In original positions in the transposition cycle,

$$\begin{aligned}
\text{distance } a\text{-}b &= \sqrt{14^2 + 3.5^2} = 14.43 \text{ ft} \\
\text{distance } a\text{-}b' &= \sqrt{14^2 + 28.5^2} = 31.75 \text{ ft} \\
\text{distance } a\text{-}a' &= \sqrt{25^2 + 28^2} = 37.54 \text{ ft} \\
D^p_{ab} &= D^p_{bc} = \sqrt[4]{14.43^2 \times 31.75^2} = 21.04 \text{ ft} \\
D_{ac} &= \sqrt[4]{25^2 \times 28^2} = 26.46 \text{ ft} \\
D_{eq} &= \sqrt[3]{21.04^2 \times 26.46} = 22.71 \text{ ft} \\
D_s &= \left[\left(\sqrt{0.0373 \times 37.54}\right)^2 \sqrt{0.0373 \times 32}\right]^{\frac{1}{3}} = 1.152 \text{ ft} \\
L &= 2 \times 10^{-7} \ln \frac{22.71}{1.152} = 5.693 \times 10^{-7} \text{ H/m} \\
&= 5.963 \times 10^{-7} \times 10^3 \times 1609 = 0.959 \text{ mH/mi} \\
X_L &= 377 \times 0.959 \times 10^{-3} = 0.362 \ \Omega\text{/mi/phase}
\end{aligned}$$

(*b*) $r = \frac{1.108}{2 \times 12} = 0.0462$ ft as in part (*a*) above, except that r is substituted for D_s:

$$D_{sC} = \left[\left(\sqrt{0.0462 \times 37.54}\right)^2 \sqrt{0.0462 \times 32}\right]^{\frac{1}{3}} = 1.282 \text{ ft}$$

From part (*a*) above, $D_{eq} = 22.71$ ft and

$$\begin{aligned}
X_C &= 2.965 \times 10^{-4} \ln \frac{22.71}{1.282} = 85,225 \ \Omega \cdot \text{mi/phase to neutral} \\
I_{chg} &= \frac{138,000/\sqrt{3}}{85,225} = 0.935 \text{ A/mi/phase} = 0.467 \text{ A/mi/conductor}
\end{aligned}$$

Chapter 6 Problem Solutions

6.1 An 18-km 60-Hz single circuit three-phase line is composed of *Partridge* conductors equilaterally spaced with 1.6 m between centers. The line delivers 2500 kW at 11 kV to a balanced load. Assume a wire temperature of 50°C.

(*a*) Determine the per-phase series impedance of the line.

(*b*) What must be the sending-end voltage when the power factor is (*i*) 80% lagging (*ii*) unity (*iii*) 90% leading?

(*c*) Determine the percent regulation of the line at the above power factors.

(*d*) Draw phasor diagrams depicting the operation of the line in each case.

Solution:

(*a*)

$$\begin{aligned}
R &= 0.3792 \times \frac{18}{1.609} = 4.242 \ \Omega \\
\text{From Table A.3, } X_a &= 0.465 \ \Omega\text{/mi}
\end{aligned}$$

and since 1.6 m = (1.6 × 100)/(2.54 × 12) = 5.25 ft,

$$\begin{aligned}
X_d &= 0.2012 \quad \text{(Table A.4, 5'-3'')} \\
X &= 0.465 + 0.2012 = 0.666\ \Omega/\text{mi} \\
\text{For 18 km, } X &= 18 \times \frac{0.666}{1.609} = 7.451\ \Omega \\
Z &= 4.242 + j7.451 = 8.57\angle 60.35^\circ\ \Omega
\end{aligned}$$

(*b*) For power factor = 1.0,

$$\begin{aligned}
I_R &= \frac{2500}{\sqrt{3} \times 11} = 131.2\ \text{A} \qquad \frac{11,000}{\sqrt{3}} = 6350\ \text{V} \\
V_S &= 6350 + 131.2\,(4.24 + j7.451) \\
&= 6906 + j977.6 = 6975\angle 8.06^\circ \\
\text{sending-end line voltage} &= \sqrt{3}\,V_S = \sqrt{3} \times 6975 = 12,081\ \text{V}
\end{aligned}$$

For power factor = 0.8 lagging,

$$\begin{aligned}
|I_R| &= \frac{2500}{\sqrt{3} \times 11 \times 0.8} = 164\ \text{A} \\
V_S &= 6350 + 164\angle -36.87^\circ \times 8.57\angle 60.35^\circ \\
&= 7639 + j5.60 = 7660\angle 4.19^\circ \\
\text{sending-end line voltage} &= \sqrt{3}\,V_S = \sqrt{3} \times 7660 = 13,268\ \text{V}
\end{aligned}$$

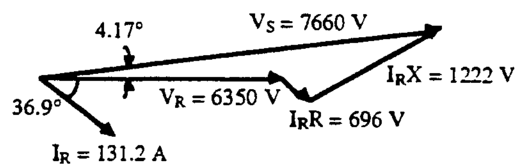
For power factor = 0.9 leading,

$$\begin{aligned}
|I_R| &= \frac{2500}{\sqrt{3} \times 11 \times 0.9} = 145.8\ \text{A} \\
V_S &= 6350 + 145.8\angle 25.84^\circ \times 8.57\angle 60.35^\circ \\
&= 6433 + j1247 = 6553\angle 10.97^\circ \\
\text{sending-end line voltage} &= \sqrt{3}\,V_S = \sqrt{3} \times 6553 = 11,350\ \text{V}
\end{aligned}$$

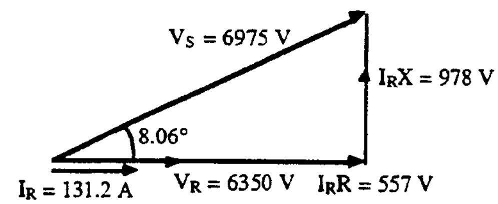
(*c*)

$$\begin{aligned}
\%\ \text{Regulation} &= \frac{|V_S| - |V_R|}{|V_R|} \times 100\% \\
\text{at p.f.} = 0.8\ \text{lagging, \% Reg.} &= \frac{7660 - 6350}{6350} \times 100\% = 20.63\,\% \\
\text{at unity p.f., \% Reg.} &= \frac{6975 - 6350}{6350} \times 100\% = 9.84\,\% \\
\text{at p.f.} = 0.9\ \text{leading, \% Reg.} &= \frac{6553 - 6350}{6350} \times 100\% = 3.20\,\%
\end{aligned}$$

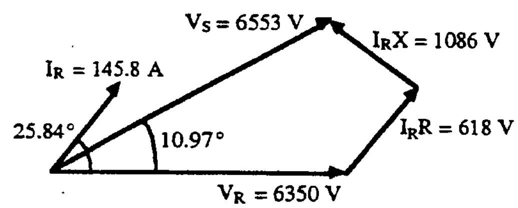
(*d*) For p.f. = 0.8 lagging,



For unity p.f.,



For p.f. = 0.9 leading,



6.2 A 100-mi, single-circuit, three-phase transmission line delivers 55 MVA at 0.8 power factor lagging to the load at 132 kV (line-to-line). The line is composed of *Drake* conductors with flat horizontal spacing of 11.9 ft between adjacent conductors. Assume a wire temperature of 50°C. Determine

(*a*) the series impedance and the shunt admittance of the line.

(*b*) the ABCD constants of the line.

(*c*) the sending-end voltage, current real and reactive powers and the power factor.

(*d*) the percent regulation of the line.

