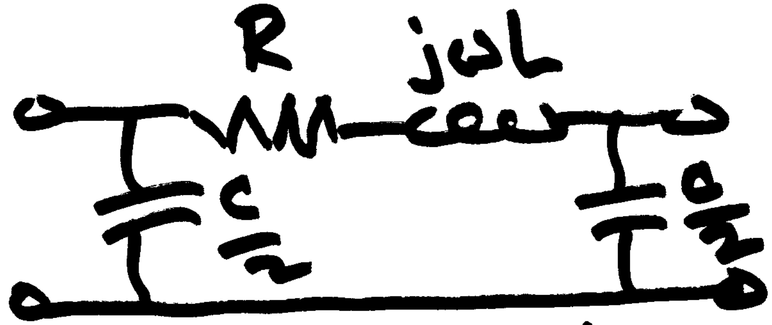
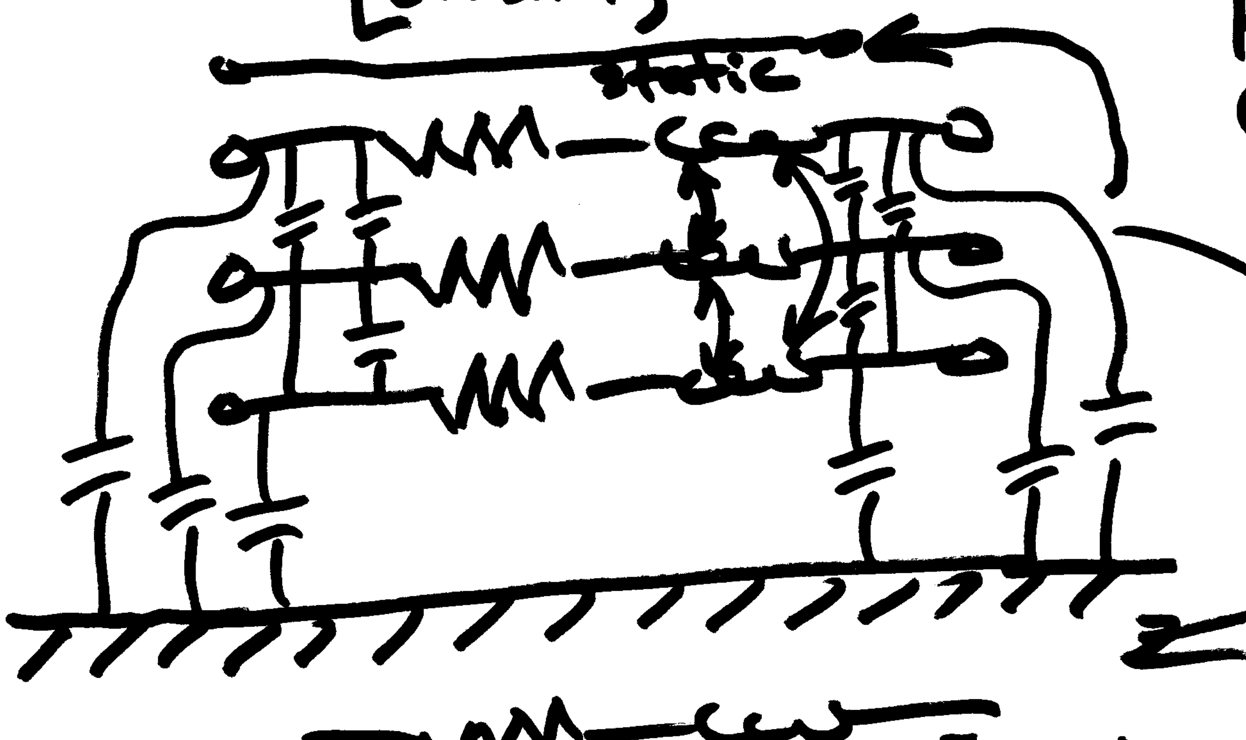
Transmission Lines:



per-phase
ΦA-N

Short-circuit
Load-Flow
Stability



Carson's Formula

Full 3-phase
Coupled L's
and C's

Return
via
earth,
static
wires

LECTURE NO. 5

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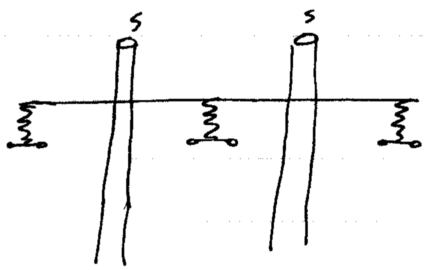
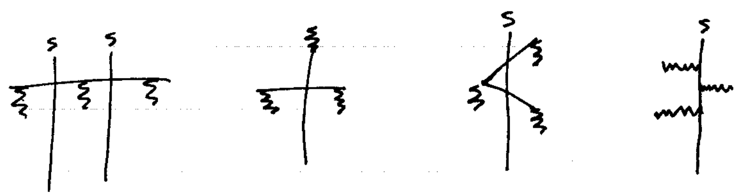
Overhead T-line Configurations

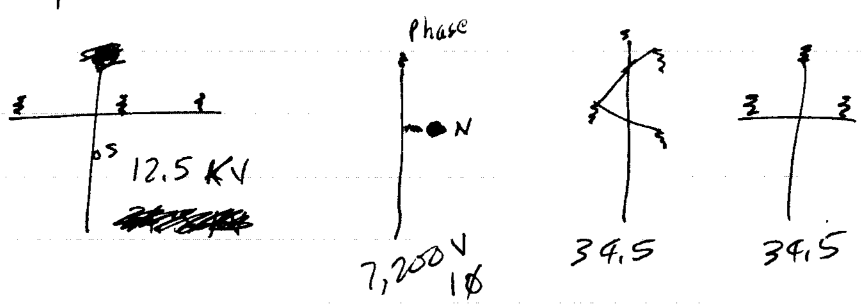


69 KV → 765 KV

SEE "EPRI RED BOOK"



34.5 KV → 115 KV

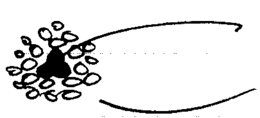


2.4 KV → 34.5 KV

CONDUCTORS - Were ~~aluminum~~ copper but are now aluminum (lighter, cheaper, less corona)

AAC : All aluminum conductor
AAA : All aluminum alloy conductor
ACSR : Aluminum Conductor Steel Reinforced
ACAR : Aluminum Conductor Alloy Reinforced
AWAC : Aluminum Wound Air Core (Expanded ACSR)

Wire is stranded for strength



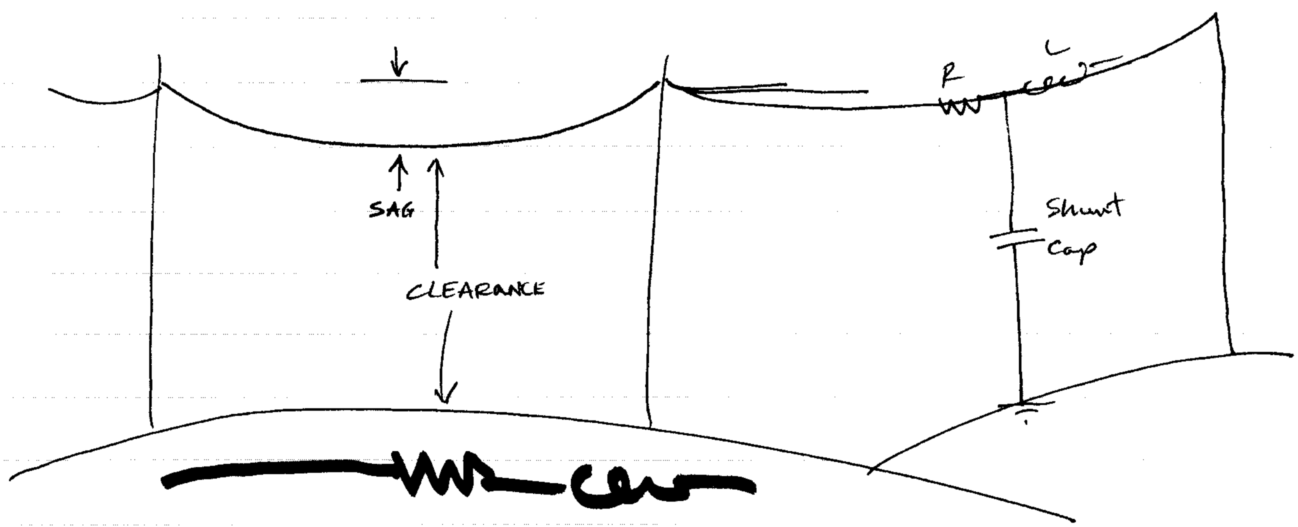
Steel carries mechanical load
Al carries current

Look at table A.1 in Stevenson (pp. 750-753)
($1\,\text{cmil} = .7854\,\text{mil}^2$)

1000 circular mills

266.8	MCM = KCM	WaxWing	18/1	2 layers Al, 1 steel
266.8	MCM	Partrige	26/7	2 layers Al, 2 steel (115-kV OTP)
795	MCM	Cuckoo	24/7	2 layers Al, 2 steel (230-kV OTP)
~~1272~~	~~MCM~~	~~Bittern~~	~~45/7~~	
1272	MCM	Bittern	45/7	3 layers Al, 2 steel (345-kV Minkota)

Total number of strands = $3x^2 - 3x + 1$ | x = no. of layers



Plus: Mutual Inductance and capacitance between lines.

