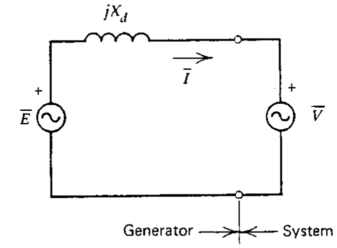

Figure 6.11. Simplified positive-sequence network.

so that equation (6.22a) becomes

$$\bar{S} = \frac{VE_f}{X_d}\underline{/90^\circ - \delta} - j\frac{V^2}{X_d} \tag{6.22c}$$

$$= \frac{VE_f}{X_d}\sin\delta + j\left[\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{X_d}\right] \tag{6.22d}$$

Therefore,

$$P = \text{Re}[\bar{S}] \tag{6.23a}$$

$$= \frac{VE_f}{X_d}\sin\delta \tag{6.23b}$$

and

$$Q = \text{Im}[\bar{S}] \tag{6.24a}$$

$$= \frac{VE_f}{X_d}\cos\delta - \frac{V^2}{X_d} \tag{6.24b}$$

We now realize an extremely important fact. The P in equation (6.23) *is* P_e, the power converted from mechanical to electrical form. Equation (6.22) is very useful in explaining generator reaction to turbine and exciter control. Let us first consider real power. Imagine adjusting *only* the main steam valve, *opening* it further. Clearly, T_m, and thus, P_m, increases. Consequently, $P_e = P$ increases. Since E_f, V, and X_d are constant, δ must increase. Study Figure 6.12. An increase from P_0 to P_1 causes a corresponding increase in δ from δ_0 to δ_1. Obviously, there is a limit to how far we can increase P. That limit P_{max} is referred to as the steady-state stability limit and is

$$P_{max} = \frac{E_f V}{X_d} \tag{6.25a}$$

The corresponding δ_c is the critical power angle

$$\delta_c = 90^\circ \tag{6.25b}$$

If P_m is further increased, the generator could no longer remain synchronized with the system. To avoid the problem, generators are normally operated at fairly small

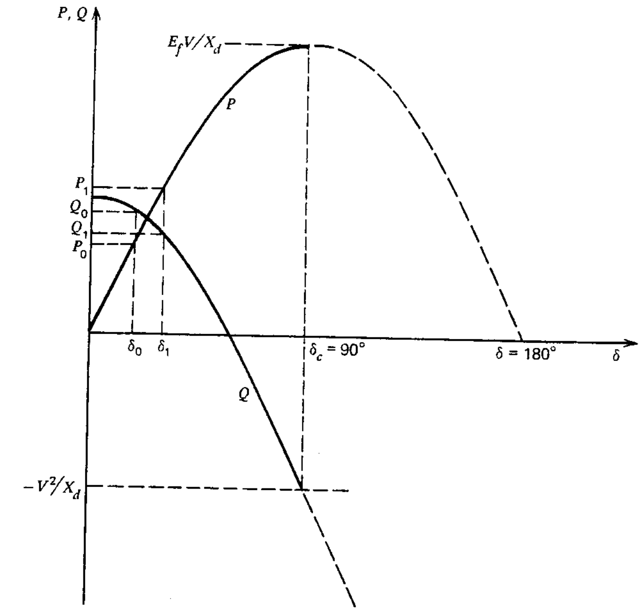

Figure 6.12. *P* and *Q* variation with delta.

power angles ($\sim 20^\circ$). The conclusion is clear; there is a strong interrelation between the steam valve setting, real power flow, and δ.

Now, consider the second control action, namely, changing the voltage regulator setting. Suppose E_f increases. If we examine equations (6.22) and assume V, X_d, and P are constant, we observe that the P, Q relationships are modified to become P' and Q', as shown in Figure 6.13. Note that Q increase from Q_0 to Q_1. The angle δ also decreases slightly. We observe a strong interrelationship between regulator setting, reactive power flow, and E_f. Example 6.3 should help our understanding.

Example 6.3

The generator in Figure 6.11 operates at rated terminal conditions with a power factor of 0.8 lagging. The reactance $X_d = 0.7$ pu (on the generator ratings).

