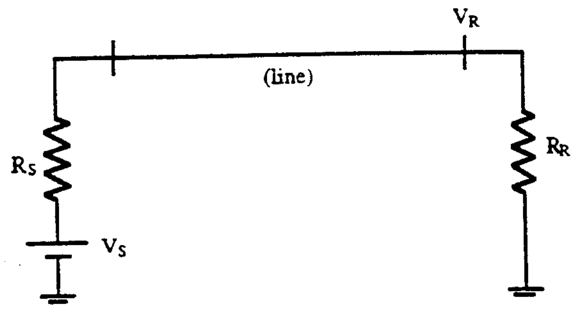


Chapter 7 Problem Solutions

7.1 Using the building-block procedure described in Sec. 7.1, determine $\mathbf{Y}_{bus}$ for the circuit of Fig. 7.18. Assume there is *no* mutual coupling between any of the branches.

Solution:

First the voltage sources are converted to current sources. Then, the building blocks are given as follows:

$$\begin{array}{c} ① \\ ① \end{array}\begin{bmatrix} 1 \end{bmatrix}(-j1.0) \qquad \begin{array}{c} ① \\ ③ \end{array}\overset{\quad ① \quad ③}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j2.0) \qquad \begin{array}{c} ① \\ ② \end{array}\overset{\quad ① \quad ②}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j2.5)$$

$$\begin{array}{c} ② \\ ③ \end{array}\overset{\quad ② \quad ③}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j4.0) \qquad \begin{array}{c} ③ \\ ④ \end{array}\overset{\quad ③ \quad ④}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j8.0) \qquad \begin{array}{c} ② \\ ⑤ \end{array}\overset{\quad ② \quad ⑤}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j5.0)$$

$$\begin{array}{c} ④ \\ ⑤ \end{array}\overset{\quad ④ \quad ⑤}{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}(-j2.0) \qquad \begin{array}{c} ⑤ \\ ⑤ \end{array}\begin{bmatrix} 1 \end{bmatrix}(-j8.0)$$

Combining these together yields

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}\begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} -j5.5 & j2.5 & j2 & j0 & j0 \\ j2.5 & -j11.5 & j4 & j0 & j5 \\ j2 & j4 & -j14 & j8 & j0 \\ j0 & j0 & j8 & -j10 & j2 \\ j0 & j5 & j0 & j2 & -j7.8 \end{bmatrix} \end{array}$$

7.2 Using the $\mathbf{Y}_{\text{bus}}$ modification procedure described in Sec. 7.4 and assuming no mutual coupling between branches, modify the $\mathbf{Y}_{\text{bus}}$ obtained in Prob. 7.1 to reflect removal of the two branches ①–③ and ②–⑤ from the circuit of Fig. 7.18.

Solution:

To remove branches ①–③ and ②–⑤, we add the following blocks to $\mathbf{Y}_{\text{bus}}$:

$$\begin{array}{c} \\ ① \\ ③ \end{array} \begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (j2.0) \qquad \begin{array}{c} \\ ② \\ ⑤ \end{array} \begin{array}{c} \begin{array}{cc} ② & ⑤ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (j5.0)$$

This results in the following modified $\mathbf{Y}_{\text{bus}}$:

$$\begin{array}{c|ccccc} & ① & ② & ③ & ④ & ⑤ \\ \hline ① & -j3.5 & j2.5 & j0 & j0 & j0 \\ ② & j2.5 & -j6.5 & j4 & j0 & j0 \\ ③ & j0 & j4 & -j12 & j8 & j0 \\ ④ & j0 & j0 & j8 & -j10 & j2 \\ ⑤ & j0 & j0 & j0 & j2 & -j2.8 \end{array}$$

7.3 The circuit of Fig. 7.18 has the linear graph shown in Fig. 7.19 with arrows indicating directions assumed for the branches a to h. Disregarding all mutual coupling between branches

(a) determine the branch-to-node incidence matrix $\mathbf{A}$ for the circuit with node 0 as reference.

(b) find the circuit $\mathbf{Y}_{\text{bus}}$ using Eq. (7.37).