Lucky to estimate T-Line Params within 5-10%.

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RESISTANCE

From Physics $R = \frac{\rho L}{A}$

ρ = Resistivity $\frac{\Omega - CM^2}{ft}$ (ΩM)
L = Length ft M
A = Area CMIL M^2

$1 \text{ Cmil} = \frac{\pi d^2}{4}$ $d = .001$ in $\ominus^1$

Area in circular mils = (dia in Mils)^2
" " square inches = (area in circular mils)$\frac{4}{\pi}$

$$\left.\begin{aligned} \rho_{Cu} &= 10.66\ \Omega \cdot \text{cmil/ft} \\ \rho_{AL} &= 17\ \Omega \cdot \text{cmil/ft} \end{aligned}\right\} \text{at } 20°C$$

Corrections to resistance:

1) Spiraling: $R = \frac{\rho L}{A}(1.02)$ (add 2% to length)

2) Temp Rise: $\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$

R_1 = Res @ t_1 ~~A~~
R_2 = Res @ t_2
T = 241 Cu
T = 228 AL
t_1 & t_2 in °C

3) Resistance & Current Density

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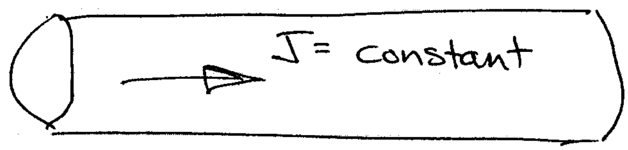
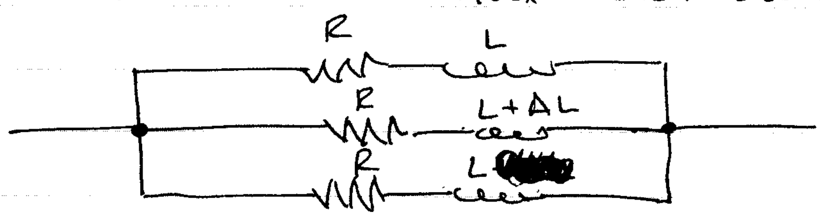
For dc current



@ dc.

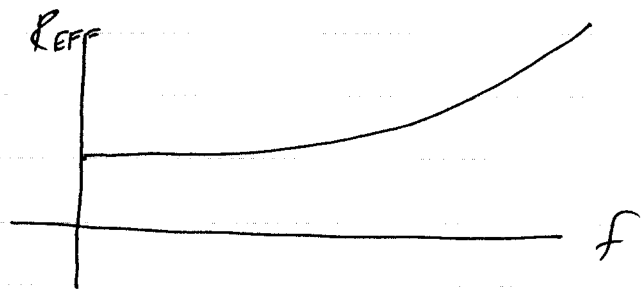
current density is uniform across cross section of conductor.

For ac, magnetic lines of force in center of conductor force current to outside surface. (Non-uniform flux distribution.)



Therefore, $R_{ac} > R_{dc}$ by about 1-10% @ 60Hz

Use tables (p. 750) to get actual measured values



$$R_{EFFECTIVE} = R_{ac} = \frac{P_{LOSS}}{|I|^2} \ \Omega$$

∴ Resistance is no problem - use tables.

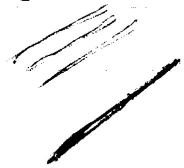
INDUCTANCE – NOT AS EASY BUT TABLES CAN BE USED IN MANY CASES.

Cases to Consider

1) Self inductance of single line.

2) Single phase groups of wires.

3) Mutual inductance with parallel phone lines, etc.



4) 3ϕ, Bundled conductors

Going back to field theory –

$$v = L \frac{di}{dt} \qquad \tilde{V} = j\omega L \tilde{I}$$

$$L = \frac{\lambda_{\text{LINKED BY } I}}{I}$$

ψ & I are in phase

λ = Flux Linked/M
I = Current (Phasor rms)
L = Inductance/unit length

$$\lambda = \int \vec{B} \cdot \vec{dA}$$

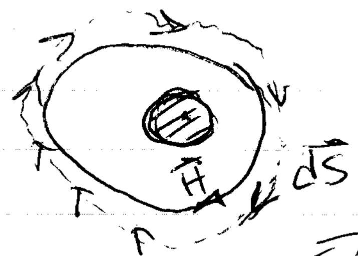
B = Magnetic Flux Density WB/m^2
dA = area in square meters

$$B = \mu H = \mu_r \mu_0 H$$

H = Magnetic field intensity $\frac{A\text{-}T}{m}$

μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H^{-1}

μ_r = relative permeability = 1 for air



$|\vec{H}||\vec{ds}|\cos\theta_{HS}$

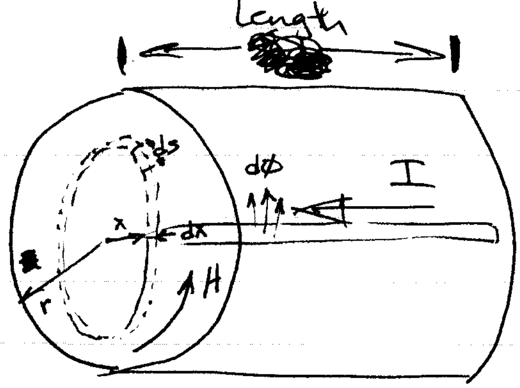
$$\oint \vec{H} \cdot \vec{ds} = I_{ENCLOSED} \quad \text{(Amperes Law)}$$

$\vec{ds}$ = incremental length (dl or dx)

Procedure to find L: (reversing preceding)

1) Find H given I
2) Find $B = \mu_0 H$
3) Find $\lambda = \psi$ linked (Wb)
4) Determine I
4a) $L = \frac{\lambda}{I}$

EX: Case I: Self (internal) inductance of single wire:



$$\oint \vec{H} \cdot \vec{ds} = I_{enclosed}$$

since $\vec{H}$ & $\vec{ds}$ are in same direction,

$$\oint \vec{H} \cdot \vec{ds} = H_x(2\pi x) = I_x$$

where I_x = current enclosed by radius x

Assuming constant current density,

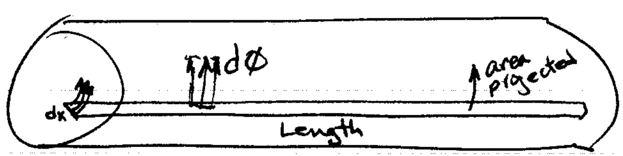
1) $I_x = \frac{\cancel{\pi} x^2}{\cancel{\pi} r^2} I$ where I = total current in conductor

2) $H_x = \frac{I_x}{2\pi x}$ from previous eqn.

$$H = \frac{I x^2}{2\pi x r^2} = \boxed{\frac{I x}{2\pi r^2} \quad \frac{A-T}{m}}$$ first part of procedure satisfied.

$$B = \mu_0 H = \boxed{\frac{\mu_0 x I}{2\pi r^2}}$$ second part of procedure satisfied.

$d\phi$ = magnetic flux/meter Wb/m



$$d\phi = \frac{\mu x I}{2\pi r^2}\, dx \quad Wb/m$$

$$\psi_{LINKED_x} = \phi \frac{\pi x^2}{\pi r^2} \quad Wb$$

$$d\psi_L = \frac{\mu_0 x I}{2\pi r^2}\, dx\, (\text{Length}) \left(\frac{x^2}{r^2}\right)$$

ψ_L = lines of flux × no. of turns linked

$$\psi_L = \int_0^r \frac{\mu_0 x^3 I\,(\text{Length})}{2\pi r^4}\, dx = \boxed{\frac{\mu_0 I\,(\text{Length})}{8\pi}}$$ third part

$$\boxed{L = \frac{\psi_L}{I} = \frac{\mu_0\,(\text{Length})}{8\pi}}$$ solution.

