Using $\mathbf{H}_{\mathbf{X}}^{(0)}$ obtained above, we have

$$\left[\mathbf{H}_{\mathbf{X}}^{(0)^T}\,\mathbf{R}^{-1}\,\mathbf{H}_{\mathbf{X}}^{(0)}\right]^{-1} =$$

$$\begin{bmatrix} 0.1884\times10^{-5} & 0.1355\times10^{-5} & 0.5578\times10^{-18} & 0.5571\times10^{-18} & 0.5583\times10^{-18} \\ & 0.1693\times10^{-5} & 0.4179\times10^{-18} & 0.4184\times10^{-18} & 0.4176\times10^{-18} \\ & & 0.4000\times10^{-3} & 0.4000\times10^{-3} & 0.4000\times10^{-3} \\ & & & 0.4019\times10^{-3} & 0.4014\times10^{-3} \\ & & & & 0.4017\times10^{-3} \end{bmatrix}$$

Finally, we have

$$\begin{bmatrix} \delta_2^{(1)} \\ \delta_3^{(1)} \\ |V_1|^{(1)} \\ |V_2|^{(1)} \\ |V_3|^{(1)} \end{bmatrix} = \begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ |V_1|^{(0)} \\ |V_2|^{(0)} \\ |V_3|^{(0)} \end{bmatrix} + \left(\mathbf{H}_{\mathbf{X}}^{(0)^T}\mathbf{R}^{-1}\mathbf{H}_{\mathbf{X}}^{(0)}\right)^{-1}\mathbf{H}_{\mathbf{X}}^{(0)^T}\mathbf{R}^{-1}\begin{bmatrix} e_1^{(0)} \\ e_2^{(0)} \\ e_3^{(0)} \\ e_4^{(0)} \\ e_5^{(0)} \\ e_6^{(0)} \\ e_7^{(0)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.00948 \\ -0.03078 \\ 0.0 \\ 0.01003 \\ -0.00379 \end{bmatrix} = \begin{bmatrix} -0.00948 \text{ radian} \\ -0.03078 \text{ radian} \\ 1.0 \text{ per unit} \\ 1.01003 \text{ per unit} \\ 0.99621 \text{ per unit} \end{bmatrix}$$

Chapter 16 Problem Solutions

16.1 A 60-Hz four-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of $H = 7.5$ MJ/MVA. Find (*a*) the kinetic energy stored in the rotor at synchronous speed and (*b*) the angular acceleration if the electrical power developed is 400 MW when the input less the rotational losses is 740,000 hp.

Solution:

(*a*) Kinetic energy $= 500 \times 7.5 = 3750$ MJ

(*b*) Input power $= 740{,}000 \times 746 \times 10^{-6} = 552$ MW. By Eq. (16.14),

$$\text{Input power} - \text{rotational loss} = \frac{7.5}{180 \times 60}\frac{d^2\delta}{dt^2} = \frac{552 - 400}{500}$$

$$\frac{d^2\delta}{dt^2} = 437.8 \text{ elec. degrees/s}^2$$

For a four-pole machine,

$$\frac{d^2\delta}{dt^2} = \frac{437.8}{2} = 218.9 \text{ mech. degrees/s}^2$$

$$\text{or } 60 \times \frac{218.9}{360} = 36.5 \text{ rpm/s}^2$$

16.2 If the acceleration computed for the generator described in Prob. 16.1 is constant for a period of 15 cycles, find the change in δ in electrical degrees in that period and the speed in revolutions per minute at the end of 15 cycles. Assume that the generator is synchronized with a large system and has no accelerating torque before the 15-cycle period begins.

