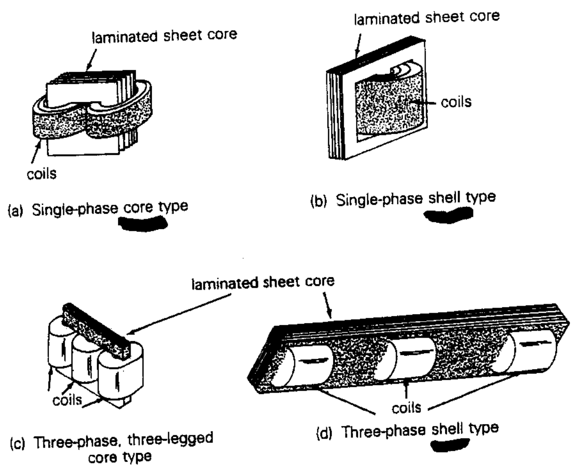
Glover & Sarma 2nd Ed.

Figure 4.15

Transformer core configurations



compared to replacement of only one phase of a three-phase bank. In either case, the equivalent circuits developed here and subsequent analyses are the same.*

SECTION 4.5

PER-UNIT SEQUENCE MODELS OF THREE-PHASE TWO-WINDING TRANSFORMERS

Figure 4.16(a) is a schematic representation of an ideal Y–Y transformer grounded through neutral impedances Z_N and Z_n. Figure 4.16(b–d) show the per-unit sequence networks of this ideal transformer. Throughout the remainder of this text per-unit quantities will be used unless otherwise indicated. Also, the subscript "p.u.," used to indicate a per-unit quantity, will be omitted in most cases.

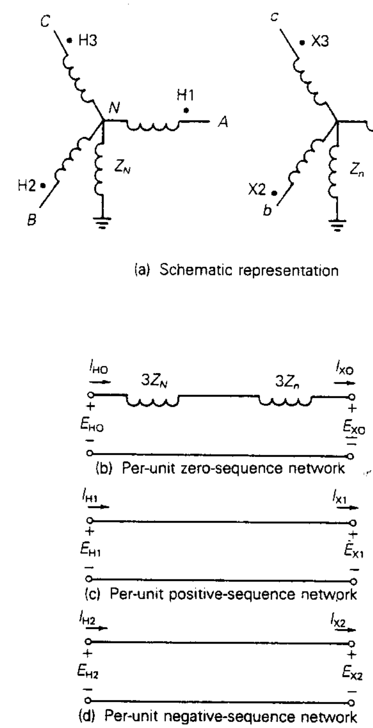
By convention, we adopt the following two rules for selecting base quantities:

1. A common S_{base} is selected for both the H and X terminals.
2. The ratio of the voltage bases V_{baseH}/V_{baseX} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.

*We note that the zero-sequence circuit of a three-phase shell-type transformer is not the same as the zero-sequence circuit of a three-phase core-type transformer [4].

Figure 4.16

Ideal Y–Y transformer



When balanced positive-sequence currents or balanced negative-sequence currents (as in Example 3.2) are applied to the transformer, the neutral currents are zero and there are no voltage drops across the neutral impedances. Therefore, the per-unit positive- and negative-sequence networks of the ideal Y–Y transformer, Figure 4.16(b) and (c), are the same as the per-unit single-phase ideal transformer, Figure 4.9(a).

Zero-sequence currents have equal magnitudes and equal phase angles. When per-unit sequence currents $I_{A0} = I_{B0} = I_{C0} = I_0$ are applied to the high-voltage windings of an ideal Y–Y transformer, the neutral current $I_N = 3I_0$ flows through the neutral impedance Z_N, with a voltage drop $(3Z_N)I_0$. Also, per-unit zero-sequence current I_0 flows in each low-voltage winding [from (4.3.9)], and therefore $3I_0$ flows through neutral impedance Z_n, with a voltage drop $(3I_0)Z_n$. The per-unit zero-sequence network, which includes the impedances $(3Z_N)$ and $(3Z_n)$, is shown in Figure 4.16(b).

Note that if either one of the neutrals of an ideal transformer is ungrounded, then no zero sequence can flow in either the high- or low-voltage windings. For example, if the high-voltage winding has an open neutral, then $I_N = 3I_0 = 0$, which in turn forces $I_0 = 0$ on the low-voltage side. This can be shown in the zero-sequence network of Figure 4.16(b) by making $Z_N = \infty$, which corresponds to an open circuit.

The per-unit sequence networks of a practical Y–Y transformer are shown in Figure 4.17(a). These networks are obtained by adding external