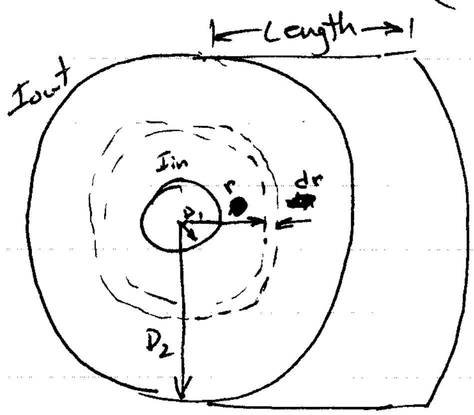
Since $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$L = \frac{4\pi \times 10^{-7}}{8\pi} = \boxed{\frac{1}{2} \times 10^{-7} \text{ H/m}} \text{ internal}$$

$$= .05 \ \mu H/m$$

Note: Internal INDUCTANCE DOES NOT DEPEND ON THE SIZE OF THE CONDUCTOR

EX: CASE II - COAX (2-WIRE SYSTEM)



Let $I_{in} = I_{out}$

$$\oint H \cdot ds = I_{ENCLOSED}$$

$$H \, 2\pi r = I$$

$$\boxed{H = \frac{I}{2\pi r}} \ \#1$$

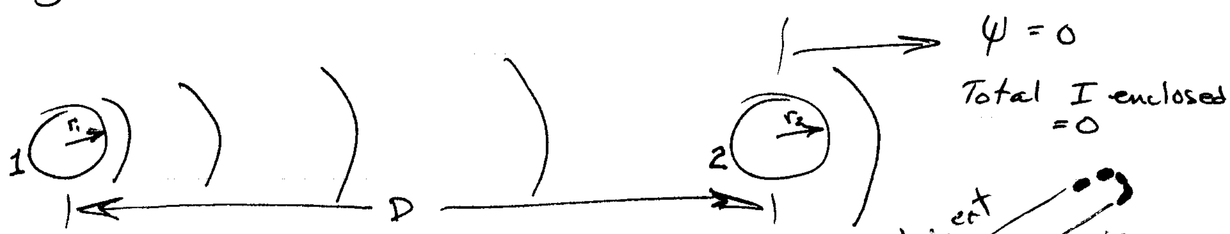
$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r} \ \#2$$

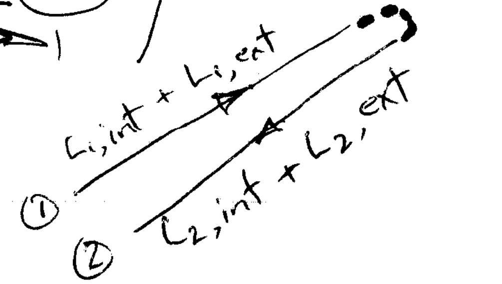
$$\psi = \int_{D_1}^{D_2} B \cdot dA = \text{Length} \int_{D_1}^{D2} \frac{\mu_0 I}{2\pi r} dr$$

$$\psi = \frac{\mu_0 I \text{ length}}{2\pi} \ln \frac{D_2}{D_1} \ \#3$$

$$L_{12} = \frac{\psi}{I} = \frac{\mu_0 \text{ Length}}{2\pi} \ln \frac{D_2}{D_1} = \boxed{2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}} \text{ Answer}$$

Two Wire System





Look at flux linked by I_1

$$L_1 = L_{\text{INTERNAL}} + \underbrace{L_{\text{EXTERNAL}}}_{\text{= mutual between 1 \& 2}} \quad \text{IF} \quad D \gg r_2 \ \& \ r_1$$

$$L_{1_{ext}} = 2\times10^{-7} \ln \frac{D}{r_1} \leftarrow \text{mutual effect of cond 2 on cond 1.}$$

$$L_{1_{int}} = \frac{1}{2}\times10^{-7} \quad H/m$$

$$L_1 = 2\times10^{-7} \ln \frac{D}{r_1} + \frac{1}{2}\times10^{-7}$$

$$= \left(\frac{1}{2} + 2 \ln \frac{D}{r_1}\right)\times10^{-7} \quad H/m$$

$$= 2\times10^{-7}\left(\frac{1}{4} + \ln \frac{D}{r_1}\right)$$

Substitute $\frac{1}{4} = \ln e^{+\frac{1}{4}}$

$$L_1 = (2\times10^{-7})\left(\ln \frac{D}{r_1 e^{-1/4}}\right) \qquad .7788 = e^{-\frac{1}{4}}$$

Substitute $r' = r_1 e^{-1/4}$ = effective radius

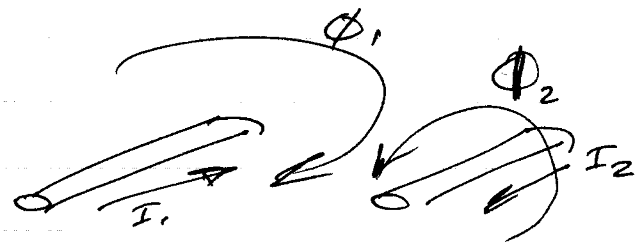
$$\boxed{L_1 = 2\times10^{-7} \ln \frac{D}{r_1'} \quad H/m}$$

$$\boxed{L_1 = .7411 \log_{10} \frac{D}{r_1'} \quad mH/mile}$$ more common form

r_1' = radius of a ficticious conductor with no internal flux but that has the same total inductance as the original conductor of radius r_1

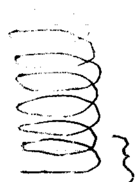
So we multiply $r_1 \times e^{-\frac{1}{4}} = r_1 \times .7788$ to get r_1' – only for solid round Conductors. Note: May have to measure for stranded conductor.

Since the currents are opposite



L_{TOTAL} for this 2 wire loop circuit is the

$$L_{TOTAL} = L_1 + L_2 = (L_{1,int} + L_{1,ext}) + (L_{2,int} + L_{2,ext})$$



where
$$L_2 = 2\times10^{-7} \ln \frac{D}{r_2'}$$

$$L_{TOTAL} = \cancel{4}\,2\times10^{-7}\left(\ln \frac{D}{r_2'} + \ln \frac{D}{r_1'}\right)$$

$$= 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}} \quad H/m$$

$\sqrt{r_1' r_2'}$ ← GMR

$$= 1.482 \log_{10} \frac{D}{\sqrt{r_1' r_2'}} \quad mH/mile.$$

EX: 3 mile solid conductor 0.5" DIA
Find X_L @ 60 Hz & 2 ft spacing

$$L = 1.482 \log_{10} \frac{2\text{ ft}}{r'} \ \frac{\text{mH}}{\text{mile}} \ (3 \text{ miles})$$

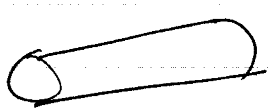
$$r' = \frac{0.5''}{2} e^{-1/4} = .1947''$$

$$r' = (.1947'')\left(\frac{1\text{ ft}}{12''}\right) = .016225\text{ ft}$$

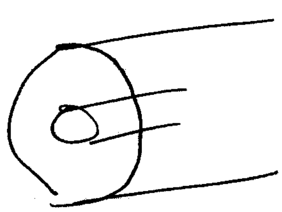
$$L = 1.482(3) \log_{10} \frac{2}{.016225} = \underline{\underline{9.296\text{ mH}}}$$

$$\boxed{\begin{aligned} X_c &= 2\pi 60 L \\ &= 377 L = 3.504\,\Omega \end{aligned}}$$

Summary



wire $L = 2\times10^{-7}$ H/m



Coax $L = 2\times10^{-7} \ln \frac{r_2}{r_1}$ H/m

ind between the conductors

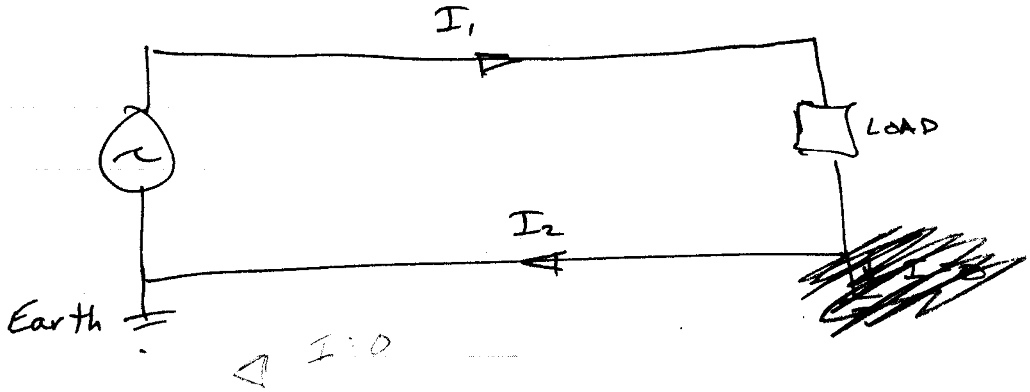
2 wires $L_{TOTAL} = 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}}$ H/m

for solid wires only

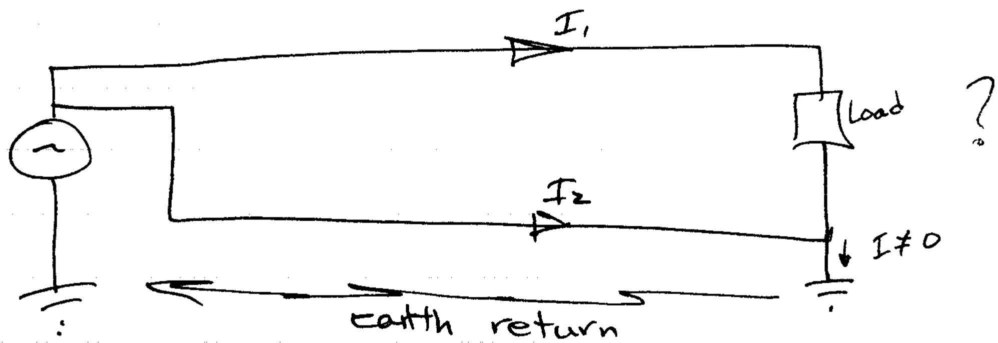
where $r_1' = r_1 e^{-1/4}$

includes self inductance

IMPORTANT - For 2 wire loop
$I_1 = I_2$



IF there is a ground fault,



The above calculated L is incorrect

EX: For 2 wire FLICKER with assumed spacing

$Z = .1945 + j.684$

For 2 wire with earth return (like zero sequence)

$Z = .652 + j2.7455$

So must not have earth return to use the method in the book.