Solution:

(a) The branch-to-node incidence matrix is found to be

$$\mathbf{A} = \begin{array}{c|ccccc} & ① & ② & ③ & ④ & ⑤ \\ \hline ⓐ & 1 & -1 & 0 & 0 & 0 \\ ⓑ & -1 & 0 & 1 & 0 & 0 \\ ⓒ & 0 & -1 & 1 & 0 & 0 \\ ⓓ & 0 & -1 & 0 & 0 & 1 \\ ⓔ & 0 & 0 & 1 & -1 & 0 \\ ⓕ & 0 & 0 & 0 & 1 & -1 \\ ⓖ & -1 & 0 & 0 & 0 & 0 \\ ⓗ & 0 & 0 & 0 & 0 & -1 \end{array}$$

(*b*) $\mathbf{Y}_{pr}$ is given by

$$\mathbf{Y}_{pr} = \begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array} \begin{array}{c} \begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\ \begin{bmatrix} -j2.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -j2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -j4 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -j5 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -j8 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -j2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -j1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -j0.8 \end{bmatrix} \end{array}$$

$$\mathbf{Y}_{\text{bus}} = \mathbf{A}^T\mathbf{Y}_{pr}\mathbf{A} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} -j5.5 & j2.5 & j2 & j0 & j0 \\ j2.5 & -j11.5 & j4 & j0 & j5 \\ j2 & j4 & -j14 & j8 & j0 \\ j0 & j0 & j8 & -j10 & j2 \\ j0 & j5 & j0 & j2 & -j7.8 \end{bmatrix} \end{array}$$

7.4 Consider that *only* the two branches ①–③ and ②–③ in the circuit of Fig. 7.18 are mutually coupled as indicated by the dots beside them and that their mutual impedance is $j0.15$ per unit (that is, ignore the dot on branch ②-⑤). Determine the circuit $\mathbf{Y}_{\text{bus}}$ by the procedure described in Sec. 7.2.

Solution:

The primitive impedance matrix for the mutually coupled branches ①–③ and ②–③ is inverted as a single entity to yield the primitive admittance matrix

$$\begin{array}{c} \\ ①\text{-}③ \\ ②\text{-}③ \end{array} \begin{array}{c} \begin{array}{cc} ①\text{-}③ & ②\text{-}③ \end{array} \\ \begin{bmatrix} j0.5 & j0.15 \\ j0.15 & j0.25 \end{bmatrix}^{-1} \end{array} = \begin{array}{c} \begin{array}{cc} ①\text{-}③ & ②\text{-}③ \end{array} \\ \begin{bmatrix} -j2.43902 & j1.46341 \\ j1.46341 & -j4.87805 \end{bmatrix} \end{array}$$

Building blocks of the two mutually coupled branches ①–③ and ②–③ are

$$\begin{array}{c} \\ ① \\ ③ \end{array} \begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j2.43902) \qquad \begin{array}{c} \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (j1.46341)$$

$$\begin{array}{c} \\ ① \\ ③ \end{array} \begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (j1.46341) \qquad \begin{array}{c} \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j4.87805)$$

Building blocks of the remaining branches are determined as

$$\begin{array}{c} \\ ① \end{array} \begin{array}{c} ① \\ \begin{bmatrix} 1 \end{bmatrix} \end{array} (-j1.0) \qquad \begin{array}{c} \\ ① \\ ② \end{array} \begin{array}{c} \begin{array}{cc} ① & ② \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j2.5) \qquad \begin{array}{c} \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cc} ③ & ④ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j8.0)$$

$$\begin{array}{c} \\ ② \\ ⑤ \end{array} \begin{array}{c} \begin{array}{cc} ② & ⑤ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j5.0) \qquad \begin{array}{c} \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{cc} ④ & ⑤ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} (-j2.0) \qquad \begin{array}{c} \\ ⑤ \end{array} \begin{array}{c} ⑤ \\ \begin{bmatrix} 1 \end{bmatrix} \end{array} (-j8.0)$$

Combining all the above building blocks gives

$$
\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}
\begin{array}{c}
\begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\
\begin{bmatrix}
-j5.93902 & j3.96341 & j0.97561 & j0 & j0 \\
j3.96341 & -j12.37805 & j3.41464 & j0 & j5 \\
j0.97561 & j3.41464 & -j12.39025 & j8 & j0 \\
j0 & j0 & j8 & -j10 & j2 \\
j0 & j5 & j0 & j2 & -j7.8
\end{bmatrix}
\end{array}
$$

7.5 Solve Prob. 7.4 using Eq. (7.37). Determine the branch-to-node incidence matrix **A** from the linear graph of Fig. 7.19 with node 0 as reference.