Solution:

(*a*)

$$\begin{aligned}
D_{eq} &= \sqrt[3]{11.9 \times 11.9 \times 2 \times 11.9} = 15 \text{ ft} \\
\text{series impedance } Z &= 100 \times (0.1284 + j0.399 + j0.3286) \\
&= 12.84 + j72.76 = 73.88\angle 80^\circ \ \Omega \\
\frac{Y}{2} &= j\frac{100}{2}\left(\frac{10^{-6}}{0.9012 + 0.0803}\right) = 2.915 \times 10^{-4}\angle 90^\circ \text{ S} \\
\text{shunt admittance } Y &= 5.83 \times 10^{-4}\angle 90^\circ \text{ S}
\end{aligned}$$

(*b*)

$$A = D = 1 + \frac{ZY}{2}$$

$$= 1 + \frac{73.88 \times 5.83 \times 10^{-4}}{2} \angle 170^\circ = 0.979 \angle 0.219^\circ$$

$$B = Z = 73.88 \angle 80^\circ \ \Omega$$

$$C = Y\left(1 + \frac{ZY}{4}\right)$$

$$= 5.83 \times 10^{-4}\left(1 + \frac{73.88 \times 5.83 \times 10^{-4}}{4} \angle 170^\circ\right) \text{ S} = 5.768 \times 10^{-4} \angle 90.108^\circ \text{ S}$$

(Check: $AD - BC = 1$ must be satisfied)

(*c*)

$$I_R = \frac{55{,}000}{\sqrt{3} \times 132}(0.8 - j0.6) = 192.4 - j144.3 \text{ A}$$

Current in series arm:

$$I_{\text{series}} = 192.4 - j144.3 + j\frac{132{,}000}{\sqrt{3}} \times 2.915 \times 10^{-4}$$

$$= 192.4 - j122.1 = 227.9 \angle -32.40^\circ$$

$$V_S = \frac{132{,}000}{\sqrt{3}} + 227.9 \angle -32.40^\circ \times 73.88 \angle 80^\circ$$

$$= 87{,}563 + j12{,}434 = 88{,}441 \angle 8.08^\circ \text{ V to neutral}$$

$$|V_S| = \sqrt{3} \times 88{,}441 = 153.2 \text{ kV, line-to-line}$$

$$I_S = 192.4 - j122.4 + j2.915 \times 10^{-4} \times (87{,}563 + j12{,}434)$$

$$= 188.8 - j96.9 = 212 \angle -27.2^\circ$$

$$|I_S| = 212 \text{ A}$$

$$\theta_S = 8.08^\circ - (-27.2^\circ) = 35.28^\circ$$

$$P_S = \left(\sqrt{3} \times 153.2 \times 212\right) \cos 35.28^\circ \text{ kW} = 45.92 \text{ MW}$$

$$Q_S = \left(\sqrt{3} \times 153.2 \times 212\right) \sin 35.28^\circ \text{ kvar} = 32.49 \text{ Mvar}$$

$$\text{(sending-end) p.f.} = \cos 35.28^\circ = 0.816 \text{ lagging}$$

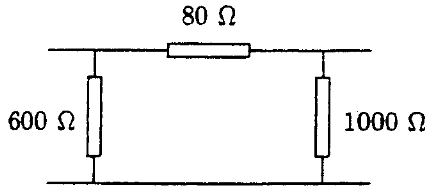
(*d*)

$$\% \text{ Reg.} = \frac{|V_S| / |A| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$$

$$= \frac{(153.2/0.979) - 132}{132} \times 100\% = 18.55\%$$

6.3 Find the *ABCD* constants of a π circuit having a 600-Ω resistor for the shunt branch at the sending end, a 1-kΩ resistor for the shunt branch at the receiving end and an 80-Ω resistor for the series branch.

Solution:



$$\begin{aligned}
V_S &= V_R + \left(I_R + \frac{V_R}{1000}\right) \times 80 \;=\; V_R + 80I_R + 0.08V_R \\
&= 1.08V_R + 80I_R \\
I_S &= I_R + \frac{V_R}{1000} + \frac{1.08V_R + 80I_R}{600} \;=\; 0.001V_R + 0.0018V_R + I_R + 0.133I_R \\
&= 0.0028V_R + 1.133I_R
\end{aligned}$$

The $ABCD$ constants are

$$\begin{aligned}
A &= 1.08 \quad & C &= 0.0028\ \text{S} \\
B &= 80\ \Omega \quad & D &= 1.133
\end{aligned}$$

6.4 The $ABCD$ constants of a three-phase transmission line are

$$\begin{aligned}
A = D &= 0.936 + j0.016 \;=\; 0.936\underline{/0.98^\circ} \\
B &= 33.5 + j138 \;=\; 142\underline{/76.4^\circ}\ \Omega \\
C &= (-5.18 + j914) \times 10^{-6}\ \ \text{S}
\end{aligned}$$

The load at the receiving end is 50 MW at 220 kV with a power factor of 0.9 lagging. Find the magnitude of the sending-end voltage and the voltage regulation. Assume the magnitude of the sending-end voltage remains constant.

Solution:

$$\begin{aligned}
I_R &= \frac{50,000}{\sqrt{3} \times 220 \times 0.9}\underline{/-25.84^\circ} \;=\; 145.8\underline{/-25.84^\circ}\ \text{A} \\
V_R &= \frac{220,000}{\sqrt{3}} \;=\; 127,000\underline{/0^\circ}\ \text{V} \\
V_S &= 0.936\underline{/0.98^\circ} \times 127,000\underline{/0^\circ} + 142\underline{/76.4^\circ} \times 145.8\underline{/-25.84^\circ} \\
&= 118,855 + j2033 + 13,153 + j15,990 \;=\; 133.23\underline{/7.77^\circ}\ \text{kV}
\end{aligned}$$

With line-to-line sending-end voltage $|V_S| = \sqrt{3} \times 133.23 = 230.8$ kV,

$$\begin{aligned}
|V_{R,NL}| &= \frac{230.8}{0.936} \;=\; 246.5\ \text{kV} \\
\%\ \text{Reg.} &= \frac{246.5 - 220}{220} \times 100 \;=\; 12.0\,\%
\end{aligned}$$

6.5 A 70 mi, single-circuit, three-phase line composed of *Ostrich* conductors is arranged in flat horizontal spacing with 15 ft between adjacent conductors. The line delivers a load of 60 MW at 230 kV with 0.8 power factor lagging.

(a) Using a base of 230 kV, 100 MVA, determine the series impedance and the shunt admittance of the line in per unit. Assume a wire temperature of 50°C. Note that the base admittance must be the reciprocal of base impedance.

(b) Find the voltage, current, real and reactive power, and the power factor at the sending end in both per unit and absolute units.

(c) What is the percent regulation of the line?

Solution:

Supporting calculations:

$$\text{Base impedance} = \frac{(230)^2}{100} = 529\ \Omega$$

$$\text{Base current} = \frac{100,000}{\sqrt{3} \times 230} = 251\ \text{A}$$

$$I_R = \frac{60,000}{\sqrt{3} \times 230 \times 0.8}\angle -36.87^\circ = 188.3\angle -36.87^\circ\ \text{A}$$

$$= 0.75\angle -36.87^\circ\ \text{per unit} = 0.6 - j0.45\ \text{per unit}$$

$$V_R = 1.0\angle 0^\circ\ \text{per unit}$$

$$D_{eq} = \sqrt[3]{2} \times 15 = 18.9\ \text{ft}$$

(a)

$$Z = 70 \times (0.3372 + j0.458 + j0.3566) = 70 \times 0.8816\angle 67.5^\circ$$

$$= 61.7\angle 67.5^\circ\ \Omega = 0.1166\angle 67.5^\circ\ \text{per unit}$$

$$\text{Then, } X_C = (0.1057 + 0.0872)/70\ \text{M}\Omega$$

$$\frac{Y}{2} = \frac{70 \times 10^{-6}}{0.1929 \times 2}\ \text{S} = 181.44 \times 10^{-6} \times 529 = 0.096\ \text{per unit}$$

$$Y = 0.192\angle 90^\circ\ \text{per unit}$$

(b)

$$I_{series} = 0.75\angle -36.87^\circ + 1.0(j0.096) = 0.6 - j0.45 + j0.96$$

$$= 0.6 - j0.546 = 0.8112\angle -42.3^\circ$$

$$V_S = 1.0 + 0.8112\angle -42.3^\circ \times 0.1166\angle 67.5^\circ = 1.0856 + j0.0403$$

$$= 1.086\angle 2.125^\circ\ \text{per unit}$$

$$= 1.086 \times 230 = 249.8\ \text{kV}$$

$$I_S = 0.6 - j0.546 + 1.086\angle 2.125^\circ \times 0.096\angle 90^\circ = 0.5961 - j0.4418$$

$$= 0.742\angle -36.54^\circ\ \text{per unit}$$

$$= 0.742 \times 251 = 186.2\ \text{A}$$

$$\text{(sending-end) p.f.} = \cos(2.125^\circ - (-36.54^\circ)) = 0.781$$

$$P_S = 1.086 \times 0.742 \times 0.781 = 0.6293\ \text{per unit}$$

$$= 100 \times 0.6293 = 62.93\ \text{MW}$$

$$Q_S = 1.086 \times 0.742 \times \sin(2.125^\circ - (-36.54^\circ)) = 0.503 \text{ per unit}$$
$$= 100 \times 0.503 = 50.3 \text{ Mvar}$$

(c)

$$A = 1 + \frac{ZY}{2} = 1 + \frac{1}{2}(0.1166\angle 67.5^\circ \times 0.192\angle 90^\circ) = 0.990\angle 0.248^\circ$$
$$\%\ \text{Reg.} = \frac{|V_S|/|A| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%$$
$$= \frac{(1.086/0.990) - 1.0}{1.0} \times 100\% = 9.73\,\%$$

6.6 A single-circuit, three-phase transmission line is composed of *Parakeet* conductors with flat horizontal spacing of 19.85 ft between adjacent conductors. Determine the characteristic impedance and the propagation constant of the line at 60 Hz and 50°C temperature.

