Chapter 3 Problem Solutions

3.1 Determine the highest speed at which two generators mounted on the same shaft can be driven so that the frequency of one generator is 60 Hz and the frequency of the other is 25 Hz. How many poles does each machine have?

Solution:

Let P = number of poles:

$$\begin{aligned} \text{speed} &= \frac{2 \times 60 \times 60}{P_{60}} = \frac{2 \times 60 \times 25}{P_{25}} \\ \frac{P_{60}}{P_{25}} &= \frac{60}{25} \end{aligned}$$

P_{60} and P_{25} must be even integral numbers lowest value where $P_{60} = 2.4P_{25}$. Thus,

$$P_{25} = 10 \qquad P_{60} = 24$$

3.2 The three-phase synchronous generator described in Example 3.1 is operated at 3600 rpm and supplies a unity power factor load. If the terminal voltage of the machine is 22 kV and the field current is 2500 A, determine the line current and the total power consumption of the load.

Solution:

Using the values in the solution of Example 3.1,

$$e_{a'_{max}} = \frac{45855}{3838} \times 2500 \text{ V} = 29869.1 \text{ V}$$

Given:

$$\begin{aligned} V_{LL} &= 22 \text{ kV} \\ v_{a_{max}} &= (\sqrt{2}/\sqrt{3}) \times 22000 \text{ V} = 17962.9 \text{ V} \end{aligned}$$

If $v_a = 17962.9 \cos \omega t$, then $i_a = i_{a_{max}} \cos \omega t$ and

$$\begin{aligned} e_{a'} &= 17962.9 \cos \omega t - 4.1484 \times 10^{-3} \times 120\pi \times i_{a_{max}} \sin \omega t \\ &= 17962.9 \cos \omega t - 1.5639\, i_{a_{max}} \sin \omega t \\ e_{a'_{max}} &= \sqrt{17962.9^2 + (1.5639 i_{a_{max}})^2} = 29869.1 \text{ V} \\ \text{Hence, } i_{a_{max}} &= 15259.4 \text{ A} \\ I_a &= i_{a_{max}}/\sqrt{2} = 10.79 \text{ kA} \\ P_{3\phi} &= \sqrt{3} \times 22 \times 10.79 \times 1 \text{ MW} = 411.2 \text{ MW} \end{aligned}$$

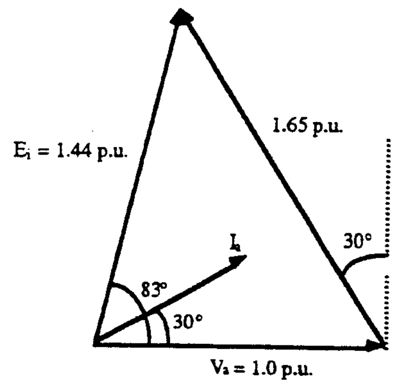
3.3 A three-phase round-rotor synchronous generator has negligible armature resistance and a synchronous reactance X_d of 1.65 per unit. The machine is connected directly to an infinite bus of voltage $1.0\angle 0^\circ$ per unit. Find the internal voltage E_i of the machine when it delivers a current of (a) $1.0\angle 30^\circ$ per unit, (*b*) $1.0\underline{/0^\circ}$ per unit and (*c*) $1.0\underline{/-30^\circ}$ per unit to the infinite bus. Draw phasor diagrams depicting the operation of the machine in each case.

Solution:

$$\begin{aligned} E_i\underline{/0^\circ} &= V_a\underline{/0^\circ} + I_a\underline{/\theta}\, X_d\underline{/90^\circ} \\ &= 1.0\underline{/0^\circ} + 1.0\underline{/\theta} \times 1.65\underline{/90^\circ} \\ &= 1.0\underline{/0^\circ} + 1.65\underline{/90^\circ + \theta} \end{aligned}$$

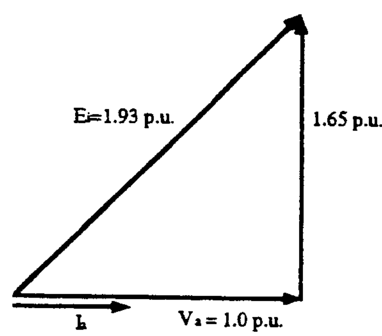
(*a*)

$$\begin{aligned} \theta &= 30^\circ \\ E_i\underline{/\delta} &= 1.0\underline{/0^\circ} + 1.65\underline{/120^\circ} \\ &= 1.44\underline{/83^\circ} \text{ per unit} \end{aligned}$$



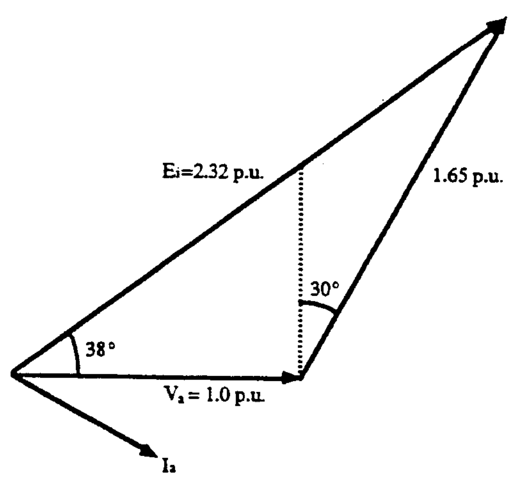
(*b*)

$$\begin{aligned} \theta &= 0^\circ \\ E_i\underline{/\delta} &= 1.0\underline{/0^\circ} + 1.65\underline{/90^\circ} \\ &= 1.93\underline{/58.8^\circ} \text{ per unit} \end{aligned}$$