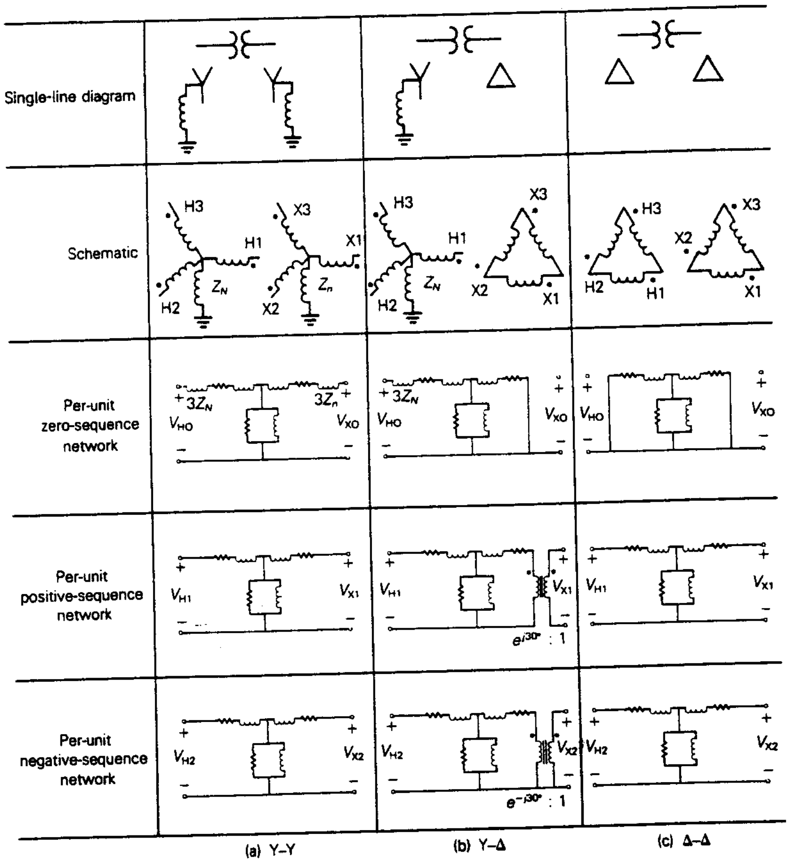


Figure 4.17 Per-unit sequence networks of practical Y–Y, Y–Δ, and Δ–Δ transformers

impedances to the sequence networks of the ideal transformer, as follows. The leakage impedances of the high-voltage windings are series impedances like the series impedances shown in Figure 3.9, with no coupling between phases ($Z_{ab} = 0$). If the phase a, b, and c windings have equal leakage impedances $Z_H = R_H + jX_H$, then the series impedances are *symmetrical* with sequence networks, as shown in Figure 3.10, where $Z_{H0} = Z_{H1} = Z_{H2} = Z_H$. Similarly, the leakage impedances of the low-voltage windings are symmetrical series impedances with $Z_{X0} = Z_{X1} = Z_{X2} = Z_X$. These series leakage impedances are shown in per-unit in the sequence networks of Figure 4.17(a).

The shunt branches of the practical Y–Y transformer, which represent exciting current, are equivalent to the Y load of Figure 3.3. Each phase in Figure 3.3 represents a core loss resistor in parallel with a magnetizing inductance. Assuming these are the same for each phase, then the Y load is *symmetrical*, and the sequence networks are shown in Figure 3.4. These shunt branches are also shown in Figure 4.17(a). Note that $(3Z_N)$ and $(3Z_n)$ have already been included in the zero-sequence network.

The per-unit positive- and negative-sequence transformer impedances of the practical Y–Y transformer in Figure 4.17(a) are identical, which is always true for nonrotating equipment. The per-unit zero-sequence network, however, depends on the neutral impedances Z_N and Z_n.

The per-unit sequence networks of the Y–Δ transformer, shown in Figure 4.17(b), have the following features:

1. The per-unit impedances do not depend on the winding connections. That is, the per-unit impedances of a transformer that is connected Y–Y, Y–Δ, Δ–Y, or Δ–Δ are the same. However, the base voltages do depend on the winding connections.

2. A phase shift is included in the per-unit positive- and negative-sequence networks. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30°. For negative sequence, the high-voltage quantities lag by 30°.

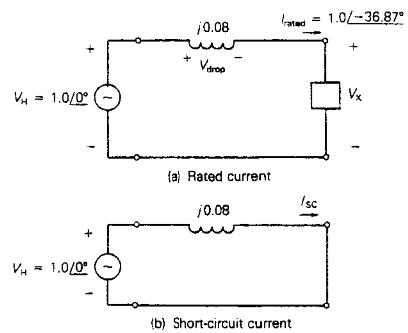
3. Zero-sequence currents can flow in the Y winding if there is a neutral connection, and corresponding zero-sequence currents flow within the Δ winding. But no zero-sequence current enters or leaves the Δ winding.

The phase shifts in the positive- and negative-sequence networks of Figure 4.17(b) are represented by the phase-shifting transformer of Figure 4.4. Also, the zero-sequence network of Figure 4.17(b) provides a path on the Y side for zero-sequence current to flow, but no zero-sequence current can enter or leave the Δ side.