Solution:

At 50°C and 60 Hz, from Table A.3, for *Parakeet* conductors,

$$r = 0.1832\ \Omega/\text{mi} \qquad X_a = 0.423\ \Omega/\text{mi}$$
$$D_{eq} = \sqrt[3]{19.85^3 \times 2}\ \text{ft} = 25\ \text{ft}$$
$$\text{At 25 ft, } X_d(\text{inductive}) = 0.3906\ \Omega/\text{mi}$$

Therefore,

$$z = 0.1832 + j(0.423 + 0.3906)\ \Omega/\text{mi}$$
$$= 0.834\angle 77.31^\circ\ \Omega/\text{mi}$$
$$X'_a = 0.0969 \times 10^{-6}\ \Omega \cdot \text{mi}$$
$$X_d(\text{capacitive}) = 0.0955 \times 10^{-6}\ \Omega \cdot \text{mi}$$
$$y = \frac{j}{X'_a + X_d} = \frac{10^{-6}\angle 90^\circ}{0.0969 + 0.0955}\ \text{S/mi}$$
$$= 5.1975 \times 10^{-6}\angle 90^\circ\ \text{S/mi}$$

Characteristic impedance:

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.834\angle 77.31^\circ}{5.1975 \times 10^{-6}\angle 90^\circ}}\ \Omega = 400.6\angle -6.345^\circ\ \Omega$$

Propagation constant:

$$\gamma = \sqrt{zy} = \sqrt{0.834 \times 5.1975 \times 10^{-6}\angle 77.31^\circ + 90^\circ}\ \text{mi}^{-1} = 2.08 \times 10^{-3}\ \text{mi}^{-1}$$

6.7 Using Eqs. (6.23) and (6.24) show that, if the receiving-end of a line is terminated by its characteristic impedance Z_c, then the impedance seen at the sending end of the line is also Z_c regardless of line length.

Solution:

If $Z_R = Z_c$, then $I_R = V_R/Z_c$, and $V_R - I_R Z_c = 0$.

$$\text{From Eq. (6.23)} \quad V_S = \frac{V_R + I_R Z_c}{2}\epsilon^{\gamma L}$$

$$\text{From Eq. (6.24)} \quad I_S = \frac{V_R + I_R Z_c}{2Z_c}\epsilon^{\gamma L}$$

where L is the length of the line. Finally,

$$Z_s = V_S/I_S = Z_c \quad \text{(which is independent of } L\text{)}$$

6.8 A 200-mi transmission line has the following parameters at 60 Hz

$$\begin{aligned} \text{resistance } r &= 0.21 \ \Omega/\text{mi per phase} \\ \text{series reactance } x &= 0.78 \ \Omega/\text{mi per phase} \\ \text{shunt susceptance } b &= 5.42 \times 10^{-6} \ \text{S/mi per phase} \end{aligned}$$

(*a*) Determine the attenuation constant α, wavelength λ and the velocity of propagation of the line at 60 Hz.

(*b*) If the line is open circuited at the receiving end and the receiving-end voltage is maintained at 100 kV line-to-line use Eqs. (6.26) and (6.27) to determine the incident and reflected components of the sending-end voltage and current.

(*c*) Hence determine the sending-end voltage and current of the line.

Solution:

(*a*)

$$\begin{aligned} r &= 0.21 \ \Omega/\text{mi} \qquad x_l = 0.78 \ \Omega/\text{mi} \\ z &= (0.21 + j0.78) \ \Omega/\text{mi} = 0.808\angle 77.31^\circ \ \Omega/\text{mi} \\ y &= 5.42 \times 10^{-6}\angle 77.31^\circ \ \text{S/mi} \\ \gamma &= \sqrt{zy} = 2.092 \times 10^{-3}\angle 82.47^\circ \ \text{mi}^{-1} \\ &= \alpha + j\beta = (2.744 \times 10^{-4} + j2.074 \times 10^{-3}) \ \text{mi}^{-1} \\ \text{Attenuation-constant } \alpha &= 2.744 \times 10^{-4} \ \text{nepers/mi} \\ \text{Wavelength } \lambda &= \frac{2\pi}{\beta} = \frac{2\pi \times 10^3}{2.074} \ \text{mi} = 3030 \ \text{mi} \\ \text{Velocity of propagation } \lambda f &= \frac{2\pi f}{\beta} = \frac{120\pi \times 10^3}{2.074} \ \text{mi/s} = 181770 \ \text{mi/s} \end{aligned}$$

(*b*)

$$\text{Characteristic impedance: } Z_c = \sqrt{\frac{z}{y}} = 386.05\angle -7.53^\circ \ \Omega$$

When the receiving end is open circuited, $I_R = 0$. Then,

$$\text{from Eq. (6.26) } V_S = \frac{V_R}{2}\epsilon^{\alpha L}\epsilon^{j\beta L} + \frac{V_R}{2}\epsilon^{-\alpha L}\epsilon^{-j\beta L}$$

$$\text{from Eq. (6.27) } I_S = \underbrace{\frac{V_R}{2Z_c}\epsilon^{\alpha L}\epsilon^{j\beta L}}_{incident} - \underbrace{\frac{V_R}{2Z_c}\epsilon^{-\alpha L}\epsilon^{-j\beta L}}_{reflected}$$

$$\text{where } L = 200 \text{ mi} = \text{length of the line}$$

$$\epsilon^{\alpha L} = 1.0564 \qquad \epsilon^{-\alpha L} = 0.9466$$

$$\beta L = 2.074 \times 10^{-3} \times 200 \times \frac{180}{\pi} \text{ deg} = 23.77^\circ$$

Hence, at the sending end (taking the receiving-end line voltage as reference), the line-to-line voltages and currents are

$$\begin{aligned}
\text{incident voltage } v_i &= \frac{100}{2}\angle 0^\circ \times 1.0564\angle 23.77^\circ \text{ kV} \\
&= 52.82\angle 23.77^\circ \text{ kV} \\
\text{reflected voltage } v_r &= \frac{100}{2}\angle 0^\circ \times 0.9466\angle -23.77^\circ \text{ kV} \\
&= 47.33\angle -23.77^\circ \text{ kV} \\
\text{incident current } I_i &= \frac{100\angle 0^\circ}{2 \times 386.05\angle -7.53^\circ} \times \frac{1.0564\angle 23.77^\circ}{\sqrt{3}\angle 30^\circ} \text{ kA} \\
&= 78.99\angle 1.3^\circ \text{ A} \\
\text{reflected current } I_r &= -\frac{100\angle 0^\circ}{2 \times 386.05\angle -7.53^\circ} \times \frac{0.9466\angle -23.77^\circ}{\sqrt{3}\angle 30^\circ} \text{ kA} \\
&= 70.78\angle 133.76^\circ \text{ A}
\end{aligned}$$

(The 30° angle in the denominator of the second fraction of the current equations above represents a phase/line V conversion.)

(*c*)

$$\begin{aligned}
V_S &= V_i + V_r = 52.82\angle 23.77^\circ + 47.33\angle -23.77^\circ \text{ kV} = 91.68\angle 1.38^\circ \text{ kV} \\
I_S &= I_i + I_r = 78.99\angle 1.3^\circ + 70.78\angle 133.76^\circ \text{ A} = 60.8370.78\angle 60.4^\circ \text{ A}
\end{aligned}$$

where all angles are expressed with respect to receiving-end *line* voltage.