(*c*)

$$\begin{aligned} \theta &= -30^\circ \\ E_i\underline{/\delta} &= 1.0\underline{/0^\circ} + 1.65\underline{/60^\circ} \\ &= 2.32\underline{/38^\circ} \text{ per unit} \end{aligned}$$



3.4 A three-phase round-rotor synchronous generator, rated 10 kV, 50 MVA has armature resistance R of 0.1 per unit and synchronous reactance X_d of 1.65 per unit. The machine operates on a 10 kV infinite bus delivering 2000 A at 0.9 power factor leading.

(*a*) Determine the internal voltage E_i and the power angle δ of the machine. Draw a phasor diagram depicting its operation.

(*b*) What is the open-circuit voltage of the machine at the same level of excitation?

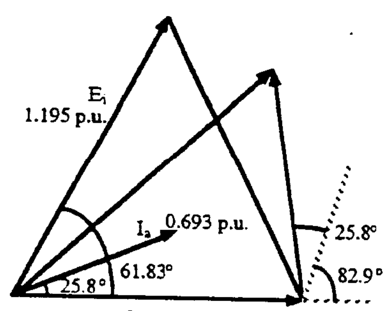
(*c*) What is the *steady-state* short-circuit current at the same level of excitation? Neglect all saturation effects.

Solution:

(*a*) Choosing $V_b = 10$ kV and $\text{MVA}_b = 50$ MVA:

$$\begin{aligned} Z_d &= (0.1 + j1.65) \text{ per unit } = 1.653\underline{/86.53^\circ} \triangleq Z_d\underline{/\alpha} \\ V &= \frac{10}{10} \text{ per unit } = 1.0 \text{ per unit} \\ I_b &= \frac{50 \times 10^3}{\sqrt{3} \times 10} \text{ A } = 2886.75 \text{ A} \\ I_a &= \frac{2000}{2886.75} \text{ per unit } = 0.693 \text{ per unit} \\ \theta &= \cos^{-1} 0.9 = 25.8^\circ \end{aligned}$$

$$\begin{aligned} E_i\underline{/\delta} &= V\underline{/0^\circ} + I_a Z_d\underline{/\alpha + \theta} \\ &= 1.0\underline{/0^\circ} + 0.693 \times 1.6530\underline{/112.37^\circ} \text{ per unit} \\ &= 1.195\underline{/61.83^\circ} = 11.95\underline{/61.83^\circ} \text{ kV} \end{aligned}$$



(*b*) Open-circuit voltage: $E_i = 11.95$ kV

(*c*) Short-circuit voltage:

$$\frac{E_i}{Z_d} = \frac{1.195}{1.653} \text{ per unit } = 0.7242 \text{ per unit } = 2090.7 \text{ A}$$

3.5 A three-phase round-rotor synchronous generator, rated 16 kV and 200 MVA, has negligible losses and synchronous reactance of 1.65 per unit. It is operated on an infinite bus having a voltage of 15 kV. The internal emf E_i and the power angle δ of the machine are found to be 24 kV (line-to-line) and 27.4°, respectively.

(*a*) Determine the line current and the three-phase real and reactive power being delivered to the system.

(*b*) If the mechanical power input and the field current of the generator are now changed so that the line current of the machine is reduced by 25% at the power factor of (*a*), find the new internal emf E_i and the power angle δ.

(*c*) While delivering the reduced line current of (*b*), the mechanical power input and the excitation are further adjusted so that the machine operates at unity power factor at its terminals. Calculate the new values of E_i and δ.

Solution:

(*a*) Using 16 kV, 200 MVA base;

$$
\begin{aligned}
V_a &= 15/16 \text{ per unit} = 0.9375 \text{ per unit} \\
E_i\angle\delta &= \frac{24}{16}\angle 27.4^\circ \text{ per unit} = 1.5\angle 27.4^\circ \text{ per unit} \\
E_i\angle\delta - V_a\angle 0^\circ &= I_a X_d\angle 90^\circ - \theta \\
1.5\angle 27.4^\circ - 0.9375\angle 0^\circ &= I_a \times 1.65\angle 90^\circ - \theta \\
I_a\angle 90^\circ - \theta &= 0.4818\angle 60.27^\circ \text{ per unit} \\
I_a\angle -\theta &= 0.4818\angle -29.73^\circ \text{ per unit} \\
\text{Base } I &= \frac{200 \times 10^3}{\sqrt{3} \times 16} \text{ kA} = 7.217 \text{ kA} \\
\text{Therefore, } I_a &= 0.4818 \times 7.217 \text{ kA} = 3.477 \text{ kA} \\
S &= 0.9375 \times 0.4818 \text{ per unit} = 0.4517 \text{ per unit} \\
&= 90.34 \text{ MVA} \\
\text{Thus, } P &= 90.34 \cos 29.73^\circ \text{ MW} = 78.45 \text{ MW} \\
Q &= 90.34 \sin 29.73^\circ \text{ Mvar} = 44.80 \text{ Mvar}
\end{aligned}
$$