The per-unit sequence networks of the Δ–Δ transformer, shown in Figure 4.17(c), have the following features:

Figure 4.20

Circuits for Example 4.9



(a) Rated current

(b) Short-circuit current

and

$$V_X = V_H - (jX_{eq})I_{rated}$$
$$= 1.0\underline{/0^\circ} - (j0.08)(1.0\underline{/-36.87^\circ})$$
$$= 1.0 - (j0.08)(0.8 - j0.6) = 0.952 - j0.064$$
$$= 0.954\underline{/-3.85^\circ} \quad \text{per unit}$$

b. As shown in Figure 4.20(b),

$$I_{SC} = \frac{V_H}{X_{eq}} = \frac{1.0}{0.08} = 12.5 \quad \text{per unit}$$

Under rated current conditions [part (a)], the 0.08 per-unit voltage drop across the transformer leakage reactance causes the voltage at the low-voltage terminals to be 0.954 per unit. Also, under three-phase short-circuit conditions [part (b)], the fault current is 12.5 times the rated transformer current. This example illustrates a compromise in the design or specification of transformer leakage reactance. A low value is desired to minimize voltage drops, but a high value is desired to limit fault currents. Typical transformer leakage reactances are given in Table A.2 in the Appendix. ■

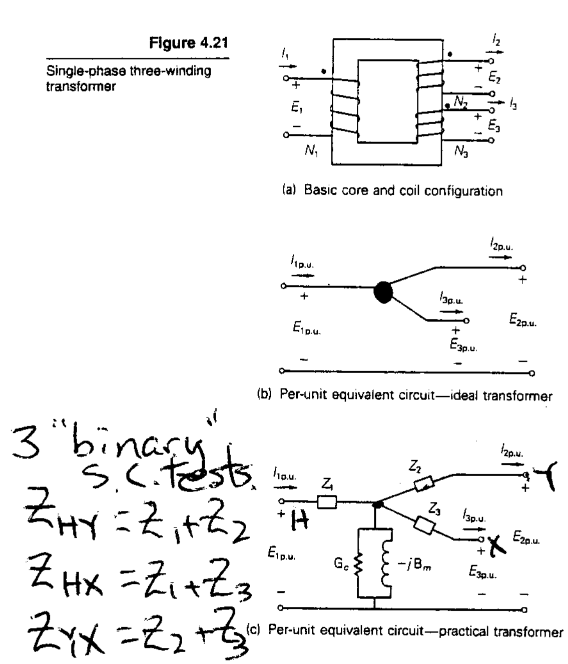
SECTION 4.6

THREE-WINDING TRANSFORMERS

Figure 4.21(a) shows a basic single-phase three-winding transformer. The ideal transformer relations for a two-winding transformer, (4.1.8) and (4.1.14), can easily be extended to obtain corresponding relations for an ideal

Figure 4.21

Single-phase three-winding transformer



(a) Basic core and coil configuration

(b) Per-unit equivalent circuit—ideal transformer

(c) Per-unit equivalent circuit—practical transformer

three-winding transformer. In actual units, these relations are:

$$N_1I_1 = N_2I_2 + N_3I_3 \tag{4.6.1}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3} \tag{4.6.2}$$

where I_1 enters the dotted terminal, I_2 and I_3 leave dotted terminals, and E_1, E_2, and E_3 have their + polarities at dotted terminals. In per-unit, (4.6.1) and (4.6.2) are:

$$I_{1p.u.} = I_{2p.u.} + I_{3p.u.} \tag{4.6.3}$$

$$E_{1p.u.} = E_{2p.u.} = E_{3p.u.} \tag{4.6.4}$$

where a common S_{base} is selected for all three windings, and voltage bases are selected in proportion to the rated voltages of the windings. These two per-unit relations are satisfied by the per-unit equivalent circuit shown in Figure

4.21(b). Also, external series impedance and shunt admittance branches are included in the practical three-winding transformer circuit shown in Figure 4.21(c). The shunt admittance branch, a core loss resistor in parallel with a magnetizing inductor, can be evaluated from an open-circuit test. Also, when one winding is left open, the three-winding transformer behaves as a two-winding transformer, and standard short-circuit tests can be used to evaluate per-unit leakage impedances, which are defined as follows:

Z_{12} = per-unit leakage impedance measured from winding 1, with winding 2 shorted and winding 3 open

Z_{13} = per-unit leakage impedance measured from winding 1, with winding 3 shorted and winding 2 open

Z_{23} = per-unit leakage impedance measured from winding 2, with winding 3 shorted and winding 1 open

From Figure 4.21(c), with winding 2 shorted and winding 3 open, the leakage impedance measured from winding 1 is, neglecting the shunt admittance branch,

$$Z_{12} = Z_1 + Z_2 \tag{4.6.5}$$

Similarly,

$$Z_{13} = Z_1 + Z_3 \tag{4.6.6}$$

and

$$Z_{23} = Z_2 + Z_3 \tag{4.6.7}$$

Solving (4.6.5)–(4.6.7),

$$Z_1 = \tfrac{1}{2}(Z_{12} + Z_{13} - Z_{23}) \tag{4.6.8}$$

$$Z_2 = \tfrac{1}{2}(Z_{12} + Z_{23} - Z_{13}) \tag{4.6.9}$$

$$Z_3 = \tfrac{1}{2}(Z_{13} + Z_{23} - Z_{12}) \tag{4.6.10}$$

Equations (4.6.8)–(4.6.10) can be used to evaluate the per-unit series impedances Z_1, Z_2, and Z_3 of the three-winding transformer equivalent circuit from the per-unit leakage impedances Z_{12}, Z_{13}, and Z_{23}, which, in turn, are determined from short-circuit tests.

Note that each of the windings on a three-winding transformer may have a *different* kVA rating. If the leakage impedances from short-circuit tests are expressed in per-unit based on winding ratings, they must first be converted to per-unit on a common S_{base} *before* they are used in (4.6.8)–(4.6.10).

