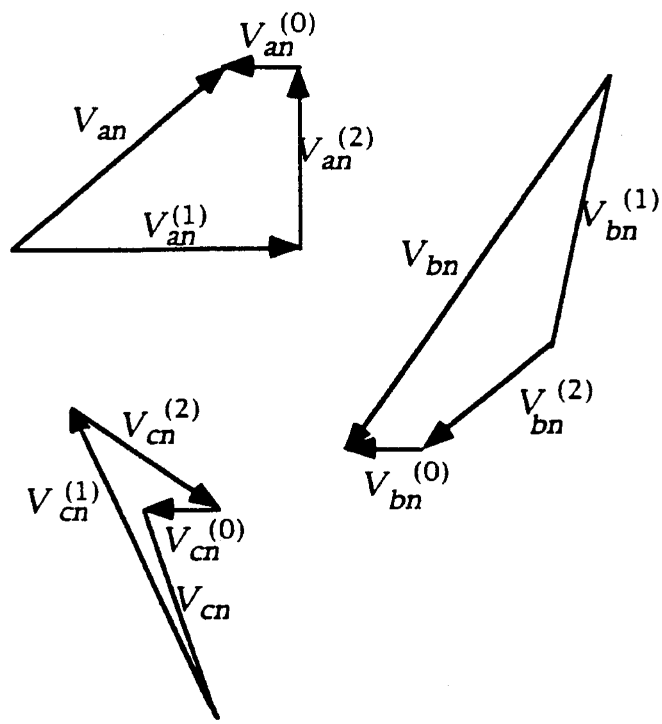
Chapter 11 Problem Solutions

11.1 If $V_{an}^{(1)} = 50\underline{/0^\circ}$, $V_{an}^{(2)} = 20\underline{/90^\circ}$ and $V_{an}^{(0)} = 10\underline{/180^\circ}$ V, determine analytically the voltages to neutral V_{an}, V_{bn} and V_{cn}, and also show graphically the sum of the given symmetrical components which determine the line-to-neutral voltages.

Solution:

$$
\begin{aligned}
V_{an} &= 50 + j20 - 10 \;=\; 40 + j20 \;=\; 44.72\underline{/26.6^\circ}\text{ V} \\
V_{bn} &= 50\underline{/240^\circ} + 20\underline{/210^\circ} - 10 \;=\; -25 - j43.33 - 17.32 - j10 - 10 \\
&= -52.32 - j53.33 \;=\; 7.47\underline{/-134.4^\circ}\text{ V} \\
V_{cn} &= 50\underline{/120^\circ} + 20\underline{/330^\circ} - 10 \;=\; -25 + j43.33 + 17.32 - j10 - 10 \\
&= -17.68 + j33.3 \;=\; 37.7\underline{/117^\circ}
\end{aligned}
$$



(11.2) When a generator has terminal *a* open and the other two terminals are connected to each other with a short circuit from this connection to ground, typical values for the symmetrical components of current in phase *a* are $I_a^{(1)} = 600\angle{-90^\circ}$, $I_a^{(2)} = 250\angle{90^\circ}$, and $I_a^{(0)} = 350\angle{90^\circ}$ A. Find the current into the ground and the current in each phase of the generator.

Solution:

$$I_a = -j600 + j250 + j350 = 0 \text{ A}$$
$$I_b^{(1)} = 600\angle{150^\circ} = -519.6 + j300$$
$$I_b^{(2)} = 250\angle{210^\circ} = -216.5 - j125$$
$$I_b^{(0)} = 350\angle{90^\circ} = 0 + j350$$
$$I_b = -736.1 + j525 = 904.1\angle{144.5^\circ} \text{ A}$$
$$I_c^{(1)} = 600\angle{30^\circ} = 519.6 + j300$$
$$I_c^{(2)} = 250\angle{330^\circ} = 216.5 - j125$$
$$I_c^{(0)} = 350\angle{90^\circ} = 0 + j350$$
$$I_c = 736.1 + j525 = 904.1\angle{35.5^\circ} \text{ A}$$

$$I_n = I_b + I_c = j1050 \text{ A}$$
or $$I_n = 3I_a^{(0)} = 3 \times j350 = j1050 \text{ A}$$

(11.3) Determine the symmetrical components of the three currents $I_a = 10\angle 0^\circ$, $I_b = 10\angle 230^\circ$, and $I_c = 10\angle 130^\circ$ A.

Solution:

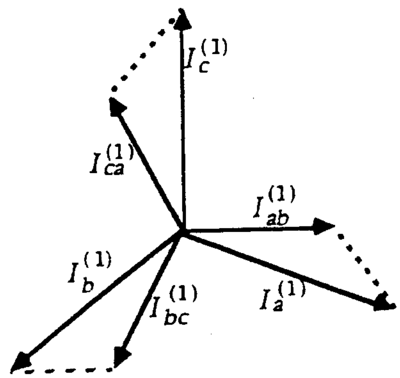
$$I_a^{(1)} = \frac{1}{3}(10\angle 0^\circ + 10\angle 350^\circ + 10\angle 370^\circ)$$
$$= \frac{1}{3}(10 + 9.848 - j1.736 + 9.848 + j1.736)$$
$$= 9.899\angle 0^\circ \text{ A}$$
$$I_a^{(2)} = \frac{1}{3}(10\angle 0^\circ + 10\angle 470^\circ + 10\angle 250^\circ)$$
$$= \frac{1}{3}(10 - 3.420 + j9.397 - 3.420 - j9.397)$$
$$= 1.053\angle 0^\circ \text{ A}$$
$$I_a^{(0)} = \frac{1}{3}(10\angle 0^\circ + 10\angle 230^\circ + 10\angle 130^\circ)$$
$$= \frac{1}{3}(10 - 6.428 - j7.66 - 6.428 + j7.66)$$
$$= 0.952\angle 180^\circ \text{ A}$$

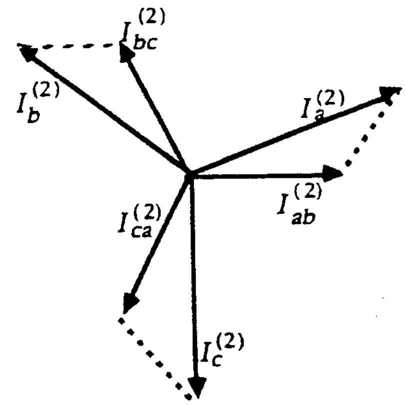
The components of I_b and I_c are easily found from $I_a^{(1)}$, $I_a^{(2)}$ and $I_a^{(0)}$.

Check: $I_a = 9.899 + 1.053 - 0.952 = 10.00$.

(11.4) The currents flowing in the lines toward a balanced load connected in Δ are $I_a = 100\angle 0^\circ$, $I_b = 141.4\angle 225^\circ$, and $I_c = 100\angle 90^\circ$. Find the symmetrical components of the given line currents and draw phasor diagrams of the positive- and negative-sequence line and phase currents. What is I_{ab} in amperes?