Solution:

$$\begin{aligned}
\text{duration of acceleration} &= \frac{15}{60} = 0.25 \text{ s} \\
\text{acceleration} &= 437.8 \text{ elec. degrees/s}^2 = 36.5 \text{ rpm/s} \\
\text{change in } \delta \text{ in 15 cycles} &= \frac{1}{2}(437.8)(0.25)^2 = 13.68 \text{ elec. degrees} \\
\text{synchronous speed} &= \frac{120 \times 60}{4} = 1800 \text{ rpm} \\
\text{After 15 cycles, speed} &= 1800 + 0.25 \times 36.5 = 1809.1 \text{ rpm}
\end{aligned}$$

16.3 The generator of Prob. 16.1 is delivering rated megavolt-amperes at 0.8 power factor lag when a fault reduces the electric power output by 40%. Determine the accelerating torque in newton-meters at the time the fault occurs. Neglect losses and assume constant power input to the shaft.

Solution:

$$\begin{aligned}
P_a &= \omega_m T_a = 0.8 \times 500 - 0.6 \times 0.8 \times 500 = 160 \text{ MW} \\
\omega_m &= \frac{2\pi f}{2} \text{ mech. radians/s} \\
T_a &= \frac{160 \times 10^6}{2\pi f/2} = 848,826 \text{ N}\cdot\text{m}
\end{aligned}$$

16.4 Determine the WR^2 of the generator of Prob. 16.1.

Solution:

$$WR^2 = \frac{S_{mach} \times H \times 10^{10}}{2.31(\text{rpm})^2} = \frac{500 \times 7.5 \times 10^{10}}{2.31(1800)^2} = 5,010,422 \text{ lb-ft}^2$$

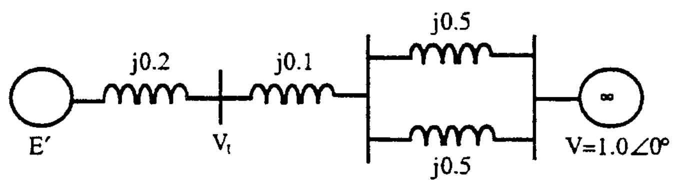
16.5 A generator having $H = 6$ MJ/MVA is connected to a synchronous motor having $H = 4$ MJ/MVA through a network of reactances. The generator is delivering power of 1.0 per unit to the motor when a fault occurs which reduces the delivered power. At the time when the reduced power delivered is 0.6 per unit determine the angular acceleration of the generator with respect to the motor.

Solution:

$$\begin{aligned}
\frac{6 \times 4}{6+4} \times \frac{1}{180f} \frac{d^2\delta_{12}}{dt^2} &= 1.0 - 0.6 \\
\frac{d^2\delta_{12}}{dt^2} &= 1800 \text{ elec. degrees/s}^2
\end{aligned}$$

16.6 A power system is identical to that of Example 16.3 except that the impedance of each of the parallel transmission lines is $j0.5$ and the delivered power is 0.8 per unit when both the terminal voltage of the machine and the voltage of the infinite bus are 1.0 per unit. Determine the power-angle equation for the system during the specified operating conditions.

Solution:



X between V_t and V is

$$j0.1 + \frac{j0.5}{2} = j0.35 \text{ per unit}$$

If $V_t = 1.0\angle\alpha$,

$$\frac{1.0 \times 1.0}{j0.35} \sin\alpha = 0.8, \quad \alpha = 16.26^\circ$$

$$I = \frac{1.0\angle 16.26^\circ - 1.0\angle 0^\circ}{0.35\angle 90^\circ} = \frac{0.96 + j0.28 - 1.0}{j0.35}$$

$$= 0.8 + j0.1143 = 0.8081\angle 8.13^\circ$$

$$E' = 1.0\angle 16.26^\circ + 0.808\angle 8.13^\circ \times 0.2\angle 90^\circ$$

