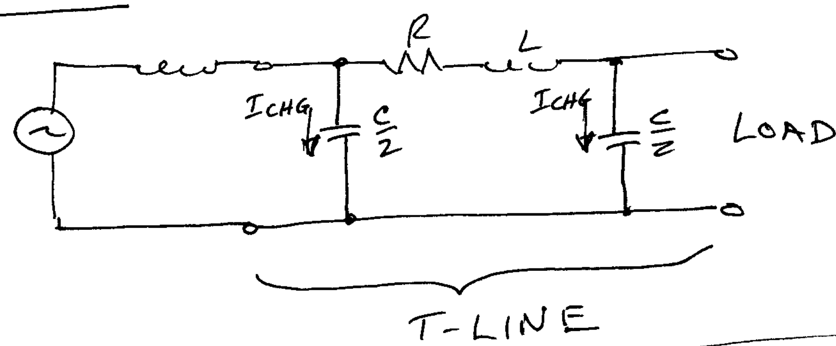
CAPACITANCE

per-phase repres.

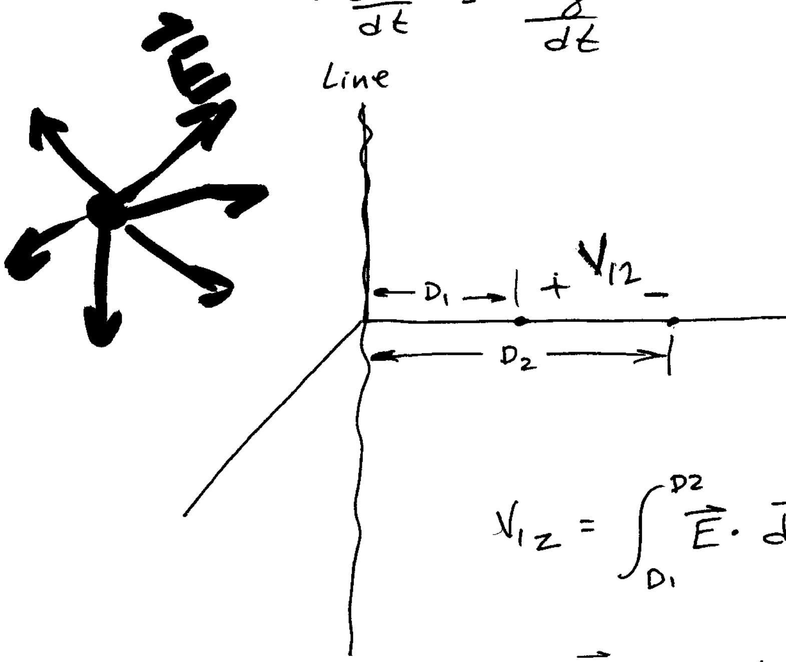


Capacitance affects voltage drop, Efficiency, PF, System stability, & transient voltages, voltage regulation

$$i = C\frac{dv}{dt} = \frac{dq}{dt}$$

$$\boxed{C = \frac{dq}{dV} \approx \frac{Q}{V}}$$

$$Q = CV$$



Q_{line} = charge/unit length

put V on line → Q

put Q on line → V

$$V_{12} = \int_{D_1}^{D_2} \vec{E} \cdot \vec{dL}$$

$\vec{E}$ = Electric Field intensity V/m

$\vec{dL}$ = Incremental Length

$\vec{D} = K\vec{E}$ = Electric Flux density C/m^2

$K = K_r K_o$

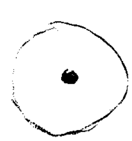
$\varepsilon = \varepsilon_r \varepsilon_o$