EXAMPLE 4.10

Three-winding single-phase transformer: per-unit impedances

The ratings of a single-phase three-winding transformer are:

winding 1: 300 MVA, 13.8 kV

winding 2: 300 MVA, 199.2 kV

winding 3: 50 MVA, 19.92 kV

The leakage reactances, from short-circuit tests, are:

$X_{12} = 0.10$ per unit on a 300-MVA, 13.8-kV base

$X_{13} = 0.16$ per unit on a 50-MVA, 13.8-kV base

$X_{23} = 0.14$ per unit on a 50-MVA, 199.2-kV base

Winding resistances and exciting current are neglected. Calculate the impedances of the per-unit equivalent circuit using a base of 300 MVA and 13.8 kV for terminal 1.

Solution $S_{base} = 300$ MVA is the same for all three terminals. Also, the specified voltage base for terminal 1 is $V_{base1} = 13.8$ kV. The base voltages for terminals 2 and 3 are then $V_{base2} = 199.2$ kV and $V_{base3} = 19.92$ kV, which are the rated voltages of these windings. From the data given, $X_{12} = 0.10$ per unit was measured from terminal 1 using the same base values as those specified for the circuit. But $X_{13} = 0.16$ and $X_{23} = 0.14$ per unit on a 50-MVA base are first converted to the 300-MVA circuit base.

$$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96 \quad \text{per unit}$$

$$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84 \quad \text{per unit}$$

Then, from (4.6.8)–(4.6.10),

$$X_1 = \tfrac{1}{2}(0.10 + 0.96 - 0.84) = 0.11 \quad \text{per unit}$$

$$X_2 = \tfrac{1}{2}(0.10 + 0.84 - 0.96) = -0.01 \quad \text{per unit}$$

$$X_3 = \tfrac{1}{2}(0.84 + 0.96 - 0.10) = 0.85 \quad \text{per unit}$$

The per-unit equivalent circuit of this three-winding transformer is shown in Figure 4.22. Note that X_2 is negative. This illustrates the fact that X_1, X_2, and X_3 are *not* leakage reactances, but instead are equivalent reactances derived from the leakage reactances. Leakage reactances are always positive.

Note also that the node where the three equivalent circuit reactances

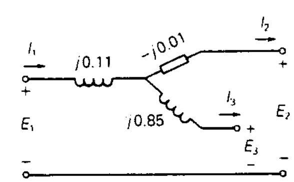


Figure 4.22 Circuit for Example 4.10

are connected does not correspond to any physical location within the transformer. Rather, it is simply part of the equivalent circuit representation. ■

Three identical single-phase three-winding transformers can be connected to form a three-phase bank. Figure 4.23 shows the general per-unit sequence networks of a three-phase three-winding transformer. Instead of labeling the windings 1, 2, and 3, as was done for the single-phase transformer, the letters H, M, and X are used to denote the high-, medium-, and low-voltage windings, respectively. By convention, a common S_{base} is selected for the H, M, and X terminals, and voltage bases V_{baseH}, V_{baseM}, and V_{baseX} are selected in proportion to the rated line-to-line voltages of the transformer.

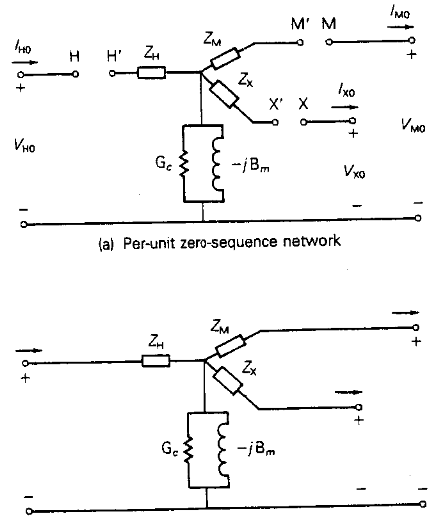
For the general zero-sequence network, Figure 4.23(a), the connection between terminals H and H′ depends on how the high-voltage windings are connected, as follows:

1. Solidly grounded Y—Short H to H′.
2. Grounded Y through Z_N—Connect $(3Z_N)$ from H to H′.
3. Ungrounded Y—Leave H–H′ open as shown.
4. Δ—Short H′ to the reference bus.

Terminals X–X′ and M–M′ are connected in a similar manner.

Figure 4.23

Per-unit sequence networks of a three-phase three-winding transformer



(a) Per-unit zero-sequence network

(b) Per-unit positive- or negative-sequence network (phase shift not shown)

The impedances of the per-unit negative-sequence network are the same as those of the per-unit positive-sequence network, which is always true for nonrotating equipment. Phase-shifting transformers, not shown in Figure 4.23(b), can be included to model phase shift between Δ and Y windings.