Solution:





The phasor diagrams for positive- and negative-sequence currents in the lines and in the Δ-connected load show the desired relations, namely:

$$I_{ab}^{(1)} = \frac{I_a^{(1)}}{\sqrt{3}} \angle +30^\circ \quad \text{and} \quad I_{ab}^{(2)} = \frac{I_a^{(2)}}{\sqrt{3}} \angle -30^\circ$$

For the given currents, we find

$$I_a^{(1)} = \frac{1}{3}(100 + 141.4\angle 345^\circ + 100\angle 330^\circ)$$
$$= 107.7 - j28.9 = 111.5\angle -15^\circ \text{ A}$$

$$I_a^{(2)} = \frac{1}{3}(100 + 141.4\angle 105^\circ + 100\angle 210^\circ)$$
$$= -7.73 + j28.9 = 29.9\angle 105^\circ \text{ A}$$

$$I_a^{(0)} = \frac{1}{3}(100 - 100 - j100 + j100)$$
$$= 0 \text{ (since zero-sequence cannot flow into the } \Delta\text{).}$$

and,

$$I_{ab}^{(1)} = \frac{111.5}{\sqrt{3}} \angle -15^\circ + 30^\circ = 64.4\angle 15^\circ = 62.2 + j16.66$$

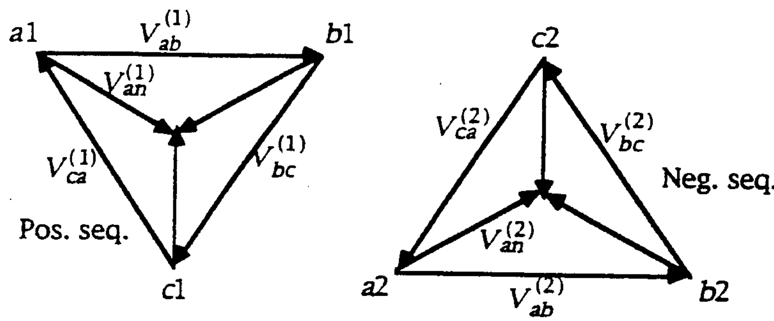
$$I_{ab}^{(2)} = \frac{29.9}{\sqrt{3}} \angle 105^\circ - 30^\circ = 17.26\angle 75^\circ = 4.47 + j16.67$$

$$I_{ab} = 66.67 + j33.33 = 74.5\angle 26.6^\circ \text{ A}$$

(11.5) The voltages at the terminals of a balanced load consisting of three 10-Ω resistors connected in Y are $V_{ab} = 100\angle 0^\circ$ $V_{bc} = 80.8\angle -121.44^\circ$, and $V_{ca} = 90\angle 130^\circ$ V. Assuming that there is no connection to the neutral of the load, find the line currents from the symmetrical components of the given line voltages.

Solution:

Phasor diagrams for the positive and negative sequence voltages are:



$$V_{an}^{(1)} = \frac{V_{ab}^{(1)}}{\sqrt{3}}\angle -30^\circ \quad \text{and} \quad V_{an}^{(2)} = \frac{V_{ab}^{(2)}}{\sqrt{3}}\angle +30^\circ$$

(no zero-sequence components)

$$V_{ab}^{(1)} = \frac{1}{3}(\,100\angle 0^\circ + 80.8\angle -1.44^\circ + 90\angle 370^\circ)$$
$$= \frac{1}{3}(100 + 80.77 - j2.03 + 88.63 + j15.63)$$
$$= 89.8 + j4.53 = 89.91\angle 2.89^\circ$$

$$V_{ab}^{(2)} = \frac{1}{3}(\,100\angle 0^\circ + 80.8\angle 118.56^\circ + 90\angle 250^\circ)$$
$$= \frac{1}{3}(100 - 38.63 + j70.97 - 30.78 - j84.57)$$
$$= 10.2 - j4.53 = 11.16\angle -23.95^\circ$$

$$V_{an}^{(1)} = \frac{89.91}{\sqrt{3}} \underline{/-27.11^\circ} = 46.21 - j23.66$$

$$V_{an}^{(2)} = \frac{11.16}{\sqrt{3}} \underline{/6.05^\circ} = 6.41 + j0.68$$

$$V_{an} = 52.62 - j22.98 = 57.42\underline{/-23.59^\circ}$$

$$I_a = \frac{57.42\underline{/-23.59^\circ}}{10} = 5.74\underline{/-23.59^\circ} \text{ A}$$

$$V_{bn}^{(1)} = \frac{89.91}{\sqrt{3}} \underline{/212.89^\circ} = -43.59 - j28.19$$

$$V_{bn}^{(2)} = \frac{11.16}{\sqrt{3}} \underline{/126.05^\circ} = -3.79 + j5.27$$

$$V_{bn} = -47.38 - j22.98 = 52.66\underline{/-154.13^\circ}$$

$$I_b = \frac{52.66\underline{/-154.13^\circ}}{10} = 5.27\underline{/-154.13^\circ} \text{ A}$$

$$V_{cn}^{(1)} = \frac{89.91}{\sqrt{3}} \underline{/92.89^\circ} = -2.62 + j51.84$$

$$V_{cn}^{(2)} = \frac{11.16}{\sqrt{3}} \underline{/246.05^\circ} = -2.62 - j5.89$$

$$V_{cn} = -5.24 + j45.95 = 46.25\underline{/96.51^\circ}$$

$$I_c = \frac{46.25\underline{/96.51^\circ}}{10} = 4.63\underline{/96.51^\circ} \text{ A}$$

Note that $I_a + I_b + I_c = 0$.

(11.6) Find the power expended in the three 10-Ω resistors of Prob. 11.5 from the symmetrical components of currents and voltages. Check the answer.

Solution:
From Prob. 11.5,

$$V_{an}^{(1)} = 51.91\underline{/-27.11^\circ}$$
$$V_{an}^{(2)} = 6.44\underline{/6.05^\circ}$$
$$V_{an}^{(0)} = 0$$

$$I_{an}^{(1)} = \frac{V_{an}^{(1)}}{10} = 5.19\underline{/-27.11^\circ}$$
$$I_{an}^{(2)} = \frac{V_{an}^{(2)}}{10} = 0.644\underline{/6.05^\circ}$$
$$I_{an}^{(0)} = 0$$

$$S = 3\left[51.91\underline{/-27.11^\circ} \quad 6.44\underline{/6.05^\circ}\right]\begin{bmatrix}5.19\underline{/-27.11^\circ}\\ 0.644\underline{/6.05^\circ}\end{bmatrix}^*$$
$$S = 3\ (51.91 \times 5.19 + 6.44 \times 0.644\ ) = 820.7\text{ W}$$

Check:
$(5.74)^2 \times 10 + (5.27)^2 \times 10 + (4.63)^2 \times 10 = 821.6$ W.