(*b*)

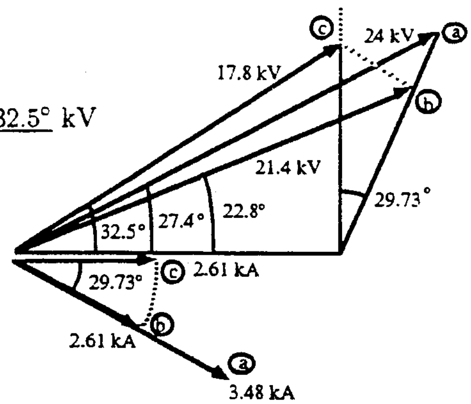
$$
\begin{aligned}
\text{New } I_a &= 0.75 \times 0.4818 \text{ per unit} = 0.3614 \text{ per unit} \\
90 - \theta &= 60.27^\circ \\
E_i\angle\delta &= V_a\angle 0^\circ + I_a X_d\angle 90^\circ - \theta \\
&= 0.9375\angle 0^\circ + 0.3614 \times 1.65\angle 60.27^\circ = 1.337\angle 22.8^\circ \text{ per unit} \\
&= 21.4\angle 22.8^\circ \text{ kV L-L}
\end{aligned}
$$

(c)

$$\text{New } I_a = 0.3614 \text{ per unit} \qquad \theta = 0^\circ$$

$$E_i\underline{/\delta} = 0.9375\underline{/0^\circ} + 0.3614 \times 1.65\underline{/90^\circ}$$

$$= 1.111\underline{/32.5^\circ} \text{ per unit} = 17.8\underline{/32.5^\circ} \text{ kV}$$



3.6 The three-phase synchronous generator of Prob. 3.5 is operated on an infinite bus of voltage 15 kV and delivers 100 MVA at 0.8 power factor lagging.

(*a*) Determine the internal voltage E_i, power angle δ and the line current of the machine.

(*b*) If the field current of the machine is reduced by 10%, while the mechanical power input to the machine is maintained constant, determine the new value of δ and the reactive power delivered to the system.

(*c*) The prime mover power is next adjusted without changing the excitation so that the machine delivers zero reactive power to the system. Determine the new power angle δ and the real power being delivered to the system.

(*d*) What is the maximum reactive power that the machine can deliver if the level of excitation is maintained as in (*b*) and (*c*)?

Draw a phasor diagram for the operation of the machine in cases (*a*), (*b*) and (*c*).

Solution:

(*a*) From Prob. 3.5,

$$V_a (\text{or } V_t) = 0.9375 \text{ per unit}$$

$$S = 0.5 \text{ per unit} \qquad X_d = 1.65 \text{ per unit} \qquad \theta = -36.9^\circ$$

$$I_a = S/V_a = 0.5/0.9375 \text{ per unit}$$

$$E_i\underline{/\delta} = V_a\underline{/0^\circ} + I_a X_d\underline{/90^\circ + \theta}$$

$$= 0.9375\underline{/0^\circ} + \frac{0.5}{0.9375} \times 1.65\underline{/90^\circ - 36.9^\circ} \text{ per unit}$$

$$= 1.6258\underline{/25.7^\circ} \text{ per unit} = 26.0\underline{/25.7^\circ} \text{ kV}$$

(*b*)

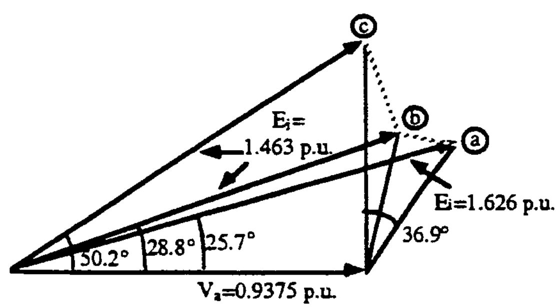
$$\begin{aligned}
\text{New } E_i &= 0.9 \times 1.6258 \text{ per unit} = 1.46322 \text{ per unit} \\
P &= 0.5 \times 0.8 \text{ per unit} = 0.4 \text{ per unit} \\
\delta &= \sin^{-1}\left(\frac{PX_d}{V_t E_i}\right) = \sin^{-1}\left(\frac{0.4 \times 1.65}{0.9375 \times 1.46322}\right) = 28.76^\circ \\
\text{New } Q &= \frac{V_t}{X_d}(E_i \cos\delta - V_t) \\
&= \frac{0.9375}{1.65}(1.46322 \cos 28.76^\circ - 0.9375) \\
&= 0.196 \text{ per unit} = 39.2 \text{ Mvar}
\end{aligned}$$

(*c*) When $Q = 0$,

$$\begin{aligned}
\delta &= \cos^{-1}\frac{V_t}{E_i} = \cos^{-1}\left(\frac{0.9375}{1.46322}\right) = 50.15^\circ \\
P &= \frac{E_i V_t}{X_d}\sin\delta = \frac{0.9375 \times 1.46322}{1.65}\sin 50.15^\circ \text{ per unit} \\
&= 0.638 \text{ per unit} = 127.65 \text{ MW}
\end{aligned}$$