K_r = relative permittivity

K_o = permittivity of air

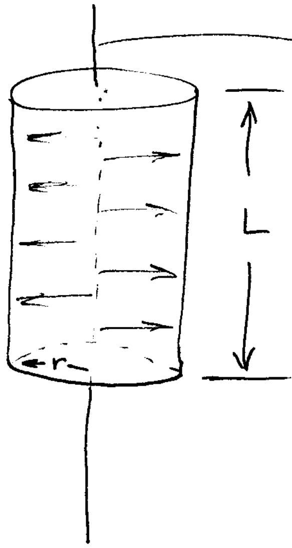
Gausses Law $\int_S \vec{D} \cdot \vec{dS} = Q_{enclosed}$ where $\vec{dS}$ = surface area

surface integral



$\epsilon_0 = 8.854\times10^{-12}$

E-Field



wire, charge $\frac{Q_{TOTAL}}{L} = Q_L$

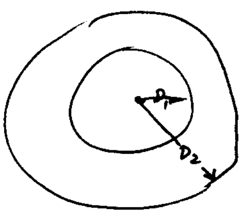
$$\int \vec{D}\cdot d\vec{S} = \vec{D}\,2\pi r L = Q_L L$$

$$D = \frac{Q_L}{2\pi r}$$

like ϵ_0

$$E = \frac{D}{K} = \frac{Q_L}{2\pi r K}$$

Top View



$$V_{12} = \int_{D_1}^{D_2} \frac{Q_L}{2\pi K r}\,dr$$

$$V_{12} = \frac{Q_L}{2\pi K r} \ln \frac{D_2}{D_1}$$

CAP of two wire system:

Q_a

$Q_b = -Q_a$

$$C = \frac{Q}{V}$$

$$V_{ab} = \underbrace{\frac{Q_a}{2\pi K} \ln \frac{D}{r_a}}_{\text{Voltage due to charge on a}} + \underbrace{\frac{Q_L}{2\pi K} \ln \frac{r_b}{D}}_{\text{voltage due to charge on b}}$$

Since $q_a = -q_b$,

$$V_{ab} = \frac{Q_a}{2\pi K} \ln \frac{D}{r_a} + \frac{Q_a}{2\pi K} \ln \frac{D}{r_b}$$

Exact Solution: $C = \frac{\pi\epsilon}{\cosh^{-1}\left(\frac{D}{r}\right)}$ F/m

$$\boxed{V_{ab} = \frac{Q_a}{2\pi K} \ln \frac{D^2}{r_a r_b}}$$

ϵ

if $r_a = r_b = r$

$$C_{ab} = \frac{Q_a}{V_{ab}} = \frac{2\pi K}{\ln \frac{D^2}{r_a r_b}} = C_{ab} = \frac{\pi K}{\ln \frac{D}{\sqrt{r_a r_b}}} = \boxed{\frac{\pi K}{\ln \frac{D}{r}}}$$

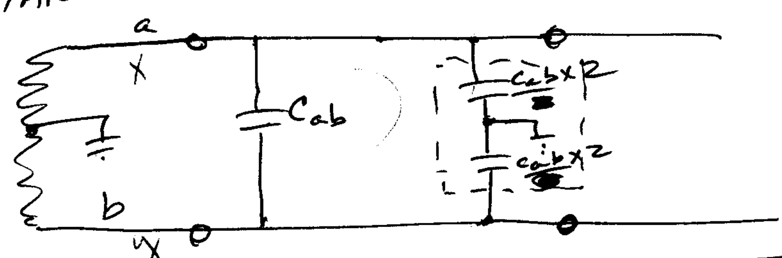
$D \gg r$

$$C_{ab} = \frac{\pi\,(8.85\times10^{-12}\ F/m)}{\ln \frac{D}{r}}$$

$$\boxed{C_{ab} = \frac{.0388}{\log_{10}\frac{D^2}{r_a r_b}}\ \mu F/mi \overset{(\text{if } r_a = r_b)}{=} \frac{.0194}{\log_{10}\frac{D}{r}}\ \mu F/m}$$

$$\boxed{= \frac{\pi\epsilon_0}{\ln\frac{D}{r}}\ F/m}$$

CAPACITANCE TO NEUTRAL:



XFMR secondary

$$\boxed{2\,\underset{xy}{C_{ab}} = \underset{xn}{C_{an}} = \underset{yn}{C_{bn}} = \frac{.0388}{\log_{10}\frac{D}{r}}\ \mu F/mi}$$

$$\boxed{= \frac{2\pi\epsilon_0}{\ln\frac{D}{r}}\ F/m}$$

$D_{sc} = r =$ outside radius of conductor – <u>not D_s</u>
same for solid or stranded

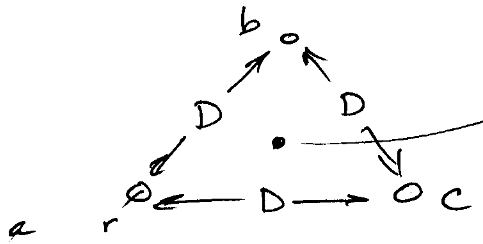
$$X_c = \frac{1}{2\pi f c} = \frac{4.10}{f}\times10^6 \log_{10}\frac{D}{r}\ \Omega\text{-mi} \quad (\text{TO NEU})$$

$$X_c = \underbrace{\frac{4.1\times10^6}{f}\log\frac{1}{r}}_{X'_a} + \underbrace{\frac{4.1\times10^6}{f}\log D}_{X'_d}\ \Omega\text{-mi}\quad(\text{TO NEU})$$

Table A.3
1' spacing

Table A.5
spacing factor

3φ LINE EQUIL SPACING



can think of neutral point having a zero charge potential.

Dist b − Dist a $$V_{ab} = \frac{1}{2\pi K}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + \cancel{q_c \ln\frac{D}{D}}\right)$$

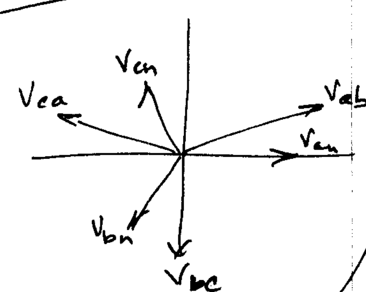
Dist c − Dist a $$V_{ac} = \frac{1}{2\pi K}\left(q_a \ln\frac{D}{r} + \cancel{q_b \ln\frac{D}{D}} + q_c \ln\frac{r}{D}\right)$$

Dist

$$V_{ab} + V_{ac} = \frac{1}{2\pi K}\left(2 q_a \ln\frac{D}{r} + (q_b + q_c)\ln\frac{r}{D}\right)$$

since $q_a + q_b + q_c = 0 \quad (q_b + q_c = -q_a)$

$$V_{ab} + V_{ac} = \frac{1}{2\pi K}\left(3 q_a \ln\frac{D}{r}\right)$$



Knowing that
$$V_{ab} = \sqrt{3}\, V_{an}\,(1\angle 30^\circ)$$
$$V_{ca} = \sqrt{3}\, V_{an}\,(1\angle +150)$$
$$V_{ac} = -V_{ca} = \sqrt{3}\, V_{an}\,(1\angle -30^\circ)$$

$$V_{ab} + V_{ac} = 2\sqrt{3}\, V_{an}\,(.866) = 3V_{an} = \frac{3 q_a}{2\pi K}\ln\frac{D}{r}$$

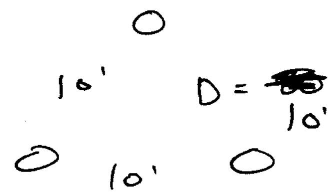
$$V_{an} = \frac{q_a}{2\pi K}\ln\frac{D}{r}$$

$$\boxed{C_n = \frac{q_a}{V_{an}} = \frac{2\pi K}{\ln\frac{D}{r}} = \frac{.0388}{\log\frac{D}{r}}\ \mu F/mi}$$