(a) Find P, Q, E_f, and δ. Draw the phasor diagram.
(b) The steam valve is opened further so that P increases 20%. Reevaluate P, Q, E_f, and δ. Draw the new phasor diagram.
(c) The system is restored to the conditions in (a). The exciter is adjusted to raise E_f 20%. Reevaluate P, Q, E_f, and δ. Draw the new phasor diagram.

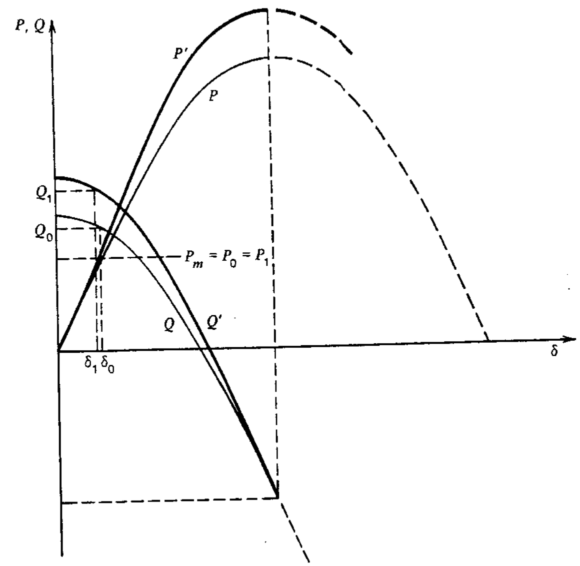

Figure 6.13. P and Q variation with E_f.

Solution

Let us use the subscript 0 to denote initial values and 1 to denote adjusted values

(a) $\bar{V} = 1\underline{/0°}$

$$\psi_0 = \cos^{-1}(0.8) = 36.9°$$

$$\therefore \quad \bar{I}_0 = 1\underline{/-36.9°} = 0.8 - j0.6$$

$$\bar{E}_{f0} = jX_d\bar{I}_0 + \bar{V}$$

$$= j0.7(0.8 - j0.6) + 1$$

$$= 1.42 + j0.56$$

$$= 1.53\underline{/21.5°}$$

$$\therefore \quad E_{f0} = 1.53$$

$$\delta_0 = 21.5°$$

$$P_0 = \frac{E_{f0}V}{X_d}\,\mathrm{Sin}\,\delta_0$$

$$= \frac{(1.53)(1.0)}{0.7}\sin(21.5°)$$

$$= 0.800$$

$$Q_0 = \frac{E_{f0}V}{X_d}\cos\delta - \frac{V^2}{X_d}$$

$$= \frac{(1.53)(1.0)}{0.7}\cos(21.5°) - \frac{(1.0)^2}{0.7}$$

$$= 0.60$$

Consult the phasor diagram shown in Figure 6.14(*a*).

(b) $P_1 = 1.2P_0 = 1.2(0.8) = 0.96$ (20% increase)

$E_{f1} = E_{f0} = 1.53$ (no change)

$$\therefore \quad \delta_1 = \sin^{-1}\left(\frac{P_1X_d}{E_{f1}V}\right)$$

$$= \sin^{-1}\left(\frac{0.96(0.7)}{(1.53)(1.0)}\right) = 26.1° \qquad (21\%\ \text{increase})$$

$$Q_1 = \frac{E_{f0}V}{X_d}\cos\delta_1 - \frac{V^2}{X_d}$$

$$= \frac{1.53(1.0)}{0.7}\cos(26.1°) - \frac{(1.0)^2}{0.7}$$

$$= 0.535 \qquad (11\%\ \text{decrease})$$

Consult the phasor diagram shown in Figure 6.14(*b*).

(c) $P_1 = P_0 = 0.8$ (no change)

$E_{f1} = 1.2\,E_{f0} = 1.2(1.53) = 1.84$ (20% increase)

$$\delta_1 = \sin^{-1}\left(\frac{P_1X_d}{E_{f1}V}\right)$$

$$= \sin^{-1}\left[\frac{0.8(0.7)}{1.84(1.0)}\right] = 17.8 \qquad (17\%\ \text{decrease})$$