(*d*) For V_t, E_i and X_d fixed, Q_{max} occurs when $\delta = 0^\circ$. Hence,

$$\begin{aligned}
Q_{max} &= \frac{V_t}{X_d}(E_i - V_t) = \frac{0.9375}{1.65}(1.46322 - 0.9375) \text{ per unit} \\
&= 0.2787 \text{ per unit} = 59.74 \text{ Mvar}
\end{aligned}$$



3.7 Starting with Eq. (3.31), modify Eq. (3.38) to show that

$$\begin{aligned}
P &= \frac{|V_t|}{R^2 + X_d^2}\{|E_i|(R\cos\delta + X_d\sin\delta) - |V_t|R\} \\
Q &= \frac{|V_t|}{R^2 + X_d^2}\{X_d(|E_i|\cos\delta - |V_t|) - R|E_i|\sin\delta\}
\end{aligned}$$

when the synchronous generator has non-zero armature resistance R.

Solution:

From Eq. (3.55), $V = E_i + I_a(R + jX_d)$ and, therefore,

$$\begin{aligned} I_a &= \frac{|E_i| \angle \delta - |V_t| \angle 0^\circ}{(R + jX_d)} \\ I_a^* &= \frac{|E_i| \angle -\delta - |V_t|}{(R - jX_d)} \\ S &= P + jQ = V_t I_a^* = \frac{|V_t|\,|E_i| \angle -\delta - |V_t|^2}{(R - jX_d)} \\ &= \frac{|V_t|\,|E_i|\,(\cos\delta - j\sin\delta) - |V_t|^2}{(R - jX_d)} \\ &= \frac{|V_t|\,|E_i|\,(\cos\delta - j\sin\delta)\,(R + jX_d) - |V_t|^2\,(R + jX_d)}{(R^2 + X_d^2)} \end{aligned}$$

Separating real and imaginary parts,

$$\begin{aligned} P &= \frac{|V_t|\,|E_i|}{(R^2 + X_d^2)}\{R\cos\delta + X_d\sin\delta\} - \frac{|V_t|^2 R}{(R^2 + X_d^2)} \\ &= \frac{|V_t|}{R^2 + X_d^2}\{|E_i|\,(R\cos\delta + X_d\sin\delta) - |V_t|\,R\} \\ Q &= \frac{|V_t|\,|E_i|}{(R^2 + X_d^2)}\{X_d\cos\delta - R\sin\delta\} - \frac{|V_t|^2 X_d}{(R^2 + X_d^2)} \\ &= \frac{|V_t|}{R^2 + X_d^2}\{X_d\,(|E_i|\cos\delta - |V_t|) - R\,|E_i|\sin\delta\} \end{aligned}$$

3.8 The three-phase synchronous generator described in Example 3.4 is now operated on a 25.2 kV infinite bus. It is found that the internal voltage magnitude $|E_i| = 49.5$ kV and that the power angle $\delta = 38.5°$. Using the loading capability diagram of Fig. 3.14, determine graphically the real and reactive power delivered to the system by the machine. Verify your answers using Eqs. (3.38).

Solution:

$$\begin{aligned} |V_t| &= 25.2 \text{ kV} = \frac{25.2}{24} \text{ per unit} = 1.05 \text{ per unit} \\ |E_i| &= 49.5 \text{ kV} = \frac{49.5}{24} \text{ per unit} = 2.0625 \text{ per unit} \end{aligned}$$

The distance n-k on the loading capability diagram is

$$\frac{|E_i|}{|V_t|\,X_d} = \frac{2.0625}{1.05 \times 1.7241} \text{ units} = 1.1393 \text{ units}$$

The angle formed by points k-n-o is 38.5°. Hence, point k is marked as shown. By reading from the chart, $P_k = 0.7$ per unit and $Q_k = 0.31$ per unit.

$$\begin{aligned} P + jQ &= |V_t|^2\,[P_k + jQ_k] = 1.05^2\,(0.7 + j0.31) \text{ per unit} \\ &= 1.05^2\,(0.7 + j0.31) \times 635 \text{ MVA} \\ P &= 490 \text{ MW} \qquad Q = 217 \text{ Mvar} \end{aligned}$$

From Eq. (3.38),

$$
\begin{aligned}
P &= \frac{|V_t|\,|E_i|}{X_d}\sin\delta \\
&= \frac{1.05 \times 2.0625}{1.7241}\sin(38.5^\circ) \times 635 \text{ MW} \;=\; 496.5 \text{ MW} \\
Q &= \frac{|V_t|}{X_d}\left(|E_i|\cos\delta - |V_t|\right) \\
&= \frac{1.05}{1.7241}\left(2.0625\cos(38.5^\circ) - 1.05\right) \times 635 \text{ Mvar} \;=\; 218.2 \text{ Mvar}
\end{aligned}
$$

3.9 A three-phase salient-pole synchronous generator with negligible armature resistance has the following values for the inductance parameters specified in Table 3.1,

$L_s = 2.7656$ mH $\quad M_f = 31.6950$ mH $\quad L_m = 0.3771$ mH

$M_s = 1.3828$ mH $\quad L_{ff} = 433.6569$ mH

During balanced steady-state operation the field current and a-phase armature current of the machine have the respective values

$$i_f = 4000 \text{ A} \qquad i_a = 20{,}000\sin(\theta_d - 30^\circ) \text{ A}$$

(a) Using Eq. (3.41), determine the instantaneous values of the flux linkages λ_a, λ_b, λ_c and λ_f when $\theta_d = 60^\circ$.