(11.7) If there is impedance in the neutral connection to ground of a Y-connected load, then show that the voltages V_a, V_b, and V_c of Eq. (11.26) must be interpreted as voltages with respect to ground.

Solution:
Power supplied by the source is:

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

Where the voltages V_a,, V_b and V_c are voltages with respect to ground.

Likewise, the power consumed by the load is:

$$S = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^* + V_n(I_a^* + I_b^* + I_c^*)$$
$$= (V_{an} + V_n)\,I_a^* + (V_{bn} + V_n)\,I_b^* + (V_{cn} + V_n)\,I_c^*$$

But $(V_{an} + V_n)$, $(V_{bn} + V_n)$ and $(V_{cn} + V_n)$ are voltages with respect to ground.

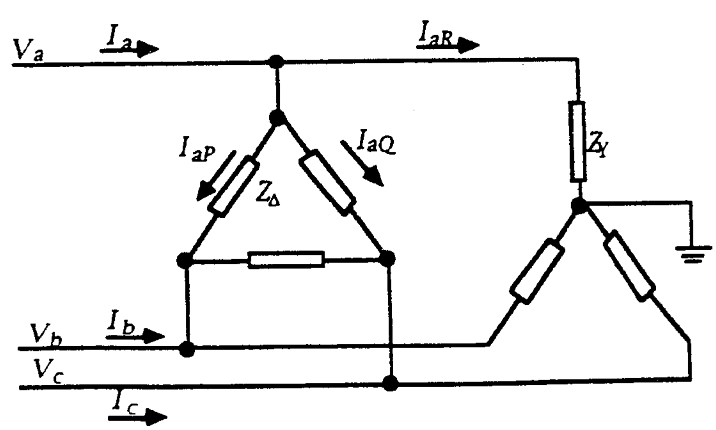
(11.8) A balanced three-phase load consists of Δ-connected impedances Z_Δ in parallel with solidly grounded Y-connected impedances Z_Y.

(*a*) Express the currents I_a, I_b, and I_c flowing in the lines from the supply source toward the load in terms of the source voltages V_a, V_b, and V_c.

(*b*) Transform the expressions of part *(a)* into their symmetrical component equivalents, and thus express $I_a^{(0)}$, $I_a^{(1)}$, and $I_a^{(2)}$ in terms of $V_a^{(0)}$, $V_a^{(1)}$ and $V_a^{(2)}$.

(*c*) Hence, draw the sequence circuit for the combined load.

Solution:



(*a*)

$$I_a = I_{aP} + I_{aQ} + I_{aR} = \frac{V_a - V_b}{Z_\Delta} + \frac{V_a - V_c}{Z_\Delta} + \frac{V_a}{Z_Y}$$

$$= \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)V_a - \frac{1}{Z_\Delta}\,[1\ 1\ 1]\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Writing similar equations for each of the three currents, and rearranging in matrix form, we have:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \frac{1}{Z_\Delta}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

where the matrix of ones is henceforth called **P**.

(*b*)

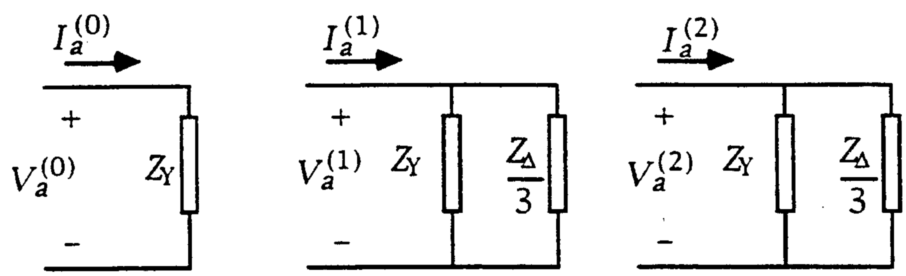
$$\mathbf{I_{abc}} = \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)\mathbf{V_{abc}} - \frac{1}{Z_\Delta}\mathbf{P}\,\mathbf{V_{abc}}$$

by premultiplying with A^{-1} and using the identity $V_{abc} = AV_{012}$, we have:

$$I_{012} = \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right) V_{012} - \frac{1}{Z_\Delta} A^{-1} P \, A V_{012}$$

$$= \begin{bmatrix} 1/Z_Y & & \\ & 3/Z_\Delta + 1/Z_Y & \\ & & 3/Z_\Delta + 1/Z_Y \end{bmatrix} V_{012}$$

(c)



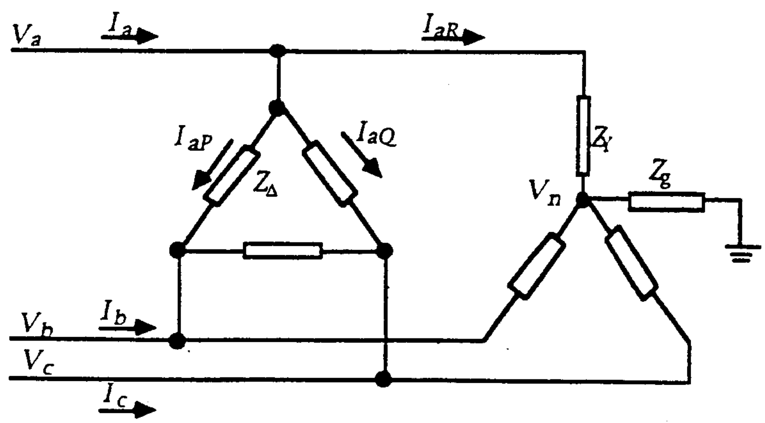
(11.9) The Y-connected impedances in parallel with the Δ-connected impedances Z_Δ of Prob. 11.8 are now grounded through an impedance Z_g.

(a) Express the currents I_a, I_b, and I_c flowing in the lines from the supply source toward the load in terms of the source voltage V_a, V_b, and V_c the voltage V_n of the neutral point.

(b) Expressing V_n in terms of $I_a^{(0)}$, $I_a^{(1)}$, $I_a^{(2)}$, and Z_g, find the equations for these currents in terms of $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$.

(*c*) Hence, draw the sequence circuit for the combined load.