Solution:

The branch-to-node incidence matrix found in Prob. 7.3 can be used here. $\mathbf{Y}_{pr}$ is obtained by inverting $\mathbf{Z}_{pr}$ as follows.

$$
\mathbf{Y}_{pr} = \mathbf{Z}_{pr}^{-1} =
\begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array}
\begin{array}{c}
\begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\
\begin{bmatrix}
j0.4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & j0.5 & j0.15 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & j0.15 & j0.25 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & j0.2 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & j0.125 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & j0.5 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & j1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & j1.25
\end{bmatrix}^{-1}
\end{array}
$$

$$
=
\begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array}
\begin{array}{c}
\begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\
\begin{bmatrix}
-j2.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -j2.43902 & j1.46341 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & j1.46341 & -j4.87805 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -j5 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -j8 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -j2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -j1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -j0.8
\end{bmatrix}
\end{array}
$$

Using Eq. (7.27), we have

$$
\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}
\begin{array}{c}
\begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\
\begin{bmatrix}
-j5.93902 & j3.96341 & j0.97561 & 0 & 0 \\
j3.96341 & -j12.37805 & j3.41464 & 0 & j5 \\
j0.97561 & j3.41464 & -j12.39025 & j8 & 0 \\
0 & 0 & j8 & -j10 & j2 \\
0 & j5 & 0 & j2 & -j7.8
\end{bmatrix}
\end{array}
$$

(7.6) Using the modification procedure of Sec. 7.4, modify the Y_{BUS} solution of Prob. 7.4 (or Prob. 7.5) to reflect removal of the branch 2-3 from the circuit.

Solution:
We first remove both branches (1-3) and (2-3) which are mutually coupled, and the new branch (1-3) is then reconnected. Thus, the following building blocks are subtracted from Y_{BUS}:

$$\begin{matrix} & \textcircled{1} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}(-j2.43902) \qquad \begin{matrix} & \textcircled{2} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}(j1.46341)$$

$$\begin{matrix} & \textcircled{1} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}(j1.46341) \qquad \begin{matrix} & \textcircled{2} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}(-j4.87805)$$

And the following building block is added to Y_{BUS}:

$$\begin{matrix} & \textcircled{1} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}(-j2.0)$$

Giving $\mathbf{Y}_{BUS}$ =

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \left[\begin{array}{ccccc} -j5.5 & j2.5 & j2 & 0 & 0 \\ j2.5 & -j7.5 & 0 & 0 & j5 \\ j2 & 0 & -j10 & j8 & 0 \\ 0 & 0 & j8 & -j10 & j2 \\ 0 & j5 & 0 & j2 & -j7.8 \end{array} \right] \end{array}$$

(7.7) Modify the Y_{BUS} determined in Example 7.3 to reflect removal of the mutually coupled branch 1-3 from the circuit of Fig. 7.11. Use the modification procedure of Sec. 7.4.

Solution:
First, both branches (1-3) and (2-3) are removed, and then branch (2-3) is reconnected. Thus, the following are subtracted from Y_{BUS}:

$$\begin{matrix} & ① & ③ \\ ① & 1 & -1 \\ ③ & -1 & 1 \end{matrix} \; (-j6.25) \qquad \begin{matrix} & ② & ③ \\ ① & 1 & -1 \\ ③ & -1 & 1 \end{matrix} \; (j3.75)$$

$$\begin{matrix} & ① & ③ \\ ② & 1 & -1 \\ ③ & -1 & 1 \end{matrix} \; (j3.75) \qquad \begin{matrix} & ② & ③ \\ ② & 1 & -1 \\ ③ & -1 & 1 \end{matrix} \; (-j6.25)$$

