From Eq. (12.27),

$$\Delta V_3^{(1)} = \frac{j0.1104 - j0.1696}{j0.15} \times 0.4050\angle 58.21^\circ = 0.1598\angle -121.79^\circ \text{ per unit}$$
$$\Delta V_3^{(2)} = \frac{j0.1104 - j0.1696}{j0.15} \times 0.1646\angle -121.79^\circ = 0.0650\angle 58.21^\circ \text{ per unit}$$
$$\Delta V_3^{(0)} = \frac{j0.0701 - j0.1999}{j0.50} \times 0.2404\angle -121.79^\circ = 0.0624\angle 58.21^\circ \text{ per unit}$$

Change in a-phase voltage at bus ③ is

$$\Delta V_3 = \Delta V_3^{(1)} + \Delta V_3^{(2)} + \Delta V_3^{(0)} = (0.1598 - 0.0650 - 0.8624)\angle -121.79^\circ = 0.0324\angle -121.79^\circ \text{ per unit}$$

Chapter 13 Problem Solutions

13.1 For a generating unit the fuel input in millions of Btu/h is expressed as a function of output P_g in megawatts by $0.032P_g^2 + 5.8P_g + 120$. Determine

(a) the equation for incremental fuel cost in dollars per megawatthour as a function of P_g in megawatts based on a fuel cost of \$2 per million Btu.

(b) the average cost of fuel per megawatthour when $P_g = 200$ MW.

(c) the approximate additional fuel cost per hour to raise the output of the unit from 200 MW to 201 MW. Also find this additional cost accurately and compare it with the approximate value.

Solution:

(a) The input-output curve in dollars per MWh is

$$f = (0.032P_g^2 + 5.8P_g + 120) \times 2$$
$$= 0.064P_g^2 + 11.6P_g + 240 \text{ \$/MWh}$$

The incremental fuel cost is

$$\frac{df}{dP_g} = 0.128P_g + 11.6 \text{ \$/MWh}$$

(b) The average cost of fuel when $P_g = 200$ MW is

$$\left.\frac{f}{P_g}\right|_{P_g=200} = \frac{0.064(202)^2 + 11.6(200) + 240}{200} = 25.6 \text{ \$/MWh}$$

(c) The approximate incremental cost for an additional 1 MW generation when $P_g = 200$ MW is

$$\left.\frac{df}{dP_g}\right|_{P_g=200} = 0.128(200) + 11.6 = 37.2 \text{ \$/h}$$

The additional cost per hour to raise the output from 200 MW to 201 MW can be calculated accurately as follows:

$$\int_{200}^{201} (0.128P_g + 11.6)\,P_g \;=\; 0.064P_g^2 + 11.6P_g\Big|_{200}^{201} \;=\; 37.264\ \$/\text{h}$$

13.2 The incremental fuel costs in \$/MWh for four units of a plant are

$$\lambda_1 = \frac{d\,f_1}{dP_{g\,1}} = 0.012P_{g\,1} + 9.0 \qquad \lambda_2 = \frac{d\,f_2}{dP_{g\,2}} = 0.0096P_{g\,2} + 6.0$$
$$\lambda_3 = \frac{d\,f_3}{dP_{g\,3}} = 0.008P_{g\,3} + 8.0 \qquad \lambda_4 = \frac{d\,f_4}{dP_{g\,4}} = 0.0068P_{g\,4} + 10.0$$

Assuming that all four units operate to meet the total plant load of 800 MW, find the incremental fuel cost λ of the plant and the required output of each unit for economic dispatch.

Solution:

$$\begin{aligned}
a_T &= \left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right)^{-1} \\
&= \left(\frac{1}{0.012}+\frac{1}{0.0096}+\frac{1}{0.008}+\frac{1}{0.0068}\right)^{-1} = 2.176\times10^{-3} \\
b_T &= a_T\left(\frac{b_1}{a_1}+\frac{b_2}{a_2}+\frac{b_3}{a_3}+\frac{b_4}{a_4}\right) \\
&= 2.176\times10^{-3}\left(\frac{9}{0.012}+\frac{6}{0.0096}+\frac{8}{0.008}+\frac{10}{0.0068}\right) = 8.368 \\
P_{gT} &= 800\ \text{MW}
\end{aligned}$$

The λ of the plant is given by

$$\lambda = a_T P_{gT} + b_T = 2.176\times10^{-3}\times 800 + 8.368 = 10.1088\ \$/\text{MWh}$$

Using Eq. (13.4), for each unit we have

$$\begin{aligned}
P_{g\,1} &= \frac{\lambda - b_1}{a_1} = \frac{10.1088-9}{0.012} = 92.4\ \text{MW} \\
P_{g\,2} &= \frac{\lambda - b_2}{a_2} = \frac{10.1088-6}{0.0096} = 428\ \text{MW} \\
P_{g\,3} &= \frac{\lambda - b_3}{a_3} = \frac{10.1088-8}{0.008} = 263.6\ \text{MW} \\
P_{g\,4} &= \frac{\lambda - b_4}{a_4} = \frac{10.1088-10}{0.0068} = 16\ \text{MW}
\end{aligned}$$

13.3 Assume that maximum load on each of the four units described in Prob. 13.2 is 200 MW, 400 MW, 270 MW and 300 MW, respectively, and that minimum load on each unit is 50 MW, 100 MW, 80 MW and 110 MW, respectively. With these maximum and minimum output limits, find the plant λ and MW output of each unit for economic dispatch.

Solution:

The solution to Prob. 13.2 shows that each unit's output would be 92.4 MW, 428 MW, 263.6 MW and 16 MW, respectively, if there were no maximum and minimum limits on unit outputs. It is seen that the output of Unit 2 violates its upper limit, and the output of Unit 4 violates its lower limit. This fact does not necessarily mean that the outputs of Units 2 and 4 should be set at their upper and lower limits, respectively. In fact, these limits should be checked individually.

First, assume that Unit 2 is operating at its upper limit of 400 MW. Using the remaining Units 1, 2 and 3, we calculate the plant λ as follows:

$$a_T = \left(\frac{1}{a_1}+\frac{1}{a_3}+\frac{1}{a_4}\right)^{-1} = \left(\frac{1}{0.012}+\frac{1}{0.008}+\frac{1}{0.0068}\right)^{-1} = 2.813793\times10^{-3}$$
$$b_T = a_T\left(\frac{b_1}{a_1}+\frac{b_3}{a_3}+\frac{b_4}{a_4}\right) = 2.813793\times10^{-3}\left(\frac{9}{0.012}+\frac{8}{0.008}+\frac{10}{0.0068}\right) = 9.062069$$

Since $P_{g\,2} = 400$ MW, the total output of Units 1, 3 and 4 should be 400 MW. Therefore,

$$\lambda = a_T P_{gT} + b_T = 2.813793\times10^{-3}\times400+9.062069 = 10.187586 \text{ \$/MWh}$$

Using this plant λ, each unit's output is

$$P_{g\,1} = \frac{\lambda-b_1}{a_1} = \frac{10.187586-9}{0.012} = 98.9655 \text{ MW}$$
$$P_{g\,3} = \frac{\lambda-b_3}{a_3} = \frac{10.187586-8}{0.008} = 273.4483 \text{ MW}$$
$$P_{g\,4} = \frac{\lambda-b_4}{a_4} = \frac{10.187586-10}{0.0068} = 27.5862 \text{ MW}$$
$$P_{g\,2} \triangleq 400 \text{ MW}$$

It is seen that the outputs of Units 3 and 4 violate their respective upper and lower limits. Consequently it is concluded that other units besides Unit 2 need be operating at their limits if the output of Unit 2 is specified to be 400 MW.