Solution:



(*a*)

$$I_a = I_{aP} + I_{aQ} + I_{aR} = \frac{V_a - V_b}{Z_\Delta} + \frac{V_a - V_c}{Z_\Delta} + \frac{V_a - V_n}{Z_Y}$$

$$= \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)V_a - \frac{1}{Z_\Delta}\,[1\ 1\ 1]\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \frac{V_n}{Z_Y}$$

Writing similar equations for each of the three currents, we get:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \frac{1}{Z_\Delta}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \frac{1}{Z_Y}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} V_n$$

(b) With the matrix of ones called **P**, the above equation is premultiplied with $\mathbf{A}^{-1}$ and after using the identity $\mathbf{V}_{abc} = \mathbf{A}\mathbf{V}_{012}$, we have:

$$\mathbf{I}_{012} = \left(\frac{3}{Z_\Delta} + \frac{1}{Z_Y}\right)\mathbf{V}_{012} - \frac{1}{Z_\Delta}\mathbf{A}^{-1}\mathbf{P}\,\mathbf{A}\mathbf{V}_{012} - \frac{1}{Z_Y}\mathbf{A}^{-1}\begin{bmatrix}1\\1\\1\end{bmatrix}V_n$$

$$= \begin{bmatrix}1/Z_Y & & \\ & 3/Z_\Delta + 1/Z_Y & \\ & & 3/Z_\Delta + 1/Z_Y\end{bmatrix}\mathbf{V}_{012} - \frac{1}{Z_Y}\begin{bmatrix}1\\0\\0\end{bmatrix}V_n$$

Since $V_n = Z_g\,(I_a + I_b + I_c) = 3I_a^{(0)}$,

$$I_a^{(0)} = \frac{1}{Z_Y}V_a^{(0)} - \frac{3Z_g}{Z_Y}I_a^{(0)}$$

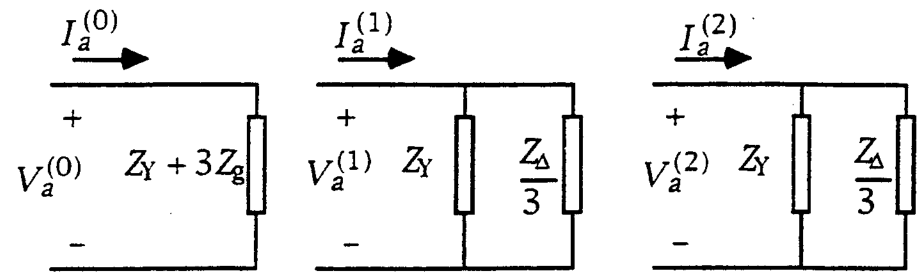
$$I_a^{(0)} = \frac{V_a^{(0)}}{Z_Y + 3Z_g}$$

Therefore, the zero-sequence admittance is

$$Y_0 = \frac{1}{Z_Y + 3Z_g}$$

and all others are unchanged.

(*c*)



(11.10) Suppose that the line-to-neutral voltages at the sending end of the line described in Example 11.5 can be maintained constant at 200-kV and that a single-phase inductive load of 420 Ω is connected between phase *a* and neutral at the receiving end

(*a*) Use Eqs. (11.51) to express numerically the receiving-end sequence voltages $V_{a'n'}^{(0)}$, $V_{a'n'}^{(1)}$, and $V_{a'n'}^{(2)}$ in terms of the load current I_L and the sequence impedances Z_0, Z_1, and Z_2 of the line.

(*b*) Hence, determine the line current I_L in amperes.

(*c*) Determine the open-circuit voltages to neutral of phases *b* and *c* at the receiving end.

(*d*) Verify your answer to part *(c)* without using symmetrical components.

Solution:

(*a*)

$$\begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} - \begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_0 I_a^{(0)} \\ Z_1 I_a^{(1)} \\ Z_2 I_a^{(2)} \end{bmatrix}$$

$I_a = I_L$, $I_b = I_c = 0$ gives $I_a^{(0)} = I_a^{(1)} = I_a^{(2)} = \frac{I_L}{3}$. Thus,

$$\begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 200\times10^3 \\ 0 \end{bmatrix} - I_L \begin{bmatrix} \frac{Z_0}{3} \\ \frac{Z_1}{3} \\ \frac{Z_2}{3} \end{bmatrix} = \begin{bmatrix} -I_L \times j\frac{160}{3} \\ 200\times10^3 - I_L \times j\frac{40}{3} \\ -I_L \times j\frac{40}{3} \end{bmatrix}$$

(*b*) Since $V_{a'n'} = V_{a'n'}^{(0)} + V_{a'n'}^{(1)} + V_{a'n'}^{(2)}$,

$$V_{a'n'} = 200\times10^3 - I_L \times \frac{j}{3}(160 + 40 + 40)$$

also, $V_{a'n'} = I_L \times j420$

Therefore,

$$I_L = \frac{200\times10^3}{j500}\,\text{A} = 400\underline{/-90^\circ}\ \text{A}$$

(*c*)

$$\begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} -400\underline{/-90^\circ} \times \frac{160}{3}\underline{/90^\circ} \\ 200\times10^3 - 400\underline{/-90^\circ} \times \frac{40}{3}\underline{/90^\circ} \\ -400\underline{/-90^\circ} \times \frac{40}{3}\underline{/90^\circ} \end{bmatrix} \text{V} = \begin{bmatrix} -\frac{64}{3}\underline{/0^\circ} \\ \frac{584}{3}\underline{/0^\circ} \\ -\frac{16}{3}\underline{/0^\circ} \end{bmatrix} \text{kV}$$

$$\begin{aligned} V_{b'n'} &= V_{a'n'}^{(0)} + 1\underline{/240^\circ}\,V_{a'n'}^{(1)} + 1\underline{/120^\circ}\,V_{a'n'}^{(2)} \\ &= -\frac{64}{3} + \frac{584}{3}\underline{/240^\circ} - \frac{16}{3}\underline{/120^\circ}\ \text{kV} \\ &= 208.46\underline{/-123.8^\circ}\ \text{kV} \end{aligned}$$

$$V_{c'n'} = V_{a'n'}^{(0)} + 1\underline{/120^\circ}V_{a'n'}^{(1)} + 1\underline{/240^\circ}V_{a'n'}^{(2)}$$
$$= -\frac{64}{3} + \frac{584}{3}\underline{/120^\circ} - \frac{16}{3}\underline{/240^\circ} \text{ kV}$$
$$= 208.46\underline{/123.8^\circ} \text{ kV}$$

(*d*) From Eq. (11.49),

$$V_{bb'} = V_{cc'} = (Z_{ab} - Z_{an})I_L - (Z_{an} - Z_{nn})I_L$$
$$= (Z_{ab} + Z_{nn} - 2Z_{an})I_L$$
$$= j(20 + 80 - 60) \times 400\underline{/-90^\circ} \text{ V}$$
$$= 16.0 \text{ kV}$$

$$V_{b'n'} = V_{bn} - V_{bb'} = 200\underline{/-120^\circ} \text{ kV} - 16\underline{/0^\circ} \text{ kV}$$
$$= 208.46\underline{/-123.8^\circ} \text{ kV}$$

$$V_{c'n'} = V_{cn} - V_{cc'} = 200\underline{/120^\circ} \text{ kV} - 16\underline{/0^\circ} \text{ kV}$$
$$= 208.46\underline{/123.8^\circ} \text{ kV}$$

(11.11) Solve Prob. 11.10 if the same 420-Ω inductive load is connected between phases *a* and *b* at the receiving end. In part (*c*) find the open-circuit voltage of phase *c* only.