EXAMPLE 4.11

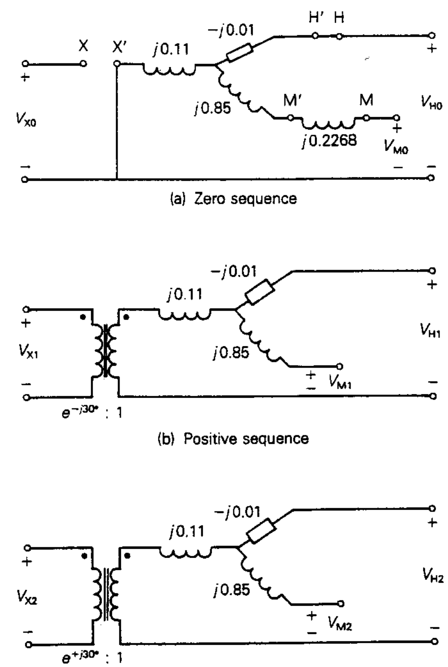
Three-winding three-phase transformer: per-unit sequence networks

Three transformers, each identical to that described in Example 4.10, are connected as a three-phase bank in order to feed power from a 900-MVA, 13.8-kV generator to a 345-kV transmission line and to a 34.5-kV distribution line. The transformer windings are connected as follows:

13.8-kV windings (X): Δ, to generator
199.2-kV windings (H): solidly grounded Y, to 345-kV line
19.92-kV windings (M): grounded Y through $Z_n = j0.10\,\Omega$, to 34.5-kV line

Figure 4.24

Per-unit sequence networks for Example 4.11



(a) Zero sequence

(b) Positive sequence

(c) Negative sequence

The positive-sequence voltages and currents of the high- and medium-voltage Y windings lead the corresponding quantities of the low-voltage Δ winding by 30°. Draw the per-unit sequence networks, using a three-phase base of 900 MVA and 13.8 kV for terminal X.

Solution The per-unit sequence networks are shown in Figure 4.24. Since $V_{baseX} = 13.8$ kV is the rated line-to-line voltage of terminal X, $V_{baseM} = \sqrt{3}(19.92) = 34.5$ kV, which is the rated line-to-line voltage of terminal M. The base impedance of the medium-voltage terminal is then

$$Z_{baseM} = \frac{(34.5)^2}{900} = 1.3225 \quad \Omega$$

Therefore, the per-unit neutral impedance is

$$Z_n = \frac{j0.10}{1.3225} = j0.07561 \quad \text{per unit}$$

and $(3Z_n) = j0.2268$ is connected from terminal M to M′ in the per-unit zero-sequence network. Since the high-voltage windings have a solidly grounded neutral, H to H′ is shorted in the zero-sequence network. Also, phase-shifting transformers are included in the positive- and negative-sequence networks. ■

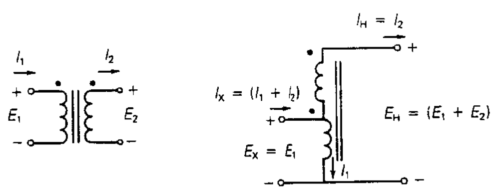
SECTION 4.7

AUTOTRANSFORMERS

A single-phase two-winding transformer is shown in Figure 4.25(a) with two separate windings, which is the usual two-winding transformer; the same transformer is shown in Figure 4.25(b) with the two windings connected in series, which is called an *autotransformer*. For the usual transformer [Figure 4.25(a)] the two windings are coupled magnetically via the mutual core flux. For the autotransformer [Figure 4.25(b)] the windings are both electrically and magnetically coupled. The autotransformer has smaller per-unit leakage impedances than the usual transformer; this results in both smaller series-voltage drops (an advantage) and higher short-circuit currents (a disadvantage). The autotransformer also has lower per-unit losses (higher efficiency), lower exciting current, and lower cost if the turns ratio is not too large. The electrical connection of the windings, however, allows transient overvoltages to pass through the autotransformer more easily.

Figure 4.25

Ideal single-phase transformers



(a) Two-winding transformer (b) Autotransformer

EXAMPLE 4.12 **Autotransformer: single-phase**

The single-phase two-winding 20-kVA, 480/120-volt transformer of Example 4.3 is connected as an autotransformer, as in Figure 4.25(b), where winding 1 is the 120-volt winding. For this autotransformer, determine (a) the voltage ratings E_X and E_H of the low- and high-voltage terminals, (b) the kVA rating, and (c) the per-unit leakage impedance.

Solution

a. Since the 120-volt winding is connected to the low-voltage terminal, $E_X = 120$ volts. When $E_X = E_1 = 120$ volts is applied to the low-voltage terminal, $E_2 = 480$ volts is induced across the 480-volt winding, neglecting the voltage drop across the leakage impedance. Therefore, $E_H = E_1 + E_2 = 120 + 480 = 600$ volts.

b. As a normal two-winding transformer rated 20 kVA, the rated current of the 480-volt winding is $I_2 = I_H = 20{,}000/480 = 41.667$ A. As an autotransformer, the 480-volt winding can carry the same current. Therefore, the kVA rating $S_H = E_H I_H = (600)(41.667) = 25$ kVA. Note also that when $I_H = I_2 = 41.667$ A, a current $I_1 = \frac{480}{120}(41.667) = 166.7$ A is induced in the 120-volt winding. Therefore, $I_X = I_1 + I_2 = 208.3$ A (neglecting exciting current) and $S_X = E_X I_X = (120)(208.3) = 25$ kVA, which is the same rating as calculated for the high-voltage terminal.