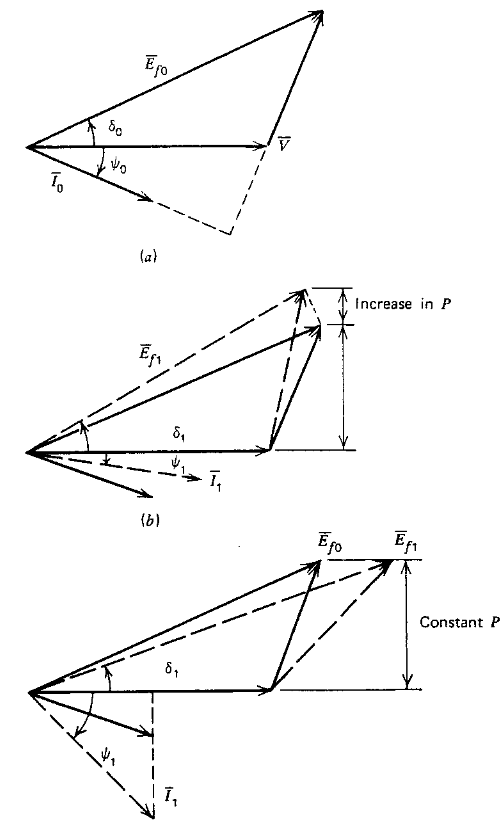

Figure 6.14. Solution to example 6.3. (*a*) Initial phasor diagram. (*b*) 20% increase in *P*. (*c*) 20% increase in E_f.

$$Q_1 = \frac{E_{f1} V}{X_d}\cos\delta - \frac{V^2}{X_d}$$

$$= \frac{(1.84)(1.0)}{0.7}\cos(17.8) - \frac{(1.0)^2}{0.7}$$

$$= 1.07 \qquad (78\,\%\text{ increase})$$

Consult the phasor diagram shown in Figure 6.14(*c*).

6.4 Operating Limits on Synchronous Generators

We are interested in the synchronous generator as a power source and, therefore, concerned with limits on its power delivery capabilities. The power system is normally operated as a constant-voltage system; that is to say, the voltage at any point in the system is held to within about $\pm 5\,\%$ of some nominal value. At fixed voltage, power is proportional to current, and we turn our attention to current capacity.

For a given stator winding conductor size, there must be an associated winding resistance. If a current I flows in the winding, there is a corresponding I^2R power loss, excluding the superconducting case. This energy must be removed, or it will raise the temperature of the conductor and its immediate environment. A major consideration in machine design is removing this waste heat. However, no matter how efficient stator cooling schemes are, there exists a current that if exceeded indefinitely, will cause stator winding temperatures to reach damaging levels. Such a current is referred to as the rated current, which may be considered as an upper limit for a short time without damaging the windings (the shorter the time, the greater the excess allowed). Associated with this value $I_{L_{\text{rated}}}$, along with $V_{L_{\text{rated}}}$, is a power rating

$$S_{3\phi_{\text{rated}}} = V_{L_{\text{rated}}} I_{L_{\text{rated}}} \sqrt{3} \tag{6.26}$$

where these quantities are in SI units. If we define $\bar{S}_{3\phi}$ as the machine output complex power, operation is constrained to

$$|\bar{S}_{3\phi}| \le S_{3\phi_{\text{rated}}} \tag{6.27}$$

If we converted equation (6.27) into per-unit with the machine's ratings taken as base values, we obtain

$$|\bar{S}| \le 1 \tag{6.28}$$

Let us interpret equation (6.28) geometrically in the $\bar{S}\,(P, Q)$ plane. It is the region inside the unit circle (*abcde*) shown in Figure 6.15. Operation is constrained to within this circle due to stator winding heating.

Consider the upper region of this curve (arc *ab*): high Q. low P. In terms of our circuit model (Figure 6.11), $\bar{I}$ is considerably out of phase with $\bar{V}$ and lagging. The phasor $jX_d\bar{I}$ is close to being in phase with $\bar{V}$ and, therefore, requires a large value for E_f. Example 6.4 illustrates this situation.

Example 6.4

A generator is modeled as shown in Figure 6.11 and has $X_d = 1.2$ pu. Calculate the required E_f at rated conditions for unity-power-factor and zero-power-factor lagging and compare.