Same as single phase-GND

$$I_{CHG} = j\omega\, C_n V_{an} \quad \text{Amps/mile}$$

EXAMPLE: ACSR Ostrich · 20 mile line



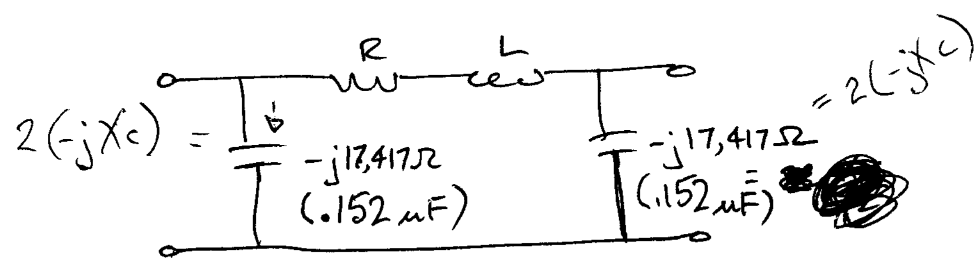
$$C_n = \frac{.0388}{\log \frac{D}{r}} = \frac{.0388}{\log \frac{(10')(12)}{(.68/2)}} = .015229 \ \frac{\mu F}{mi}$$

for 20 miles, $C_n = .304588\ \mu F$

$$X_c = \frac{1}{377\,C} = 8708.75\ \Omega$$

$$Z = -jX_c = -j\,8.7\,K\Omega$$

Normally, $\frac{1}{2}$ of capacitance is put at each end



PI section

With the tables,

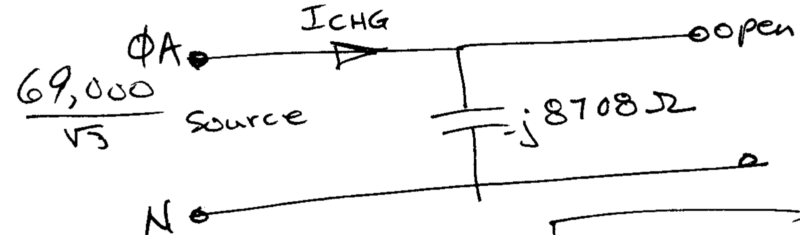
$$X_c = \underset{\uparrow\ \text{Table A.3}}{X_a'} + \underset{\nwarrow\ \text{Table A.5}}{X_d}$$

$$X_c = \frac{.1057 \times 10^6\ \Omega\text{-mi} + .0683 \times 10^6\ \Omega\text{-mi}}{20\ \text{miles}} = \frac{.1740 \times 10^6\ \Omega\text{-miles}}{20\ \text{miles}}$$

$$\boxed{\approx 8700\ \Omega}$$

Charging Current: use X_c

EX 69 KV OSTRICH 10 ft. spacing



$$I_{CH} = \frac{69\,KV}{\sqrt{3}\,(-j\,8708)} = \boxed{j\,4.57 \text{ Amps}}$$

- Disconnect Sw.
- Circuit Bkrs

Must be able to disconnect.

UNSYMMETRICAL SPACING

= Average cap/phase assumming transposition

$$C_n = \frac{.0388}{\log_{10} \frac{D_{eq}}{r}} \ \mu F/mi$$

where $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

EX: OSTRICH - 10 miles

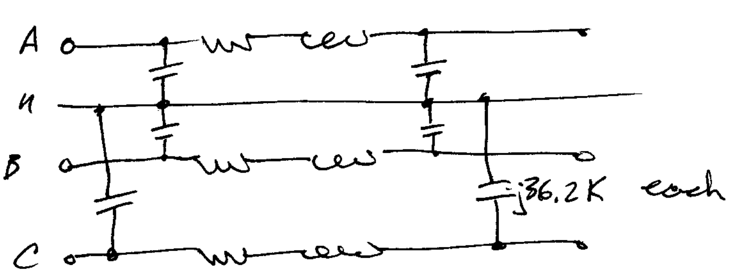
a ← 10' → b ← 10' → c

$$D_{eq} = \sqrt[3]{(10)^2\, 20} = 12.6 \text{ ft}$$

$$X_c = .1057 + .075 = .181 \ M\Omega \cdot mi$$

$$X_c = \frac{.181}{10} = \boxed{18.1\ K\Omega}$$

$$X_c = \frac{1}{2\pi f c} = \frac{\log \frac{12.6\,(12)}{.134}}{2\pi\,(60)(.0388) \times 10^{-6}\,(10\ mi} = \boxed{18.1\ K\Omega}$$