This time assume that Unit 4 is operating at its lower limit of 110 MW. Using Units 1, 2 and 3 only, the plant λ is calculated as follows:

$$a_T = \left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012}+\frac{1}{0.0096}+\frac{1}{0.008}\right)^{-1} = 3.2\times10^{-3}$$
$$b_T = a_T\left(\frac{b_1}{a_1}+\frac{b_2}{a_2}+\frac{b_3}{a_3}\right) = 3.2\times10^{-3}\left(\frac{9}{0.012}+\frac{6}{0.0096}+\frac{8}{0.008}\right) = 7.6$$

Since $P_{g\,4} = 110$ MW, the total output of Units 1, 2 and 3 should be 690 MW. Therefore,

$$\lambda = a_T P_{gT} + b_T = 3.2\times10^{-3}\times690+7.6 = 9.808 \text{ \$/MWh}$$

Using this plant λ, each unit's output is

$$P_{g\,1} = \frac{\lambda-b_1}{a_1} = \frac{9.808-9}{0.012} = 67.3333 \text{ MW}$$

$$P_{g2} = \frac{\lambda - b_2}{a_2} = \frac{9.808 - 6}{0.0096} = 396.6667 \text{ MW}$$
$$P_{g3} = \frac{\lambda - b_3}{a_3} = \frac{9.808 - 8}{0.008} = 226 \text{ MW}$$
$$P_{g4} \triangleq 110 \text{ MW}$$

Apparently there are no limit violations here. Therefore, economic dispatch requires that the output of Unit 4 be set to its lower limit of 110 MW and that the outputs of the remaining units be those obtained above.

13.4 Solve Prob. 13.3 when the minimum load on Unit 4 is 50 MW rather than 110 MW.

Solution:

It was shown in Prob. 13.3 that if the output of Unit 2 is set to its maximum limit of 400 MW, some other units will also have to be operating at their limits. We now examine whether load limit constraints will be violated if Unit 4 is operating at its new lower limit of 50 MW.

Using Units 1, 2 and 3, the plant λ is calculated as follows:

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.0096} + \frac{1}{0.008}\right)^{-1} = 3.2 \times 10^{-3}$$
$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3}\right) = 3.2 \times 10^{-3}\left(\frac{9}{0.012} + \frac{6}{0.0096} + \frac{8}{0.008}\right) = 7.6$$

Since $P_{g4} = 50$ MW, the total output of Units 1,2 and 3 should be 750 MW. Therefore,

$$\lambda = a_T P_{gT} + b_T = 3.2 \times 10^{-3} \times 750 + 7.6 = 10 \text{ \$/MWh}$$

Each unit's output is calculated as follows:

$$P_{g1} = \frac{\lambda - b_1}{a_1} = \frac{10 - 9}{0.012} = 83.3333 \text{ MW}$$
$$P_{g2} = \frac{\lambda - b_2}{a_2} = \frac{10 - 6}{0.0096} = 416.6667 \text{ MW}$$
$$P_{g3} = \frac{\lambda - b_3}{a_3} = \frac{10 - 8}{0.008} = 250 \text{ MW}$$

It is noted that the output of Unit 2 exceeds its maximum load limit.

It follows from the above analysis that both Units 2 and 4 should be operating at their upper and lower limits, respectively. Therefore, let $P_{g2} \triangleq 400$ MW and $P_{g4} \triangleq 50$ MW, and find the plant λ as follows:

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.008}\right)^{-1} = 4.8 \times 10^{-3}$$
$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_3}{a_3}\right) = 4.8 \times 10^{-3}\left(\frac{9}{0.012} + \frac{8}{0.008}\right) = 8.4$$

The plant λ in this case is

$$\lambda = a_T P_{gT} + b_T = 4.8 \times 10^{-3} \times 350 + 8.4 = 10.08 \text{ \$/MWh}$$

The outputs of Units 1 and 3 are calculated to be

$$P_{g1} = \frac{\lambda - b_1}{a_1} = \frac{1008 - 9}{0.012} = 90 \text{ MW}$$
$$P_{g3} = \frac{\lambda - b_3}{a_3} = \frac{1008 - 8}{0.008} = 260 \text{ MW}$$

In summary,

$$P_{g1} = 90 \text{ MW} \qquad P_{g3} = 260 \text{ MW}$$
$$P_{g2} = 400 \text{ MW} \qquad P_{g4} = 50 \text{ MW}$$

13.5 The incremental fuel costs for two units of a plant are

$$\lambda_1 = \frac{d f_1}{dP_{g1}} = 0.012P_{g1} + 8.0 \qquad \lambda_2 = \frac{d f_2}{dP_{g2}} = 0.008P_{g2} + 9.6$$

where f is in dollars per hour and P_g is in megawatts. If both units operate at all times and maximum and minimum loads on each unit are 550 MW and 100 MW, plot λ of the plant in \$/MWh versus plant output in MW for economic dispatch as total load varies from 200 to 1100 MW.

Solution:

At their lower limit of 100 MW, the incremental costs of the units are calculated as

$$\frac{d f_1}{dP_{g1}} = 0.012P_{g1} + 8.0\Big|_{P_{g1}=100} = 9.2$$
$$\frac{d f_2}{dP_{g2}} = 0.008P_{g2} + 9.6\Big|_{P_{g2}=100} = 10.4$$

As the plant output exceeds 200 MW, initially the incremental fuel cost λ of the plant is determined by Unit 1 alone and the additional power should come from Unit 1. This will continue until the incremental fuel cost of Unit 1 becomes \$10.4/MWh, (i.e., $0.012P_{g1}+8.0 = 10.4$) from which the values of $P_{g1} = 200$ MW. Therefore, for $200 \le P_{gT} \le 300$,

$$\lambda = 0.012P_{g1} + 8.0 = 0.012(P_{gT} - 100) + 8.0 = 0.012P_{gT} + 6.8$$

For $P_{gT} > 300$, both units will increase their outputs simultaneously. To determine which unit will reach its upper limit first, we calculate incremental costs at the upper limit as follows:

$$\frac{d f_1}{dP_{g1}}\Big|_{P_{g1}=550} = 0.012P_{g1} + 8.0\Big|_{P_{g1}=550} = 14.6$$
$$\frac{d f_2}{dP_{g2}}\Big|_{P_{g2}=550} = 0.008P_{g2} + 9.6\Big|_{P_{g2}=550} = 14.0$$

The result shows that Unit 2 will reach its maximum load limit earlier than Unit 1. The value of P_{g1} for which the incremental cost becomes \$14.0/MWh is computed from $0.012P_{g1}+8.0 = 14.0$ which yields $P_{g1} = 500$ MW. For $300 \le P_{gT} \le 1050$, the plant λ is calculated. Since the incremental fuel costs of Units 1 and 2 should be the same, we have

$$0.012P_{g1} + 8.0 = 0.008P_{g2} + 9.6$$

from which $P_{g2} = 1.5P_{g1} - 200$. Since $P_{g1} + P_{g2} = P_{gT}$, P_{g1} can be represented in terms of P_{gT} as $P_{g1} + 1.5P_{g1} - 200 = P_{gT}$ from which $P_{g1} = 0.4P_{gT} + 80$. The plant λ is then given by

$$\lambda = 0.012P_{g1} + 8.0 = 0.012\,(0.4P_{gT} + 80) + 8.0 = 0.0048P_{gT} + 8.96$$

For $P_{gT} > 1050$, only Unit 1 will have an excess capacity, and the plant λ is determined by Unit 1 alone as

$$\lambda = 0.012P_{g1} + 8.0 = 0.012\,(P_{gT} - 550) + 8.0 = 0.012P_{gT} + 1.4$$

The results are summarized as follows:

$$\begin{array}{ll} \text{For } 200 \le P_{gT} \le 300 & \lambda = 0.012P_{gT} + 6.8 \\ \text{For } 300 \le P_{gT} \le 1050 & \lambda = 0.0048P_{gT} + 8.96 \\ \text{For } 1050 \le P_{gT} \le 1100 & \lambda = 0.012P_{gT} + 1.4 \end{array}$$

13.6 Find the savings in \$/h for economic dispatch of load between the units of Prob. 13.5 compared with their sharing the output equally when the total plant output is 600 MW.