Solution:

(*a*)

$$\begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} - \begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_0 I_a^{(0)} \\ Z_1 I_a^{(1)} \\ Z_2 I_a^{(2)} \end{bmatrix}$$

$I_a = -I_b = I_L$, $I_c = 0$ gives

$$I_a^{(0)} = 0; \; I_a^{(1)} = \frac{I_L}{\sqrt{3}} \angle -30^\circ \; ; \; I_a^{(2)} = \frac{I_L}{\sqrt{3}} \angle +30^\circ$$

Thus,

$$\begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 200\times10^3 \\ 0 \end{bmatrix} - I_L \begin{bmatrix} 0 \times Z_0 \\ \frac{Z_1}{\sqrt{3}} \angle -30^\circ \\ \frac{Z_2}{\sqrt{3}} \angle +30^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 200\times10^3 - \frac{40 I_L}{\sqrt{3}} \angle 60^\circ \\ -\frac{40 I_L}{\sqrt{3}} \angle 120^\circ \end{bmatrix} \text{V}$$

(b) Therefore,

$$\begin{aligned} V_{a'b'} &= V_{a'n'} - V_{b'n'} \\ &= (V_{a'n'}^{(0)} + V_{a'n'}^{(1)} + V_{a'n'}^{(2)}) - (V_{a'n'}^{(0)} + a^2 V_{a'n'}^{(1)} + a V_{a'n'}^{(2)}) \\ &= \sqrt{3}\, V_{a'n'}^{(1)} \angle +30^\circ + \sqrt{3}\, V_{a'n'}^{(2)} \angle -30^\circ \\ &= 200\times10^3 \sqrt{3} \angle +30^\circ - 80 I_L \angle 90^\circ \end{aligned}$$

also,

$$V_{a'b'} = j420 \times I_L$$

and so

$$I_L = \frac{200\times10^3 \sqrt{3} \angle 30^\circ}{500 \angle 90^\circ} = 400\sqrt{3} \angle -60^\circ \text{ A}$$

(c)

$$\begin{bmatrix} V_{a'n'}^{(0)} \\ V_{a'n'}^{(1)} \\ V_{a'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 200\times10^3 - \frac{40}{\sqrt{3}}\underline{/+60^\circ}\times400\sqrt{3}\underline{/-60^\circ} \\ -\frac{40}{\sqrt{3}}\underline{/120^\circ}\times400\sqrt{3}\underline{/-60^\circ} \end{bmatrix} \text{V} = \begin{bmatrix} 0 \\ 184\underline{/0^\circ} \\ -16\underline{/60^\circ} \end{bmatrix} \text{kV}$$

$$\begin{aligned} V_{c'n'} &= V_{a'n'}^{(0)} + V_{a'n'}^{(1)}\underline{/120^\circ} + V_{a'n'}^{(2)}\underline{/240^\circ} \\ &= 0 + 184\underline{/120^\circ} - 16\underline{/300^\circ} \\ &= 200\underline{/120^\circ}\ \text{kV} \end{aligned}$$

(d) From Eq. (11.49),

$$\begin{aligned} V_{cc'} &= (Z_{aa} - Z_{an}) \times 0 \ + \ (Z_{ab} - Z_{an})\ (I_L - I_L) + (Z_{an} - Z_{nn}) \times 0 \\ &= 0 \end{aligned}$$

Therefore,

$$V_{c'n'} = V_{cn} = 200\underline{/120^\circ}\ \text{kV}$$

(11.12) A Y-connected synchronous generator has sequence reactances $X_0 = 0.09$, $X_1 = 0.22$, and $X_2 = 0.36$, all in per unit. The neutral point of the machine is grounded through a reactance of 0.09 per unit. The machine is running on no load with rated terminal voltage when it suffers and unbalanced fault. The fault currents out of the machine are $I_a = 0$, $I_b = 3.75\underline{/150^\circ}$, and $I_c = 3.75\underline{/30^\circ}$, all in per unit with respect to phase a line-to-neutral voltage. Determine

(a) The terminal voltages in each phase of the machine with respect to ground,

(b) The voltage of the neutral point of the machine with respect to ground, and

(c) The nature (type) of the fault from the results of part *(a)*.

Solution:

(a)

$Z_1 = j0.22$ p.u., $Z_2 = 0.36$ p.u.,
$Z_0 = Z_{g0} + 3Z_n = j0.09 + 3\times j0.09 = 0.36$ p.u.

$I_a = 0$, $I_b = 3.75\angle 0^\circ$ p.u., and $I_c = 3.75\angle 0^\circ$ p.u.

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ 3.75\angle 150^\circ \\ 3.75\angle 30^\circ \end{bmatrix} = \begin{bmatrix} j1.25 \\ -j2.5 \\ j1.25 \end{bmatrix}$$

Hence,

$$V_a^{(0)} = -I_a^{(0)} Z_0 = -j1.25 \times j0.36 = 0.45 \text{ p.u.}$$

$$V_a^{(1)} = E_{an} - I_a^{(1)} Z_1 = 1\angle 0^\circ - (-j2.5 \times j0.22)$$
$$= 0.45 \text{ p.u.}$$

$$V_a^{(2)} = -I_a^{(2)} Z_2 = -j1.25 \times j0.36 = 0.45 \text{ p.u.}$$

and,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0.45\angle 0^\circ \\ 0.45\angle 0^\circ \\ 0.45\angle 0^\circ \end{bmatrix} = \begin{bmatrix} 1.35\angle 0^\circ \\ 0 \\ 0 \end{bmatrix} \text{ p.u.}$$

(b)

$$V_n = -3I_a^{(0)} \times j0.09 \text{ p.u.}$$
$$= -3 \times j1.25 \times j0.09 \text{ p.u.}$$
$$= 0.3375 \text{ p.u.}$$

(c) since $V_b = V_c = 0$, it is a double-line-to-ground fault.

(11.13) Solve Prob. 11.12 if the fault currents in per unit are $I_a = 0$, $I_b = -2.986\angle 0^\circ$, and $I_c = 2.986\angle 0^\circ$.

Solution:

(*a*)

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -2.986 \\ 2.986 \end{bmatrix} = \begin{bmatrix} 0 \\ -j1.724 \\ j1.724 \end{bmatrix}$$

$$V_a^{(0)} = -I_a^{(0)} Z_0 = 0$$

$$V_a^{(1)} = E_{an} - I_a^{(1)} Z_1 = 1\angle 0^\circ - (-j1.724)(j0.22)$$
$$= 0.621 \text{ p.u.}$$

$$V_a^{(2)} = -I_a^{(2)} Z_2 = -(-j1.724)(j0.36)$$
$$= 0.621 \text{ p.u.}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0.621 \\ 0.621 \end{bmatrix} = \begin{bmatrix} 1.242\angle 0^\circ \\ -0.621\angle 0^\circ \\ -0.621\angle 0^\circ \end{bmatrix} \text{ p.u.}$$

(*b*) Since $I_a^{(0)} = 0$, $V_n = 0$.