(b) Using Park's Transformation given by Eqs. (3.42) and (3.43), determine the instantaneous values of the flux linkages λ_d, λ_q and λ_0, and the currents i_d, i_q and i_0 when $\theta_d = 60^\circ$.

(c) Verify results using Eqs. (3.45) – (3.46)

Solution:

(a) From Table 3.1,

$$
\begin{aligned}
L_{abc} &\triangleq \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{aa} & L_{ab} & L_{ac} \\ L_{aa} & L_{ab} & L_{ac} \end{bmatrix} = \begin{bmatrix} L_s & -M_s & -M_s \\ -M_s & L_s & -M_s \\ -M_s & -M_s & L_s \end{bmatrix} \\
&+ L_m \begin{bmatrix} \cos 2\theta_d & -\cos 2(\theta_d + 30^\circ) & -\cos 2(\theta_d + 150^\circ) \\ -\cos 2(\theta_d + 30^\circ) & \cos 2(\theta_d - 120^\circ) & -\cos 2(\theta_d - 90^\circ) \\ -\cos 2(\theta_d + 150^\circ) & \cos 2(\theta_d - 90^\circ) & \cos 2(\theta_d + 120^\circ) \end{bmatrix} \\
L_{abc} &= \begin{bmatrix} 2.7656 & -1.3828 & -1.3828 \\ -1.3828 & 2.7656 & -1.3828 \\ -1.3828 & -1.3828 & 2.7656 \end{bmatrix} \\
&+ 0.3771 \begin{bmatrix} \cos 120^\circ & -\cos 180^\circ & -\cos 420^\circ \\ -\cos 180^\circ & \cos(-120^\circ) & -\cos(-60^\circ) \\ -\cos 420^\circ & -\cos(-60^\circ) & \cos 360^\circ \end{bmatrix} \text{ mH}
\end{aligned}
$$

$$= \begin{bmatrix} 2.57705 & -1.0057 & -1.57135 \\ -1.0057 & 2.57705 & -1.57135 \\ -1.57135 & -1.57135 & 3.1427 \end{bmatrix} \text{ mH}$$

$$L_{abc,\,f} \triangleq \begin{bmatrix} L_{af} \\ L_{bf} \\ L_{cf} \end{bmatrix} = M_f \begin{bmatrix} \cos\theta_d \\ \cos(\theta_d - 120^\circ) \\ \cos(\theta_d - 240^\circ) \end{bmatrix}$$

$$= 31.695 \begin{bmatrix} \cos 60^\circ \\ \cos(-60^\circ) \\ \cos(-180^\circ) \end{bmatrix} \text{ mH} = \begin{bmatrix} 15.8475 \\ 15.8475 \\ -31.695 \end{bmatrix} \text{ mH}$$

$$i_{abc} \triangleq \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = 20000 \begin{bmatrix} \sin(30^\circ) \\ \sin(-90^\circ) \\ \sin(-210^\circ) \end{bmatrix} \text{ A} = \begin{bmatrix} 10 \\ -20 \\ 10 \end{bmatrix} \text{ kA}$$

With $i_f = 4$ kA and $L_{ff} = 433.6569$ mH,

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

$$= \begin{bmatrix} 2.57705 & -1.0057 & -1.57135 & 15.8475 \\ -1.0057 & 2.57705 & -1.57135 & 15.8475 \\ -1.57135 & -1.57135 & 3.1427 & -31.6950 \\ 15.8475 & 15.8475 & -31.6950 & 433.6569 \end{bmatrix} \begin{bmatrix} 10 \\ -20 \\ 10 \\ 4 \end{bmatrix} \text{ Wb-T}$$

$$= \begin{bmatrix} 93.5610 \\ -13.9215 \\ -79.6395 \\ 1259.2026 \end{bmatrix} \text{ Wb-T}$$

(*b*) When $\theta_d = 60^\circ$,

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 93.5610 \\ -13.9215 \\ -79.6395 \end{bmatrix}}_{\lambda_{abc}} = \begin{bmatrix} 97.5381 \\ 76.0016 \\ 0 \end{bmatrix} \text{ Wb-T}$$

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 10 \\ -20 \\ 10 \end{bmatrix}}_{i_{abc}} = \begin{bmatrix} -12.2474 \\ 21.2132 \\ 0 \end{bmatrix} \text{ kA}$$

(*c*)

$$L_d = L_s + M_s + \tfrac{3}{2} L_m = 4.71405 \text{ mH}$$

$$
\begin{aligned}
L_q &= L_s + M_s - \tfrac{3}{2}L_m \;=\; 3.58275 \text{ mH} \\
\sqrt{\tfrac{3}{2}}\,M_f &= 38.8183 \text{ mH} \\
\lambda_d &= L_d i_d + \sqrt{\tfrac{3}{2}}\,M_f i_f \;=\; 4.71405 \times (-12.2474) + 38.8183 \times 4 \text{ Wb-T} \\
&= 97.5381 \text{ Wb-T} \\
\lambda_q &= L_q i_q \;=\; 3.58275 \times 21.2132 \text{ Wb-T} \;=\; 76.0016 \text{ Wb-T} \\
\lambda_0 &= L_0 i_0 \;=\; 0 \quad (\text{since } i_0 = 0) \\
\lambda_f &= \sqrt{\tfrac{3}{2}}\,M_f i_d + L_{ff} i_f \;=\; 38.8183 \times (-12.2474) + 433.6569 \times 4 \text{ Wb-T} \\
&= 1259.20 \text{ Wb-T}
\end{aligned}
$$