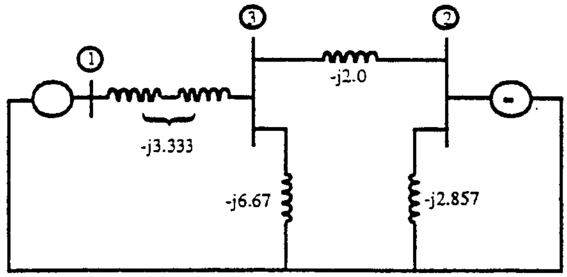
$$= 0.96 + j0.28 - 0.023 + j0.16 = 1.0352\angle 25.15^\circ$$

$$P_e = \frac{1.0352 \times 1.0}{0.35 + 0.20} \sin\delta = 1.882 \sin\delta$$

16.7 If a three-phase fault occurs on the power system of Prob. 16.6 at a point on one of the transmission lines at a distance of 30% of the line length away from the sending-end terminal of the line, determine (a) the power-angle equation during the fault and (b) the swing equation. Assume the system is operating under the conditions specified in Prob. 16.6 when the fault occurs. Let $H = 5.0$ MJ/MVA as in Example 16.4.

Solution:

The circuit diagram with admittances marked in per unit and the fault as described is shown below:



$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j3.333 & 0 & j3.333 \\ 0 & -j4.857 & j2.0 \\ j3.333 & j2.0 & -j12.0 \end{bmatrix}$$

After elimination of node 3 by the usual method, in row 1, column 2 of the new $\mathbf{Y}_{\text{bus}}$ matrix,

$$\begin{aligned} Y_{12} &= \frac{j2.0 \times j3.333}{-j12} = j0.556 \\ P_e &= 1.0352 \times 1.0 \times 0.556 \sin\delta = 0.575 \sin\delta \\ \frac{5}{180f}\frac{d^2\delta}{dt^2} &= 0.8 - 0.575 \sin\delta \\ \frac{d^2\delta}{dt^2} &= 36f(0.8 - 0.575 \sin\delta) \end{aligned}$$

16.8 Series resistance in the transmission network results in positive values for P_c and γ in Eq. (16.80). For a given electrical power output, show the effects of resistance on the synchronizing coefficient S_p, the frequency of rotor oscillations and the damping of these oscillations.

Solution:

Equation (16.80) is $P_e = P_c + P_{max}\sin(\delta - \gamma)$ and Eq. (16.47) defines

$$S_p = \left.\frac{dP_e}{d\delta}\right|_{\delta=\delta_0}$$

So, if the network is resistive

$$S_p = P_{max}\cos(\delta_0 - \gamma)$$

This S_p is greater than that for a purely reactive network where $\gamma = 0$. Hence, by Eq. (16.50) which shows

$$f_n = \sqrt{\frac{S_p\omega_s}{2H}}$$

wherein f_n is correspondingly larger. We now define $\delta' = \delta - \gamma$ and $P'_m = P_m - P_c$ so that the swing equation becomes

$$\frac{2H}{\omega_s} \times \frac{d^2\delta'}{dt^2} = P'_m - P_{max}\sin\delta'$$

which must have a solution reflecting undamped oscillations (see footnote in Sec. 16.5) as in a purely reactive network. Consequently, series resistance cannot introduce damping of mechanical oscillations.

16.9 A generator having $H = 6.0$ MJ/MVA is delivering power of 1.0 per unit to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 per unit. When the fault is cleared the original network conditions again exist. Determine the critical clearing angle and critical clearing time.

Solution:

$$\begin{aligned}
2.5\sin\delta_0 &= 1.0 \\
\delta_0 &= 23.58^\circ \text{ or } 0.4115 \text{ rad} \\
\text{By Eq. (16.70), } \delta_{cr} &= \cos^{-1}\left[(\pi - 0.823)\sin 23.58^\circ - \cos 23.58^\circ\right] \\
&= \cos^{-1}(0.9275 - 0.9165) = 89.27^\circ = 1.560 \text{ rad} \\
\text{By Eq. (16.72), } t_{cr} &= \sqrt{\frac{4 \times 6\,(1.395 - 0.4115)}{2\pi 60 \times 1.0}} = 0.270 \text{ s}
\end{aligned}$$

16.10 A 60-Hz generator is supplying 60% of P_{max} to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 400%. When the fault is cleared the maximum power that can be delivered is 80% of the original maximum value. Determine the critical clearing angle for the condition described.