Solution:

Economic dispatch for $P_{gT} = 600$ MW requires that

$$0.012P_{g1} + 8.0 = 0.008P_{g2} + 9.6 \quad \text{and} \quad P_{g1} + P_{g2} = 600$$

Solving the two equations for P_{g1} and P_{g2} yields

$$P_{g1} = 320 \text{ MW} \qquad P_{g2} = 280 \text{ MW}$$

The savings calculations are as follows:

$$\begin{aligned} \text{Net savings} &= \int_{320}^{300} \frac{df_1}{dP_{g1}}\,dP_{g1} + \int_{280}^{300} \frac{df_2}{dP_{g2}}\,dP_{g2} \\ &= \int_{320}^{300} (0.012P_{g1} + 8)\,dP_{g1} + \int_{280}^{300} (0.008P_{g2} + 9.6)\,dP_{g2} \\ &= (0.006P_{g1}^2 + 8P_{g1})\Big|_{320}^{300} + (0.004P_{g2}^2 + 9.6P_{g2})\Big|_{280}^{300} \\ &= -234.4 + 238.4 = 4.0 \text{ \$/h} \end{aligned}$$

13.7 A power system is supplied by three plants, all of which are operating on economic dispatch. At the bus of Plant 1 the incremental cost is \$10.0 per MWh, at Plant 2 it is \$9.0 per MWh and at Plant 3 it is \$11.0 per MWh. Which plant has the highest penalty factor and which one has the lowest penalty factor? Find the penalty factor of Plant 1 if the cost per hour to increase the total delivered load by 1 MW is \$12.0.

Solution:

Since the system λ should satisfy the equation

$$\lambda = 10.0L_1 = 9.0L_2 = 11.0L_3$$

it must be the case that $L_2 > L_1 > L_3$. Given the system λ of \$12.0/MWh, the penalty factor of plant 1, L_1, can be calculated from $10.0L_1 = 12.0$ from which we have

$$L_1 = 1.2$$

13.8 A power system has two generating plants and B-coefficients corresponding to Eq. (13.37) which are given in per unit on a 100 MVA base by

$$\left[\begin{array}{cc|c} 5.0 & -0.03 & 0.15 \\ -0.03 & 8.0 & 0.20 \\ \hline 0.15 & 0.20 & 0.06 \end{array}\right] \times 10^{-3}$$

The incremental fuel costs in \$/MWh of the generating units at the two plants are

$$\lambda_1 = \frac{df_1}{dP_{g1}} = 0.012P_{g1} + 6.6 \qquad \lambda_2 = \frac{df_2}{dP_{g2}} = 0.0096P_{g2} + 6.0$$

If Plant 1 presently supplies 200 MW and Plant 2 supplies 300 MW, find the penalty factors of each plant. Is the present dispatch most economical? If not, which plant output should be increased and which one should be decreased? Explain why.

Solution:

The power loss P_L is given by the equation

$$P_L = \left[\begin{array}{cc|c} P_{g1} & P_{g2} & 1 \end{array}\right] \left[\begin{array}{cc|c} 5\times10^{-3} & -0.03\times10^{-3} & 0.15\times10^{-3} \\ -0.03\times10^{-3} & 8\times10^{-3} & 0.2\times10^{-3} \\ \hline 0.15\times10^{-3} & 0.2\times10^{-3} & 0.06\times10^{-3} \end{array}\right] \left[\begin{array}{c} P_{g1} \\ P_{g2} \\ \hline 1 \end{array}\right]$$

$$= 5\times10^{-3}P_{g1}^2 - 2\left(0.03\times10^{-3}\right)P_{g1}P_{g2} + 8\times10^{-3}P_{g2}^2 + 0.15\times10^{-3}P_{g1} + 0.2\times10^{-3}P_{g2} + 0.06\times10^{-3}$$

where P_{g1} and P_{g2} are in per unit on the 100 MVA base. Penalty factors are calculated as

$$L_1 = \frac{1}{1-\frac{\partial P_L}{\partial P_{g1}}} = \frac{1}{1-\left\{2\times(5\times10^{-3})P_{g1} - 2(0.03\times10^{-3})P_{g2} + 0.15\times10^{-3}\right\}\Big|_{\substack{P_{g1}=2\\P_{g2}=3}}}$$

$$= \frac{1}{1-19.97\times10^{-3}} = 1.0203769$$

$$L_2 = \frac{1}{1-\frac{\partial P_L}{\partial P_{g2}}} = \frac{1}{1-\left\{2\times(8\times10^{-3})P_{g2} - 2(0.03\times10^{-3})P_{g1} + 0.2\times10^{-3}\right\}\Big|_{\substack{P_{g1}=2\\P_{g2}=3}}}$$

$$= \frac{1}{1-48.08\times10^{-3}} = 1.0505084$$

The incremental fuel costs at the two plant buses are calculated to be

$$\left.\frac{df_1}{dP_{g1}}\right|_{P_{g1}=200} = 0.012 \times 200 + 6.6 = 9.0,$$
$$\left.\frac{df_2}{dP_{g2}}\right|_{P_{g2}=300} = 0.0096 \times 300 + 6.0 = 8.88$$

when penalty factors are incorporated into the incremental fuel costs, we have

$$L_1\frac{df_1}{dP_{g1}} = 9.1833921 \qquad L_2\frac{df_2}{dP_{g2}} = 9.3285146$$

Since $L_1\,(df_1/dP_{g1})$ is smaller than $L_2\,(df_2/dP_{g2})$, the output of plant 1 should be increased while that of plant 2 should be decreased to achieve economic dispatch.

13.9 Using \$10.0/MWh as the starting value of system λ in Example 13.4, perform the necessary calculations during the first iteration to obtain an updated λ.

Solution:

Step 1: $P_D = 5$ per unit.

Step 2: $\lambda^{(1)} = 10.0$ is given.

Step 3: It follows from Eq. (13.43) that the following two equations should be solved for P_{g1} and P_{g2}.

$$(a_1P_{g1} + b_1) - \lambda + \lambda(2B_{11}P_{g1} + 2B_{12}P_{g2} + B_{10}) = 0$$
$$(a_2P_{g2} + b_2) - \lambda + \lambda(2B_{22}P_{g2} + 2B_{21}P_{g1} + B_{20}) = 0$$

which are rewritten as

$$(a_1 + 2\lambda B_{11})P_{g1} + (2\lambda B_{12})P_{g2} = (-b_1 + \lambda - \lambda B_{10})$$
$$(2\lambda B_{21})P_{g1} + (a_2 + 2\lambda B_{22})P_{g2} = (-b_2 + \lambda - \lambda B_{20})$$

Upon substituting proper values, we have

$$\left(0.8 + 2 \times 10 \times 8.383183 \times 10^{-3}\right)P_{g1} + \left(2 \times 10 \times (-0.049448) \times 10^{-3}\right)P_{g2} = -8 + 10 - 10 \times 0.750164 \times 10^{-3}$$
$$\left(2 \times 10 \times (-0.049448) \times 10^{-3}\right)P_{g1} + \left(0.96 + 2 \times 10 \times 5.963568 \times 10^{-3}\right)P_{g2} = -6.4 + 10 - 10 \times 0.38994 \times 10^{-3}$$

which are rewritten as

$$0.9676636P_{g1} - 0.0009890P_{g2} = 1.9924984$$
$$-0.0009890P_{g1} + 1.0792714P_{g2} = 3.5961006$$