6.9 Evaluate $\cosh\theta$ and $\sinh\theta$ for $\theta = 0.5\angle 82^\circ$.

Solution:

$$\begin{aligned}
0.5\angle 82^\circ &= 0.0696 + j0.4951 \\
0.4951 \text{ radian} &= 28.37^\circ \\
\cosh\theta &= \frac{1}{2}\left(\epsilon^{0.0696}\angle 28.37^\circ + \epsilon^{-0.0696}\angle -28.37^\circ\right) \\
&= \frac{1}{2}(0.9433 + j0.5094 + 0.8207 - j0.4432) = 0.8820 + j0.0331 \\
\sinh\theta &= \frac{1}{2}(0.9433 + j0.5094 - 0.8207 + j0.4432) = 0.0613 + j0.4763
\end{aligned}$$

6.10 Using Eqs. (6.1), (6.2), (6.10) and (6.37) show that the generalized circuit constants of all three transmission line models satisfy the condition that

$$AD - BC = 1$$

Solution:

Short-line model (from Eq. (6.1) and (6.2)):

$$\begin{aligned} A &= D = 1 \qquad B = Z \qquad C = 0 \\ AD - BC &= 1 - Z \times 0 = 1 \end{aligned}$$

Medium-length line model (from Eq. (6.10)):

$$\begin{aligned} A &= D = \left(1 + \frac{ZY}{2}\right) \qquad B = Z \qquad C = Y\left(1 + \frac{ZY}{4}\right) \\ AD - BC &= \left(1 + \frac{ZY}{2}\right)^2 - Z \times Y\left(1 + \frac{ZY}{4}\right) = 1 + ZY + \frac{Z^2Y^2}{4} - ZY - \frac{Z^2Y^2}{4} = 1 \end{aligned}$$

Long-line model (from Eq. (6.37)):

$$\begin{aligned} A &= D = \cosh \gamma l \qquad B = Z_c \sinh \gamma l \\ C &= \frac{\sinh \gamma l}{Z_c} \\ AD - BC &= \cosh^2 \gamma l - Z_c \sinh \gamma l \left(\frac{\sinh \gamma l}{Z_c}\right) \\ &= \cosh^2 \gamma l - \sinh^2 \gamma l = \left(\frac{\epsilon^{\gamma l} + \epsilon^{-\gamma l}}{2}\right)^2 - \left(\frac{\epsilon^{\gamma l} - \epsilon^{-\gamma l}}{2}\right)^2 \\ &= \frac{\epsilon^{2\gamma l} + 2 + \epsilon^{-2\gamma l}}{4} - \frac{\epsilon^{2\gamma l} - 2 + \epsilon^{-2\gamma l}}{4} = 1 \end{aligned}$$

6.11 The sending-end voltage, current and power factor of the line described in Example 6.3 are found to be 260 kV (line-to-line), 300 A and 0.9 lagging, respectively. Find the corresponding receiving-end voltage, current and power factor.

Solution:

From Example 6.3,

$$\begin{aligned} A &= D = \cosh \gamma l = 0.8904\underline{/1.34^\circ} \\ B &= Z_c \sinh \gamma l = 406.4\underline{/-5.48^\circ} \times 0.4597\underline{/84.93^\circ}\ \Omega = 186.82\underline{/79.45^\circ}\ \Omega \\ C &= \frac{\sinh \gamma l}{Z_c} = \frac{0.4597\underline{/84.93^\circ}}{406.4\underline{/-5.48^\circ}}\ \text{S} = 1.131 \times 10^{-3}\underline{/90.41^\circ}\ \text{S} \\ \begin{bmatrix} V_R \\ I_R \end{bmatrix} &= \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix} \\ V_R &= DV_S - BI_S \qquad I_R = -CV_S + AI_S \\ V_S &= \frac{260}{\sqrt{3}}\underline{/0^\circ}\ \text{kV} = 150.11\underline{/0^\circ}\ \text{kV} \end{aligned}$$

$$
\begin{aligned}
I_S &= 300\angle{-\cos^{-1}0.9} \text{ A} = 300\angle{-25.84^\circ} \text{ A} \\
V_R &= 0.8904\angle{1.34^\circ} \times 150.11\angle{0^\circ} - \frac{186.82\angle{79.45^\circ} \times 300\angle{-25.84^\circ}}{1000} \text{ kV} \\
&= 108.85\angle{-22.76^\circ} \text{ kV} \\
|V_R| &= \sqrt{3} \times 108.85 \text{ kV} = 188.5 \text{ kV line-to-line} \\
I_R &= -1.131 \times 10^{-3}\angle{90.41^\circ} \times 150.11 \times 10^{-3}\angle{0^\circ} + 0.8904\angle{1.34^\circ} \times 300\angle{-25.84^\circ} \text{ A} \\
&= 372.0\angle{-48.95^\circ} \text{ A} \\
|I_R| &= 372 \text{ A}
\end{aligned}
$$

The receiving-end power factor is then

$$\text{p.f.} = \cos(-22.76^\circ + 48.95^\circ) = 0.897 \text{ lagging}$$

6.12 A 60 Hz three-phase transmission is 175 mi long. It has a total series impedance of $35 + j140\ \Omega$ and a shunt admittance of $930 \times 10^{-6}\angle{90^\circ}$ S. It delivers 40 MW at 220 kV, with 90% power factor lagging. Find the voltage at the sending end by (a) the short-line approximation, (b) the nomial-π approximation and (c) the long-line equation.

Solution:

$$
\begin{aligned}
l &= 175 \text{ mi} \\
Z &= 35 + j40 = 144.3\angle{75.96^\circ}\ \Omega \\
Y &= 930 \times 10^{-6} \text{ S} \\
I_R &= \frac{40,000}{\sqrt{3} \times 220 \times 0.9} = 116.6\angle{-25.84^\circ} \text{ A}
\end{aligned}
$$

(a) Using the short-line approximation,

$$
\begin{aligned}
V_S &= 127,017 + 116.6\angle{-25.84^\circ} \times 144.3\angle{75.96^\circ} = 127,017 + 10,788 + j12,912 \\
&= 138,408\angle{5.35^\circ} \text{ V} \\
|V_S| &= \sqrt{3} \times 138,408 = 239.73 \text{ kV}
\end{aligned}
$$

(b) Using the nominal-π approximation and Eq. (6.5),

$$
\begin{aligned}
V_S &= 127,017\left(\frac{0.1342}{2}\angle{165.96^\circ} + 1\right) + 144.3\angle{75.96^\circ} \times 116.6\angle{-25.84^\circ} \\
&= 127,017(0.935 + j0.0163) + 10,788 + j12,912 = 129,549 + j14,982 \\
&= 130,412\angle{6.6^\circ} \\
|V_S| &= \sqrt{3} \times 130,412 = 225.88 \text{ kV}
\end{aligned}
$$

(c) Using the long-line equation,

$$
\begin{aligned}
Z_c &= \left(\frac{144.3\angle{75.96^\circ}}{930 \times 10^{-6}\angle{90^\circ}}\right)^{\frac{1}{2}} = 394\angle{-7.02^\circ} \\
\gamma l &= \sqrt{144.3 \times 930 \times 10^{-6}\angle{165.96^\circ}} = 0.3663\angle{83.0^\circ} = 0.0448 + j0.364 \\
\epsilon^{0.0448}\epsilon^{j0.364} &= 1.0458\angle{20.86^\circ} = 0.9773 + j0.3724
\end{aligned}
$$

$$
\begin{aligned}
\epsilon^{-0.0448}\epsilon^{-j0.364} &= 0.9562\angle{-20.86^\circ} = 0.8935 - j0.3405 \\
\cosh\gamma l &= (0.9773 + j0.3724 + 0.8935 - j0.3405)/2 = 0.9354 + j0.0160 \\
\sinh\gamma l &= (0.9773 + j0.3724 - 0.8935 + j0.3405)/2 = 0.0419 + j0.3565 \\
V_S &= 127{,}017\,(0.9354 + j0.0160) + 116.6\angle{-25.84^\circ} \times 394\angle{-7.02^\circ}\,(0.0419 + j0.3565) \\
&= 118{,}812 + j2{,}032 + 10{,}563 + j12{,}715 = 129{,}315 + j14{,}747 \\
&= 130{,}153\angle{6.5^\circ}\ \text{V} \\
|V_S| &= \sqrt{3} \times 130{,}153 = 225.4\ \text{kV}
\end{aligned}
$$

6.13 Determine the voltage regulation for the line described in Prob. 6.12. Assume that the sending-end voltage remains constant.

Solution:

By Problem 6.12, volt-to-neutral results,

$$V_S = 130.15\ \text{kV} \qquad V_R = 127.02\ \text{kV}$$

For $I_R = 0$, $V_S = V_R \cosh\gamma l$,

$$
\begin{aligned}
|V_{R,\,NL}| &= \frac{130.15}{|0.9354 + j0.0161|} = 139.12\ \text{kV} \\
\%\ \text{Reg.} &= \frac{139.12 - 127.02}{127.02} \times 100 = 9.53\,\%
\end{aligned}
$$

6.14 A three-phase 60-Hz transmission line is 250 mi long. The voltage at the sending end is 220 kV. The parameters of the line are $R = 0.2\ \Omega/\text{mi}$, $X = 0.8\ \Omega/\text{mi}$ and $Y = 5.3\ \mu\text{S/mi}$. Find the sending-end current when there is no load on the line.