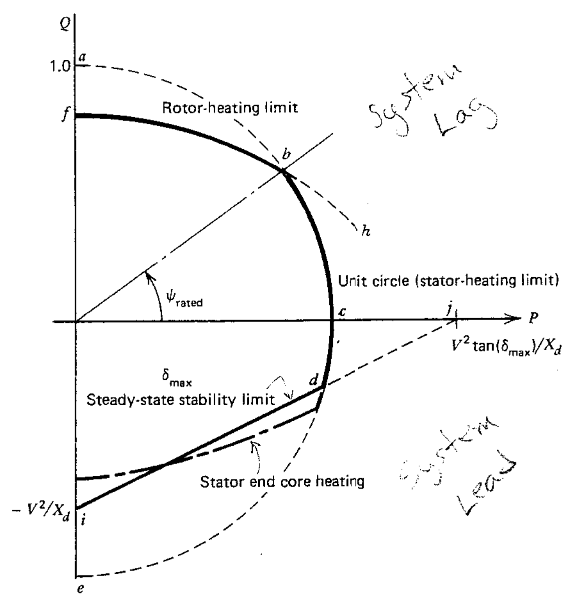

Figure 6.15. Generator-operating limits.

Solution

Unity-power-factor: $\bar{I} = 1 + j0$

$$\bar{E}_f = jX_d\bar{I} + \bar{V}$$
$$= j1.2(1) + 1 = 1.56\underline{/50.2°}$$
$$\therefore \quad E_f = 1.56$$

Zero-power-factor lagging: $\bar{I} = 0 - j1$

$$\bar{E}_f = jX_d\bar{I} + \bar{V}$$
$$= j1.2(-j) + 1 = 2.20\underline{/0°}$$
$$\therefore \quad E_f = 2.20$$

A substantially larger E_f is required in the lagging case. Recall that E_f is generated by the rotor field, which in turn is created by the rotor field current I_f. Because of $I_f{}^2R_f$ heating in the rotor windings, I_f cannot be increased indefinitely.

Therefore, there is a practical upper limit on E_f. We could, of course, design the machine to provide the appropriate E_f for even the zero-power-factor lagging case. However, this condition is rarely required (who needs a generator that delivers no P?) and does not justify the added expense of allowing for it. Therefore, as we move up the P, Q curve from c to b, we require increasingly larger values of E_f until at point b we reach $E_{f\max}$, corresponding to $I_{f\max}$. From this point on, E_f is no longer adjustable, but fixed at $E_{f\max}$. This point (b) corresponds to the rated power factor of the machine, such that

$$\text{pf}_{\text{rated}} = \cos(\psi_{\text{rated}}) \tag{6.29}$$

Now, what is the operating-constraint curve? To answer this question, recall equation (6.22) with $E_f = E_{f\max}$

$$\bar{S} = \frac{VE_{f\max}}{X_d}\sin\delta + j\left(\frac{VE_{f\max}}{X_d}\cos\delta - \frac{V^2}{X_d}\right) \tag{6.30}$$

This is the equation of a circle in the $\bar{S}$ plane with center $[0, -(V^2/X_d)]$ and radius $VE_{f\max}/X_d$. A segment of this circle fbh appears in Figure 6.15. Operation is constrained to the region within the circle due to rotor field winding heating.

Now consider the lower region of the circle $abcde$, cde. Note that Q is negative, implying a leading power factor situation. As we move from c to d, E_f decreases, which seems to be desirable. Suppose we examine a specific situation with an example.

Example 6.5

Continue example 6.4 by calculating $\bar{E}_f$ for a power factor of 0.553 leading at rated-terminal conditions. Comment.

Solution

$$\psi = \cos^{-1}(0.553) = 56.4°$$
$$\bar{I} = 1\underline{/56.4°}$$
$$\bar{E}_f = j\bar{I}X_d + \bar{V}$$
$$= j1.2(1\underline{/56.4°}) + 1$$
$$= j0.664$$
$$= 0.664\underline{/90°}$$
$$E_f = 0.664 \qquad \delta = 90°$$

The $\delta = 90°$ value indicates that operation at this power factor is completely unacceptable. We are on the verge of instability; if any additional P_m is suddenly supplied by the prime mover, we shall lose synchronism.