Solution:

$$\begin{aligned}
P_{max}\sin\delta_0 &= 0.6P_{max} \\
\delta_0 &= 36.87^\circ,\ 0.6435 \text{ rad} \\
r_1 &= 0.25 \qquad r_2 = 0.8 \\
r_2 P_{max}\sin\delta_{max} &= P_m \quad \text{(Fig. 16.11)} \\
\frac{P_m}{P_{max}} &= 0.6 \quad \text{(given)} \\
\sin\delta_{max} &= \frac{0.6}{0.8} = 0.75 \\
\delta_{max} &= 180^\circ - 48.59^\circ = 131.41^\circ = 2.294 \text{ rad} \\
\cos\delta_{cr} &= \frac{0.6(2.294 - 0.6435) + 0.8\cos 131.4^\circ - 0.25\cos 36.87^\circ}{0.8 - 0.25} = 0.475 \\
\delta_{cr} &= \cos^{-1} 0.475 = 61.64^\circ
\end{aligned}$$

16.11 If the generator of Prob. 16.10 has an inertia constant of $H = 6$ MJ/MVA and P_m (equal to 0.6 P_{max}) is 1.0 per-unit power, find the critical clearing time for the condition of Prob. 16.10. Use $\Delta t = 0.05$ to plot the necessary swing curve.

Solution:

From Prob. 16.10, $\delta_{cr} = 61.64°$ and t_{cr} can be read from the swing curve for a sustained fault

$$P_{max} = \frac{1.0}{0.6} = 1.667 \text{ per unit}$$
$$P_e = 1.667/4 = 0.4167 \text{ during fault}$$
$$k = \frac{180 \times 60}{6}(0.05)^2 = 4.5$$
$$\delta_0 = 36.87° \qquad P_m = 1.0 \qquad P_c = 0 \qquad Y = 0$$

Values in the table below were found by a computer program and rounded off only for tabulation.

t	P_e	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	1.0	0			36.87°
0+	0.250	0.75			36.87°
0 av		0.375	1.688		36.87°
				1.688°	
0.05	0.260	0.740	3.331		38.56°
				5.019°	
0.10	0.287	0.713	3.207		43.58°
				8.226°	
0.15	0.328	0.673	3.026		51.81°
				11.252°	
0.20					63.05°

Problem 16.11 Solution Data

By linear interpolation,

$$t_c \cong 0.15 + 0.05\left(\frac{61.64 - 51.81}{63.05 - 51.81}\right)$$
$$\cong 0.15 + 0.044 = 0.194 \text{ s or } 11.6 \text{ cycles}$$

16.12 For the system and fault conditions described in Probs. 16.6 and 16.7 determine the power-angle equation if the fault is cleared by the simultaneous opening of breakers at both ends of the faulted line at 4.5 cycles after the fault occurs. Then plot the swing curve of the generator through $t = 0.25$ s.

Solution:

From Prob. 16.6 and 16.7 $E' = 1.0352\angle 25.15°$ per unit and before the fault

$$P_e = 1.882 \sin\delta \qquad P_m = 0.8 \qquad \delta_0 = 25.15°$$

During the fault,

$$P_e = 0.575 \sin\delta$$

after clearing,

$$Y_{12} = \frac{1}{j0.3 + j0.5} = -j1.25 \text{ per unit}$$

and

$$\begin{aligned} P_e &= 1.0352 \times 1.0 \times 1.25 \sin\delta = 1.294 \sin\delta \\ k &= \frac{180 \times 60}{5}(0.05)^2 = 5.4 \\ 4.5 \text{ cycles} &= 0.075 \text{ s (middle of interval)} \end{aligned}$$