c. From Example 4.3, the leakage impedance is $0.0729\underline{/78.13°}$ per unit as a normal, two-winding transformer. As an autotransformer, the leakage impedance *in ohms* is the same as for the normal transformer, since the core and windings are the same for both (only the external winding connections are different). But the base impedances are different. For the high-voltage terminal, using (4.3.4),

$$Z_{baseHold} = \frac{(480)^2}{20{,}000} = 11.52 \quad \Omega \quad \text{as a normal transformer}$$

$$Z_{baseHnew} = \frac{(600)^2}{25{,}000} = 14.4 \quad \Omega \quad \text{as an autotransformer}$$

Therefore, using (4.3.10),

$$Z_{p.u.new} = (0.0729\underline{/78.13°})\left(\frac{11.52}{14.4}\right) = 0.05832\underline{/78.13°} \quad \text{per unit}$$

For this example, the rating is 25 kVA, 120/600 volts as an autotransformer versus 20 kVA, 120/480 volts as a normal transformer. The autotransformer has both a larger kVA rating and a larger voltage ratio for the same cost.

Also, the per-unit leakage impedance of the autotransformer is smaller. However, the increased high-voltage rating as well as the electrical connection of the windings may require more insulation for both windings. ■

SECTION 4.8

TRANSFORMERS WITH OFF-NOMINAL TURNS RATIOS

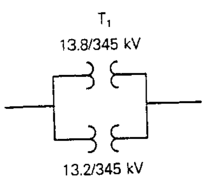


Figure 4.26 Two transformers connected in parallel

It has been shown that models of transformers that use per-unit quantities are simpler than those that use actual quantities. The ideal transformer winding is eliminated when the ratio of the selected voltage bases equals the ratio of the voltage ratings of the windings. In some cases, however, it is impossible to select voltage bases in this manner. For example, consider the two transformers connected in parallel in Figure 4.26. Transformer T_1 is rated 13.8/345 kV and T_2 is rated 13.2/345 kV. If we select $V_{baseH} = 345$ kV, then transformer T_1 requires $V_{baseX} = 13.8$ kV and T_2 requires $V_{baseX} = 13.2$ kV. It is clearly impossible to select the appropriate voltage bases for both transformers.

To accommodate this situation, we will develop a per-unit model of a transformer whose voltage ratings are not in proportion to the selected base voltages. Such a transformer is said to have an "off-nominal turns ratio." Figure 4.27(a) shows a transformer with rated voltages V_{1rated} and V_{2rated}, which satisfy

$$V_{1rated} = a_t V_{2rated} \tag{4.8.1}$$

where a_t is assumed, in general, to be either real or complex. Suppose the selected voltage bases satisfy

$$V_{base1} = b V_{base2} \tag{4.8.2}$$

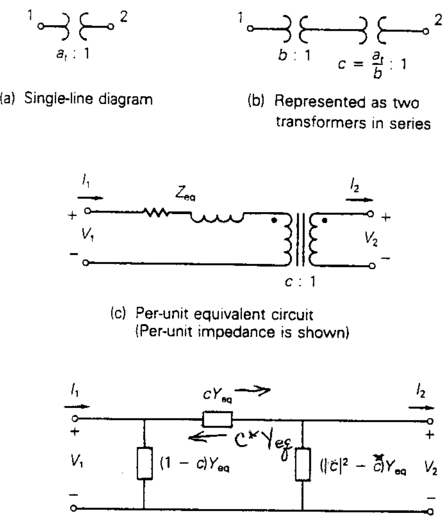
Defining $c = a_t/b$, (4.8.1) can be rewritten as

$$V_{1rated} = b\left(\frac{a_t}{b}\right) V_{2rated} = bc\, V_{2rated} \tag{4.8.3}$$

Equation (4.8.3) can be represented by two transformers in series, as shown in Figure 4.27(b). The first transformer has the same ratio of rated winding voltages as the ratio of the selected base voltages, b. Therefore this transformer has a standard per-unit model, as shown in Figure 4.9 or 4.17. We will assume that the second transformer is ideal, all real and reactive losses being associated with the first transformer. The resulting per-unit model is shown in Figure 4.27(c), where, for simplicity, the shunt-exciting branch is neglected. Note that if $a_t = b$, then the ideal transformer winding shown in this figure can be eliminated, since its turns ratio $c = (a_t/b) = 1$.

The per-unit model shown in Figure 4.27(c) is perfectly valid, but it is not suitable for some of the computer programs presented in later chapters because these programs do not accommodate ideal transformer windings. An

Figure 4.27 Transformer with off-nominal turns ratio



(a) Single-line diagram

(b) Represented as two transformers in series

(c) Per-unit equivalent circuit (Per-unit impedance is shown)

(d) π circuit representation for ~~real~~ c (Per-unit admittances are shown; $Y_{eq} = \frac{1}{Z_{eq}}$)

alternative representation can be developed, however, by writing nodal equations for this figure as follows:

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{4.8.4}$$

where both I_1 and $-I_2$ are referenced *into* their nodes in accordance with the nodal equation method (Section 2.4). Recalling two-port network theory, the admittance parameters of (4.8.4) are, from Figure 4.27(c),