Solution:

$$
\begin{aligned}
Z &= (0.2 + j0.8) \times 250 = 206.1\angle{75.96^\circ} \\
Y &= 250 \times 5.3 \times 10^{-6} = 1.325 \times 10^{-3}\angle{90^\circ} \\
\gamma l &= \sqrt{ZY} = \sqrt{206.1 \times 1.325 \times 10^{-3}\angle{165.96^\circ}} = 0.5226\angle{82.98^\circ} \\
&= 0.0639 + j0.5187 \\
Z_c &= \sqrt{Z/Y} = \sqrt{\frac{206.1\angle{75.96^\circ}}{1.325 \times 10^{-3}\angle{90^\circ}}} = 394\angle{-7.02^\circ}\ \Omega
\end{aligned}
$$

By Eq. (6.39) for $I_R = 0$,

$$
\begin{aligned}
I_S &= (V_S/Z_c)\,\frac{\sinh\gamma l}{\cosh\gamma l} \\
\beta l &= 0.5187\ \text{rad} = 29.72^\circ \\
\epsilon^{\alpha l}\epsilon^{j\beta l} &= 0.9258 + j0.5285 \\
\epsilon^{-\alpha l}\epsilon^{-j\beta l} &= 0.8147 - j0.4651 \\
\cosh\gamma l &= \frac{1}{2}(0.9258 + 0.8147 + j0.5285 - j0.4651) = 0.8709\angle{2.086^\circ}
\end{aligned}
$$

$$\sinh \gamma l = \frac{1}{2}[0.9258 - 0.8147 + j(0.5285 + 0.4651)] = 0.4999\angle 83.61^\circ$$

$$I_S = \frac{220,000/\sqrt{3}}{394\angle -7.02^\circ} \times \frac{0.4999\angle 83.61^\circ}{0.8709\angle 2.086^\circ} = 185.0\angle 88.54^\circ \text{ A}$$

6.15 If the load on the line described in Prob. 6.14 is 80 MW at 220 kV, with unity power factor, calculate the current, voltage and power at the sending end. Assume that the sending-end voltage is held constant and calculate the voltage regulation of the line for the load specified above.

Solution:

$$V_R = \frac{220}{\sqrt{3}} = 127 \text{ kV} \qquad I_R = \frac{80,000}{\sqrt{3} \times 220} = 209.95 \text{ A}$$

With values of $\cosh \gamma l$ and $\sinh \gamma l$ from Problem 6.14,

$$\begin{aligned} V_S &= 127,017\,(0.8703 + j0.0317) + 209.95 \times 394\angle -7.02^\circ \times 0.4999\angle 83.61^\circ \\ &= 110,528 + j4,026 + 9,592 + j40,232 = 128,014\angle 20.23^\circ \text{ V to neutral} \\ |V_S| &= \sqrt{3} \times 128,014 = 221.7 \text{ kV} \\ I_S &= 209.95\,(0.8703 + j0.0317) + \frac{127,000}{394\angle -7.02^\circ} \times 0.4999\angle 83.61^\circ \\ &= 182.72 + j6.66 - 1.77 + j161.13 = 246.8\angle 42.84^\circ \text{ A} \\ P_S &= \sqrt{3} \times 221.7 \times 246.8 \cos(20.3^\circ - 42.84^\circ) = 87,486 \text{ kW} \quad \text{(or 87.5 MW)} \end{aligned}$$

At $I_R = 0$,

$$\begin{aligned} |V_R| &= \frac{127,000}{0.8709} = 145,826 \text{ V to neutral} \\ \%\text{ Reg.} &= \frac{145.8 - 127}{127} = 14.8\,\% \end{aligned}$$

6.16 A three-phase tranmission line is 300 mi long and serves a load of 400 MVA, 0.8 lagging power factor at 345 kV. The $ABCD$ constants of the line are

$$\begin{aligned} A &= D = 0.8180\angle 1.3^\circ \\ B &= 172.2\angle 84.2^\circ \ \Omega \\ C &= 0.001933\angle 90.4^\circ \ \text{S} \end{aligned}$$

(a) Determine the sending-end line-to-neutral voltage, the sending-end current and the percent voltage drop at full load.

(b) Determine the receiving-end line-to-neutral voltage at no load, the sending-end current at no load and the voltage regulation.

Solution:

$$V_R = \frac{345,000}{\sqrt{3}} = 199,186\angle 0^\circ \text{ V} \qquad I_R = \frac{400,000}{\sqrt{3} \times 345} = 669.4\angle -36.87^\circ \text{ A}$$

(*a*)

$$\begin{aligned} V_S &= 0.8180\angle 1.3^\circ \times 199,186\angle 0^\circ + 172.2\angle 84.2^\circ \times 669.4\angle -36.87^\circ \\ &= 256,738\angle 20.15^\circ \text{ V} \\ I_S &= 0.001933\angle 90.4^\circ \times 199,186\angle 0^\circ + 0.8180\angle 1.3^\circ \times 669.4\angle -36.87^\circ \\ &= 447.7\angle 8.54^\circ \text{ A} \\ \text{Voltage drop} &= \frac{256,738 - 199,186}{256,738} \times 100 = 22.4\,\% \end{aligned}$$

(*b*)

$$\begin{aligned} V_{R,NL} &= \frac{256,738\angle 20.15^\circ}{0.8180\angle 1.3^\circ} = 313,861\angle 18.85^\circ \text{ V} \\ I_{S,NL} &= 0.001933\angle 90.4^\circ \times 313,861\angle 18.85^\circ = 606.7\angle 109.25^\circ \end{aligned}$$

(*c*)

$$\%\text{ Reg.} = \frac{313,861 - 199,186}{199,186} = 57.6\,\%$$

6.17 Justify Eq. (6.50) by substituting for the hyperbolic functions the equivalent exponential expressions.

Solution:

$$\begin{aligned} \text{left hand side} &= \tanh\gamma\frac{l}{2} = \frac{\sinh\gamma l/2}{\cosh\gamma l/2} = \frac{\epsilon^{\gamma l/2} - \epsilon^{-\gamma l/2}}{\epsilon^{\gamma l/2} + \epsilon^{-\gamma l/2}} \\ \text{right hand side} &= \frac{\cosh\gamma l - 1}{\sinh\gamma l} = \frac{\frac{1}{2}\left(\epsilon^{\gamma l} + \epsilon^{-\gamma l}\right) - 1}{\frac{1}{2}\left(\epsilon^{\gamma l} - \epsilon^{-\gamma l}\right)} = \frac{\epsilon^{\gamma l} - 2 + \epsilon^{-\gamma l}}{\epsilon^{\gamma l} - \epsilon^{-\gamma l}} \\ &= \frac{\left(\epsilon^{\gamma l/2} - \epsilon^{-\gamma l/2}\right)^2}{\left(\epsilon^{\gamma l/2} + \epsilon^{-\gamma l/2}\right)\left(\epsilon^{\gamma l/2} - \epsilon^{-\gamma l/2}\right)} = \frac{\epsilon^{\gamma l/2} - \epsilon^{-\gamma l/2}}{\epsilon^{\gamma l/2} + \epsilon^{-\gamma l/2}} \\ \text{Therefore, left hand side} &= \text{right hand side} \end{aligned}$$

6.18 Determine the equivalent-π circuit for the line of Prob. 6.12.

Solution:

By Eq. (6.46) and Problem 6.12,

$$\begin{aligned} \sinh\gamma l &= 0.0419 + j0.3565 = 0.359\angle 83.3^\circ \\ Z' &= 144.3\angle 75.96^\circ \times \frac{0.359\angle 83.3^\circ}{0.3663\angle 83.0^\circ} = 141.4\angle 75.99^\circ\ \Omega \end{aligned}$$

By Eq. (6.49) and Problem 6.12,

$$\frac{Y'}{2} = \frac{1}{394\angle -7.02^\circ} \times \frac{0.9354 + j0.016 - 1}{0.359\angle 83.3^\circ} = 471 \times 10^{-6}\angle 89.8^\circ \text{ S}$$

6.19 Use Eqs. (6.1) and (6.2) to simplify Eqs. (6.57) and (6.58) for the short transmission line with (*a*) series reactance X and resistance R and (*b*) series reactance X and negligible resistance.