And the building block for the new branch (2-3) is added to Y_{BUS}:

$$\begin{matrix} & ② & ③ \\ ② & 1 & -1 \\ ③ & -1 & 1 \end{matrix} \; (-j4.0)$$

Giving Y_{BUS} =

$$\begin{matrix} & ① & ② & ③ & ④ \\ ① & -j10.5 & j8 & 0 & j2.5 \\ ② & j8 & -j17 & j4 & j5 \\ ③ & 0 & j4 & -j4.8 & 0 \\ ④ & j2.5 & j5 & 0 & -j8.3 \end{matrix}$$

(7.8) A new branch having a self-impedance of $j0.2$ per unit is added between nodes 2 and 3 in the circuit of Fig. 7.11. Mutual impedance of $j0.1$ per unit couples this new branch to the branch already existing between nodes 2 and 3. Modify the Y_{BUS} obtained in Example 7.3 to account for the addition of the new branch.

Solution:
Since the existing branch (2-3) is mutually coupled to branch (1-3), we remove these simultaneously. Then we reconnect these branches along with a new branch between nodes 2 and 3.

Thus, we subtract the following from Y_{BUS}:

$$\begin{array}{c} \\ ① \\ ③ \end{array}\!\!\begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}(-j6.25) \qquad \begin{array}{c} \\ ① \\ ③ \end{array}\!\!\begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}(j3.75)$$

$$\begin{array}{c} \\ ② \\ ③ \end{array}\!\!\begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}(j3.75) \qquad \begin{array}{c} \\ ② \\ ③ \end{array}\!\!\begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}(-j6.25)$$

And, to reconnect these branches, along with a new mutually coupled branch between nodes 2 and 3, the primitive admittance matrix is first found:

$$\begin{array}{c} \\ 1\text{-}3 \\ 2\text{-}3 \\ 2\text{-}3\text{new} \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1\text{-}3 & 2\text{-}3 & 2\text{-}3\text{new} \end{array} \\ \begin{bmatrix} j0.25 & j0.15 & 0 \\ j0.15 & j0.25 & j0.1 \\ 0 & j0.1 & j0.2 \end{bmatrix}^{-1} \end{array} = \begin{array}{c} \begin{array}{ccc} 1\text{-}3 & 2\text{-}3 & 2\text{-}3\text{new} \end{array} \\ \begin{bmatrix} -j7.27273 & j5.45455 & -j2.72727 \\ j5.45455 & -j9.09091 & j4.54545 \\ -j2.72727 & j4.54545 & -j7.27273 \end{bmatrix} \end{array}$$

And from these, the additional building blocks:

$$\begin{array}{c|cc|} & \textcircled{1} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(-j7.27273) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(j5.45455) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(-j2.72727)$$

$$\begin{array}{c|cc|} & \textcircled{1} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(j5.45455) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(-j9.09091) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(j4.54545)$$

$$\begin{array}{c|cc|} & \textcircled{1} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(-j2.72727) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(j4.54545) \quad \begin{array}{c|cc|} & \textcircled{2} & \textcircled{3} \\ \textcircled{2} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{array}(-j7.27273)$$

Giving $\mathbf{Y}_{BUS}$ =

$$\begin{bmatrix} -j17.77273 & j10.72728 & j4.54545 & j2.5 \\ & -j20.27274 & j4.54546 & j5.0 \\ & & -j9.89091 & 0 \\ & \text{Symmetric} & & -j8.3 \end{bmatrix}$$

(7.9) Suppose that mutual coupling exists pairwise between branches 1-3 and 2-3, and *also* between branches 2-3 and 2-5 of Fig. 7.18, as shown by the dots in that figure. The mutual impedance between the former pair of branches is $j0.15$ per unit (the same as in Prob. 7.4) and between the latter pair is $j0.1$ per unit. Use the procedure of Sec. 7.2 to find $\mathbf{Y}_{BUS}$ for the overall circuit including the *three* mutually coupled branches.