Values in the table below were found by a computer program and rounded off only for tabulation.

t	P_e	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	0.8	0.0			25.15°
0+	0.244	0.556	3.000		25.15°
0 av			1.500		25.15°
				1.500°	
0.05	0.258	0.542	2.927		26.65°
				4.427°	
0.10	0.668	0.132	0.713		31.08°
				5.140°	
0.15	0.765	0.035	0.191		36.22°
				5.332°	
0.20	0.858	−0.058	−0.315		41.55°
				5.017°	
0.25					46.57°

Problem 16.12 Solution Data

Note: If the table is continued a maximum value of δ will be found equal to 56.20° at $t = 0.45$ s. At 0.55 s, $\delta = 52.56°$.

16.13 Extend Table 16.6 to find δ at $t = 1.00$ s.

Solution:

Continuing the computer program used to generate Table 16.6 and tabulating values only to the fourth decimal place we obtain:

t	$\delta_n - \gamma$	$P_{max}\sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0.85	16.9591	1.8940	−0.2244	−0.7575		17.8061°
					−3.2292°	
0.90	13.7299	1.5412	0.1284	0.4334		14.5769°
					−2.7957°	
0.95	10.9342	1.2317	0.4379	1.4780		11.7812°
					−1.3177°	
1.0						10.4634°

Problem 16.13 Solution Data

Note: At $t = 1.05$, $\delta = 11.1196°$.

Sample calculation (at $t = 0.85$ s):

$$\begin{aligned}
\delta_n - \gamma &= 17.8061 - 0.847 = 16.9591° \\
P_{max}\sin(\delta - \gamma) &= 6.4934 \sin 16.9591° = 1.8940 \\
P_a &= P_m - P_c - P_{max}\sin(\delta - \gamma) = 1.6696 - 1.8940 = -0.2244 \\
kP_a &= -0.7574 \\
\Delta\delta_n &= \Delta\delta_{n-1} - kP_a = -2.4716 - (-0.7574) = -3.2292°
\end{aligned}$$

16.14 Calculate the swing curve for machine 2 of Examples 16.9 – 16.11 for fault clearing at 0.05 s by the method described in Sec. 16.9. Compare the results with the values obtained by the production-type program and listed in Table 16.7.

Solution:

Using the computer programmed to obtain δ vs. t showing intermediate steps in the calculation and rounding off only for tabulation we have

t	$\delta_n - \gamma$	$P_{max}\sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0−			0.000			16.19°
0+	15.435	1.4644	0.2310			16.19°
0 av			0.1155	0.3898		16.19°
					0.3898°	
0.05−	15.8248	1.5005	0.1950			16.5798°
0.05+	15.7328	1.7607	−0.0911			
0.05 av			0.0520	0.1753		
					0.5653°	
0.10	16.2983	1.8223	−0.1527	−0.5153		17.1453°
					0.0500°	
0.15	16.3483	1.8227	−0.1581	−0.5337		17.1953°
					−0.4837°	
0.20	15.8685	1.7751	−0.1055	−0.3559		16.7155°
					−0.8396°	
0.25	15.0249	1.6833	−0.0137	−0.0464		15.8719°
					−0.8860°	
0.30	14.1389	1.5862	0.0834	0.2816		14.9859°
					−0.6044°	
0.35	13.5345	1.5197	0.1499	0.5061		14.3815°
					−0.0983°	
0.40	13.4361	1.5088	0.1608	0.5427		14.2831°
					0.4443°	
0.45	13.8804	1.5577	0.1119	0.3775		14.7274°
					0.8218°	
0.50						15.5493°
0.55						16.444°
0.60						17.0813°
0.65						17.2267°

Problem 16.14 Solution Data

Note: Collecting student prepared computer programs is suggested.