Solution:

From Eq. (6.1) and (6.2), it follows that, for a short line

$$A = D = 1 \qquad B = Z = R + jX \triangleq |Z| \angle \phi \qquad C = 0$$

(*a*)

$$\text{From Eq. (6.57):}\ P_R = \frac{|V_S|\,|V_R|}{|Z|}\cos(\phi - \delta) - \frac{|V_R|^2}{|Z|}\cos\phi$$

$$\text{From Eq. (6.58):}\ Q_R = \frac{|V_S|\,|V_R|}{|Z|}\sin(\phi - \delta) - \frac{|V_R|^2}{|Z|}\sin\phi$$

(*b*) If $R = 0$, then $B = Z = X \angle 90°$ and

$$P_R = \frac{|V_S|\,|V_R|}{X}\sin\delta$$

$$Q_R = \frac{|V_S|\,|V_R|}{X}\cos\delta - \frac{|V_R|^2}{X}$$

6.20 Rights of way for transmission circuits are difficult to obtain in urban areas and existing lines are often upgraded by reconductoring the line with larger conductors or by reinsulating the line for operation at higher voltage. Thermal considerations and maximum power which the line can transmit are the important considerations. A 138-kV line is 50 km long and is composed of *Partridge* conductors with flat horizontal spacing of 5 m between adjacent conductors. Neglect resistance and find the percent increase in power which can be transmitted for constant $|V_S|$ and $|V_R|$ while δ is limited to 45°

(*a*) if the *Partridge* conductor is replaced by *Osprey* which has more than twice the area of aluminum in square millimeters,

(*b*) if a second *Partridge* conductor is placed in a two-conductor bundle 40 cm from the original conductor and a center-to-center distance between bundles of 5 m and

(*c*) if the voltage of the original line is raised to 230 kV with increased conductor spacing of 8 m.

Solution:

Length of 50 km is a short line and with resistance neglected the generalized circuit constants are $A = 1\angle 0°$ and $B = X\angle 90°$. Then, since resistance is neglected conductor heating is disregarded; and from Eq. (6.57),

$$P_R = \frac{|V_S||V_R|}{X}\cos 45°$$

or, inversely proportional to X if we assume constant $|V_S|$ and $|V_R|$. Additionally,

$$D_{eq} = \sqrt[3]{5 \times 5 \times 10} = 6.30 \text{ m, or } 6.30/0.3048 = 20.67 \text{ ft}$$

(*a*)

$$\text{For } Partridge\text{: } X = 0.0754 \ln \frac{20.67}{0.0217} = 0.5172\ \Omega/\text{km}$$
$$\text{For } Osprey\text{: } X = 0.0754 \ln \frac{20.67}{0.0284} = 0.4969\ \Omega/\text{km}$$

Ratio of P_R (new/old):

$$\frac{0.5172}{0.4969} = 1.041 \quad (4.1\% \text{ increase})$$

(*b*)

$$D_s = \sqrt{0.0217 \times (0.4/0.3048)} = 0.1688 \text{ ft}$$
$$X = 0.0754 \ln \frac{20.67}{0.1688} = 0.3625\ \Omega/\text{km}$$
$$\frac{0.5172}{0.3625} = 1.427 \quad (42.7\% \text{ increase})$$

(*c*) P_R increases by factor of $\left(\frac{230}{138}\right)^2 = 2.78$ due to increased V. P_R decreases due to increase of X.

$$D_{eq} = \sqrt[3]{8 \times 8 \times 15} = 33.07 \text{ ft}$$
$$X = 0.0754 \ln \frac{33.07}{0.0217} = 0.5526 \text{ km}$$
$$\text{Decrease factor} = \frac{0.5172}{0.5526}$$
$$\text{Resultant factor of increase} = 2.78 \times \frac{0.5172}{0.5526} = 2.602$$
$$\text{Increase} = 160.3\,\%$$

However, in addition to the increase in conductor spacing and insulation, larger conductors will probably be required since current will increase by a factor of about 230/138 and $|I|^2R$ loss in the line by a factor of about 2.78 for the increase in load at the same power factor.

6.21 Construct a receiving-end power-circle diagram similar to Fig. 6.11 for the line of Prob. 6.12. Locate the point corresponding to the load of Prob. 6.12 and locate the center of circles for various values of $|V_S|$ if $|V_R| = 220$ kV. Draw the circle passing through the load point. From the measured radius of the latter circle determine $|V_S|$ and compare this value with the values calculated for Prob. 6.12.

Solution:

Use scale of 1″ = 50 MVA. By comparing the work in Problem 6.12(*c*) with the equation $V_S = AV_R + BI_R$ we find

$$
\begin{aligned}
A &= 0.9354 + j0.0160 = 0.936\angle 0.98^\circ \\
B &= 394\angle -7.02^\circ\,(0.0419 + j0.3565) = 141.4\angle 76.28^\circ\ \Omega \\
\beta - \alpha &= 76.28^\circ - 0.98^\circ = 75.3^\circ \\
\frac{|A|\,|V_R|^2}{|B|} &= \frac{0.9354 \times 220^2}{141.4} = 320.2 \text{ MVA}
\end{aligned}
$$

Use above data to construct load line through origin at $\cos^{-1} 0.9 = 25.8^\circ$ in the first quadrant. Draw a vertical line at 40 MW. The load point is at the intersection of this line and the load line. The radius of the circle through the load point is 7.05″.

$$
\begin{aligned}
7.05 \times 50 &= 352.5 \\
\frac{|V_S|\,|V_R|}{|B|} &= 352.5 \\
|V_S| &= \frac{352.5 \times 141.4}{220} = 226.5 \text{ kV}
\end{aligned}
$$

6.22 A synchronous condenser is connected in parallel with the load described in Prob. 6.12 to improve the overall power factor at the receiving end. The sending-end voltage is always adjusted so as to maintain the receiving-end voltage fixed at 220 kV. Using the power-circle diagram constructed for Prob. 6.21, determine the sending-end voltage and the reactive power supplied by the synchronous condenser when the overall power factor at the receiving end is (*a*) unity (*b*) 0.9 leading.

Solution:

On the diagram for Problem 6.21 draw a new load line in the fourth quadrant at $\cos^{-1} 0.9$ with the horizontal axis. Draw power circles at radii $|V_S||V_R|/|B| = 311$, 327, 342, 358, 373 and 389 MVA for $|V_S| = 200$, 210, 220, 230, 240 and 250 kV, respectively. This provides the power circle diagram that we can use for parts (*a*) and (*b*).

For p.f. = 1.0 read $|V_S| = 214$ kV at 40 MW on the horizontal axis. The vertical distance between the horizontal axis and the load line in the first quadrant respresents the kvar of the capacitors needed. The value is 19.3 kvar.

For p.f. = 0.9 leading, read $|V_S| = 202$ kV where the vertical line through 40 MW intersects the load line in the fourth quadrant. The vertical distance between the two load lines at 40 MW represents the kvar of capacitors needed. The value is 38.6 kvar.

6.23 A series capacitor bank having a reactance of 146.6 Ω is to be installed at the midpoint of the 300-mi line of Prob. 6.16. The *ABCD* constants for each 150 mi portion of line are

$$
\begin{aligned}
A &= D = 0.9534\angle 0.3^\circ \\
B &= 90.33\angle 84.1^\circ\ \Omega \\
C &= 0.001014\angle 90.1^\circ\ \text{S}
\end{aligned}
$$

(*a*) Determine the equivalent *ABCD* constants of the cascade combination of the line-capacitor-line. (See Table A.6 in the Appendix.)

(*b*) Solve Prob. 6.16 using these equivalent *ABCD* constants.

Note to Instructor: This problem is somewhat long, but the solution is interesting to show that the *ABCD* constants of networks in series as given in Table A.6 can be calculated by matrix multiplication. The problem also shows the large reduction in voltage accomplished by series capacitors in the middle of the line. Compare results of Problems 6.16 and 6.23.

Solution:

(*a*)

$$\text{Let } \bar{\mathbf{A}} = \begin{bmatrix} 0.9534\angle 0.3^\circ & 90.33\angle 84.1^\circ \\ 0.001014\angle 90.1^\circ & 0.9534\angle 0.3^\circ \end{bmatrix}$$

$$\bar{\mathbf{A}} \times \begin{bmatrix} 1.0\angle 0^\circ & 1.46\angle -90^\circ \\ 0 & 1.0\angle 0^\circ \end{bmatrix} \times \bar{\mathbf{A}} =$$

$$\begin{bmatrix} 0.9534\angle 0.3^\circ & 50.91\angle -78.65^\circ \\ 0.001014\angle 90.1^\circ & 1.1022\angle 0.27^\circ \end{bmatrix} \times \begin{bmatrix} 0.9534\angle 0.3^\circ & 90.33\angle 84.1^\circ \\ 0.001014\angle 90.1^\circ & 0.9534\angle 0.3^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.9597\angle 1.18^\circ & 42.30\angle 64.5^\circ \\ 0.002084\angle 90.4^\circ & 0.9597\angle 1.18^\circ \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{new}}$$

(*b*) For V_R and I_R from Problem 6.16,

$$V_S = 0.9597\angle 1.18^\circ \times 199,186\angle 0^\circ + 42.30\angle 64.5^\circ \times 669.4\angle -36.87^\circ$$

$$= 216,870\angle 4.5^\circ$$

$$I_S = 0.002084\angle 90.4^\circ \times 199,186\angle 0^\circ + 0.9597\angle 1.18^\circ \times 669.4\angle -36.87^\circ$$

$$= 520.4\angle 4.44^\circ$$

$$\text{Voltage drop} = \frac{216,870 - 199,186}{216,870} \times 100 = 8.15\,\%$$

(Compare this voltage drop with that of Problem 6.16)

$$V_{R,NL} = \frac{216,870\angle 4.5^\circ}{0.9597\angle 1.18^\circ} = 225,977\angle 3.32^\circ \text{ V}$$

$$I_{S,NL} = 0.002084\angle 90.4^\circ \times 225,977\angle 3.32^\circ = 470.9\angle 93.7^\circ \text{ A}$$

$$\%\text{ Reg.} = \frac{225,977 - 199,186}{199,186} \times 100 = 13.45\,\%$$

(without capacitors 57.6%)

6.24 The shunt admittance of a 300-mi transmission line is

$$y_c = 0 + j6.87 \times 10^{-6} \text{ S/mi}$$

Determine the $ABCD$ constants of a shunt reactor that will compensate for 60% of the total shunt admittance.