3.10 The armature of a three-phase salient-pole generator carries the currents

$$
\begin{aligned}
i_a &= \sqrt{2} \times 1000 \sin(\theta_d - \theta_a) \;\; \text{A} \\
i_b &= \sqrt{2} \times 1000 \sin(\theta_d - 120^\circ - \theta_a) \;\; \text{A} \\
i_c &= \sqrt{2} \times 1000 \sin(\theta_d - 240^\circ - \theta_a) \;\; \text{A}
\end{aligned}
$$

(*a*) Using the **P**-Transformation matrix of Eq. (3.42), find the direct-axis current i_d and the quadrature-axis current i_q. What is the zero-sequence current i_0?

(*b*) Suppose that the armature currents are

$$
\begin{aligned}
i_a &= \sqrt{2} \times 1000 \sin(\theta_d - \theta_a) \;\; \text{A} \\
i_b &= i_c \;=\; 0
\end{aligned}
$$

Determine i_d, i_q and i_0.

Solution:

(*a*)

$$
\begin{aligned}
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} &= P \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \\
&= \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \times 1000\sqrt{2} \begin{bmatrix} \sin(\theta_d - \theta_a) \\ \sin(\theta_d - \theta_a - 120^\circ) \\ \sin(\theta_d - \theta_a - 240^\circ) \end{bmatrix} \;\; \text{A} \\
&= 1000\sqrt{3} \begin{bmatrix} \sin(\theta_d - \theta_a - 60^\circ) \\ \sin(\theta_d - \theta_a + 30^\circ) \\ 0 \end{bmatrix} \;\; \text{A}
\end{aligned}
$$

(b)

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \times 1000\sqrt{2} \begin{bmatrix} \sin(\theta_d - \theta_a) \\ 0 \\ 0 \end{bmatrix} \text{ A}$$

$$= \frac{2000}{\sqrt{3}} \begin{bmatrix} \frac{1}{2}\sin(\theta_d - \theta_a) \\ \frac{\sqrt{3}}{2}\sin(\theta_d - \theta_a) \\ \frac{1}{\sqrt{2}}\sin(\theta_d - \theta_a) \end{bmatrix} \text{ A}$$

3.11 Calculate the direct-axis synchronous reactance X_d, the direct-axis transient reactance X_d' and the direct-axis subtransient reactance X_d'' of the 60 Hz salient-pole synchronous machine with the following parameters:

$L_s = 2.7656$ mH	$L_{ff} = 433.6569$ mH	$L_D = 4.2898$ mH
$M_s = 1.3828$ mH	$M_f = 31.6950$ mH	$M_D = 3.1523$ mH
$L_m = 0.3771$ mH	$M_r = 37.0281$ mH	

Solution:

$$L_d = L_s + M_s - \frac{3}{2}L_m = 2.7656 + 1.3828 - \frac{3}{2} \times 0.3771 \text{ mH} = 4.71405 \text{ mH}$$

$$X_d = 120\pi \times 4.71405 \times 10^{-3}\,\Omega = 1.777\,\Omega$$

$$L_d' = L_d - \frac{3}{2}\frac{M_f^2}{L_{ff}} = 4.71405 - \frac{3}{2} \times \frac{31.6950^2}{433.6569} \text{ mH} = 1.2393 \text{ mH}$$

$$X_d' = 120\pi \times 1.2393 \times 10^{-3}\,\Omega = 0.467\,\Omega$$

$$L_d'' = L_d - \frac{3}{2}\left(\frac{M_f^2 L_D + M_D^2 L_{ff} - 2M_f M_D M_r}{L_{ff}L_D - M_r^2}\right)$$

$$= 4.71405 - \frac{3}{2}\left(\frac{31695^2 \times 4.2898 + 3.1523^2 \times 433.6569 - 2 \times 31.6950 \times 3.1523 \times 37.0281}{433.6569 \times 4.2898 - 37.0281^2}\right) \text{ mH}$$

$$= 0.9748 \text{ mH}$$

$$X_d'' = 120\pi \times 0.9748 \times 10^{-3}\,\Omega = 0.367\,\Omega$$

3.12 The single-line diagram of an unloaded power system is shown in Fig. 3.22. Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are rated as follows:

Generator 1:	20 MVA, 13.8 kV, $X''_d = 0.20$ per unit
Generator 2:	30 MVA, 18 kV, $X''_d = 0.20$ per unit
Generator 3:	30 MVA, 20 kV, $X''_d = 0.20$ per unit
Transformer T_1:	25 MVA, 220Y/13.8Δ kV, $X = 10\%$
Transformer T_2:	Single-phase units each rated 10 MVA,127/18 kV, $X = 10\%$
Transformer T_3:	35 MVA, 220Y/22Y kV, $X = 10\%$

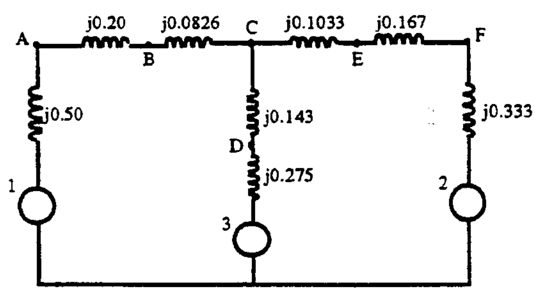
(*a*) Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the single-line diagram. Choose a base of 50 MVA, 13.8 kV in the circuit of Generator 1.