The values of P_{g1} and P_{g2} which solve the above equations are

$$P_{g1} = 2.062489 \qquad P_{g2} = 3.333861$$

Step 4: The transmission loss is computed to be

$$\begin{aligned} P_L &= B_{11}P_{g1}^2 + 2B_{12}P_{g1}P_{g2} + B_{22}P_{g2}^2 + B_{10}P_{g1} + B_{20}P_{g2} + B_{00} \\ &= 8.383183 \times 10^{-3} \times (2.062489)^2 + 2 \times (-0.049448) \times 10^{-3} \times 2.062489 \times 3.333861 \\ &\quad + 5.963568 \times 10^{-3} \times (3.333861)^2 + 0.750164 \times 10^{-3} \times 2.062489 \\ &\quad + 0.38994 \times 10^{-3} \times 3.333861 + 0.090121 \times 10^{-3} = 0.104201 \text{ per unit} \end{aligned}$$

Step 5: Since $P_D = 5$ per unit,

$$\begin{aligned} P_D + P_L - \left(P_{g1}^{(1)} + P_{g2}^{(1)}\right) &= 5 + 0.104201 - (2.062489 + 3.333861) \\ &= -0.292148 \text{ per unit} \end{aligned}$$

The incremental change in λ is calculated from Eq. (13.58) as follows:

$$\begin{aligned} \Delta\lambda^{(1)} &= \left(\lambda^{(1)} - \lambda^{(0)}\right) \frac{P_D + P_L^{(1)} - \left(P_{g1}^{(1)} + P_{g2}^{(1)}\right)}{\left(P_{g1}^{(1)} + P_{g2}^{(1)}\right) - \left(P_{g1}^{(0)} + P_{g2}^{(0)}\right)} \\ &= (10 - 0)\frac{-0.292149}{5.396350 - 0} = -0.5413814 \end{aligned}$$

and the updated λ becomes

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 10 - 0.5413826 = 9.4586174$$

13.10 Suppose that bus ② of a four-bus system is a generator bus and at the same time a load bus. By defining both a generation current and a load current at bus ② as shown in Fig. 13.5c, find the transformation matrix **C** for this case in the form shown in Eq. (13.31).

Solution:

Let the generator and load currents at bus ② be denoted by I_2^g and I_2^d, respectively. The total system load current is given by

$$I_D = I_2^d + I_3 + I_4$$

Constants d_2, d_3 and d_4 are then obtained to be

$$d_2 = \frac{I_2^d}{I_D} \qquad d_3 = \frac{I_3}{I_D} \qquad d_4 = \frac{I_4}{I_D}$$

Since the net current injection at bus ②, I_2, is $I_2^g + I_2^d$, from Eq. (13.23) we have

$$\begin{aligned} V_{1n} &= Z_{11}I_1 + Z_{12}\left(I_2^g + I_2^d\right) + Z_{13}I_3 + Z_{14}I_4 \\ &= Z_{11}I_1 + Z_{12}I_2^g + (d_2Z_{12} + d_3Z_{13} + d_4Z_{14})\, I_D \end{aligned}$$

from which we also get

$$\begin{aligned} I_D &= \frac{-Z_{11}}{d_2Z_{12} + d_3Z_{13} + d_4Z_{14}} I_1 + \frac{-Z_{12}}{d_2Z_{12} + d_3Z_{13} + d_4Z_{14}} I_2^g + \frac{-Z_{11}}{d_2Z_{12} + d_3Z_{13} + d_4Z_{14}} I_n^0 \\ &\triangleq -t_1I_1 - t_2I_2^g - t_1I_n^0 \end{aligned}$$

where $I_n^0 = -V_{1n}/Z_{11}$. Now the load currents can be represented in terms of generator currents and the no-load current as follows:

$$\begin{aligned} I_2^d &= -d_2t_1I_1 - d_2t_2I_2^g - d_2t_1I_n^0 \\ I_3 &= -d_3t_1I_1 - d_3t_2I_2^g - d_3t_1I_n^0 \\ I_4 &= -d_4t_1I_1 - d_4t_2I_2^g - d_4t_1I_n^0 \end{aligned}$$

The transformation $\mathbf{C}$ of old currents I_1, $I_2^g + I_2^d$, I_3 and I_4 to the generator and no-load currents I_1, I_2^g and I_n^0 is defined as

$$\begin{bmatrix} I_1 \\ I_2^g + I_2^d \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d_2t_1 & 1-d_2t_2 & -d_2t_1 \\ -d_3t_1 & d_3t_2 & -d_3t_1 \\ -d_4t_1 & d_4t_2 & -d_4t_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2^g \\ I_n^0 \end{bmatrix} \triangleq \mathbf{C} \begin{bmatrix} I_1 \\ I_2^g \\ I_n^0 \end{bmatrix}$$

13.11 The four-bus system depicted in Fig. 13.5 has bus and line data given in Table 13.2. Suppose that the bus data is slightly modified such that at bus ②, P-generation is 4.68 per unit, and P-load and Q-load are 1.5 per unit and 0.9296 per unit, respectively. Using the results of Table 13.3, find the power-flow solution corresponding to this modified bus data. Using the solution to Prob. 13.10, also find the B-coefficients of this modified problem in which there is load as well as generation at bus ②.

Solution:

The power flow solution should be the same as that of Table 13.3 except that P- and Q-generation at bus ② needs to be modified to account for the load at that bus. Using P- and Q-generation of 3.8 per unit and 1.325439 per unit as shown in Table 13.3, and adding to those P- and Q-load at bus ②, we get

$$\begin{aligned} P_{g2} &= 3.18 + 1.5 = 4.68 \text{ per unit} \\ Q_{g2} &= 1.325439 + 0.9296 = 2.255039 \text{ per unit} \end{aligned}$$

The bus voltages and P- and Q-generation at bus ① should remain the same. The load currents are calculated from the power-flow results as follows:

$$\begin{aligned} I_2^d &= \frac{P_2^d - jQ_2^d}{V_2^*} = \frac{-1.5 + j0.9296}{1.0\underline{/-2.43995^\circ}} = -1.5382150 + j0.8648990 \\ I_3 &= \frac{P_3 - jQ_3}{V_3^*} = \frac{-2.2 + j1.36340}{0.96051\underline{/1.07932^\circ}} = -2.2633193 + j1.4623529 \\ I_4 &= \frac{P_4 - jQ_4}{V_4^*} = \frac{-2.8 + j1.73520}{0.94304\underline{/2.62658^\circ}} = -2.881685 + j1.9741431 \end{aligned}$$

Constants d_2, d_3 and d_4 are then found to be

$$\begin{aligned} d_2 &= \frac{I_2^d}{I_2^d + I_3 + I_4} = 0.2216412 + j0.0132372 \\ d_3 &= \frac{I_3}{I_2^d + I_3 + I_4} = 0.3390421 - j0.0005983 \\ d_4 &= \frac{I_4}{I_2^d + I_3 + I_4} = 0.4393167 - j0.0126389 \end{aligned}$$

Constants t_1 and t_2 defined in Prob. 13.10 are calculated to be

$$t_1 = \frac{Z_{11}}{d_2 Z_{12} + d_3 Z_{13} + d_4 Z_{14}} = 0.9930664 + j0.0013435$$
$$t_2 = \frac{Z_{12}}{d_2 Z_{12} + d_3 Z_{13} + d_4 Z_{14}} = 1.0020780 - j0.0004610$$