(*c*) Since $V_b = V_c$, it is a line-to-line fault.

(11.14) Assume that the currents specified in Prob. 11.4 are flowing toward a load from lines connected to the Y side of a Δ-Y transformer rated 10 MVA, 13.2Δ/66Y kV. Determine the currents flowing in the lines on the Δ side by converting the symmetrical components of the currents to per unit on the base of the transformer rating and by shifting the components according to Eq. (11.88). Check the results by computing the currents in each phase of the Δ windings in amperes directly from the currents on the Y side by multiplying by the turns ratio of the windings. Complete the check by computing the line currents from the phase currents on the Δ side.

Solution:
Capital letters are here used for currents to the load since the load is on the high tension side of the transformer.

Base line currents are:

$$\text{Y-side:} \quad \frac{10{,}000}{66\times\sqrt{3}} = 87.5 \text{ A}$$

$$\Delta\text{-side:} \quad \frac{10{,}000}{13.2\times\sqrt{3}} = 437.4 \text{ A}$$

Line-currents on Y-side are:

$$I_A^{(1)} = \frac{111.5\underline{/-15^\circ}}{87.5} = 1.274\underline{/-15^\circ} \text{ p.u.}$$

$$I_A^{(2)} = \frac{29.9\underline{/105^\circ}}{87.5} = 0.342\underline{/105^\circ} \text{ p.u}$$

and on the Δ-side:

$$I_a^{(1)} = jI_A^{(1)} = 1.274\underline{/75^\circ} = 0.330 + j1.231 \text{ p.u.}$$

$$I_a^{(2)} = -jI_A^{(2)} = 0.342\underline{/15^\circ} = 0.330 + j0.089 \text{ p.u}$$

thus $I_a = 0.660 + j1.320 = 1.476\underline{/63.4^\circ}$ p.u.

$$I_b^{(1)} = I_a^{(1)}\underline{/240^\circ} = 1.274\underline{/315^\circ} = 0.901 - j0.901 \text{ p.u.}$$

$$I_b^{(2)} = I_a^{(2)}\underline{/120^\circ} = -0.242 + j0.242 \text{ p.u}$$

thus $I_b = 0.659 - j0.659 = 0.932\underline{/45^\circ}$ p.u. and

$$I_c^{(1)} = I_a^{(1)}\underline{/120^\circ} = -1.231 - j0.330$$
$$I_c^{(2)} = I_a^{(2)}\underline{/240^\circ} = -0.089 - j0.330$$

and $I_c = -1.320 - j0.660 = 1.476\underline{/206.6^\circ}$ p.u.

In terms of actual currents:

$$I_a = 437.4 \times 1.476\underline{/63.4^\circ} = 645.6\underline{/63.4^\circ}\text{ A}$$
$$I_b = 407.7\underline{/-45^\circ}\text{ A}$$
$$I_c = 645.6\underline{/206.6^\circ}\text{ A}$$

The turns ratio of the transformer is $(66/\sqrt{3})/13.2 = 2.89$. Check:

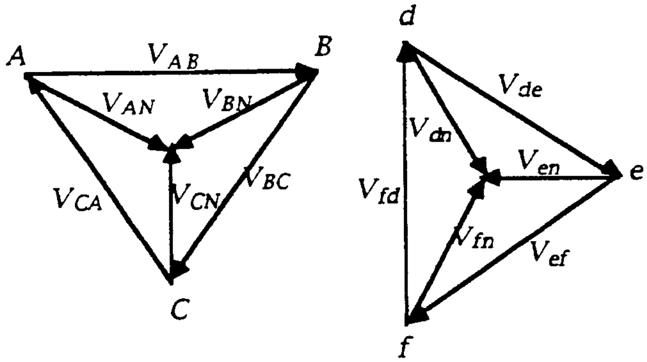
$$I_{ab} = 2.89I_C = 2.89\times 100\underline{/90^\circ} = j289$$
$$I_{bc} = 2.89I_A = 2.89\times 100\underline{/0^\circ} = 289$$
$$I_{ca} = 2.89I_B = 2.89\times 141.4\underline{/225^\circ} = -289 - j289$$

$$I_a = I_{ab} - I_{ca} = 289 + j578 = 646\underline{/63.4^\circ}$$
$$I_b = I_{bc} - I_{ab} = 289 - j289 = 408.7\underline{/-45^\circ}$$
$$I_c = I_{ca} - I_{bc} = -578 - j289 = 646\underline{/206.6^\circ}$$

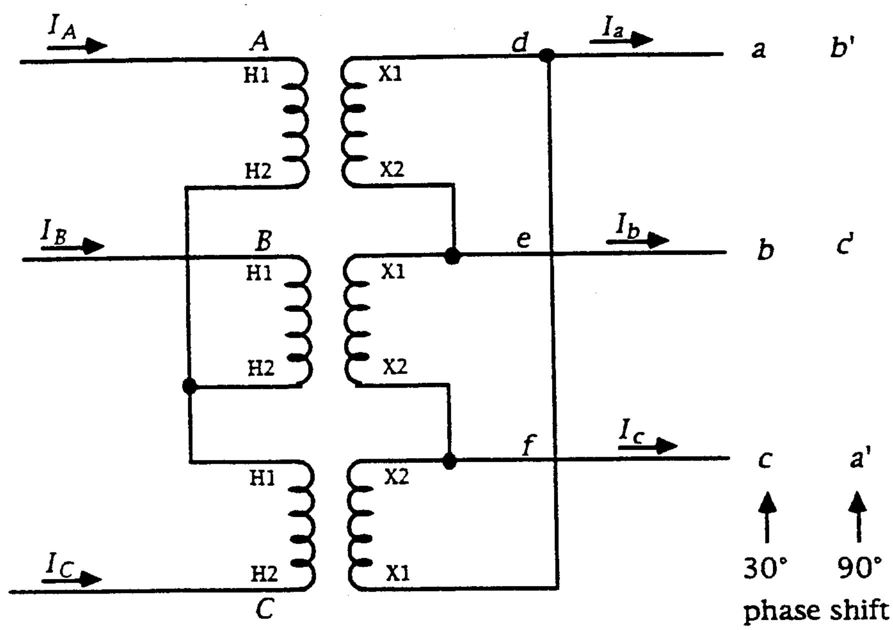
(11.15) Three single-phase transformers are connected as shown in Fig. 11.29 to form a Y-Δ transformer. The high-voltage windings are Y-connected with polarity marks as indicated. Magnetically coupled windings are drawn in parallel directions. Determine the correct placement of polarity marks on the low-voltage windings. Identify the numbered terminals on the low-voltage side (*a*) with the letters *a*, *b*, and *c*, where $I_A^{(1)}$ leads $I_a^{(1)}$ by 30°, and (*b*) with the letters *a'*, *b'*, and *c'* so that $I_a^{(1)}$ is 90° out of phase with $I_A^{(1)}$.