More Detailed Explanation of "Lecture No. 5"

1

Inductance

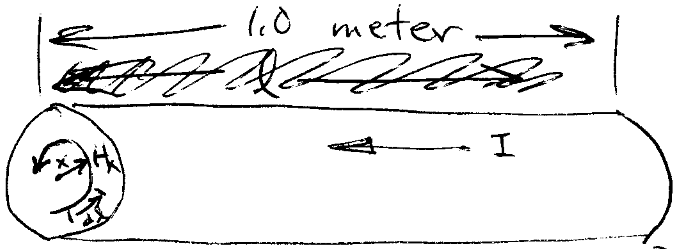
$$L = \frac{\lambda}{i}$$

4 Cases:

1) Self-inductance of single line
2) Single phase groups of wires.
3) Mutual inductances { parallel lines, phone lines
4) 3ϕ, Bundled Conductors

Field Theory Basics

Current flowing in conductor produces magnetic field H (right hand rule)



$$\vec{B} = \underbrace{\mu_r}_{\substack{1.0 \text{ for air or} \\ \text{nonmagnetic mat'ls}}} \underbrace{\mu_0}_{4\pi \times 10^{-7}\ H/m} \vec{H}$$

B = Magnetic flux density (Wb/m²)
H = Magnetic field intensity (A/m)

To find H, use Ampere's Circuital Law

$$\oint \vec{H} \cdot \vec{dl} = I \text{ enclosed} = \int \vec{J} \cdot \vec{dS}$$

Stoke's Theorem

$$\text{in general} \quad I_{ENCL} = \underbrace{2\pi x}_{\text{circumference}} H_x$$

STEP 1

$$\boxed{H_x = \frac{I_x}{2\pi x} \text{ Amps/meter}}$$

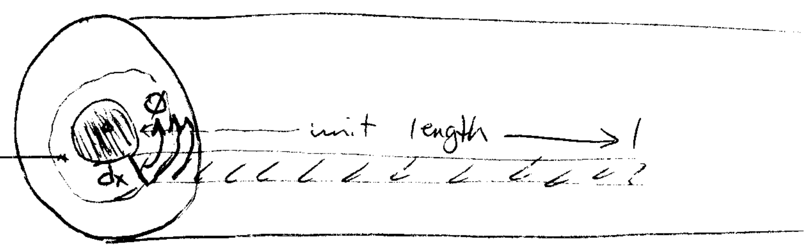
For a nonmagnetic conductor,

$$B_x = \mu_0 H_x = \frac{\mu_0 I_x}{2\pi x} \ \mathrm{Wb/m^2}$$

But $I_x = \left(\frac{x}{r}\right)^2 I$

STEP 2 $$\boxed{B_x = \frac{\mu_0 x I}{2\pi r^2} \ \mathrm{Wb/m^2}}$$ flux density

Looking at flux ϕ flowing thru a differential cross section of conductor



Like single turn

flux crossing strip dx × unit length,

$$d\phi = B_x dx \ \mathrm{Wb/m}$$

$$\boxed{\lambda = N\phi_{enclosed}}$$

The flux linked (contained) by concentric flux path of radius x is:

STEP 3 $$d\lambda = \underbrace{\left(\frac{x}{r}\right)^2}_{\text{Portion of area for } r = x} d\phi = \frac{\mu_0 I}{2\pi r^4} x^3 \, dx$$

integrating

$$\lambda_{\substack{\text{contained}\\ \text{by wire}}} = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 \, dx$$

$$= \frac{\mu_0 I}{2\pi r^4}\left(\frac{r^4}{4}\right) = \frac{\mu_0 I}{8\pi} \ \frac{\mathrm{Wb \cdot T}}{\mathrm{m}}$$

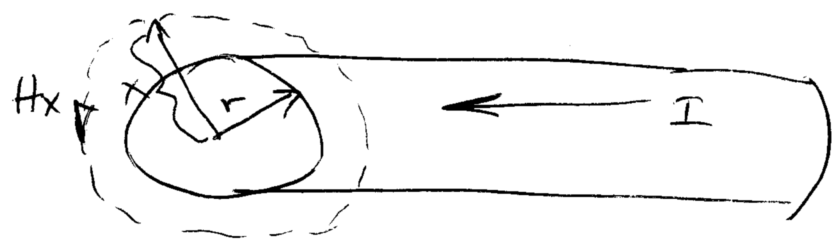
$$\boxed{\lambda = \frac{4\pi \times 10^{-7}\, I}{8\pi} = \frac{1}{2}\times 10^{-7}\, I}$$

Then,

$$\boxed{L = \frac{\lambda}{I} = \frac{1}{2}\times 10^{-7}\ H/m} = .05\ \mu H/m$$

Note: **Internal Inductance does not depend on size of conductor.**

For a conductor of radius r,
H_x outside the conductor at radius x



Step 1 $\left(H_x = \left.\dfrac{I}{2\pi x}\right| \; x \geq r \right.$

step 2 $\left(B_x = \mu_0 H_x = \dfrac{\overset{4\pi\times10^{-7}}{\mu_0}\, I}{2\pi x} = 2\times 10^{-7}\,\dfrac{I}{x}\ \text{wb}/m^2 \right.$

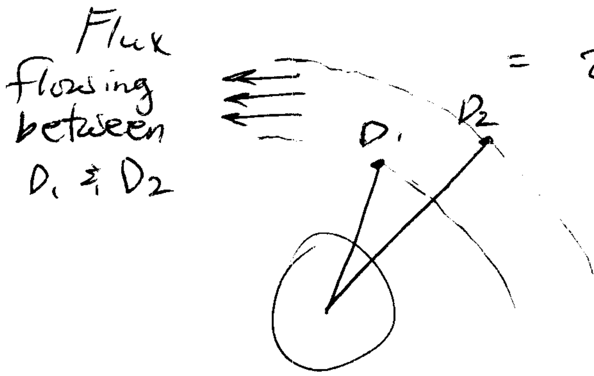
$$\cancel{d\phi = B_x\, dx = 2\times10^{-7}\,\frac{I}{x}\,dx\ \text{wb}/m}$$

$$d\lambda = B_x\, dx = 2\times 10^{-7}\,\frac{I}{x}\,dx = \frac{\mu_0 I}{2\pi x}\,dx$$

flux linked between 2 external points at distance D_1 & D_2 from center is then:

$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2\times10^{-7} I \int_{D_1}^{D_2} \frac{dx}{x}$$

$$= 2\times10^{-7} I \left(\ln D_2 - \ln D_1\right)$$



D_1 is closer (usually $= r$)

D_2 is distance to next conductor.