Using the constants d_2, d_3, d_4, t_1 and t_2, the transformation $\mathbf{C}$ defined in Prob. 13.10 is obtained:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ -0.2200866 - j0.0134432 & 0.7778921 - j0.0131625 & -0.2200866 - j0.0134432 \\ -0.3366921 + j0.0001386 & -0.3397464 + j0.0007558 & -0.3366921 + j0.0001386 \\ -0.4362876 + j0.0119610 & -0.4402238 + j0.0128677 & -0.4362876 + j0.0119610 \end{bmatrix}$$

Using $\mathbf{R}_{\text{bus}}$ given in Example 13.3, we then find

$$\mathbf{C}^T \mathbf{R}_{\text{bus}} \mathbf{C}^* = \begin{bmatrix} 4.543134 + j0 & -0.892927 - j0.076535 & 0.938793 - j0.023045 \\ -0.892927 + j0.076535 & 3.075414 + j0 & 0.194991 + j0.054548 \\ 0.938793 + j0.023045 & 0.194991 - j0.054548 & 0.246415 + j0 \end{bmatrix} \times 10^{-3}$$

The no-load current is calculated, as given in Example 13.3, to be

$$I_n^0 = \frac{-V_1}{Z_{11}} = -0.000436 - j0.387164$$

Also using the power-flow solution, we have

$$\alpha_1 = \frac{1 - js_1}{V_1^*} = \frac{1 - j\left(\frac{1.872240}{1.913152}\right)}{1.0\angle 0^\circ} = 1.0 - j0.978615$$
$$\alpha_2 = \frac{1 - js_2}{V_2^*} = \frac{1 - j\left(\frac{2.255039}{4.68}\right)}{1.0\angle -2.43995^\circ} = 1.0196070 - j0.4388369$$

The matrix $\mathbf{T}_\alpha$ of Eq. (13.36) is then calculated to be

$$\mathbf{T}_\alpha = \begin{bmatrix} 8.894036 + j0 & -1.336685 + j0.388211 & 0.364217 + j0.355146 \\ -1.336685 - j0.388211 & 3.789452 + j0 & 0.011499 + j0.086254 \\ 0.364217 - j0.355146 & 0.011499 - j0.086254 & 0.036937 + j0 \end{bmatrix} \times 10^{-3}$$

The B-coefficients are the real parts of the matrix $\mathbf{T}_\alpha$. Finally, the power loss is calculated as follows:

$$P_L = \left[\ 1.913152 \quad 4.68 \mid 1\ \right] \left[\ \mathbf{B}\ \right] \begin{bmatrix} 1.913152 \\ 4.68 \\ \hline 1 \end{bmatrix} = 0.093153 \text{ per unit}$$

13.12 Three generating units operating in parallel at 60 Hz have ratings of 300 MW, 500 MW, and 600 MW and have speed-droop characteristics of 5%, 4% and 3%, respectively. Due to a change in load, an increase in system frequency of 0.3 Hz is experienced before any supplementary control action occurs. Determine the amount of the change in system load, and also the amount of the change in generation of each unit to absorb the load change.

Solution:

Using Eq. (13.65), the change in the system load is calculated to be

$$\begin{aligned}\Delta P &= -\left(\frac{S_{R1}}{R_{1u}}+\frac{S_{R2}}{R_{2u}}+\frac{S_{R3}}{R_{3u}}\right)\frac{\Delta f}{f_R}\\ &= -\left(\frac{300}{0.05}+\frac{500}{0.04}+\frac{600}{0.03}\right)\frac{0.3}{60} = -192.5 \text{ MW}\end{aligned}$$

The change in the outputs of the units can be calculated by Eq. (13.63) or Eq. (13.67) as

$$\begin{aligned}\Delta P_{g1} &= -\frac{S_{R1}}{R_{1u}}\frac{\Delta f}{f_R} = -\frac{300}{0.05}\frac{0.3}{60} = -30 \text{ MW}\\ \Delta P_{g2} &= -\frac{S_{R2}}{R_{2u}}\frac{\Delta f}{f_R} = -\frac{500}{0.04}\frac{0.3}{60} = -62.5 \text{ MW}\\ \Delta P_{g3} &= -\frac{S_{R3}}{R_{3u}}\frac{\Delta f}{f_R} = -\frac{600}{0.03}\frac{0.3}{60} = -100 \text{ MW}\end{aligned}$$

13.13 A 60-Hz system consisting of the three generating units described in Prob. 13.12 is connected to a neighboring system via a tie line. Suppose that a generator in the neighboring system is forced out of service, and that the tie-line flow is observed to increase from the scheduled value of 400 MW to 631 MW. Determine the amount of the increase in generation of each of the three units and find the ACE of this system whose frequency-bias setting is −58 MW/0.1 Hz.

Solution:

The increase in the total generation resulted in the increase in the tie line flow by 231 MW. Therefore, it follows from Eq. (13.65) that

$$231 = -\left(\frac{300}{0.05}+\frac{500}{0.04}+\frac{600}{0.03}\right)\frac{\Delta f}{60}$$

from which we have

$$\Delta f = -231 \times 60 = -0.36 \text{ Hz}$$

The three units should have increased their outputs according to Eq. (13.63) as follows:

$$\begin{aligned}\Delta P_{g1} &= -\frac{300}{0.05}\frac{(-0.36)}{60} = 36 \text{ MW}\\ \Delta P_{g2} &= -\frac{500}{0.04}\frac{(-0.36)}{60} = 75 \text{ MW}\\ \Delta P_{g3} &= -\frac{600}{0.03}\frac{(-0.36)}{60} = 120 \text{ MW}\end{aligned}$$

The ACE of the system is determined from Eq. (13.68) as

$$\text{ACE} = (631-400) - 10(-58)(-0.36) = 22.2 \text{ MW}$$

13.14 Suppose that it takes 5 minutes for the AGC of the power system of Prob. 13.13 to command the three units to increase their generation to restore system frequency to 60 Hz. What is the time error in seconds incurred during this 5-minute period? Assume that the initial frequency deviation remains the same throughout this restoration period.

Solution:

The frequency error in per unit is

$$\frac{-0.36}{60} = -6 \times 10^{-3} \text{ per unit}$$

Therefore, the time error incurred during the 5-minute period is

$$-6 \times 10^{-3} \times 5 \times 60 = -1.8 \text{ s}$$

13.15 Solve Example 13.8 when the system load level is 1300 MW.

Solution:

Among the four combinations x_1, x_2, x_3 and x_9, combination x_9 is infeasible since the total generation from units 1 and 2 cannot exceed 1250 MW.