16.15 If the three-phase fault on the system of Example 16.9 occurs on line ④–⑤ at bus ⑤ and is cleared by simultaneous opening of breakers at both ends of the line at 4.5 cycles after the fault occurs prepare a table like that of Table 16.6 to plot the swing curve of machine 2 through $t = 0.30$ s.

Solution:

Before the fault and after clearing, the conditions are the same as in Examples 16.9 and 16.11. During the fault P_m is still 1.85 per unit for machine 2, but $P_e = 0$. So, $P_a = 1.85$ per unit. After clearing, $P_m - P_c = 1.6696$, $P_{max} = 6.4934$, $Y = 0.847°$. Clearing in 4.5 cycles, or $t = 0.075$ s. Values in the table below were obtained by a computer program and rounded off for tabulation only.

t	$P_{max}\sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	1.85	0	0	0°	16.19°
0+	0	1.850	6.244		16.19°
0 av		0.925	3.122		16.19°
				3.122°	
0.05	0	1.85	6.244		19.31°
				9.366°	
0.10	3.031	−1.362	−4.596		28.68°
				4.769°	
0.15	3.498	−1.829	−6.172		33.45°
				−1.403°	
0.20	3.363	−1.694	−5.717		32.04°
				−7.120°	
0.25	2.649	−0.979	−3.306		24.92°
				−10.425°	
0.30	1.533	0.137	0.463		14.50°
				−9.963°	
0.35	0.418	1.252	4.225		4.54°
				−5.738°	
0.40	−0.232	1.902	6.419		−1.20°
				0.681°	
0.45	−0.155	1.825	6.158		−0.52°
				6.839°	
0.50	0.619	1.051	3.546		6.32°
				10.385°	
0.55					16.70°

Problem 16.15 Solution Data

Note: Although the problem does not ask for values beyond $t = 0.30$ s, the table has been extended to show the extent of the variation of δ.

16.16 By applying the equal-area criterion to the swing curves obtained in Examples 16.9 and 16.10 for machine 1, (a) derive an equation for the critical clearing angle, (b) solve the equation by trial and error to evaluate δ_{cr} and (c) use Eq. (16.72) to find the critical clearing time.

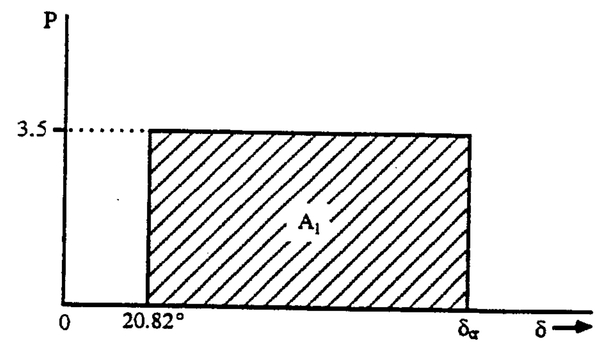
Solution:

Note: Students may need guidance in starting this problem which determines the critical clearing time for machine 1 for the fault specified in Example 16.9. This time must, of course, be less than 0.225 s as is evident from examination of Fig. 16.15 and Table 16.7.

(*a*) From Example 16.9 for machine 1:

$$P_m = 3.5 \text{ per unit} \quad \text{(Table 16.3)}$$
$$E_1' = 1.100\underline{/20.82^\circ}$$
$$\text{Thus, } \delta_0 = 20.82^\circ = 0.3634 \text{ rad}$$

Since the impedance between E_1' and the three-phase fault is pure inductive reactance, $P_e = 0$ during the fault and $P_a = P_m - P_e = 3.5$. The area A_1 for the equal-area criterion is shown below.