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} = \frac{1}{Z_{eq}} = Y_{eq} \tag{4.8.5}$$

$$Y_{22} = \frac{-I_2}{V_2}\bigg|_{V_1 = 0} = \frac{1}{Z_{eq}/|c|^2} = |c|^2 Y_{eq} \tag{4.8.6}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0} = \frac{-cV_2/Z_{eq}}{V_2} = -cY_{eq} \tag{4.8.7}$$

$$Y_{21} = \frac{-I_2}{V_1}\bigg|_{V_2 = 0} = \frac{-c^* I_1}{V_1} = -c^* Y_{eq} \tag{4.8.8}$$

Equations (4.8.4)–(4.8.8) with real or complex c are convenient for representing transformers with off-nominal turns ratios in the computer programs presented later. Note that when c is complex, Y_{12} is not equal to Y_{21}, and the preceding admittance parameters cannot be synthesized with a passive RLC circuit. However, the π network shown in Figure 4.27(d), which has the same admittance parameters as (4.8.4)–(4.8.8), can be synthesized for real c. Note also that when $c = 1$, the shunt branches in this figure become open circuits (zero per unit mhos), and the series branch becomes Y_{eq} per unit mhos (or Z_{eq} per unit ohms).

EXAMPLE 4.13 **Tap-changing three-phase transformer: per-unit positive-sequence network**

A three-phase generator step-up transformer is rated 1000 MVA, 13.8 kV Δ/345 kV Y with $Z_{eq} = j0.10$ per unit. The transformer high-voltage winding has ±10% taps. The system base quantities are

$$S_{base3\phi} = 500 \quad \text{MVA}$$

$$V_{baseXLL} = 13.8 \quad \text{kV}$$

$$V_{baseHLL} = 345 \quad \text{kV}$$

Determine the per-unit positive-sequence equivalent circuit for the following tap settings:

a. Rated tap

b. −10% tap (providing a 10% voltage decrease for the high-voltage winding)

Neglect transformer winding resistance, exciting current, and phase shift.

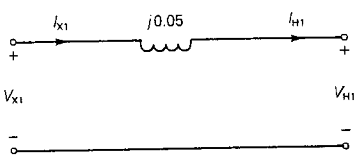
Solution **a.** Using (4.8.1) and (4.8.2) with the low-voltage winding denoted winding 1,

$$a_t = \frac{13.8}{345} = 0.04 \qquad b = \frac{V_{baseXLL}}{V_{baseHLL}} = \frac{13.8}{345} = a_t \qquad c = 1$$

From (4.3.11)

$$Z_{p.u.new} = (j0.10)\left(\frac{500}{1000}\right) = j0.05 \quad \text{per unit}$$

The positive-sequence equivalent circuit, not including winding resistance, exciting current, and phase shift is:



(Per-unit impedance is shown)

b. Using (4.8.1) and (4.8.2),

$$a_t = \frac{13.8}{345(0.9)} = 0.04444 \quad b = \frac{13.8}{345} = 0.04 \quad c = \frac{a_t}{b} = \frac{0.04444}{0.04} = 1.1111$$

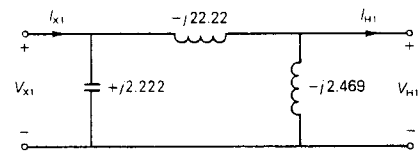
From Figure 4.27(d),

$$cY_{eq} = 1.1111\left(\frac{1}{j0.05}\right) = -j22.22 \quad \text{per unit}$$

$$(1-c)Y_{eq} = (-0.11111)(-j20) = +j2.222 \quad \text{per unit}$$

$$(|c|^2 - c)Y_{eq} = (1.2346 - 1.1)(-j20) = -j2.469 \quad \text{per unit}$$

The per-unit positive-sequence network is:



(Per-unit admittances are shown) ■

The three-phase regulating transformers shown in Figures 4.28 and 4.29 can be modeled as transformers with off-nominal turns ratios. For the voltage-magnitude-regulating transformer shown in Figure 4.28, adjustable voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn}, which have equal magnitudes ΔV and which are in phase with the phase voltages V_{an}, V_{bn}, and V_{cn}, are placed in the series

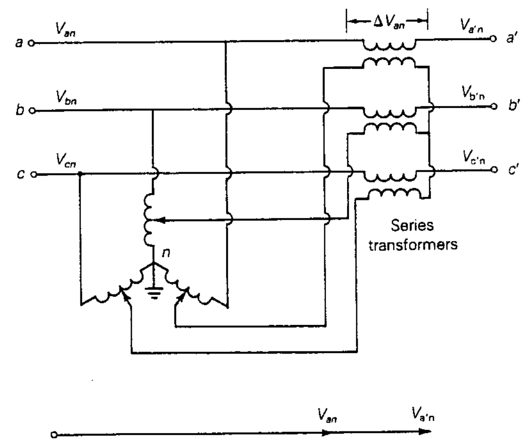


Figure 4.28 An example of a voltage-magnitude-regulating transformer

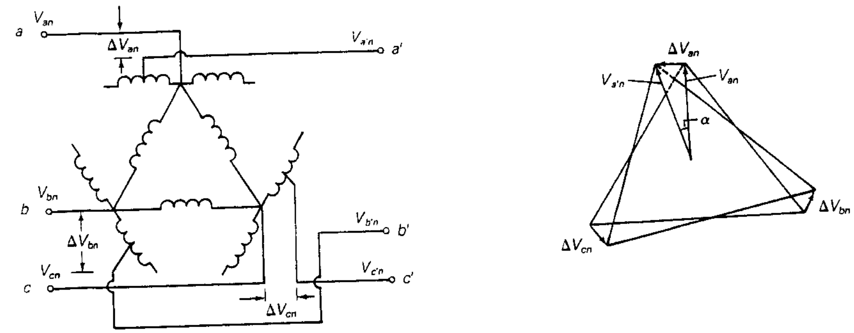


Figure 4.29 An example of a phase-angle-regulating transformer. Windings drawn in parallel are on the same core