(i) Combination x_3:

With units 1, 2 and 4 operating, we use Eqs. (13.7) and (13.8) to calculate the coefficients

$$\begin{aligned} a_T &= \left(a_1^{-1} + a_2^{-1} + a_4^{-1}\right)^{-1} = \left(0.008^{-1} + 0.0096^{-1} + 0.011^{-1}\right)^{-1} = 3.1243 \times 10^{-3} \\ b_T &= a_t\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_4}{a_4}\right) = a_T\left(\frac{8}{0.008} + \frac{6.4}{0.0096} + \frac{7.5}{0.011}\right) = 7.3374 \end{aligned}$$

The incremental fuel cost at the load level of 1300 MW is

$$\lambda = a_T P_{gT} + b_T = 3.1243 \times 10^{-3} \times 1300 + 7.3374 = 11.3990$$

The corresponding economic dispatch outputs are

$$\begin{aligned} P_{g1} &= \frac{\lambda - b_1}{a_1} = \frac{11.3990 - 8.0}{0.008} = 424.88 \cong 425 \text{ MW} \\ P_{g2} &= \frac{\lambda - b_2}{a_2} = \frac{11.3990 - 6.4}{0.0096} = 520.73 \cong 521 \text{ MW} \\ P_{g4} &= \frac{\lambda - b_4}{a_4} = \frac{11.3990 - 7.5}{0.011} = 354.45 \cong 354 \text{ MW} \end{aligned}$$

The hourly production costs of the three units are calculated to be

$$\begin{aligned} f_1 &= 0.004P_{g1}^2 + 8.0P_{g1} + 500\Big|_{P_{g1}=425} = 4623 \text{ \$/h} \\ f_2 &= 0.0048P_{g2}^2 + 6.4P_{g2} + 400\Big|_{P_{g2}=521} = 5037 \text{ \$/h} \\ f_4 &= 0.0055P_{g4}^2 + 7.5P_{g4} + 400\Big|_{P_{g4}=354} = 3744 \text{ \$/h} \end{aligned}$$

(ii) Combination x_2:

Using the values of a_T and b_T found in Example 13.8, λ is calculated to be

$$\lambda = a_T P_{gT} + b_T = 3.038 \times 10^{-3} \times 1350 + 7.4634 = 11.4128$$

The corresponding economic dispatch outputs are

$$\begin{aligned}
P_{g1} &= \frac{\lambda - b_1}{a_1} = \frac{11.4128 - 8.0}{0.008} = 426.60 \cong 427 \text{ MW} \\
P_{g2} &= \frac{\lambda - b_2}{a_2} = \frac{11.4128 - 6.4}{0.0096} = 522.17 \cong 522 \text{ MW} \\
P_{g3} &= \frac{\lambda - b_3}{a_3} = \frac{11.4128 - 7.9}{0.01} = 351.28 \cong 351 \text{ MW}
\end{aligned}$$

The hourly production costs of the three units are calculated to be

$$\begin{aligned}
f_1 &= 0.004P_{g1}^2 + 8.0P_{g1} + 500\Big|_{P_{g1}=427} = 4645 \text{ \$/h} \\
f_2 &= 0.0048P_{g2}^2 + 6.4P_{g2} + 400\Big|_{P_{g2}=522} = 5049 \text{ \$/h} \\
f_3 &= 0.005P_{g3}^2 + 7.9P_{g3} + 600\Big|_{P_{g3}=351} = 3989 \text{ \$/h}
\end{aligned}$$

(iii) Combination x_1:

Using the values of a_T and b_T found in Example 13.8, λ is calculated to be

$$\lambda = a_T P_{gT} + b_T = 2.3805 \times 10^{-3} \times 1300 + 7.4712 = 10.56585$$

The corresponding economic dispatch outputs are

$$\begin{aligned}
P_{g1} &= \frac{\lambda - b_1}{a_1} = \frac{10.56585 - 8.0}{0.008} = 320.73 \cong 321 \text{ MW} \\
P_{g2} &= \frac{\lambda - b_2}{a_2} = \frac{10.56585 - 6.4}{0.0096} = 433.94 \cong 434 \text{ MW} \\
P_{g3} &= \frac{\lambda - b_3}{a_3} = \frac{10.56585 - 7.9}{0.01} = 266.59 \cong 266 \text{ MW} \\
P_{g4} &= \frac{\lambda - b_4}{a_4} = \frac{10.56585 - 7.5}{0.11} = 278.71 \cong 279 \text{ MW}
\end{aligned}$$

The hourly production costs of the four units are calculated to be

$$\begin{aligned}
f_1 &= 0.004P_{g1}^2 + 8.0P_{g1} + 500\Big|_{P_{g1}=321} = 3480 \text{ \$/h} \\
f_2 &= 0.0048P_{g2}^2 + 6.4P_{g2} + 400\Big|_{P_{g2}=434} = 4082 \text{ \$/h} \\
f_3 &= 0.005P_{g3}^2 + 7.9P_{g3} + 600\Big|_{P_{g3}=266} = 3055 \text{ \$/h} \\
f_4 &= 0.0055P_{g4}^2 + 7.5P_{g4} + 400\Big|_{P_{g4}=279} = 2921 \text{ \$/h}
\end{aligned}$$

13.16 If the start-up costs of the four units of Example 13.9 are changed to \$2500, \$3000, \$3400, and \$2600, and the shut-down costs are changed to \$1500, \$1200, \$1000, and \$1400, respectively, find the optimal unit commitment policy. Assume that all other conditions remain unchanged.

Solution:

At stage 6, the condition remains the same. Therefore,

$$F_9(6) = \$\,45,868$$

At stage 5, by using different transition costs T, we have

$$\begin{aligned}
F_1(5) &= \{P_1(5) + T_{1,9}(5) + F_9(6)\} = [58,428 + 1000 + 1400 + 45,868] = \$\,106,696 \\
F_2(5) &= \{P_2(5) + T_{2,9}(5) + F_9(6)\} = [59,356 + 1000 + 45,868] = \$\,106,224 \\
F_3(5) &= \{P_3(5) + T_{3,9}(5) + F_9(6)\} = [58,236 + 1400 + 45,868] = \$\,105,504
\end{aligned}$$

At stage 4,

$$\begin{aligned}
F_1(4) &= \min[P_1(4) + T_{1,1}(4) + F_1(5); P_1(4) + T_{1,2}(4) + F_2(5); P_1(4) + T_{1,3}(4) + F_3(5)] \\
&= \min[76,472 + 0 + 106,696; 76,472 + 1400 + 106,224; 76,472 + 1000 + 105,504] \\
&= \min[183,168; 184,096; 182,976] = \$\,182,976 \\
F_2(4) &= \min[P_2(4) + T_{2,1}(4) + F_1(5); P_2(4) + T_{2,2}(4) + F_2(5); P_2(4) + T_{2,3}(4) + F_3(5)] \\
&= \min[79,184 + 2600 + 106,696; 79,184 + 0 + 106,224; 79,184 + 1000 + 2600 + 105,504] \\
&= \min[188,480; 185,408; 188,288] = \$\,185,408
\end{aligned}$$

At stage 3,

$$\begin{aligned}
F_1(3) &= \min[P_1(3) + T_{1,1}(3) + F_1(4); P_1(3) + T_{1,2}(3) + F_2(4)] \\
&= \min[70,908 + 0 + 182,976; 70,908 + 1400 + 185,408] \\
&= \min[253,884; 257,716] = \$\,253,884 \\
F_2(3) &= \min[P_2(3) + T_{2,1}(3) + F_1(4); P_2(3) + T_{2,2}(3) + F_2(4)] \\
&= \min[68,976 + 2600 + 182,976; 68,976 + 0 + 185,408] \\
&= \min[254,552; 254,384] = \$\,254,384 \\
F_3(3) &= \min[P_3(3) + T_{3,1}(3) + F_1(4); P_3(3) + T_{3,2}(3) + F_2(4)] \\
&= \min[67,856 + 3400 + 182,976; 67,856 + 3400 + 1400 + 185,408] \\
&= \min[254,232; 258,064] = \$\,254,232
\end{aligned}$$