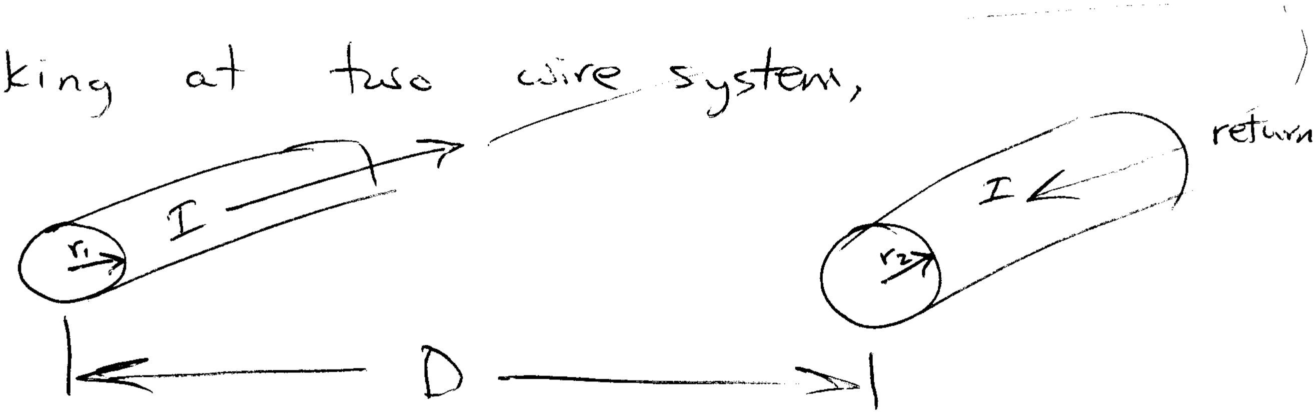
step 3 $\boxed{\lambda_{12} = 2\times10^{-7} I \ln\left(\frac{D_2}{D_1}\right)}$ wb-T/m

$$\boxed{L_{12} = \frac{\lambda_{12}}{I} = 2\times10^{-7} \ln\left(\frac{D_2}{D_1}\right) \text{ H}}$$

if $D_1 = r$ and $D_2 = D$ then

$$L_{12} = 2\times10^{-7} \ln\left(\frac{D}{r}\right)$$

Looking at two wire system,



L due to conductor 1

Assumes $D >> r_1$

$$L_1 = L_{INT_1} + L_{EXT_1}$$

$$= \frac{1}{2} \times 10^{-7} \text{ H/m} + 2 \times 10^{-7} \ln\left(\frac{D}{r_1}\right) \text{ H/m}$$

Combining,

$$L_1 = 2 \times 10^{-7} \left[\frac{1}{4} + \ln\left(\frac{D}{r_1}\right)\right] \text{ H/m}$$

Substitute $\frac{1}{4} = \ln e^{+1/4}$

$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r_1 e^{-1/4}} \text{ H/m}$$

r_1' = effective resistance which includes effect of L_{INT}

$$\boxed{L_1 = 2 \times 10^{-7} \ln \frac{D}{r_1'} \text{ H/m} \quad \text{or,} \quad L_1 = .7411 \log \frac{D}{r_1'} \text{ H/m}}$$

$$\boxed{r_1' = .7788\, r_1}$$

only for solid conductor. (use tables A-4, A-3 otherwise)

Inductance increases with D
decreases with r

→ Do ex. p. 7

Inductance for conductor 2 is

$$L_2 = 2\times10^{-7} \ln \frac{D}{r_2'} \ \text{H/m}$$

for entire circuit,

$$L_{TOTAL} = L_1 + L_2$$

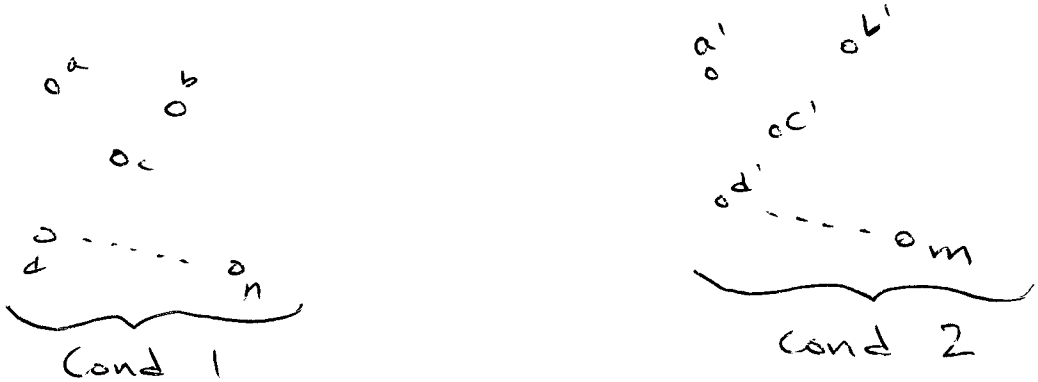
$$= 2\times10^{-7}\left[\ln\frac{D}{r_1'} + \ln\frac{D}{r_2'}\right]$$

$$= 2\times10^{-7} \ln \frac{D^2}{r_1' r_2'}$$

$$\boxed{\underset{\substack{\text{2-wire circuit}\\\text{Loop Inductance}}}{L} = 4\times10^{-7} \ln \frac{D}{\sqrt{r_1' r_2'}}} \ \text{H/m} = 1.482 \log_{10} \frac{D}{\sqrt{r_1' r_2'}} \ \text{mH/mi}$$

Note: Geometric mean radius

IN GENERAL,
for single phase multi-conductor line,



$$L = 4\times10^{-7} \ln \frac{GMD}{GMR}$$

$$GMD = \sqrt[mn]{(D_{aa'} D_{ab'} \cdots D_{am}) \cdots (D_{na'} D_{nb'} \cdots D_{nm})}$$

$$GMR_1 = \sqrt[n^2]{(\underset{\uparrow \atop r_a'}{D_{aa}} D_{ab} \cdots D_{am}) \cdots (D_{ma} D_{mb} \cdots \underset{\uparrow \atop r_m'}{D_{mm}})}$$

$$GMR_2 = \sqrt[m^2]{(\underset{\uparrow \atop r_{a'}'}{D_{a'a'}} D_{a'b'} \cdots D_{a'm}) \cdots (D_{ma'} D_{mb'} \cdots \underset{\uparrow \atop r_m'}{D_{mm}})}$$

$$L = 2\times10^{-7} \ln \frac{GMD}{GMR_1} + 2\times10^{-7} \ln \frac{GMD}{GMR_2}$$

$$L = 4\times10^{-7} \ln \frac{GMD}{\underbrace{\sqrt{GMR_1\, GMR_2}}_{GMR}}$$

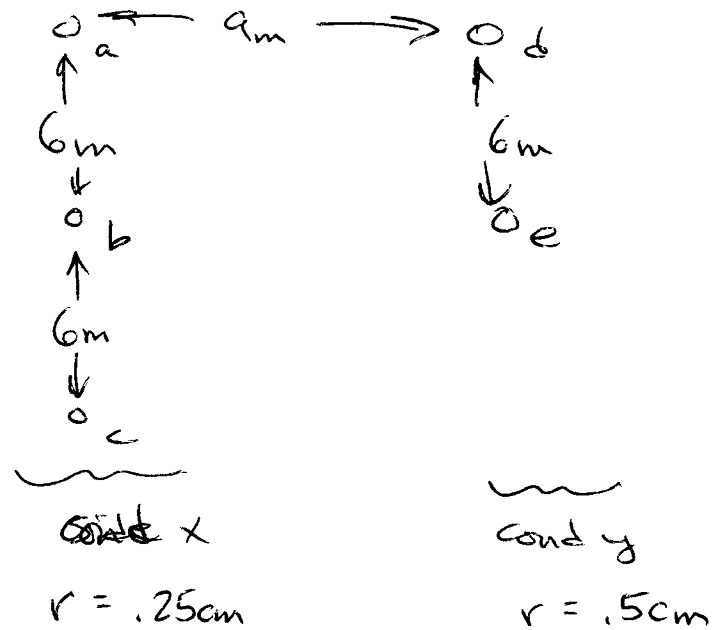
<u>Do examples</u>

<u>Ex:</u> 3 mile solid cond 0.5" DIA, 2-ft spacing.

$$L = 3\left[1.482 \log_{10} \frac{2 \text{ feet}}{\underbrace{\left(\frac{0.5}{2}\right)\left(\frac{1}{12}\right)\left(e^{-1/4}\right)}_{r' \text{ in feet}}}\right] = \underline{9.296\,\text{mH}}$$

$$X_c = 2\pi 60 L = 377L = \underline{\underline{3.504\,\Omega}}$$

Ex:



$$GMD = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

$$D_{ad} = D_{be} = 9m$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2+9^2} = \sqrt{177}\,m$$

$$D_{cd} = \sqrt{9^2+12^2} = 15m$$

3 4 5

$$GMD = \sqrt[6]{9^2 \cdot 177^{3/2} \cdot 15} = \underline{10.743m}$$

$$GMR_1 = \sqrt[9]{(.0025\, e^{-1/4})^3 \times 6^4 \times 12^2} = \underline{\underline{0.481m}}$$

$$GMR_2 = \sqrt[4]{(.005\, e^{-1/4})^2 \times 6^2} = \underline{\underline{0.153m}}$$

$$L_{xy} = 4\times10^{-7} \ln \frac{10.743}{\sqrt{(.481)(.153)}} = 1.472\ \mu H/m$$

$$(= 2.37\ mH/mi)$$

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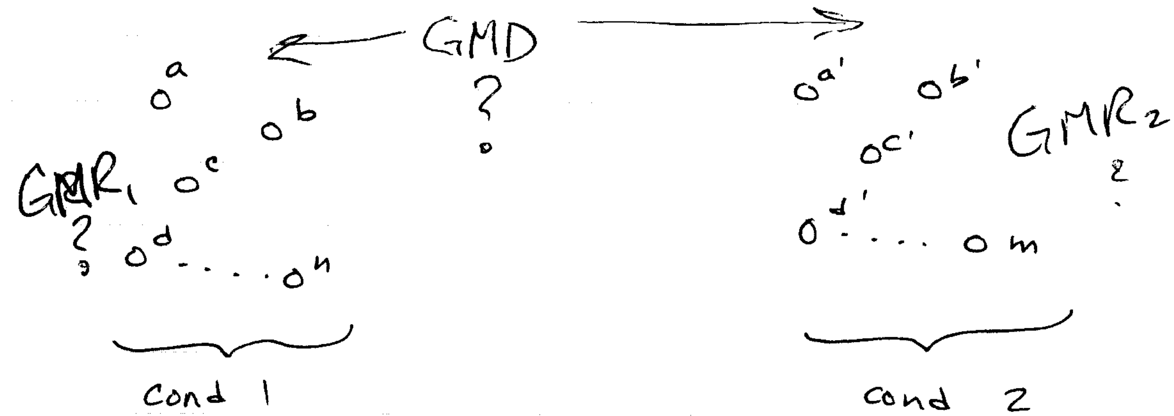
~~Bundled conductors~~