link between buses a–a', b–b', and c–c'. Modeled as a transformer with an off-nominal turns ratio (see Figure 4.27), $c = (1 + \Delta V)$ for a voltage-magnitude increase toward bus abc, or $c = (1 + \Delta V)^{-1}$ for an increase toward bus $a'b'c'$.

For the phase-angle-regulating transformer in Figure 4.29, the series voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn} are $\pm 90°$ out of phase with the phase voltages V_{an}, V_{bn}, and V_{cn}. The phasor diagram in Figure 4.29 indicates that each of the bus voltages $V_{a'n}$, $V_{b'n}$, and $V_{c'n}$, has a phase shift that is approximately proportional to the magnitude of the added series voltage. Modeled as a transformer with an off-nominal turns ratio (see Figure 4.27), $c \approx 1\underline{/\alpha}$ for a phase increase toward bus abc or $c \approx 1\underline{/-\alpha}$ for a phase increase toward bus $a'b'c'$.

EXAMPLE 4.14 **Voltage-regulating and phase-shifting three-phase transformers**

Two buses abc and $a'b'c'$ are connected by two parallel lines L1 and L2 with positive-sequence series reactances $X_{L1} = 0.25$ and $X_{L2} = 0.20$ per unit. A regulating transformer is placed in series with line L1 at bus $a'b'c'$. Determine the 2×2 positive-sequence bus admittance matrix when the regulating transformer (a) provides a 0.05 per-unit increase in voltage magnitude toward bus $a'b'c'$ and (b) advances the phase 3° toward bus $a'b'c'$. Assume that the regulating transformer is ideal. Also, the series resistance and shunt admittance of the lines are neglected.

Solution The circuit is shown in Figure 4.30.

a. For the voltage-magnitude-regulating transformer, $c = (1 + \Delta V)^{-1} = (1.05)^{-1} = 0.9524$ per unit. From (4.8.5)–(4.8.8), the admittance parameters of the regulating transformer in series with line L1 are

$$Y_{11L1} = \frac{1}{j0.25} = -j4.0$$

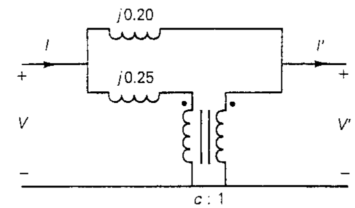


Figure 4.30 Positive-sequence circuit for Example 4.14

$$Y_{22L1} = (0.9524)^2(-j4.0) = -j3.628$$

$$Y_{12L1} = Y_{21L1} = (-0.9524)(-j4.0) = j3.810$$

For line L2 alone,

$$Y_{11L2} = Y_{22L2} = \frac{1}{j0.20} = -j5.0$$

$$Y_{12L2} = Y_{21L2} = -(-j5.0) = j5.0$$

Combining the above admittances in parallel,

$$Y_{11} = Y_{11L1} + Y_{11L2} = -j4.0 - j5.0 = -j9.0$$

$$Y_{22} = Y_{22L1} + Y_{22L2} = -j3.628 - j5.0 = -j8.628$$

$$Y_{12} = Y_{21} = Y_{12L1} + Y_{12L2} = j3.810 + j5.0 = j8.810 \quad \text{per unit}$$

b. For the phase-angle-regulating transformer, $c = 1\underline{/-\alpha} = 1\underline{/-3°}$. Then, for this regulating transformer in series with line L1,

$$Y_{11L1} = \frac{1}{j0.25} = -j4.0$$

$$Y_{22L1} = |1.0\underline{/-3°}|^2(-j4.0) = -j4.0$$

$$Y_{12L1} = -(1.0\underline{/-3°})(-j4.0) = 4.0\underline{/87°} = 0.2093 + j3.9945$$

$$Y_{21L1} = -(1.0\underline{/-3°})^*(-j4.0) = 4.0\underline{/93°} = -0.2093 + j3.9945$$

The admittance parameters for line L2 alone are given in part (a) above. Combining the admittances in parallel,

$$Y_{11} = Y_{22} = -j4.0 - j5.0 = -j9.0$$

$$Y_{12} = 0.2093 + j3.9945 + j5.0 = 0.2093 + j8.9945$$

$$Y_{21} = -0.2093 + j3.9945 + j5.0 = -0.2093 + j8.9945 \quad \text{per unit}$$

■

Note that a voltage-magnitude-regulating transformer controls the *reactive* power flow in the series link in which it is installed, whereas a phase-angle-regulating transformer controls the *real* power flow (see Problem 4.31).