At stage 2,

$$\begin{aligned}
F_1(2) &= \min[P_1(2) + T_{1,1}(2) + F_1(3); P_1(2) + T_{1,2}(2) + F_2(3); P_1(2) + T_{1,3}(2) + F_3(3)] \\
&= \min[58,428 + 0 + 253,884; 58,428 + 1400 + 254,384; 58,428 + 1000 + 254,232] \\
&= \min[312,312; 314,212; 313,660] = \$\,312,312 \\
F_2(2) &= \min[P_2(2) + T_{2,1}(2) + F_1(3); P_2(2) + T_{2,2}(2) + F_2(3); P_2(2) + T_{2,3}(2) + F_3(3)] \\
&= \min[59,356 + 2600 + 253,884; 59,356 + 0 + 254,384; 59,356 + 1000 + 2600 + 254,232] \\
&= \min[315,840; 313,740; 317,188] = \$\,313,740 \\
F_3(2) &= \min[P_3(2) + T_{3,1}(2) + F_1(3); P_3(2) + T_{3,2}(2) + F_2(3); P_3(2) + T_{3,3}(2) + F_3(3)] \\
&= \min[58,236 + 3400 + 253,884; 58,236 + 3400 + 1400 + 254,384; 58,236 + 0 + 254,232] \\
&= \min[315,520; 317,420; 312,468] = \$\,312,468
\end{aligned}$$

At stage 1,

$$
\begin{aligned}
F_9(1) &= \min\left[P_9(1)+T_{9,1}(1)+F_1(2); P_9(1)+T_{9,2}(1)+F_2(2); P_9(1)+T_{9,3}(1)+F_3(2)\right] \\
&= \min\left[45,868+3400+2600+312,312; 45,868+3400+313,740; 45,868+2600+312,468\right] \\
&= \min\left[364,180; 363,008; 360,936\right] = \$\,360,936
\end{aligned}
$$

When the least cost path is retraced, the optimal unit commitment is found to be the same as that of Example 13.9. The associated total operating cost in this case is \$360,936, which is \$600 less than \$361,536 obtained in Example 13.9.

13.17 Due to a 400 MW short-term purchase request from the neighboring utility, the demand during the second interval of the day is expected to increase from 1400 MW to 1800 MW for the system described in Example 13.9. Assuming that other conditions remain unchanged, find the optimal unit commitment policy and the associated total operating cost for the day.

Solution:

In applying dynamic programming to this problem, the process up to stage 3 should be the same as that given in Example 13.9. At stage 2 no other combinations besides x_1 and x_2 have sufficient capacity to serve the increased load of 1800 MW; therefore, we only have to consider combinations x_1 and x_2. It was found at stage 4 that to serve the load of 1800 MW, the minimum production costs of combinations x_1 and x_2 would be \$76,472 and \$79,184, respectively. Stage 2 can now be handled as follows:

$$
\begin{aligned}
F_1(2) &= \min\left[P_1(2)+T_{1,1}(2)+F_1(3); P_1(2)+T_{1,2}(2)+F_2(3); P_1(2)+T_{1,3}(2)+F_3(3)\right] \\
&= \min\left[76,472+0+254,484; 76,472+1500+254,884; 76,472+1500+254,432\right] \\
&= \min\left[330,956; 332,856; 332,404\right] = \$\,330,956 \\
F_2(2) &= \min\left[P_2(2)+T_{2,1}(2)+F_1(3); P_2(2)+T_{2,2}(2)+F_2(3); P_2(2)+T_{2,3}(2)+F_3(3)\right] \\
&= \min\left[79,184+3000+254,484; 79,184+0+254,884; 79,184+4500+254,432\right] \\
&= \min\left[336,668; 334,068; 338,116\right] = \$\,334,068
\end{aligned}
$$

At stage 1,

$$
\begin{aligned}
F_9(1) &= \min\left[P_9(1)+T_{9,1}(1)+F_1(2); P_9(1)+T_{9,2}(1)+F_2(2)\right] \\
&= \min\left[45,868+6000+330,956; 45,868+3000+334,068\right] \\
&= \min\left[382,824; 382,936\right] = \$\,382,824
\end{aligned}
$$

The optimal unit commitment is found by retracing the least cost path in the forward direction, and is given by

Stage	Optimal combination	Load level
1	x_9	1100 MW
2	x_1	1800 MW
3	x_1	1600 MW
4	x_1	1800 MW
5	x_3	1400 MW
6	x_9	1100 MW

From $F_9(1)$, the total operating cost is found to be \$382,824.

13.18 Suppose Unit 4 of Example 13.9 will have to be taken off line for 8 hours beginning at the fifth interval of the day to undergo minor repair work. Determine the optimal unit commitment policy to serve the system load of Fig. 13.11 and the increase in the operating cost for the day.

Solution:

Combinations x_1 or x_3 are no longer a viable option during the 5^{th} and 6^{th} intervals. Therefore, the only $F_i(k)$ that needs to be evaluated at stage 5 is $F_2(5) = 106,724$ which was given in Figure 13.14. Since x_2 is the only combination available at stage 5, $F_i(4)$, for $i = 1$ and $i = 2$, are to be recalculated (using the already available information in Figure 13.14) as

$$F_1(4) = 184,696 \qquad F_2(4) = 185,908$$

Now, recalculation of $F_i(k)$ is required from stage 3 onwards. At stage 3,

$$\begin{aligned}
F_1(3) &= \min[P_1(3) + T_{1,1}(3) + F_1(4); P_1(3) + T_{1,2}(3) + F_2(4)] \\
&= \min[70,908 + 0 + 184,696; 70,908 + 1500 + 185,908] \\
&= \min[255,604; 258,316] = \$\,255,604 \\
F_2(3) &= \min[P_2(3) + T_{2,1}(3) + F_1(4); P_2(3) + T_{2,2}(3) + F_2(4)] \\
&= \min[68,976 + 3000 + 184,696; 68,976 + 0 + 185,908] \\
&= \min[256,672; 254,884] = \$\,254,884 \\
F_3(3) &= \min[P_3(3) + T_{3,1}(3) + F_1(4); P_3(3) + T_{3,2}(3) + F_2(4)] \\
&= \min[67,856 + 3000 + 184,696; 67,856 + 4500 + 185,908] \\
&= \min[255,552; 258,264] = \$\,255,552
\end{aligned}$$

At stage 2,

$$\begin{aligned}
F_1(2) &= \min[P_1(2) + T_{1,1}(2) + F_1(3); P_1(2) + T_{1,2}(2) + F_2(3); P_1(2) + T_{1,3}(2) + F_3(3)] \\
&= \min[58,428 + 0 + 255,604; 58,428 + 1500 + 254,884; 58,428 + 1500 + 255,552] \\
&= \min[314,032; 314,812; 315,480] = \$\,314,032 \\
F_2(2) &= \min[P_2(2) + T_{2,1}(2) + F_1(3); P_2(2) + T_{2,2}(2) + F_2(3); P_2(2) + T_{2,3}(2) + F_3(3)] \\
&= \min[59,356 + 3000 + 255,604; 59,356 + 0 + 254,884; 59,356 + 4500 + 255,552] \\
&= \min[317,960; 314,240; 319,408] = \$\,314,240 \\
F_3(2) &= \min[P_3(2) + T_{3,1}(2) + F_1(3); P_3(2) + T_{3,2}(2) + F_2(3); P_3(2) + T_{3,3}(2) + F_3(3)] \\
&= \min[58,236 + 3000 + 255,604; 58,236 + 4500 + 254,884; 59,236 + 0 + 255,552] \\
&= \min[316,840; 317,620; 313,788] = \$\,313,788
\end{aligned}$$

At stage 1,

$$\begin{aligned}
F_9(1) &= \min[P_9(1) + T_{9,1}(1) + F_1(2); P_9(1) + T_{9,2}(1) + F_2(2); P_9(1) + T_{9,3}(1) + F_3(2)] \\
&= \min[45,868 + 6000 + 314,032; 45,868 + 3000 + 314,240; 45,868 + 3000 + 313,788] \\
&= \min[365,900; 363,108; 362,656] = \$\,362,656
\end{aligned}$$

Tracing the process in the forward direction, we find the optimal policy as (x_9 x_3 x_3 x_1 x_2 x_9) from stage 1 to stage 6. The increase in the operating cost is

$$362,656 - 361,536 = \$\,1120$$

13.19 A diagram similar to Fig. 13.14 is shown in Fig. 13.15 in which directed branches represent transitions from one state, represented by a node, to another. Associated with each directed branch (i, j) is the cost $f_{ij}(k)$, as defined in Eq. (13.72). The values of $f_{ij}(k)$ are given in Table 13.7. Note that index k of $f_{ij}(k)$ does not play any role here, and consequently will now be omitted. If the value of f_{ij} is interpreted to be the distance between states i and j, then the unit commitment problem becomes that of finding the *shortest path* from the origin, represented by node ①, to the destination, represented by node ⑩. The problem of this nature is called the *stagecoach problem*. Write the backward recurrence equation similar to Eq. (13.75), and solve the problem by commencing calculations at the destination and then moving toward the origin.