Grouped conductors (Bundled)

What Happens if:



Before, we needed D = distance between conductors

$$L_1 = 2\times10^{-7} \ln \frac{D}{r'}$$

r' = radius of conductor

$$L_1 = 2\times10^{-7} \ln \frac{GMD}{GMR} = 2\times10^{-7} \ln \frac{D_M}{D_s}$$

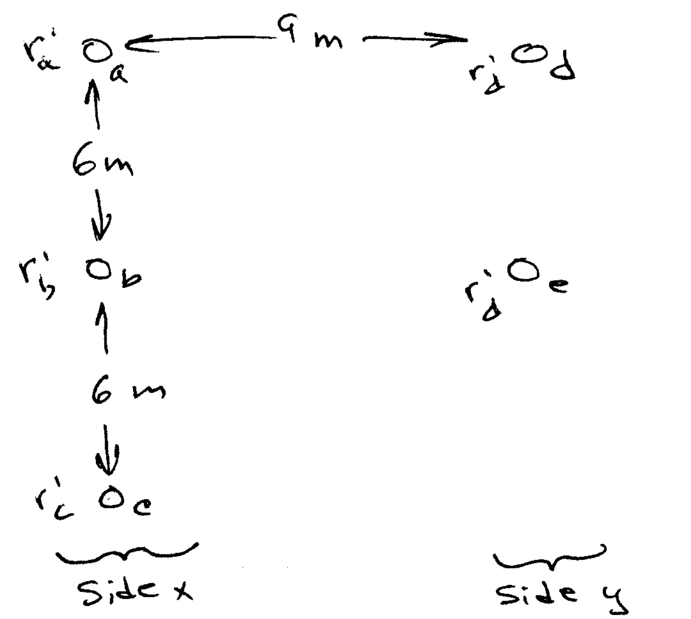
$$GMD = \sqrt[mn]{(D_{aa'} D_{ab'} D_{ac'} \cdots D_{am'})(D_{ba'} D_{bb'} D_{bc'} \cdots D_{bm})(\cdots)(D_{na'} D_{nb'} \cdots D_{nm})}$$

$$GMR_1 = \sqrt[n^2]{(D_{aa} D_{ab} D_{ac} \cdots D_{an})(D_{ba} D_{bb} \cdots D_{bn})(\cdots)(D_{na} D_{nb} D_{nc} \cdots D_{nn})}$$

D_M = GMD = Geometric Mean ~~Diameter~~ Distance

D_s = GMR = Geometric Mean Radius $\boxed{D_{aa} = r'_a}$

Ex:



r = .25 cm   r = .5 cm

$$D_m = GMD = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

Same for sides x & y

$$D_{ad} = D_{be} = 9\,m$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2+9^2} = \sqrt{177}\,m$$

$$D_{cd} = \sqrt{9^2+12^2} = 15\,m$$

$$D_M = \sqrt[6]{9^2 \times 15 \times 177^{3/2}} = \underline{10.743\,m}$$

side x

$$D_s = GMR = \sqrt[9]{(D_{aa} D_{ab} D_{ac})(D_{ba} D_{bb} D_{bc})(D_{ca} D_{cb} D_{cc})}$$

$$= \sqrt[9]{(.25 \times .7788 \times 10^{-2})^3 \times 6^4 \times 12^2} = \underline{0.481\,m}$$

side y

$$D_s = GMR = \sqrt[4]{(.5 \times .7788 \times 10^{-2})^2 \times 6^2} = \underline{0.153\,m}$$

$$L_x = 2 \times 10^{-7} \ln \frac{10.743}{.481} = \boxed{6.212 \times 10^{-7}\,H/m}$$

$$L_y = 2\times10^{-7} \ln \frac{10.743}{.153} = \boxed{8.503\times10^{-7}\ H/m}$$

$$\boxed{\begin{aligned} L_{xy} &= L_x + L_y = 14.715\times10^{-7}\ H/m \\ &= 2.37\ mH/mi \end{aligned}}$$

tighter bundling gives higher L
decreased phase spacing gives lower L

Tables

Usually we want inductance in ohms per mile

$$X_L = 2\pi f L = 2\pi f L \times 2\times10^{-7} \ln \frac{D_m}{D_s}\ H/m$$

$$= 2.022\times10^{-3} f \ln \frac{D_m}{D_s}\ \text{ohms/mile}$$

where D_s is listed in table A.1 for 60 Hz

$$\boxed{X_L = .2794 \log_{10} \frac{D_m}{D_s}\ \Omega/mi \quad \text{for } 60\text{ Hz}}$$

$$X_L = .2794 \log_{10} \frac{1}{D_s} + .2794 \log_{10} \frac{D_m}{1}\ \Omega/\text{mile}$$

From Table A4

or

$$X_L = X_a + X_d$$

From Table A.2

X_a: inductive reactance at one foot spacing

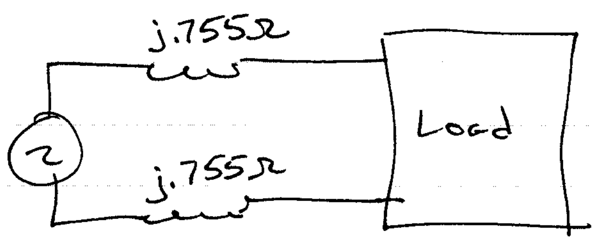
X_d: inductive reactance spacing factor

Ex: Waxwing at 10 ft Spacing - single phase

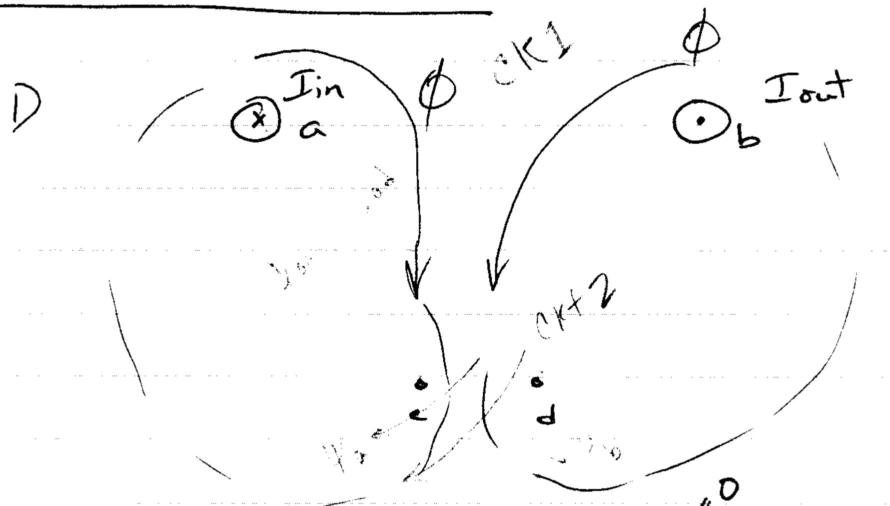
$$X_L = .476 + .2794 \ \Omega/\text{mile}$$

$$= .7554 \ \Omega/\text{mile} \quad (\text{one cond only})$$

For 10 miles of line, (1ϕ)



Mutual Inductance



$I_a = -I_b$

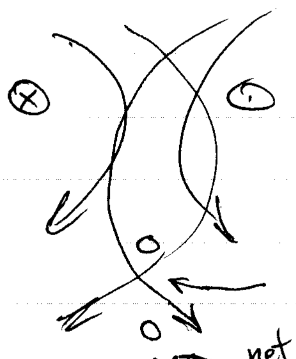
$V_{cd} = j\omega M I_a$ want = 0

I_a & I_b produce same flux (same I)

$$\sqrt{K_{12}K_{21}}L_1L_2 = M_{12} = \frac{\psi_{12}}{I_1} = 2 \times .7411 \log_{10} \frac{D_{ad}}{D_{ac}} \ \mu H/mi$$