(*b*) Suppose that the system is unloaded and that the voltage throughout the system is 1.0 per unit on bases chosen in part (*a*). If a three-phase short circuit occurs from bus C to ground, find the phasor value of the short-circuit current (in amperes) if each generator is represented by its subtransient reactance.

(*c*) Find the megavoltamperes supplied by each generator under the conditions of part (*b*).

Solution:

(*a*)



$$\text{Gen 1: } X'' = 0.2 \times \frac{50}{20} = 0.50 \text{ per unit}$$

$$3\phi \text{ rating } T_2 = 220/18 \text{ kV, } 30 \text{ MVA}$$

$$\text{Base in trans. line: } 220 \text{ kV, } 50 \text{ MVA}$$

$$\text{Base for Gen 2} = 18 \text{ kV}$$

$$\text{Gen 2: } X'' = 0.2 \times \frac{50}{30} = 0.333 \text{ per unit}$$

$$\text{Base for Gen 3} = 22 \text{ kV}$$

$$\text{Gen 3: } X'' = 0.2\left(\frac{20}{22}\right)^2 \times \frac{50}{30} = 0.275 \text{ per unit}$$

$$\text{Transformer } T_1\text{: } X = .01 \times \frac{50}{25} = 0.20 \text{ per unit}$$

$$\text{Transformer } T_2\text{: } X = .01 \times \frac{50}{30} = 0.167 \text{ per unit}$$

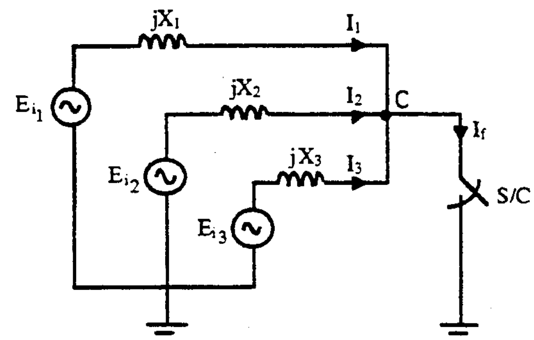
$$\text{Transformer } T_3\text{: } X = .01 \times \frac{50}{35} = 0.143 \text{ per unit}$$

Transmission lines:

$$\text{Base } Z = \frac{220^2}{50} = 968\ \Omega$$

$$\frac{80}{968} = 0.0826 \text{ per unit} \qquad \frac{100}{968} = 0.1033 \text{ per unit}$$

(*b*)



$$\begin{aligned}
E_{i_1} &= E_{i_2} = E_{i_3} = 1.0\angle 0^\circ \text{ per unit} \\
X_1 &= 0.50 + 0.20 + 0.0826 \text{ per unit} = 0.7826 \text{ per unit} \\
X_2 &= 0.333 + 0.167 + 0.1033 \text{ per unit} = 0.6033 \text{ per unit} \\
X_3 &= 0.143 + 0.275 \text{ per unit} = 0.418 \text{ per unit} \\
I_1 &= \frac{E_{i_1}}{jX_1} = \frac{1}{0.7826}\angle -90^\circ \text{ per unit} = 1.278\angle -90^\circ \text{ per unit} \\
I_2 &= \frac{E_{i_2}}{jX_2} = \frac{1}{0.6033}\angle -90^\circ \text{ per unit} = 1.658\angle -90^\circ \text{ per unit} \\
I_3 &= \frac{E_{i_3}}{jX_3} = \frac{1}{0.418}\angle -90^\circ \text{ per unit} = 2.392\angle -90^\circ \text{ per unit} \\
I_f &= I_1 + I_2 + I_3 = (1.278 + 1.658 + 2.392)\angle -90^\circ \text{ per unit} = 5.328\angle -90^\circ \text{ per unit} \\
I_{\text{base}} \text{ at C} &= \frac{50 \times 10^6}{\sqrt{3} \times 220 \times 10^3} \text{ A} = 131.22 \text{ A} \\
|I_f| &= 5.328 \times 131.22 \text{ A} = 699 \text{ A}
\end{aligned}$$

(*c*)

$$\begin{aligned}
|S_1| &= E_{i_1} I_1 = 1.0 \times 1.278 \times 50 \text{ MVA} = 63.9 \text{ MVA} \\
|S_2| &= E_{i_2} I_2 = 1.0 \times 1.658 \times 50 \text{ MVA} = 82.9 \text{ MVA} \\
|S_3| &= E_{i_3} I_3 = 1.0 \times 2.392 \times 50 \text{ MVA} = 119.6 \text{ MVA}
\end{aligned}$$

3.13 The ratings of the generators, motors and transformers of Fig. 3.23 are:

Generator 1:	20 MVA, 18 kV, $X_d'' = 20\%$
Generator 2:	20 MVA, 18 kV, $X_d'' = 20\%$
Synchronous motor 3:	30 MVA, 13.8 kV, $X_d'' = 20\%$
Three-phase Y-Y transformers:	20 MVA, 138Y/20Y kV, $X = 10\%$
Three-phase Y-Δ transformers:	15 MVA, 138Y/13.8Δ kV, $X = 10\%$