Solution:

The correct voltage diagram if V_{AB} is reference are shown.



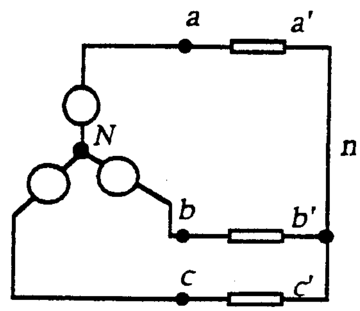
The solutions for both cases are:



(11.16) Balanced three-phase voltages of 100 V line to line are applied to a Y-connected load consisting of three resistors. The neutral of the load is not grounded. The resistance in phase *a* is 10 Ω, in phase *b* is 20 Ω, and in phase *c* is 30 Ω. Select voltage to neutral of the three-phase line as reference and determine the current in phase *a* and the voltage V_{an}.

Solution:

If an ideal generator represents the source, the circuit diagram is:



We see that $V_{aN} = V_{an} + V_{Nn}$. Voltage V_{Nn} can only be a zero-sequence voltage, and therefore $V_{aa'} = V_{an}$ has no negative sequence components. Also, $I_a^{(0)} = 0$ since node n is isolated.

With V_{aN} as reference,

$$V_{aa'}^{(1)} = \frac{100}{\sqrt{3}} \angle 0° = 57.7\angle 0° ;$$

$$V_{aa'}^{(2)} = 0$$

In order to calculate the voltage drops in the unbalanced network, the coupled sequence-impedances are determined:

$$\frac{1}{3}(Z_a + Z_b + Z_c) = 20\angle 0° \ \Omega$$
$$\frac{1}{3}(Z_a + aZ_b + a^2 Z_c) = 5.77\angle 210° \ \Omega$$
$$\frac{1}{3}(Z_a + a^2 Z_b + aZ_c) = 5.77\angle 150° \ \Omega$$

The voltage drops due to only the positive- and negative-sequence currents are:

$$\begin{bmatrix} 57.7 \\ 0 \end{bmatrix} = \begin{bmatrix} 20\angle 0^\circ & 5.77\angle 150^\circ \\ 5.77\angle 210^\circ & 20\angle 0^\circ \end{bmatrix} \begin{bmatrix} I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

And so the currents are calculated as

$$\begin{bmatrix} I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \begin{bmatrix} 20\angle 0^\circ & 5.77\angle 150^\circ \\ 5.77\angle 210^\circ & 20\angle 0^\circ \end{bmatrix}^{-1} \begin{bmatrix} 57.7 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.147 \\ 0.908\angle 30^\circ \end{bmatrix} \text{A}$$

Giving $I_a = 3.933 + j0.454$ A $= 3.959\angle 6.58^\circ$ A. The value of V_n is obtained from

$$V_{aa'}^{(0)} = 3.147 \times 5.77\angle 150^\circ + 0.908\angle 30^\circ + 5.77\angle 210^\circ$$
$$= -18.346 + j4.542$$

$$V_n = V_{aa'}^{(1)} + V_{aa'}^{(0)} = 57.7 - 18.346 + j4.4542$$
$$= 39.61\angle 6.58^\circ \text{ V}$$

or by using $I_a Z_a$.

(11.17) Draw the negative- and zero-sequence impedance networks for the power system of Prob. 3.12. Mark the values of all reactances in per unit on a base of 50 MVA, 13.8 kV in the circuit of generator 1. Letter the networks to correspond to the single-line diagram. The neutrals of generators 1 and 3 are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Each generator has negative- and zero-sequence reactances of 20 and 5%, respectively, on its own rating as base. The zero-sequence reactance of the transmission line is 210 Ω from B to C and 250 Ω from C to E.

Solution:

The negative-sequence network is the same as the positive-sequence network for the power system of Prob. 3.12 *with the EMF's short circuited* since the values for Z_2 are the same as Z_1 in all parts of the network. Consequently, it will not be drawn here. Z_0 for the transformers is the same as Z_1. The new values needed are:

Gen. 1: $Z_{0g} = j0.05 \times \frac{50}{20} = j0.125$ p.u. ;

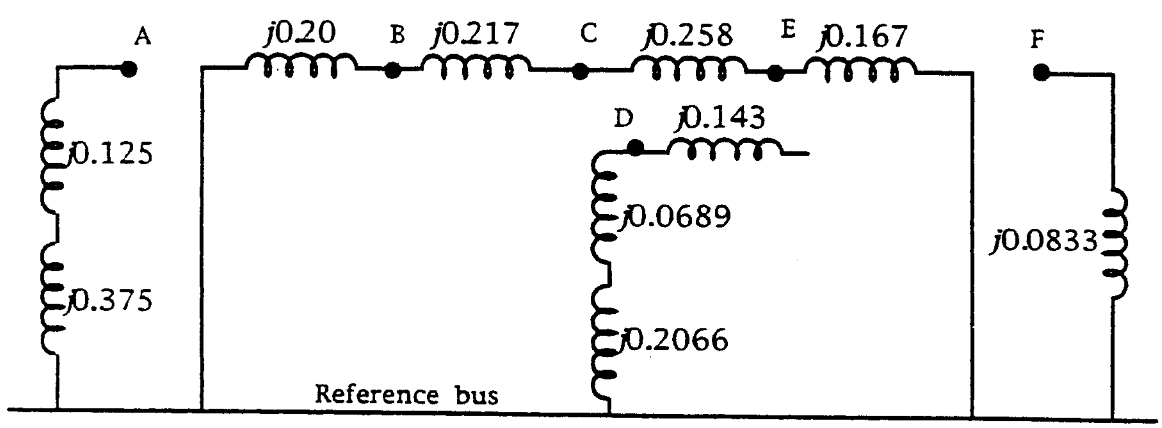
$3Z_n = 3 \times j0.05 \times \frac{50}{20} = j0.375$ p.u.

Gen. 2: $Z_{0g} = j0.05 \times \frac{50}{30} = j0.0833$ p.u. ;

Gen. 3: $Z_{0g} = j0.05 \times \frac{50}{30} \times \left(\frac{20}{22}\right)^2 = j0.0689$ p.u. ;

$3Z_n = 3 \times j0.05 \times \frac{50}{30} \times \left(\frac{20}{22}\right)^2 = j0.2066$ p.u.

And the zero-sequence reactance diagram is:



(11.18) Draw the negative- and zero-sequence impedance networks for the power system of Prob. 3.13. Choose a base of 50 MVA, 138 kV in the 40-Ω transmission line and mark all reactances in per unit. The negative-sequence reactance of each synchronous machine is equal to its subtransient reactance. The zero-sequence reactance of each machine is 8% based on its own rating. The neutrals of the machines are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Assume that the zero-sequence reactances of the transmission lines are 300% of their positive-sequence reactances.