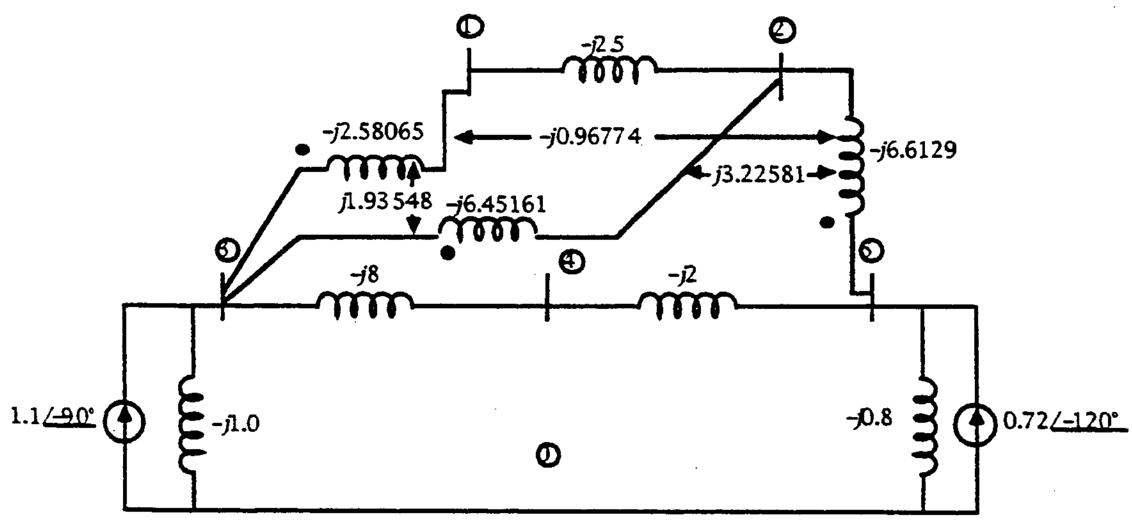
Solution:

The primitive impedance matrix is inverted as a single entity to yield:

$$\begin{array}{c} \\ 3\text{-}1 \\ 3\text{-}2 \\ 5\text{-}2 \end{array}\begin{array}{c} \begin{array}{ccc} 3\text{-}1 & 3\text{-}2 & 5\text{-}2 \end{array} \\ \begin{bmatrix} j0.5 & j0.15 & 0 \\ j0.15 & j0.25 & j0.1 \\ 0 & j0.1 & j0.2 \end{bmatrix}^{-1} \end{array} = \begin{array}{c} \begin{array}{ccc} 3\text{-}1 & 3\text{-}2 & 5\text{-}2 \end{array} \\ \begin{bmatrix} -j2.58065 & j1.93548 & -j0.96774 \\ j1.93548 & -j6.45161 & j3.22581 \\ -j0.96774 & j3.22581 & -j6.61290 \end{bmatrix} \end{array}$$

All other admittances are determined on an element-by-element basis.

The resulting admittance diagram is shown below:



The building blocks are:

$$\begin{array}{c} \\ \textcircled{3} \\ \textcircled{1} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{1} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2.58065) \quad \begin{array}{c} \\ \textcircled{3} \\ \textcircled{1} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(j1.93548) \quad \begin{array}{c} \\ \textcircled{3} \\ \textcircled{1} \end{array}\begin{array}{cc} \textcircled{5} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j0.96774)$$

$$\begin{array}{c} \\ \textcircled{3} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{1} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(j1.93548) \quad \begin{array}{c} \\ \textcircled{3} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j6.45161) \quad \begin{array}{c} \\ \textcircled{3} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{5} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(j3.22581)$$

$$\begin{array}{c} \\ \textcircled{5} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{1} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j0.96774) \quad \begin{array}{c} \\ \textcircled{5} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(j3.22581) \quad \begin{array}{c} \\ \textcircled{5} \\ \textcircled{2} \end{array}\begin{array}{cc} \textcircled{5} & \textcircled{2} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j6.6129)$$

$$\begin{array}{c} \\ \textcircled{2} \\ \textcircled{1} \end{array}\begin{array}{cc} \textcircled{2} & \textcircled{1} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2.5) \quad \begin{array}{c} \\ \textcircled{3} \\ \textcircled{4} \end{array}\begin{array}{cc} \textcircled{3} & \textcircled{4} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j8)$$