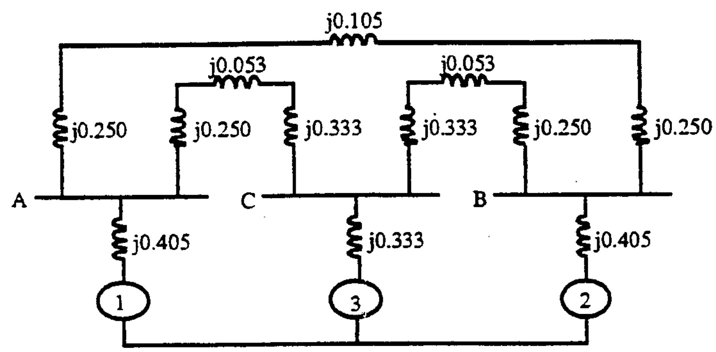
(a) Draw the impedance diagram for the power system. Mark impedances in per unit. Neglect resistance and use a base of 50 MVA, 138 kV in the 40-Ω line.

(b) Suppose that the system is unloaded and that the voltage throughout the system is 1.0 per unit on bases chosen in part (a). If a three-phase short circuit occurs from bus C to ground, find the phasor value of the short-circuit current (in amperes) if each generator is represented by its subtransient reactance.

(c) Find the megavoltamperes supplied by each synchronous machine under the conditions of part (b).

Solution:

(a)



Base voltages are:

40 Ω lines	138 kV
20 Ω lines	138 kV
Gen. 1 & 2	20 kV
Motor 3	13.8 kV

$$\text{Base impedance in lines} = \frac{138^2}{50} = 381\ \Omega$$

$$40\ \Omega \text{ line: } Z = \frac{40}{381} = 0.105 \text{ per unit}$$

$$20\ \Omega \text{ line: } Z = \frac{20}{381} = 0.053 \text{ per unit}$$

Transformers:

$$\text{Y-Y} = 0.1 \times \frac{50}{20} = 0.250 \text{ per unit}$$

$$\text{Y-}\Delta = 0.1 \times \frac{50}{15} = 0.333 \text{ per unit}$$

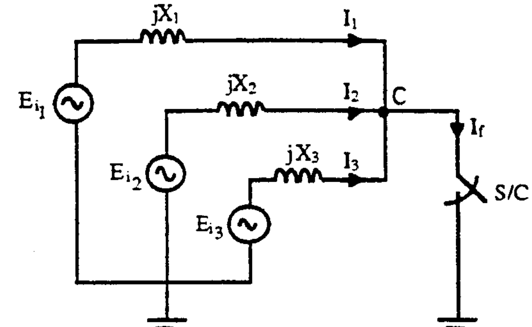
Gens. 1 & 2:

$$X'' = 0.20 \times \left(\frac{18}{20}\right)^2 \times \frac{50}{20} = 0.405 \text{ per unit}$$

Motor 3:

$$X'' = 0.20 \times \frac{50}{30} = 0.333 \text{ per unit}$$

(*b*) If a fault occurs at C, by symmetry equal currents are input from generators 1 and 2. Moreover, no current should exist between busses A and B through the $j0.105$ per unit branch. If this branch is omitted from the circuit, the system simplifies to



$$\begin{aligned}
E_{i_1} &= E_{i_2} = E_{i_3} = 1.0\angle 0^\circ \text{ per unit} \\
X_1 &= X_2 = 0.405 + 0.250 + 0.053 + 0.333 \text{ per unit} = 1.041 \text{ per unit} \\
X_3 &= 0.333 \text{ per unit} \\
|I_1| &= |I_2| = \frac{|E_{i_1}|}{|X_1|} = \frac{1.0}{1.041} \text{ per unit} = 0.9606 \text{ per unit} \\
|I_3| &= \frac{|E_{i_3}|}{|X_3|} = \frac{1.0}{0.333} \text{ per unit} = 3.0 \text{ per unit} \\
|I_f| &= 4.9212 \text{ per unit} \\
I_{\text{base}} \text{ at C} &= \frac{50 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} \text{ A} = 2091.8 \text{ A} \\
|I_f| &= 4.9212 \times 2091.8 \text{ A} = 10.29 \text{ kA}
\end{aligned}$$

(*c*)

$$\begin{aligned}
|S_1| &= |S_2| = 1.0 \times 50 \times 0.9606 \text{ MVA} = 48.03 \text{ MVA} \\
|S_3| &= 1.0 \times 50 \times 3.0 \text{ MVA} = 150 \text{ MVA}
\end{aligned}$$

Chapter 4 Problem Solutions

4.1 The all-aluminum conductor identified by the code word *Bluebell* is composed of 37 strands each having a diameter of 0.1672 in. Tables of characteristics of all-aluminum conductors list an area of 1,033,500 cmil for this conductor (1 cmil = $(\pi/4) \times 10^{-6}$ in^2). Are these values consistent with each other? Find the overall area of the strands in square millimeters.

Solution:

$$\text{diameter} = 0.1672 \times 1000 = 167.2 \text{ mils/strand}$$