In forward recurrence the process starts with the origin and moves toward the destination. Write the forward recurrence equation, solve the problem and check the result with that of the backward dynamic programming procedure.

Table 13.7 Matrix of costs (or distances) f_{ij} between states (or nodes) ⓘ and ⓙ of Fig. 13.15

		ⓙ								
		②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
	①	20	15	17						
	②				35	31	38			
	③				39	42	33			
	④				36	40	34			
ⓘ	⑤							26	22	
	⑥							29	25	
	⑦							41	44	
	⑧									15
	⑨									18

Solution:

The backward recurrence equation can be written as

$$F_i(N-1) = \min_j \{f_{ij} + F_j(N)\}$$

Starting with node 10, the destination, we have at stage 5,

$$F_{10}(5) = 0$$

At stage 4,

$$F_8(4) = \min[f_{8\,10} + F_{10}(5)] = \min[15+0] = 15$$
$$F_9(4) = \min[f_{9\,10} + F_{10}(5)] = \min[18+0] = 18$$

At stage 3,

$$F_5(3) = \min[f_{5\,8} + F_8(4); f_{5\,9} + F_9(4)] = \min[26+15; 22+18]$$
$$= \min[41; 40] = 40$$
$$F_6(3) = \min[f_{6\,8} + F_8(4); f_{6\,9} + F_9(4)] = \min[29+15; 25+18]$$
$$= \min[44; 43] = 43$$
$$F_7(3) = \min[f_{7\,8} + F_8(4); f_{7\,9} + F_9(4)] = \min[41+15; 44+18]$$
$$= \min[56; 62] = 56$$

At stage 2,

$$F_2(2) = \min[f_{2\,5} + F_5(3); f_{2\,6} + F_6(3); f_{2\,7} + F_7(3)] = \min[35+40; 31+43; 38+56]$$
$$= \min[75; 74; 94] = 74$$
$$F_3(2) = \min[f_{3\,5} + F_5(3); f_{3\,6} + F_6(3); f_{3\,7} + F_7(3)] = \min[39+40; 42+43; 33+56]$$
$$= \min[79; 85; 89] = 79$$
$$F_4(2) = \min[f_{4\,5} + F_5(3); f_{4\,6} + F_6(3); f_{4\,7} + F_7(3)] = \min[36+40; 40+43; 34+56]$$
$$= \min[76; 83; 90] = 76$$

At stage 1,

$$F_1(1) = \min[f_{1\,2} + F_2(2); f_{1\,3} + F_3(2); f_{1\,4} + F_4(2)] = \min[20+74; 15+79; 17+76]$$
$$= \min[94; 94; 93] = 93$$

Retracing the path in the forward direction, the shortest path is found to be

$$1 \rightarrow 4 \rightarrow 5 \rightarrow 9 \rightarrow 10$$

and the shortest distance is 93.

The forward recurrence equation can be written as

$$F_j(N) = \min_i \{f_{ij} + F_i(N-1)\}$$

Starting with node 1, we have at stage 1,

$$F_1(1) = 0$$

At stage 2,

$$F_2(2) = \min[f_{1\,2} + F_1(1)] = \min[20+0] = 20$$
$$F_3(2) = \min[f_{1\,3} + F_1(1)] = \min[15+0] = 15$$
$$F_4(2) = \min[f_{1\,4} + F_1(1)] = \min[17+0] = 17$$

At stage 3,

$$F_5(3) = \min[f_{2\,5} + F_2(2); f_{3\,5} + F_3(2); f_{4\,5} + F_4(2)]$$
$$= \min[35+20; 39+15; 36+17] = \min[55; 54; 53] = 53$$
$$F_6(3) = \min[f_{2\,6} + F_2(2); f_{3\,6} + F_3(2); f_{4\,6} + F_4(2)]$$
$$= \min[31+20; 42+15; 40+17] = \min[51; 57; 57] = 51$$
$$F_7(3) = \min[f_{2\,7} + F_2(2); f_{3\,7} + F_3(2); f_{4\,7} + F_4(2)]$$
$$= \min[38+20; 33+15; 34+17] = \min[58; 58; 51] = 51$$

At stage 4,

$$F_8(4) = \min[f_{58} + F_5(3); f_{68} + F_6(3); f_{78} + F_7(3)]$$
$$= \min[26+53; 29+51; 41+51] = \min[79; 80; 92] = 79$$
$$F_9(4) = \min[f_{59} + F_5(3); f_{69} + F_6(3); f_{79} + F_7(3)]$$
$$= \min[22+53; 25+51; 44+51] = \min[75; 76; 95] = 75$$

At stage 5,

$$F_{10}(5) = \min[f_{8\,10} + F_8(4); f_{9\,10} + F_9(4)]$$
$$= \min[15+79; 18+75] = \min[94; 93] = 93$$

Retracing the process from node 10 in the backward direction, the shortest path is identified to be

$$10 \rightarrow 9 \rightarrow 5 \rightarrow 4 \rightarrow 1$$

and the distance of the corresponding path is 93. This result is identical to that by the backward dynamic programming procedure.

Chapter 14 Problem Solutions

14.1 A four-bus system with $\mathbf{Z}_{\text{bus}}$ given in per unit by

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} j0.041 & j0.031 & j0.027 & j0.018 \\ j0.031 & j0.256 & j0.035 & j0.038 \\ j0.027 & j0.035 & j0.158 & j0.045 \\ j0.018 & j0.038 & j0.045 & j0.063 \end{bmatrix} \end{array}$$

has bus voltages $V_1 = 1.0\angle 0^\circ$, $V_2 = 0.98\angle 0^\circ$, $V_3 = 0.96\angle 0^\circ$ and $V_4 = 1.04\angle 0^\circ$. Using the compensation current method, determine the change in voltage at bus ② due to the outage of line ①–③ with series impedance $j0.3$ per unit.

Solution:

$$\mathbf{Z}_{\text{bus}}^{(1-3)} = \mathbf{Z}_{\text{bus}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} j(0.041-0.027) \\ j(0.031-0.035) \\ j(0.027-0.158) \\ j(0.018-0.045) \end{bmatrix} = \begin{bmatrix} j0.014 \\ -j0.004 \\ -j0.131 \\ -j0.027 \end{bmatrix}$$

By Eq. (14.14),

$$I_{13} = \frac{(V_1 - V_3)}{Z_{\text{th},13} + z_{13}} = \frac{(1.0 - 0.96)}{j0.014 - j(-0.131) - j0.3} = \frac{2.581}{-j}$$

By Eq. (14.7),

$$\Delta V_2 = -\left(\frac{2.581}{-j}\right) \times (-j0.004) = -0.0103 \text{ per unit}$$