~~Dimension~~
~~Integer Alph1, Alph2~~
~~Data ALPH1/~~

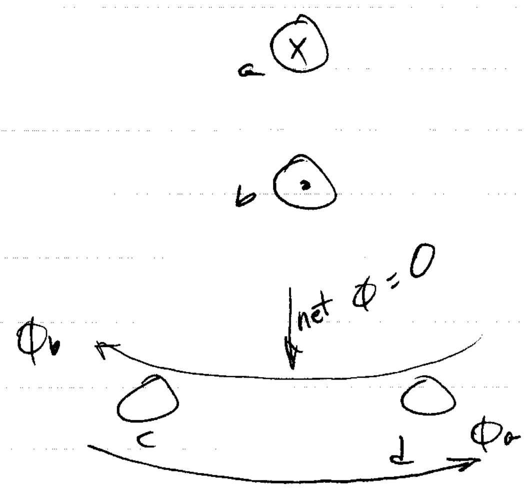
2) Cancel out



net flux = 0

cancel $M = 0$

3)



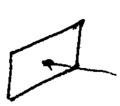
net $\phi = 0$

$m = 0$

each one cancels itself out

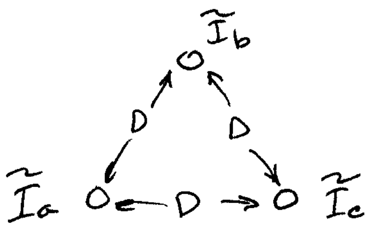
4)

$m \neq 0$



ϕ pass through loop to induce voltage.

3 PHASE EQUIL SPACING



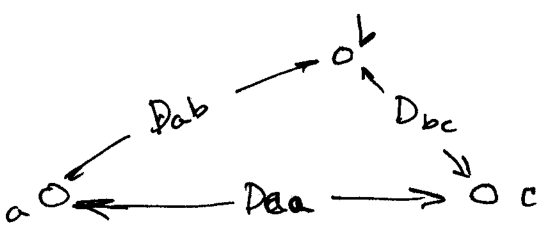
$$\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = 0$$

$$\tilde{I}_a = -(\tilde{I}_b + \tilde{I}_c)$$

So same situation as before w/ 2wire circuit

$$\boxed{L = .7411 \log_{10} \frac{D}{D_s} \ \ mH/mi \quad \text{(per phase)}}$$

3 PHASE UNSYMM SPACING



$$\tilde{I}_a + \tilde{I}_b + I_c = 0$$

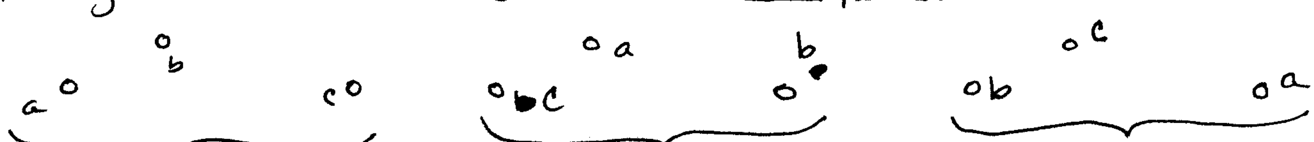
But flux cut by $I_a \neq$ Flux cut by $I_b \neq$ flux cut by I_c

$\boxed{\text{But close enough}}$

$$\boxed{L_a \cong L_b \cong L_c \cong .7411 \log_{10} \frac{D_{eq}}{D_s} \ \ mH/mi}$$

where $D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$

To get $L_a = L_b = L_c$ we <u>transpose</u> the lines

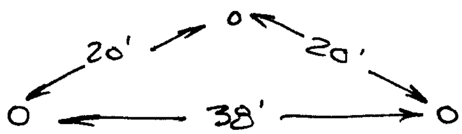


First 10 miles | Next 10 miles | Next 10 miles

The unbalance then averages out.

EX:

3.4



60 Hz
ACSR DRAKE

Find the inductive reactance per mile per phase

From table A.1 $D_s = GMR = \underline{0.0373 \text{ ft}}$

$$D_{eq} = \sqrt[3]{20 \times 20 \times 38} = \underline{24.8 \text{ ft}}$$

$$L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m} = .7411 \log_{10} \frac{D_{eq}}{D_s} \text{ mH/mi}$$

$$= 13 \times 10^{-7} \text{ H/m} = 2.092 \text{ mH/mile}$$

$$X_L = 377 L = 377 (2.092) \times 10^{-3} \ \Omega/\text{mile}$$

$$\boxed{X_L = .789 \ \Omega/\text{mile}}$$

From Tables $X = X_a + X_d$

$$X_a = 0.399 \ \Omega/\text{mi}$$

$$X_d = .2794 \log_{10} D_{eq} = .389 \ \Omega/\text{mi}$$

$$\boxed{X = X_a + X_d = .399 + .389 = .788 \ \Omega/\text{mi}}$$

Go To Tables - interpolate from A.2 for 24.8 ft.

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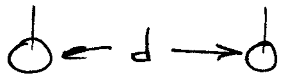
BUNDLED CONDUCTORS

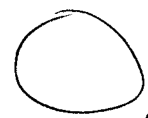


Used at 345-KV and above to reduce corona

For same voltage on a wire, say 100 KV

ρ_s is large

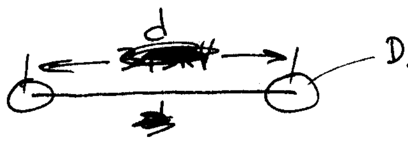
ρ_s = charge density



ρ_s is small

E field is less

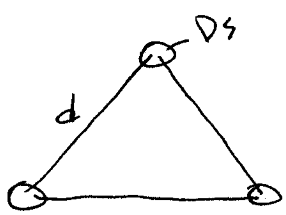
Corona loss - blue haze around conductors
bundling reduces corona



345-KV

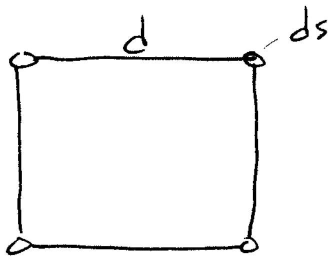
$$D_s^b = GMR = \sqrt[4]{(D_s d)^2} = \sqrt{D_s d}$$

D_s from tables



500 KV

$$\boxed{D_s^b = GMR = \sqrt[9]{(D_s d d)^3} = \sqrt[3]{D_s d^2}}$$



765 KV

$$\boxed{D_s^b = GMR = \sqrt[16]{(D_s d d \sqrt{2} d)^4} = 1.09\sqrt[4]{D_s \times d^3}}$$

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Bundled Conductors

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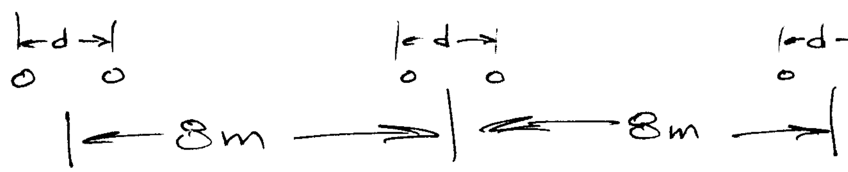
EXAMPLE 3.5

ACSR Pheasant $d = 45$ cm



From table A.1 $D_s = .0466$

$$D_s^b = \sqrt{D_s d} = \sqrt{(.0466)(45)(}$$

$$= \sqrt{(.0466\ \text{ft})\left(\frac{.3048\ \text{m}}{\text{ft}}\right)(.45)}$$

$$\underline{\underline{D_s^b = .08\ \text{m}}}$$

$$D_{eq} = \sqrt[3]{(8)(8)(16)} = 10.08\ \text{m}$$

$$L = 2\times10^{-7} \ln \frac{D_{EQ}}{D_s^b} = 2\times10^{-7} \ln \frac{10.08}{.08} = .967\ \mu H/m$$

$$X_L = (377)(.967\times10^{-6})\ H/m = .365\ \Omega/Km \left(\frac{1.609\ Km}{m}\right)$$

$$= \boxed{.587\ \Omega/mi}$$

Find p.u. series reactance if Line = 160 Km and Base is 100 MVA 345 KV

$$X_L = (160\ Km)\left(\frac{.365\ \Omega}{Km}\right) = 58.4\ \Omega$$

$$= \frac{345{,}000/\sqrt{3}}{167.35}$$

$$I_{BASE} = \frac{100\times10^3\ KVA}{\sqrt{3}(345\ KV)} = 167.35\ A$$

$$Z_{BASE} = 1190\ \Omega$$

$$\boxed{X_L = \frac{58.4}{1190} = .049\ p.u.}$$

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Ex:

For line in previous example, if no Bundling

$$X_L = (377)(2\times10^{-7}) \ln \frac{10.08}{.0466} (160{,}000\,m)$$

$$\boxed{= 64.86\ \Omega}$$

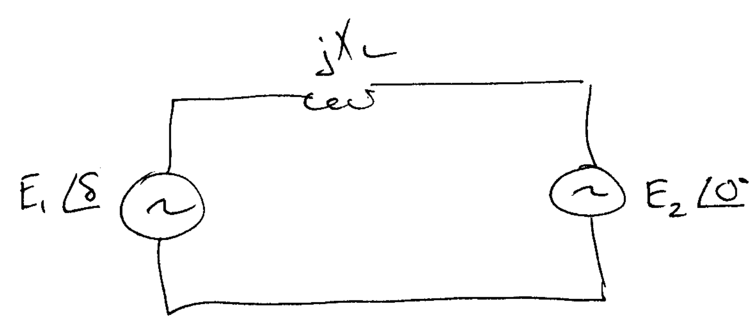
Bundled

$$P_{MAX} = \frac{E_1E_2}{X}\cancel{\sin\delta}^{\,1} = \frac{E^2}{58.4} = \underline{\underline{.017}}$$

$\boxed{\text{11.2\% more power}}$

Not Bundled

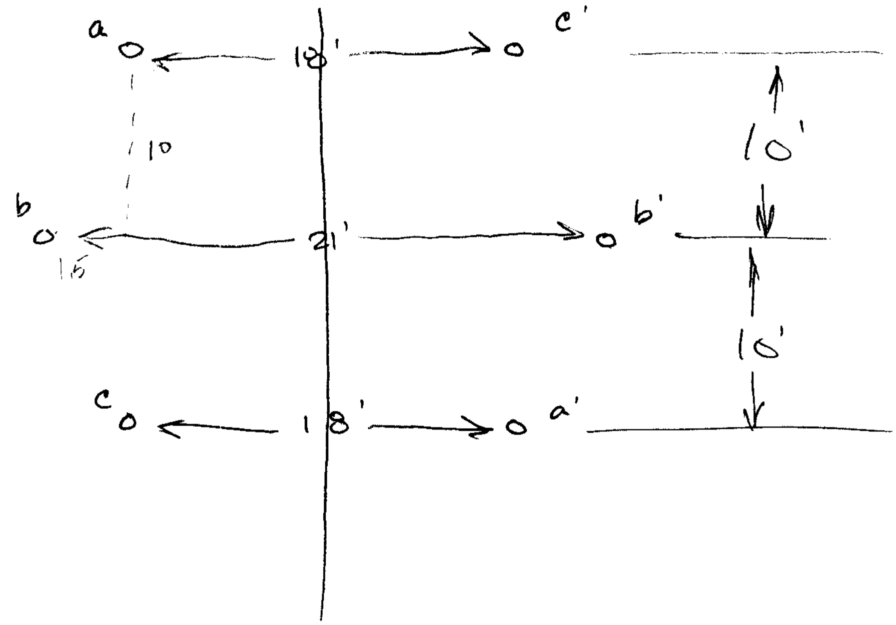
$$P_{MAX} = \frac{E_1E_2}{64.86} = \frac{E^2}{64.86} = \underline{\underline{.0154}}$$



$$P = \frac{E_1E_2}{X}\sin\delta$$

DOUBLE CIRCUIT LINES - 3Ø

300 MCM
OSTRICH 26/7



$$L = .7411 \log_{10} \frac{GMD}{GMR} \ \frac{mH}{mi}$$

FIND D_{eq} using GMD method

$$D_{eq} = \sqrt[3]{D^p_{ab} D^p_{bc} D^p_{ca}}$$

where $D^p_{ab} = \sqrt[4]{d_{ab}\, d_{ab'}\, d_{a'b}\, d_{a'b'}}$

$D^p_{bc} = \sqrt[4]{d_{bc}\, d_{bc'}\, d_{b'c}\, d_{b'c'}}$

$D^p_{ca} = \sqrt[4]{d_{ca}\, d_{ca'}\, d_{c'a}\, d_{c'a'}}$

$d_{ab} = \sqrt{10^2 + 1.5^2} = 10.11$ ft

$d_{ab'} = \sqrt{10^2 + 19.5^2} = 21.91$ ft

$$D^p_{ab} = \sqrt[4]{(10.11)^2 (21.91)^2} = 14.88'$$

$$D^p_{bc} = \sqrt[4]{(10.11)^2 (21.91)^2} = 14.88'$$

$$D^p_{ca} = \sqrt[4]{(20)^2 (18)^2} = 18.97'$$

$$GMD = \boxed{D_{eq} = \sqrt[3]{(14.88)^2 (18.97)} = 16.13'}$$

$GMR = ?$ Like bundled conductors

$D_s = .0229$

$d_{aa'} = \sqrt{18^2 + 20^2} = 26.91'$

$$GMR_{aa'} = \sqrt{(d_{aa'}) D_s}$$

$$\boxed{GMR_{aa'} = \sqrt{(.0229)(26.91)} = \underline{.785 \text{ ft}}}$$

$$GMR_{bb'} = \sqrt{(.0229)(21)} = \underline{.693 \text{ ft}}$$

$$GMR_{cc'} = \sqrt{(.0229)(26.91)} = .785 \text{ ft} = GMR_{aa'}$$

$$GMR = \boxed{D_s^P = \sqrt[3]{(.785)^2 (.693)} = .753 \text{ ft}}$$

$$L = .7411 \log_{10} \frac{D_{eq}^P}{D_s^P} = .7411 \log_{10} \frac{16.13}{.753}$$

$$= .986 \text{ mH/mi} \quad \text{per phase}$$

$$\boxed{X_L = (377)(.986 \times 10^{-3}) = .372 \ \Omega/\text{mile} \quad \text{per phase}}$$

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SUMMARY OF INDUCTANCE CALCULATIONS

Three Phase

$$L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s} \quad H/m \quad \text{per phase}$$

$$L = .7411 \log_{10} \frac{D_{eq}}{D_s} \quad mH/mi \quad \text{per phase}$$

$$X_L = .0754 \ln \frac{D_{eq}}{D_s} \quad \Omega/Km \quad \text{per phase}$$

$$X_L = .1213 \ln \frac{D_{eq}}{D_s} \quad \Omega/mi \quad \text{per phase}$$

D_{eq} & D_s in same units

Single conductors: D_s from table A.1

D_{eq} = GMD = $\sqrt[3]{\text{prod of sep. distances}}$

Bundled Conductors: $GMR = \sqrt{d_{aa}\, d_{bundling}} = D_s^b$

GMD = D_{eq} between centers of bundles $= D_{eq}^b$

Parallel Circuits: GMR: do like bundled by take geometric mean of the three phases $= D_s^p$

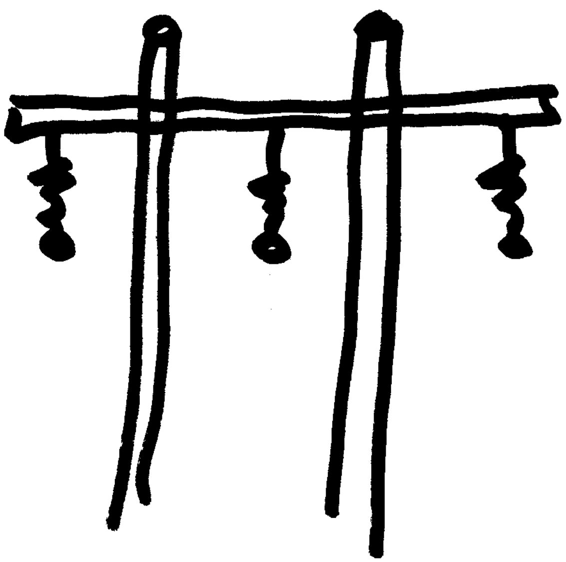
GMD: = ~~Geometric mean~~ Geometric mean of the Geometric means distances between conductors $= D_{eq}^p$

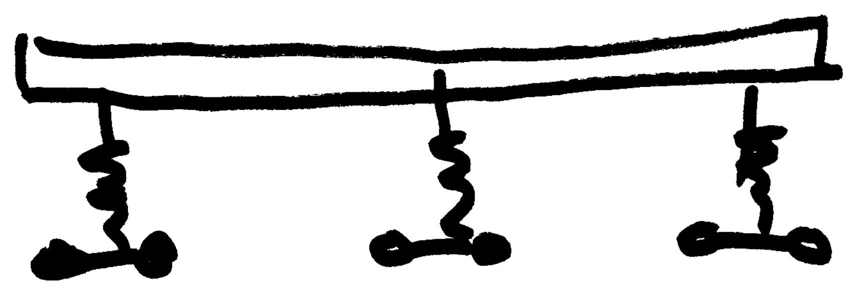
Inductances:

$$L \propto \ln \frac{D}{r} \quad \text{or} \quad \propto \ln \frac{GMD}{GMR}$$

As phase spacing $\uparrow$, $L \uparrow$

As conductor radius $\uparrow$, $L \downarrow$





For same phase spacing, bundled conductors will result in <u>smaller</u> L.