We realize that as we moved from *c* to *d*, E_f decreased, but δ *increased*. How far is it reasonable to allow δ to increase? If we conclude that it is prudent to keep a 10% reserve on real-power delivery capability, $\delta_{max} = \sin^{-1}(0.9) = 64.2°$. For a given δ_{max}, we can calculate P and Q as E_f varies

$$P = \frac{E_f V}{X_d} \sin \delta_{max} \qquad (6.31a)$$

$$Q = \frac{E_f V}{X_d} (\cos \delta_{max}) - \frac{V^2}{X_d} \qquad (6.31b)$$

But from equation (6.23)

$$\frac{E_f V}{X_d} = \frac{P}{\sin \delta_{max}} \qquad (6.32)$$

and substituting equation (6. 31b)

$$Q = \frac{1}{\tan \delta_{max}} P - \frac{V^2}{X_d} \qquad (6.33)$$

which is a straight line with Q intercept $-V^2/X_d$ and P intercept $V^2 \tan \delta_{max}/X_d$. This is the equation of the line *idj*. To avoid steady-state instability, generator operation is constrained to the region above the line *idj*.

There is another limit to be considered when operating at leading power factors. The stator leakage flux is at right angles to the stator laminations at the ends of the stator core, Thus, there is excessive core-loss-type heating in the stator iron in this region. This excessive heating is avoided by also reducing the negative Q output at low leading power factors. This effect will modify the lower contour *idf* somewhat.

For some types of units, it is undesirable to allow the real power output to go completely to zero unless we intend to take the unit off line. An example would be a fossil fuel steam plant, where constant boiler temperatures are required to prevent slagging problems.

Generator operation is, therefore, confined to the P, Q interior region bounded by *fbcdi*, as shown in Figure 6.15. For a specific machine, these curves can be accurately calculated and steady-state operation limited to within the enclosed region, by either an operator, or automatically, or a combination of both. We can simplify the situation by "squaring off" the operating characteristic, creating the region shown in Figure 6.16. Such a characteristic is particularly useful for computer-control applications. Assuming the real power P is specified, we need to know Q_{max} and Q_{min} to define the operating region.

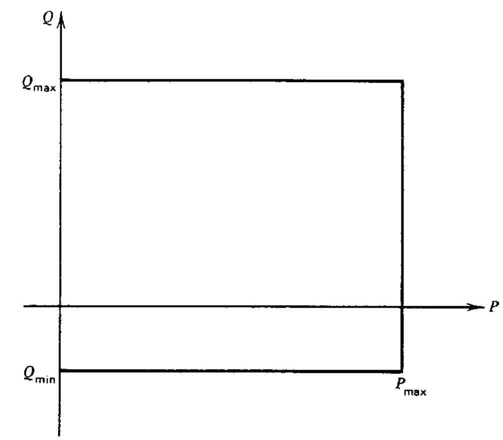

Figure 6.16. Simplified generator-operating characteristic.

To summarize, there are three major effects that limit the output power of a generator: P_{max} (stator heating), Q_{max} (rotor heating), and Q_{min} (steady-state stability). In a particular installation, additional considerations are important for refinement to these results. The machine's saturation characteristics should be included. The machine realistically is not terminated in an ideal source, but in an actual system whose impedance is significant. The exciter and regulator dynamic characteristics can be important.

6.5 The Salient-Pole Synchronous Machine

The simple circuit in Figure 6.11 is applicable to only the nonsalient-pole, or round-rotor, synchronous machine. The reason is that the IX_d voltage is produced by a rotating magnetic field that is caused by I (actually, the combined effect of all three stator currents). It happens that if the phasor diagram representing stator quantities is referenced to the physical rotor axis in a certain way, the *phase* position of I corresponds to the *spatial* position of the revolving magnetomotive force (mmf) caused by I. This was unimportant in the nonsalient rotor case, because the machine's air gap (the space between the rotor and the inner stator surface) was uniform in all directions, and thus the coefficient of I was the same for all phase orientations of I.