Solution:

Capacitive susceptance: $B_C = 6.87 \times 10^{-6} \times 300 = 0.002061$ S
Inductive susceptance: $B_L = 0.6 \times 0.002061 = 0.001237$ S
For the shunt reactor, $A = D = 1.0\angle 0^\circ$
$B = 0$
$C = -j0.001237$ S

6.25 A 250-Mvar, 345-kV shunt reactor whose admittance is $0.0021\angle -90^\circ$ S is connected to the receiving end of the 300-mi line of Prob. 6.16 at no load.

(a) Determine the equivalent $ABCD$ constants of the line in series with the shunt reactor. (See Table A.6 in the Appendix.)

(b) Rework part (b) of Prob. 6.16 using these equivalent $ABCD$ constants and the sending-end volage found in Prob. 6.16.

Solution:

For the shunt reactor

$$A = D = 1.0 \qquad B = 0 \qquad C = -j0.0021 \text{ S}$$

(a)

$$A_{eq} = 0.818\angle 1.3^\circ + 172.2\angle 84.2^\circ \times 0.0021\angle -90^\circ = 1.1777\angle -0.88^\circ$$
$$B_{eq} = 172.2\angle 84.2^\circ \qquad D_{eq} = 0.818\angle 1.3^\circ$$
$$C_{eq} = 0.001933\angle 90.4^\circ + 0.0021\angle -90^\circ \times 0.818\angle 1.3^\circ = 0.000217\angle 83.25^\circ$$

(b) From Problem 6.16, $V_S = 256,738\angle 20.15^\circ$. So,

$$V_{R,NL} = \frac{256,738\angle 20.15^\circ}{1.1777\angle -0.88^\circ} = 217,999\angle 21.03^\circ \text{ V}$$
$$I_{S,NL} = 0.000217\angle 83.25^\circ \times 217,999\angle 21.03^\circ = 47.3\angle 104.28^\circ \text{ A}$$

Recall that the shunt reactor is in the circuit only at no load. So, from Problem 6.16,

$$V_{R,FL} = 199,186\angle 0^\circ \text{ V}$$

and

$$\% \text{ Reg.} = \frac{217,999 - 199,186}{199,186} = 9.45\,\%$$

(compare with Problems 6.16 and 6.24)

6.26 Draw the lattice diagram for current and plot current versus time at the sending end of the line of Example 6.8 for the line terminated in (a) an open circuit (b) a short circuit.

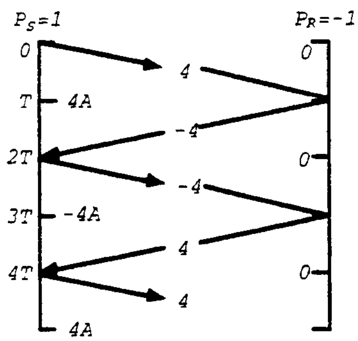
Solution:

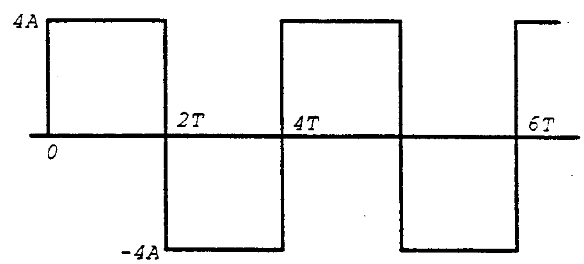
(a) $Z_R = \infty$, and for current

$$\rho_R = -\frac{Z_R - Z_c}{Z_R + Z_c} = -\frac{1 - \frac{Z_c}{Z_R}}{1 + \frac{Z_c}{Z_R}} = -1$$

$$\rho_s = -\frac{0 - Z_c}{0 + Z_c} = +1$$

$$\text{Initially } i^+ = \frac{120}{30} = 4 \text{ A}$$



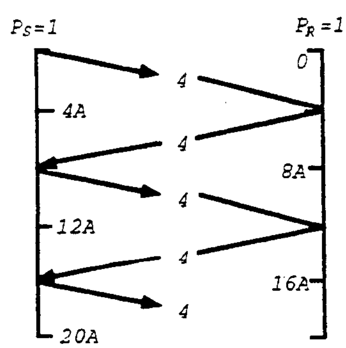


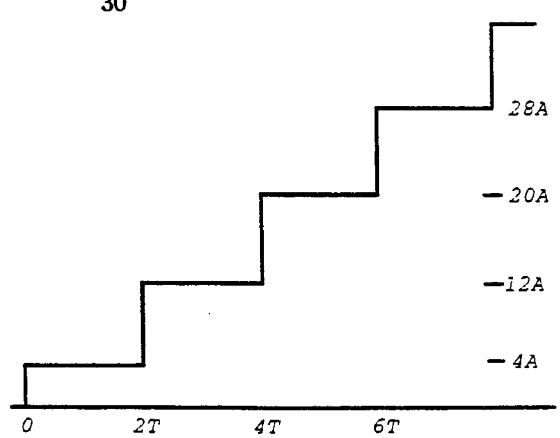
(b) $Z_R = 0$, and for current

$$\rho_R = -\frac{0 - Z_c}{0 + Z_c} = 1$$

$$\rho_s = -\frac{0 - Z_c}{0 + Z_c} = 1$$

$$\text{Initially } i^+ = \frac{120}{30} = 4 \text{ A}$$

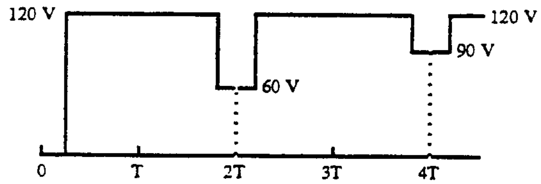




6.27 Plot voltage versus time for the line of Example 6.8 at a point distant from the sending end equal to one-fourth of the length of the line if the line is terminated in a resistance of 10 Ω.

Solution:

Imagine a vertical line on the diagram of Fig. 6.15(b) at one-fourth the line length from the sending end toward the receiving end. Intersections of this line and the slant lines occur at $T = 0.25T$, $1.75T$, $2.25T$, $3.75T$, etc. Changes in voltage occur at these times. The sum of the incident and reflected voltages are shown between slanted lines and determine the values plotted below.



6.28 Solve Example 6.8 is a resistance of 54 Ω is in series with the source.

Solution:

For voltage,

$$\rho_s = \frac{54-30}{54+30} = \frac{2}{7}$$

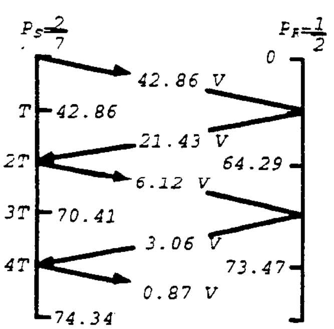
$$\rho_R = \frac{90-30}{90+30} = \frac{1}{2}$$

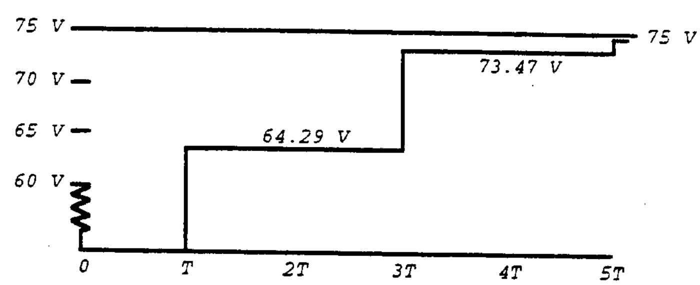
Initial voltage impressed on line:

$$\frac{30}{30+54} \times 120 = 42.86 \text{ V}$$

Final value:

$$120 \times \frac{90}{90+54} = 75 \text{ V}$$





6.29 Voltage from a dc source is applied to an overhead transmission line by closing a switch. The end of the overhead line is connected to an underground cable. Assume both the line and the cable are lossless and that the initial voltage along the line is v^+. If the characteristic impedances of the line and cable are 400 Ω and 50 Ω, respectively, and the end of the cable is open-circuited, find in terms of v^+

(*a*) the voltage at the junction of the line and cable immediately after the arrival of the incident wave and

(*b*) the voltage at the open end of the cable immediately after arrival of the first voltage wave.

