctance terms and are caused by rotor saliency. It is interesting to note that even if  $E_f = 0$ , which represents loss of excitation, the machine still has some  $P$  generation capability. Also observe that  $P_{\max}$  is larger, and occurs at a smaller  $\delta_c$ , compared with the nonsalient case. Observe that if  $X_d = X_q$ , equation (6.43) reduce to equation (6.31).

Typical plots of  $P$  and  $Q$  versus  $\delta$  for the salient-pole case are shown in Figure 6.18. Example 6.6 illustrates this case.

### Example 6.6

A salient-pole generator has  $X_d = 1.0$  and  $X_q = 0.7$ . It operates at rated conditions with a pf = 0.8 lagging.

All values are in per unit on generator bases.

- Find  $E_f$  and  $\delta$ .
- Calculate  $P$  and  $Q$  using equations 6.43.
- Calculate  $P_{\max}$  and  $\delta_c$ .

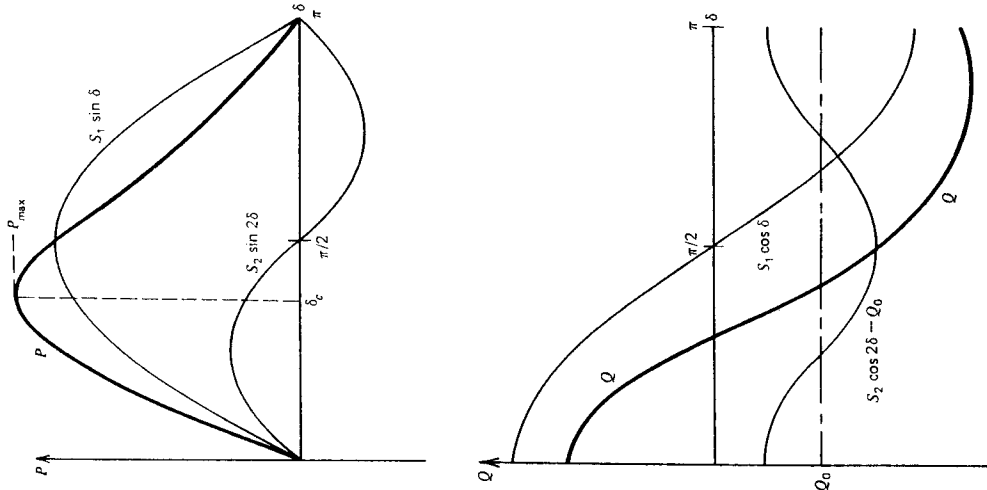


Figure 6.18.  $P$ - $Q$  versus  $\delta$  curves for a salient machine.

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**Solution**

$$(a) \bar{V} = 1 \angle 0^\circ$$

$$\psi = \pm \cos^{-1}(0.8) = +36.9^\circ$$

$$\bar{I} = 1 \angle -36.9^\circ$$

$$\begin{aligned} \bar{E}_q &= \bar{V} + j\bar{I}X_q \\ &= 1 \angle 0^\circ + 0.7 \angle 53.1^\circ = 1.5264 \angle 21.5^\circ \end{aligned}$$

$$\therefore \delta = 21.5^\circ$$

$$I_d = I \sin(\delta + \psi) = 0.8517$$

$$\begin{aligned} E_f &= E_q + (X_d - X_q)I_d \\ &= 1.5264 + (0.3)(0.8517) \\ &= 1.7819 \end{aligned}$$

$$(b) S_1 = \frac{1(1.7819)}{1} = 1.7819$$

$$S_2 = \frac{1}{2(1)(0.7)}(1 - 0.7) = 0.2143$$

$$Q_0 = \frac{1}{1.4}(1.7) = 1.2143$$

$$P = 1.7819 \sin(21.5^\circ) + 0.2143 \sin(43.0^\circ) = 0.8000$$

$$Q = 1.7819 \cos(21.5^\circ) + 0.2143 \cos(43.0^\circ) - 1.2143 = 0.6000$$

**Check**

$$\begin{aligned} \bar{S} &= \bar{V}\bar{I}^* \\ &= (1 \angle 0^\circ)(1 \angle -36.9^\circ)^* = 0.8 + j0.6 \end{aligned}$$

**(c) For maximum power,**

$$\frac{\partial P}{\partial \delta} = S_1 \cos \delta_c + 2S_2 \cos 2\delta_c = 0$$

$$S_1 \cos \delta_c + 2S_2(2 \cos^2 \delta_c - 1) = 0$$

$$\cos^2 \delta_c + \frac{S_1}{4S_2} \cos \delta_c - \frac{1}{2} = 0$$

$$\cos \delta_c = \frac{-S_1 \pm \sqrt{S_1^2 + 32S_2^2}}{8S_2} = \frac{-2.2965}{8} + 0.2177$$

We want the first-quadrant solution

$$\delta_c = \cos^{-1}(0.2177) = 77.4^\circ$$

$$\begin{aligned} P_{\max} &= 1.7819 \sin(77.4^\circ) + 0.2143 \sin(154.8^\circ) \\ &= 1.8302 \end{aligned}$$

## 6.6 Synchronous-Machine Electrical Transient Performance

There exists an important class of problems that require us to model the machine operating under suddenly switched conditions. The most important of these problems are described as fault studies. A fault is a suddenly applied short circuit. Our knowledge of circuit theory would suggest that we now must abandon our sinusoidal steady-state approach in favor of a more general transient treatment. Rigorously, this is correct, but experimental and field tests made over a number of years, and for a variety of conditions, show that the general electrical behavior of the machine is still basically ac. The students' first reaction to the term "transient ac" may be that the author is playing without a full deck. Nonetheless, the term accurately describes the conventional analytical approach to the problem, and it is reasonable. Let us consider an experimental approach.

Imagine a synchronous machine running at constant angular speed with its field excited by a constant-voltage source. Now, consider a three-phase short circuit suddenly applied to the stator terminals, which were previously opened. The recorded current wave forms appear in Figure 6.19. Observe that initially the currents are considerably larger than they are several cycles later (the frequency of the alternating current shown is 60 Hz). If you look closely, you will also observe that each wave form is offset, that is not symmetric about the time axis. This latter effect is predicted by the straightforward solution to a simple  $R$ - $L$  circuit problem. Consider Example 6.7.

### Example 6.7

In the circuit in Figure 6.20,  $e = V_m \sin(\omega t + \alpha)$ , and the switch is closed at  $t = 0$ . Solve for  $i(t)$ .

### Solution

Kirchhoff's voltage law produces

$$e = L \frac{di}{dt} + iR \quad (6.44a)$$