The rotor d axis is the magnetic axis of the rotor field and centered on the air gap at its narrowest point. The positive-axis orientation is out of the north pole. The q

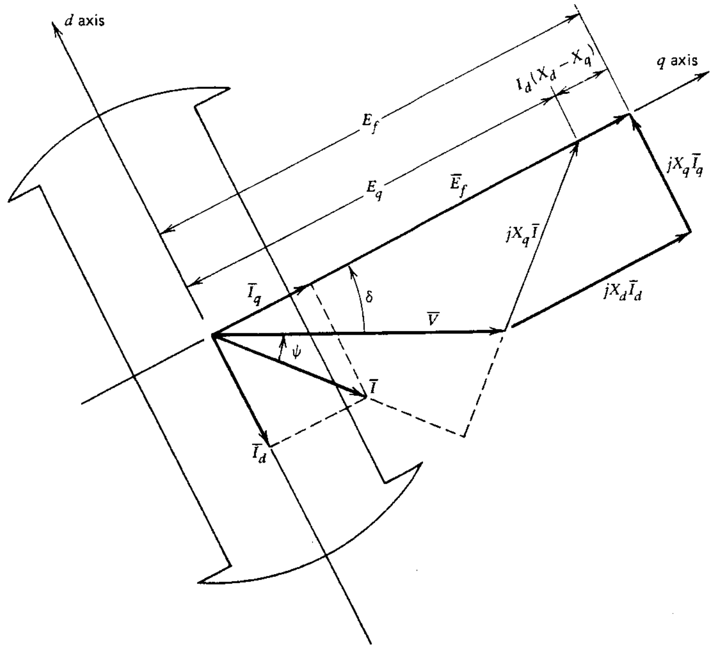

Figure 6.17. Phasor diagram for a salient pole synchronous machine.

axis lags the d axis by 90 electrical deg (i.e., sweeps past a reference point on the stator one-quarter voltage cycle after the d axis), as illustrated in Figure 6.17. The stator phasor diagram is placed on the rotor d, q axes such that E_f falls along the positive q axis. Thus, the revolving stator field can be broken into two components d and q. The relative magnitudes of the d, q components are proportional to I_d and I_q. The corresponding induced voltages are calculated as $I_d X_d$ and $I_q X_q$; their correct phase orientation are shown in Figure 6.17. The parameters X_d and X_q are routinely available for synchronous machines. Neglecting stator resistance the terminal voltage, current, and E_f are interrelated as follows:

$$\bar{E}_f = jX_d\bar{I}_d + jX_q\bar{I}_q + \bar{V} \qquad (6.34)$$

where

$\bar{V} = V\underline{/0^\circ}$ = terminal voltage.
$\bar{I} = I\underline{/-\psi}$ = terminal current.
$\bar{E}_f = E_f\underline{/\delta}$ = internal stator voltage.
$\bar{I}_d = I_d\underline{/\delta - 90^\circ}$ = direct-axis component of $\bar{I}$.
$\bar{I}_q = I_q\underline{/\delta}$ = quadrature-axis component of $\bar{I}$.

Now, consider the problem of finding $\bar{E}_f$, given the generator terminal voltage and load condition (S and pf) for a machine of known parameters X_d and X_q. Referring to Figure 6.17, locate $\bar{V} = V\underline{/0^\circ}$ (no generality is lost by assigning the phase of $\bar{V}$ to be 0°). Furthermore,

$$I = \frac{S}{V}$$

$$\psi = \pm\cos^{-1}(\text{pf})$$

$$\bar{I} = I\underline{/-\psi}$$

There is an apparent dilemma. We need to break $\bar{I}$ into $\bar{I}_d$, $\bar{I}_q$ components, but in order to do this we must locate the d, q axes (i.e., δ must be found). How can we efficiently determine δ? Recall

$$\bar{I}_q = \bar{I} - \bar{I}_d \qquad (6.35)$$

$$\bar{E}_f = j(X_d - X_q)\bar{I}_d + jX_q\bar{I} + \bar{V} \qquad (6.36a)$$

$$= j(X_d - X_q)\bar{I}_d + \bar{E}_q \qquad (6.36b)$$

where

$$\bar{E}_q = jX_q\bar{I} + \bar{V} \qquad (6.36c)$$

The voltage $\bar{E}_q$ is easily calculated, since we know all right-hand terms. Now, observe that

$$j(X_d - X_q)\bar{I}_d = (X_d - X_q)I_d\underline{/90^\circ + \delta - 90^\circ} \qquad (6.37a)$$