$$\begin{array}{c} \\ \textcircled{5} \\ \textcircled{4} \end{array}\begin{array}{cc} \textcircled{5} & \textcircled{4} \\ \end{array}\!\!\!\! \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}(-j2) \quad \begin{array}{c} \\ \textcircled{1} \end{array}\begin{array}{c} \textcircled{1} \\ {[1]} \end{array}(-j1.0) \quad \begin{array}{c} \\ \textcircled{5} \end{array}\begin{array}{c} \textcircled{5} \\ {[1]} \end{array}(-j0.8)$$

Giving Y_{BUS} =

$$\begin{bmatrix} -j6.08065 & j3.46774 & j0.64517 & 0 & j0.96774 \\ & -j9.11289 & j2.25806 & 0 & j3.38709 \\ & & -j13.16130 & j8 & j2.25807 \\ & \text{Symmetric} & & -j10 & j2 \\ & & & & -j9.41290 \end{bmatrix}$$

(7.10) Solve for the Y_{BUS} of Prob. 7.9 using Eq. (7.37). Use the linear graph of Fig. 7.19 with reference node 0 to determine the branch-to-node incidence matrix **A**.

Solution:

(a) The branch-to-node matrix is:

$$\mathbf{A} = \begin{array}{c} ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array} \overset{\begin{matrix} ① & ② & ③ & ④ & ⑤ \end{matrix}}{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}}$$

(b) The primitive admittance matrix is:

$$Y_{pr} = Z_{pr}^{-1} = \begin{array}{c} ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array} \overset{\begin{matrix} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{matrix}}{\begin{bmatrix} j0.4 & & & & & & & \\ & j0.5 & j0.15 & & & & & \\ & j0.15 & j0.25 & j0.1 & & & & \\ & & j0.1 & j0.2 & & & & \\ & & & & j0.125 & & & \\ & & & & & j0.5 & & \\ & & & & & & j1.0 & \\ & & & & & & & j1.25 \end{bmatrix}^{-1}}$$

$$
= \begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array}
\begin{array}{c}
\begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\
\begin{bmatrix}
-j2.5 & & & & & & & \\
 & -j2.58065 & j1.93548 & -j0.96774 & & & & \\
 & j1.93548 & -j6.45161 & j3.22581 & & & & \\
 & -j0.96774 & j3.22581 & -j6.61290 & & & & \\
 & & & & -j8 & & & \\
 & & & & & -j2 & & \\
 & & & & & & -j1.0 & \\
 & & & & & & & -j0.8
\end{bmatrix}
\end{array}
$$

And $\mathbf{Y}_{BUS} = \mathbf{A}^T\mathbf{Y}_{pr}\mathbf{A} =$

$$
\begin{bmatrix}
-j6.08065 & j3.46774 & j0.64517 & 0 & j0.96774 \\
 & -j9.11289 & j2.25806 & 0 & j3.38709 \\
 & & -j13.16130 & j8 & j2.25807 \\
 & \text{Symmetric} & & -j10 & j2 \\
 & & & & -j9.41290
\end{bmatrix}
$$

(as in Prob. 7.9.)

(7.11). Suppose that the direction of branch *d* in Fig. 7.19 is reversed so that it is now directed from node 2 to node 5. Find the branch-to-node incidence matrix **A** of this modified graph and then solve for the Y_{BUS} of Prob. 7.9 using Eq. (7.37).