Solution:

(*a*) The initial wave of voltage v_0^+ arriving at the juction with the cable "sees" the Z_c of the cable. So, at the end of the overhead line:

$$\rho_R = \frac{50 - 400}{50 + 400} = -0.777$$

and the voltage at the juction is

$$(1 - 0.777)\, v^+ = 0.223 v_0^+$$

which is the refracted voltage wave travelling along the cable.

(*b*) At the end of the cable $\rho_R = 1.0$ and

$$v_R = (0.223 + 0.223)\, v^+ = 0.446 v_0^+$$

6.30 A dc source of voltage V_S and internal resistance R_S is connected through a switch to a lossless line having characteristic impedance R_c. The line is terminated in a resistance R_R. The travelling time of a voltage wave across the line is T. The switch closes at $t = 0$.

(*a*) Draw a lattice diagram showing the voltage of the line during the period $t = 0$ to $t = 7T$. Indicate the voltage components in terms of V_S and the reflection coefficients ρ_R and ρ_s.

(*b*) Determine the receiving-end voltage at $t = 0$, $2T$, $4T$ and $6T$, and hence at $t = 2nT$ where n is any non-negative integer.

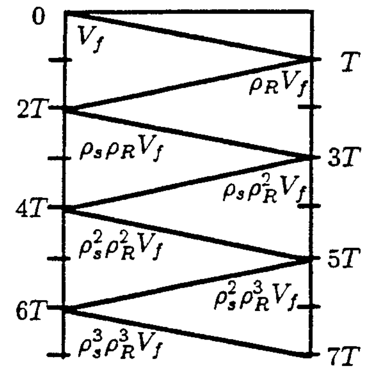
(*c*) Hence determine the steady state voltage at the receiving end of the line in terms of V_S, R_S, R_R and R_c.

(*d*) Verify the result in Part (*c*) by analyzing the system as a simple dc circuit in the steady state. (Note that the line is lossless and remember how inductances and capacitances behave as short circuits and open circuits to dc.)

Solution:

(*a*)

$$V_f = V_S \left(\frac{R_c}{R_c + R_S} \right) \qquad \rho_R = \frac{R_R - R_c}{R_R + R_c} \qquad \rho_s = \frac{R_S - R_c}{R_S + R_c}$$



(b)

$$
\begin{aligned}
t = 0' \Rightarrow \quad V_R(0) &= 0 \\
t = 2T \Rightarrow V_R(2T) &= V_R(0) + V_f + \rho_R V_f \;=\; (1+\rho_R)V_f \\
t = 4T \Rightarrow V_R(4T) &= V_R(2T) + \rho_s\rho_R V_f + \rho_s\rho_R^2 V_f \\
&= (1+\rho_R)V_f + \rho_s\rho_R(1+\rho_R)V_f \\
&= (1+\rho_R)(1+\rho_s\rho_R)V_f \\
t = 6T \Rightarrow V_R(6T) &= V_R(4T) + \rho_s^2\rho_R^2 V_f + \rho_s^2\rho_R^3 V_f \\
&= (1+\rho_R)(1+\rho_s\rho_R)V_f + \rho_s^2\rho_R^2(1+\rho_R)V_f \\
&= (1+\rho_R)\left[1 + \rho_s\rho_R + (\rho_s\rho_R)^2\right] V_f
\end{aligned}
$$

Hence at any given $t = 2nT$,

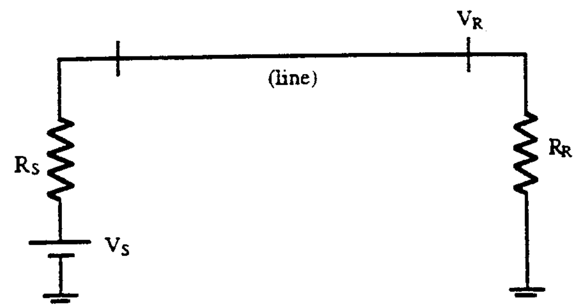
$$
V_R(2nT) = (1+\rho_R)\left\{\sum_{j=0}^{n-1} (\rho_s\rho_R)^j\right\} V_f = (1+\rho_R)\,\frac{1-(\rho_s\rho_R)^n}{1-\rho_s\rho_R}\cdot V_f
$$

(c) At the steady state, $n \to \infty$. If R_S or $R_R \neq 0$, $|\rho_s\rho_R| < 1$ and $(\rho_s\rho_R)^n \to 0$ as $n \to \infty$. Hence,

$$
\begin{aligned}
V_R(\infty) &= (1+\rho_R)\,\frac{V_f}{1-\rho_s\rho_R} \\
\text{Since, } \rho_R &= \frac{R_R - R_c}{R_R + R_c} \quad \text{and} \quad \rho_s = \frac{R_S - R_c}{R_S + R_c} \\
1 + \rho_R &= \frac{2R_R}{R_R + R_c} \\
\frac{1}{1-\rho_s\rho_R} &= \frac{(R_S + R_c)(R_R + R_c)}{2R_c\,(R_S + R_R)} \\
V_f &= \frac{R_c}{R_S + R_c}\cdot V_S \\
V_R(\infty) &= \frac{2R_R}{R_R + R_c} \times \frac{(R_S + R_c)(R_R + R_c)}{2R_c\,(R_S + R_R)} \times \frac{R_c}{R_S + R_c}\cdot V_S = \frac{R_R}{R_S + R_R}\cdot V_S
\end{aligned}
$$

(d)

$$
V_R = V_S\left(\frac{R_R}{R_R + R_S}\right)
$$



Chapter 7 Problem Solutions

7.1 Using the building-block procedure described in Sec. 7.1, determine $\mathbf{Y}_{bus}$ for the circuit of Fig. 7.18. Assume there is *no* mutual coupling between any of the branches.

Solution:

First the voltage sources are converted to current sources. Then, the building blocks are given as follows:

$$\begin{array}{c} \textcircled{1} \\ \textcircled{1}\ [\,1\,](-j1.0) \end{array} \qquad \begin{array}{cc} & \textcircled{1}\quad\textcircled{3} \\ \begin{array}{c}\textcircled{1}\\ \textcircled{3}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2.0) \end{array} \qquad \begin{array}{cc} & \textcircled{1}\quad\textcircled{2} \\ \begin{array}{c}\textcircled{1}\\ \textcircled{2}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2.5) \end{array}$$

$$\begin{array}{cc} & \textcircled{2}\quad\textcircled{3} \\ \begin{array}{c}\textcircled{2}\\ \textcircled{3}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j4.0) \end{array} \qquad \begin{array}{cc} & \textcircled{3}\quad\textcircled{4} \\ \begin{array}{c}\textcircled{3}\\ \textcircled{4}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j8.0) \end{array} \qquad \begin{array}{cc} & \textcircled{2}\quad\textcircled{5} \\ \begin{array}{c}\textcircled{2}\\ \textcircled{5}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j5.0) \end{array}$$

$$\begin{array}{cc} & \textcircled{4}\quad\textcircled{5} \\ \begin{array}{c}\textcircled{4}\\ \textcircled{5}\end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2.0) \end{array} \qquad \begin{array}{c} \textcircled{5} \\ \textcircled{5}\ [\,1\,](-j8.0) \end{array}$$

Combining these together yields

$$\begin{array}{cc} & \begin{array}{ccccc} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} \end{array} \\ \begin{array}{c}\textcircled{1}\\ \textcircled{2}\\ \textcircled{3}\\ \textcircled{4}\\ \textcircled{5}\end{array} & \begin{bmatrix} -j5.5 & j2.5 & j2 & j0 & j0 \\ j2.5 & -j11.5 & j4 & j0 & j5 \\ j2 & j4 & -j14 & j8 & j0 \\ j0 & j0 & j8 & -j10 & j2 \\ j0 & j5 & j0 & j2 & -j7.8 \end{bmatrix} \end{array}$$