Solution:

The negative-sequence network is the same as the positive-sequence network for the power system of Prob. 3.13 *with the EMF's short circuited* since the values for Z_2 are the same as Z_1 in all parts of the network. Consequently, it will not be drawn here. Calculations for the zero sequence network require:

G_1: $3Z_n = 3j\,0.05\left(\frac{50}{20}\right)\left(\frac{18}{20}\right)^2 = j0.304$ p.u.

G_2: $3Z_n = j0.304$ p.u.

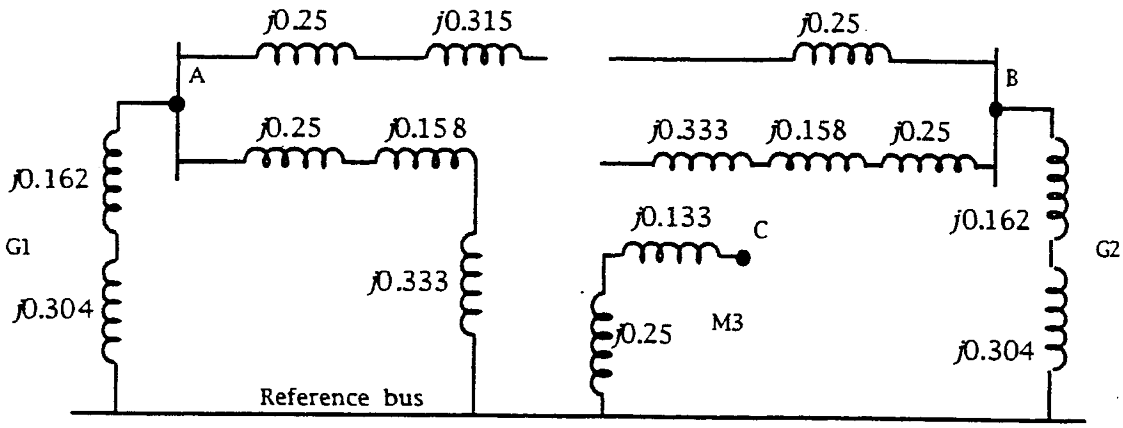
M_3: $3Z_n = 3j\,0.05\left(\frac{50}{30}\right) = j0.25$ p.u.

X_0 is three times the positive sequence reactance in the lines;

G_1 & G_2: $Z_{0g} = j\,0.08\left(\frac{50}{20}\right)\left(\frac{18}{20}\right)^2 = j0.162$ p.u.

M_3: $Z_{0g} = j\,0.08\left(\frac{50}{30}\right) = j0.1333$ p.u.

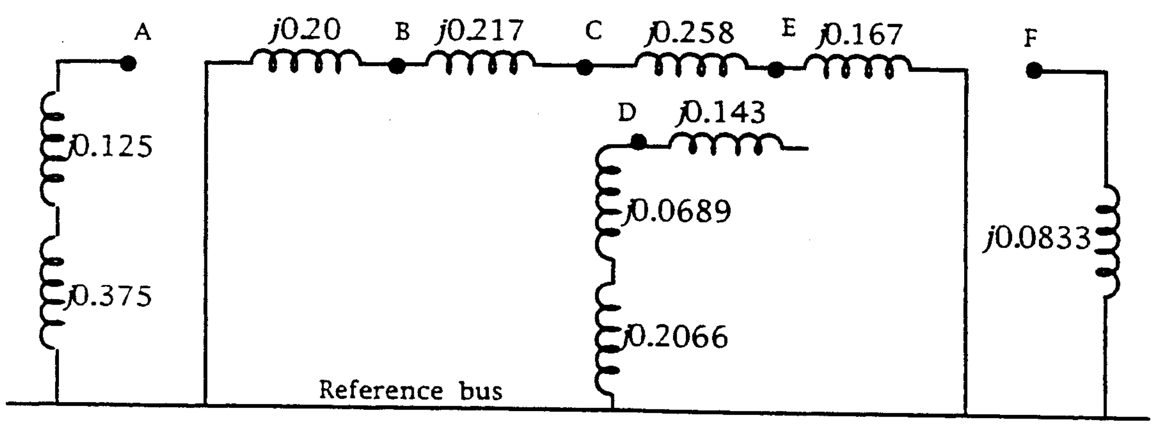
The zero-sequence reactance diagram is then:



(11.19) Determine the zero-sequence Thévénin impedance seen at bus c of the system described in Prob. 11.17 if transformer T_3 has (a) one ungrounded and one solidly grounded neutral, as shown in Fig. 3.23, and (b) both neutrals are solidly grounded.

Solution:

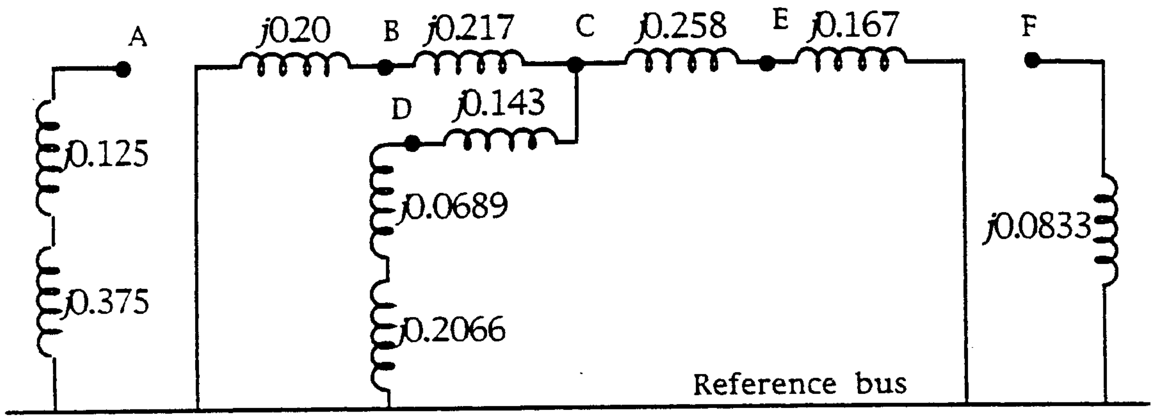
The zero-sequence network is:



(a) The zero-sequence Thévénin impedance is:

$$Z = (j0.20 + j0.217) \parallel (j0.258 + j0.167)$$
$$= j0.417 \parallel j0.425 = j0.210 \text{ p.u.}$$

(*b*) If both neutrals of T3 are solidly grounded, then the zero-sequence network becomes:



and the zero-sequence Thévénin impedance becomes:

$$Z = j0.417 \parallel j0.425 \parallel (j0.143 + j0.0689 + j0.2066)$$
$$= j0.417 \parallel j0.425 \parallel j0.425 = j0.140 \text{ p.u.}$$