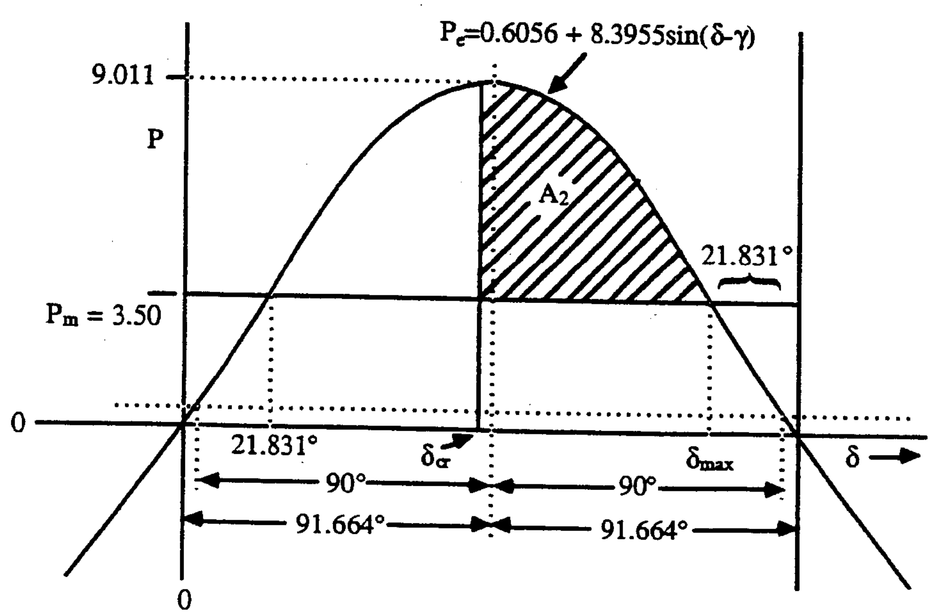


$$\text{where} \quad A_1 = 3.5(\delta_{cr} - 0.3634) = 3.5\delta_{cr} - 1.2719$$

From Example 16.10, the post-fault power-angle curve is given by

$$P_e = 0.6056 + 8.3955\sin(\delta - 1.664^\circ)$$

The curve, P_e vs. δ, is shown below:



Where P_m intercepts the fault curve,

$$3.5 = 0.6056 + 8.3955\sin(\delta - 1.664^\circ)$$

$$
\begin{aligned}
\delta &= 21.8309^\circ \\
P_{e,\,max} &= 0.6056 + 8.3955 \quad \text{where } \delta = 90^\circ + 1.664^\circ = 91.664^\circ \\
\delta_{max} &= 2 \times 91.664^\circ - 21.8309^\circ = 161.497^\circ = 2.8187 \text{ rad} \\
\text{Area } A_2 &= \int_{\delta_{cr}}^{\delta_{max}} \left[0.6056 + 8.3955 \sin(\delta - 1.664^\circ)\right] d\delta - 3.50\,(\delta_{max} - \delta_{cr}) \\
&= (0.6056 - 3.5)\,(\delta_{max} - \delta_{cr}) + 8.3955\left[\cos(\delta_{cr} - 1.664^\circ) - \cos(\delta_{max} - 1.664^\circ)\right] \\
&= -2.8944\,(2.8187 - \delta_{cr}) + 8.3955\left[\cos(\delta_{cr} - 1.664^\circ) - \cos(161.497^\circ - 1.664^\circ)\right] \\
&= -0.2776 + 2.8944\delta_{cr} + 8.3955\cos(\delta_{cr} - 1.664^\circ)
\end{aligned}
$$

Equating A_1 and A_2 yields

$$0.6056\delta_{cr} - 8.3955\cos(\delta_{cr} - 1.664^\circ) = 0.9943$$

(*b*) By trial and error we find

$$\delta_{cr} \cong 91.83^\circ = 1.6027 \text{ rad}$$

(*c*) The critical clearing time can be found from Eq. (16.72) since $P_e = 0$ during the fault:

$$t_{cr} = \sqrt{\frac{4 \times 11.2\,(1.6027 - 0.3644)}{377 \times 3.5}} = 0.205 \text{ s}$$