$$= (X_d - X_q)I_d\underline{/\delta} \qquad (6.37b)$$

That is, $j(X_d - X_q)\bar{I}_d$ lies along the q axis. But $\bar{E}_f = E_f\underline{/\delta}$ also lies on the q axis! Therefore, $\bar{E}_q$ lies on the q axis or

$$\delta = \text{Arg}\,[\bar{E}_q] \qquad (6.38)$$

Now that δ is known

$$I_d = I\sin(\delta + \psi) \qquad (6.39)$$

$$E_f = E_q + (X_d - X_q)I_d \qquad (6.40a)$$

and

$$\bar{E}_f = E_f\underline{/\delta} \qquad (6.40b)$$

Now, consider power relationships

$$\bar{S} = \bar{V}\bar{I}^* \tag{6.41a}$$

$$= V[I_q\underline{/\delta} - jI_d\underline{/\delta}]^* \tag{6.41b}$$

$$= V\underline{/-\delta}[I_q + jI_d] \tag{6.41c}$$

$$= (V\cos\delta - jV\sin\delta)\left(\frac{V\sin\delta}{X_q} + j\frac{E_f - V\cos\delta}{X_d}\right) \tag{6.41d}$$

$$= \frac{E_f V}{X_d}\sin\delta + \frac{V^2 \sin 2\delta}{2X_d X_q}(X_d - X_q) + j\left[\frac{E_f V}{X_d}\cos\delta - \frac{V^2}{2X_d X_q}(X_d + X_q) + \frac{V^2\cos 2\delta}{2X_d X_q}(X_d - X_q)\right] \tag{6.41e}$$

If we define

$$S_1 = \frac{E_f V}{X_d} \tag{6.42a}$$

$$S_2 = \frac{V^2}{2X_d X_q}(X_d - X_q) \tag{6.42b}$$

$$Q_0 = \frac{V^2}{2X_d X_q}(X_d + X_q) \tag{6.42c}$$

Then,

$$P = S_1 \sin\delta + S_2 \sin 2\delta \tag{6.43a}$$

$$Q = S_1\cos\delta + S_2\cos 2\delta - Q_0 \tag{6.43b}$$

The 2δ terms are sometimes called reluctance terms and are caused by rotor saliency. It is interesting to note that even if $E_f = 0$, which represents loss of excitation, the machine still has some P generation capability. Also observe that P_{max} is larger, and occurs at a smaller δ_c, compared with the nonsalient case. Observe that if $X_d = X_q$, equation (6.43) reduce to equation (6.31).

Typical plots of P and Q versus δ for the salient-pole case are shown in Figure 6.18. Example 6.6 illustrates this case.

Example 6.6

A salient-pole generator has $X_d = 1.0$ and $X_q = 0.7$. It operates at rated conditions with a pf = 0.8 lagging.

All values are in per unit on generator bases.

(a) Find E_f and δ.
(b) Calculate P and Q using equations 6.43.
(c) Calculate P_{max} and δ_c.

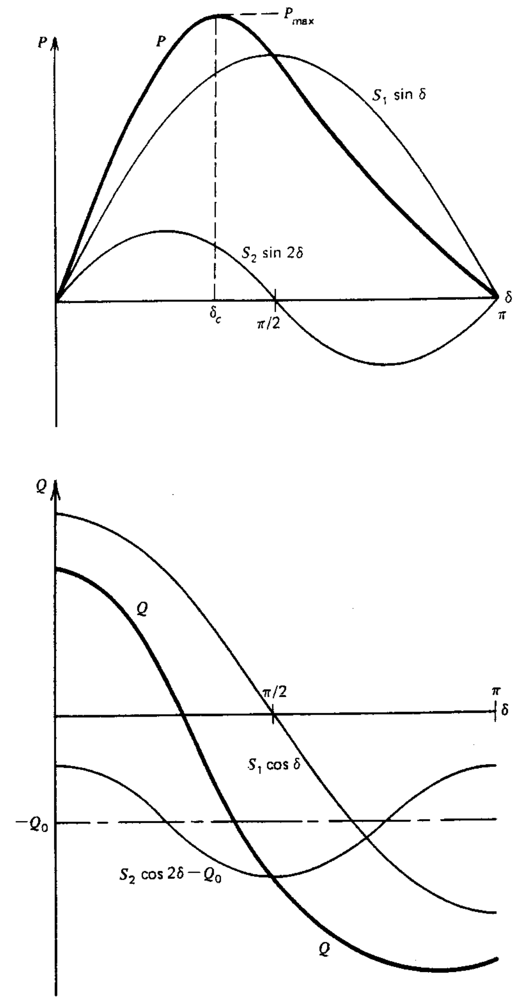

Figure 6.18. *P-Q* versus δ curves for a salient machine.