Assumes "continuous transposition"

$C_{ab} = C_{ba}$ $D = 10'$ $[X_{Cab} = X_{Cbc}]$ ← X_{ca}

$C_{bc} = C_{cb}$ $D = 10'$ (X_{Cac} is larger)

$C_{ac} = C_{ca}$ $D = 20'$

DS HCD

r_{ac} @ 25° & 50°

? are they dc

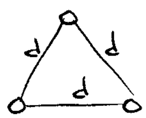
so $$C = \frac{.0388}{\log_{10} \frac{D_{eq}}{r}} \ \frac{\mu F}{mi}$$

actual r $\boxed{\text{not } r'}$

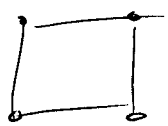
$$D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$$

Bundled Conductors: r is replaced by D^b_{sc}

$D^b_{sc} = \sqrt{rd}$



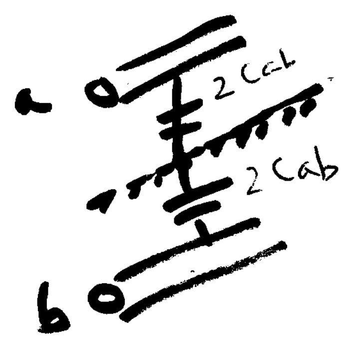
$D^b_{sc} = \sqrt[3]{rd^2}$

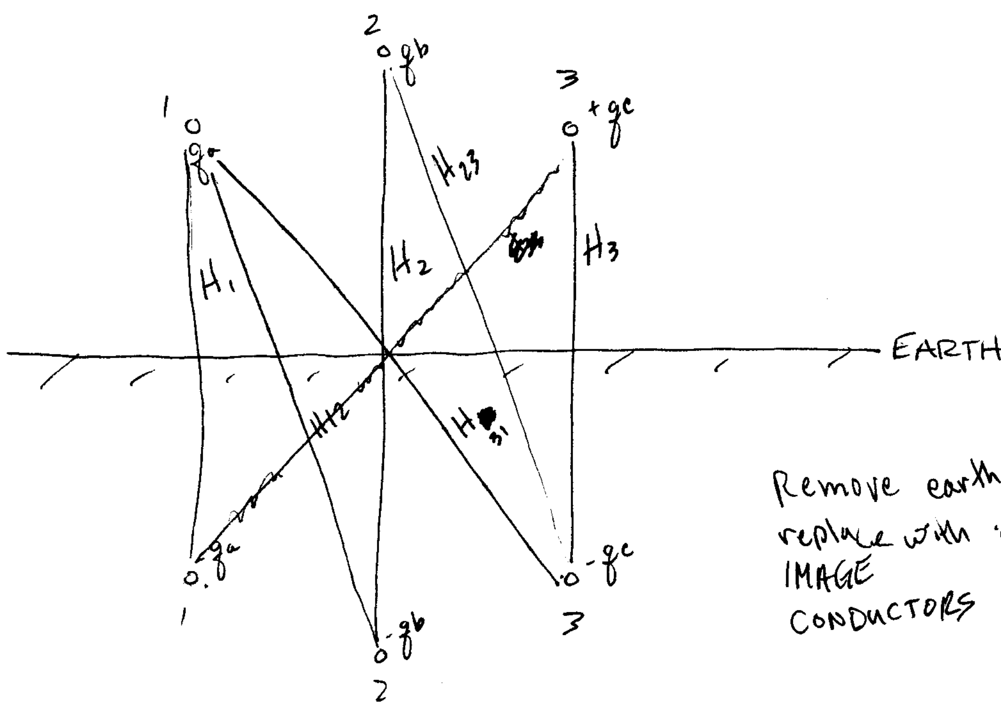


$D^b_{sc} = \sqrt[4]{rd^3\sqrt{2}}$

r is actual radius $\boxed{\text{Not } r'}$

Effect of earth on capacitance:





Remove earth and replace with imaginary IMAGE CONDUCTORS → same E field above ground line

$$V_{ab} = \frac{1}{2\pi K}\left[q_a \left(\ln \frac{D_{12}}{r} - \ln \frac{H_{12}}{H_1} \right) + q_b \left(\ln \frac{r}{D_{12}} - \ln \frac{H_2}{H_{12}} \right) + q_c \left(\ln \frac{D_{23}}{D_{31}} - \ln \frac{H_{23}}{H_{31}} \right) \right]$$

Proceding with similar derrivation as before, (Vac, etc)

$$\boxed{C_n = \frac{.0388}{\log \frac{D_{eq}}{r} - \underbrace{\log \frac{\sqrt[3]{H_{12} H_{23} H_{31}}}{\sqrt[3]{H_1 H_2 H_3}}}_{\text{This term is subtracted from denominator}}} \ \mu F/mi \text{ to neu}}$$

So:

If conductors are very high above ground,

$H_1 \simeq H_{12}$

$H_2 \cong H_{23}$

$H_3 \cong H_{31}$

and correction is very small. We can usually ignore.

If conductors are close to the ground

$$\left.\begin{aligned} H_{12} &> H_1 \\ H_{23} &> H_2 \\ H_{31} &> H_3 \end{aligned}\right\} \text{How close? See printouts included in notebook....}$$

This decreases value of denominator and results in increased C

∴ - Earth increases capacitance
- reduction of ground clearance increases capacitance.
- Increase in phase spacing increases capacitance
- reduction in conductor radius decreases capacitance

~~BUNDLED CONDUCTORS~~

$$\cancel{C_n = \frac{.0388}{\log\left(\frac{D_{eq}}{D^b_{sc}}\right)}}$$ ~~where~~ $\cancel{D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}}$

$\cancel{D^b_{sc}}$

Parallel Circuit Three Phase Lines

(Double Circuit)

$$C_n = \frac{.0388}{\log_{10} \frac{D_{eq}}{D^p_{sc}}} \ \mu F/mi$$

D_{eq} = same as for inductance calculations

D^p_{sc} = same except use r instead of D_s

EX:

10 Mile Line - Ostrich

o ←10'→ o ←10'→ o

$R = 3.372\ \Omega$ (Table A.1 @ 50°C)

$D_{eq} = \sqrt[3]{10^2(20)} = 12.6$ ft

$X_L = \ = 7.64\ \Omega$

$X_c = \ = 18.1\ K\Omega$

$D = .68'' \Rightarrow .057/2 = r = .028$

$D_s = .0229'$

Bundled: 6" spacing:

$R = 1.686\ \Omega$

$X_L = 5.79\ \Omega$

$X_c = 13.866\ K\Omega$

$C = .1913\ \mu F$

$D^b_s = \sqrt{(.0229)(.5)} = .107'$

$D^b_{sc} = \sqrt{(.028)(.5)} = .118$

Summary

	Single phase 2 wire	Mult Conductor	3ϕ EQUIL	3ϕ UNSYM	3ϕ Bundled
R	R_{ac}	R of parallel Combination	R	R	R/N
L	$.7411 \log \frac{D}{r_1'} \frac{mH}{mi}$	$.7411 \log \frac{GMD}{GMR}$	$.7411 \log \frac{D}{D_s}$	$.7411 \log \frac{D_{EQ}}{D_s}$	$.7411 \log \frac{D_{eq}}{D_s^b}$
C_n	$\frac{.0388}{\log \frac{D}{r}} \frac{\mu F}{mi}$		$\frac{.0388}{\log \frac{D}{r}} \frac{\mu F}{mi}$	$\frac{.0388}{\log \frac{D_{eq}}{r}} \frac{\mu F}{mi}$	$\frac{.0388 \frac{\mu F}{mi}}{\log \frac{D_{eq}}{D_{sc}}}$

3