Solution

(a) $\bar{V} = 1\underline{/0^\circ}$

$\psi = \pm\cos^{-1}(0.8) = +36.9^\circ$

$\bar{I} = 1\underline{/-36.9^\circ}$

$\bar{E}_q = \bar{V} + j\bar{I}X_q$

$= 1\underline{/0^\circ} + 0.7\underline{/53.1^\circ} = 1.5264\underline{/21.5^\circ}$

$\therefore \quad \delta = 21.5^\circ$

$I_d = I \sin(\delta + \psi) = 0.8517$

$E_f = E_q + (X_d - X_q)I_d$

$= 1.5264 + (0.3)(0.8517)$

$= 1.7819$

(b) $S_1 = \dfrac{1(1.7819)}{1} = 1.7819$

$$S_2 = \frac{1}{2(1)(0.7)}(1 - 0.7) = 0.2143$$

$$Q_0 = \frac{1}{1.4}(1.7) = 1.2143$$

$P = 1.7819 \sin(21.5^\circ) + 0.2143 \sin(43.0^\circ) = 0.8000$

$Q = 1.7819 \cos(21.5^\circ) + 0.2143 \cos(43.0^\circ) - 1.2143 = 0.6000$

Check

$$\bar{S} = \bar{V}\bar{I}^*$$

$$= (1\underline{/0^\circ})(1\underline{/-36.9^\circ})^* = 0.8 + j0.6$$

(c) For maximum power,

$$\frac{\partial P}{\partial \delta} = S_1 \cos\delta_c + 2S_2 \cos 2\delta_c = 0$$

$$S_1 \cos\delta_c + 2S_2(2\cos^2\delta_c - 1) = 0$$

$$\cos^2\delta_c + \frac{S_1}{4S_2}\cos\delta_c - \tfrac{1}{2} = 0$$

$$\cos\delta_c = \frac{-S_1 \pm \sqrt{S_1^2 + 32S_2^2}}{8S_2} = -2.2965, +0.2177$$

We want the first-quadrant solution

$\delta_c = \cos^{-1}(0.2177) = 77.4^\circ$

$P_{max} = 1.7819 \sin(77.4^\circ) + 0.2143 \sin(154.8^\circ)$

$= 1.8302$

6.6 Synchronous-Machine Electrical Transient Performance

There exists an important class of problems that require us to model the machine operating under suddenly switched conditions. The most important of these problems are described as fault studies. A fault is a suddenly applied short circuit. Our knowledge of circuit theory would suggest that we now must abandon our sinusoidal steady-state approach in favor of a more general transient treatment. Rigorously, this is correct, but experimental and field tests made over a number of years, and for a variety of conditions, show that the general electrical behavior of the machine is still basically ac. The students' first reaction to the term "transient ac" may be that the author is playing without a full deck. Nonetheless, the term accurately describes the conventional analytical approach to the problem, and it is reasonable. Let us consider an experimental approach.

Imagine a synchronous machine running at constant angular speed with its field excited by a constant-voltage source. Now, consider a three-phase short circuit suddenly applied to the stator terminals, which were previously opened. The recorded current wave forms appear in Figure 6.19. Observe that initially the currents are considerably larger than they are several cycles later (the frequency of the alternating current shown is 60 Hz). If you look closely, you will also observe that each wave form is offset, that is not symmetric about the time axis. This latter effect is predicted by the straightforward solution to a simple *R-L*circuit problem. Consider Example 6.7.

Example 6.7

In the circuit in Figure 6.20, $e = V_m \sin(\omega t + \alpha)$, and the switch is closed at $t = 0$. Solve for $i(t)$.

Solution

Kirchhoff's voltage law produces

$$e = L\frac{di}{dt} + iR \tag{6.44a}$$