Solution:
A is given by:

	①	②	③	④	⑤
ⓐ	1	-1	0	0	0
ⓑ	-1	0	1	0	0
ⓒ	0	-1	1	0	0
ⓓ	0	-1	0	0	1
ⓔ	0	0	1	-1	0
ⓕ	0	0	0	1	-1
ⓖ	-1	0	0	0	0
ⓗ	0	0	0	0	-1

The primitive impedance matrix can be written as follows where the sign of the mutual impedance between branches c and d is changed since the directions of the branch c to d does not match the polarities indicated by the dots of Fig. 7.18.

$$
Y_{pr} = Z_{pr}^{-1} = \begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array}
\begin{array}{c} \begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\
\left[\begin{array}{cccccccc}
j0.4 & & & & & & & \\
 & j0.5 & j0.15 & & & & & \\
 & j0.15 & j0.25 & -j0.1 & & & & \\
 & & -j0.1 & j0.2 & & & & \\
 & & & & j0.125 & & & \\
 & & & & & j0.5 & & \\
 & & & & & & j1.0 & \\
 & & & & & & & j1.25
\end{array} \right]^{-1} \end{array}
$$

$$
\begin{array}{c} \\ ⓐ \\ ⓑ \\ ⓒ \\ ⓓ \\ ⓔ \\ ⓕ \\ ⓖ \\ ⓗ \end{array}
\begin{array}{c} \begin{array}{cccccccc} ⓐ & ⓑ & ⓒ & ⓓ & ⓔ & ⓕ & ⓖ & ⓗ \end{array} \\
\left[\begin{array}{cccccccc}
-j2.5 & & & & & & & \\
 & -j2.58065 & j1.93548 & j0.96774 & & & & \\
 & j1.93548 & -j6.45161 & -j3.22581 & & & & \\
 & j0.96774 & -j3.22581 & -j6.61290 & & & & \\
 & & & & -j8 & & & \\
 & & & & & -j2 & & \\
 & & & & & & -j1.0 & \\
 & & & & & & & -j0.8
\end{array} \right] \end{array}
$$

And $\mathbf{Y}_{BUS} = \mathbf{A}^T\mathbf{Y}_{pr}\mathbf{A} =$

$$
\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}
\begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\
\left[\begin{array}{ccccc}
-j6.08065 & j3.46774 & j0.64517 & 0 & j0.96774 \\
 & -j9.11289 & j2.25806 & 0 & j3.38709 \\
 & & -j13.16130 & j8 & j2.25807 \\
 & \text{Symmetric} & & -j10 & j2 \\
 & & & & -j9.41290
\end{array} \right] \end{array}
$$

Note that regardless of the directions of the branches in the linear graph, a proper primitive impedance matrix can be chosen to give the correct result.

(7.12) Using the Y_{BUS} modification procedure described in Sec. 7.4, remove branch 2-3 from the Y_{BUS} solution obtained in Prob. 7.9 (or Prob. 7.10 or Prob. 7.11).

Solution:
Since line (2-3) is mutually coupled to lines (1-3) and (2-5), we first remove all three mutually coupled lines. The we reconnect lines (1-3) and (2-5) resulting in the removal of line (2-3) only.

The following building blocks are subtracted from Y_{BUS}:

$$\begin{matrix} & \textcircled{3} & \textcircled{1} \\ \textcircled{3} & 1 & -1 \\ \textcircled{1} & -1 & 1 \end{matrix}\;(-j2.58065) \qquad \begin{matrix} & \textcircled{3} & \textcircled{2} \\ \textcircled{3} & 1 & -1 \\ \textcircled{1} & -1 & 1 \end{matrix}\;(j1.93548) \qquad \begin{matrix} & \textcircled{5} & \textcircled{2} \\ \textcircled{3} & 1 & -1 \\ \textcircled{1} & -1 & 1 \end{matrix}\;(-j0.96774)$$

$$\begin{matrix} & \textcircled{3} & \textcircled{1} \\ \textcircled{3} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(j1.93548) \qquad \begin{matrix} & \textcircled{3} & \textcircled{2} \\ \textcircled{3} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(-j6.45161) \qquad \begin{matrix} & \textcircled{5} & \textcircled{2} \\ \textcircled{3} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(j3.22581)$$

$$\begin{matrix} & \textcircled{3} & \textcircled{1} \\ \textcircled{5} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(-j0.96774) \qquad \begin{matrix} & \textcircled{3} & \textcircled{2} \\ \textcircled{5} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(j3.22581) \qquad \begin{matrix} & \textcircled{5} & \textcircled{2} \\ \textcircled{5} & 1 & -1 \\ \textcircled{2} & -1 & 1 \end{matrix}\;(-j6.6129)$$

And the following building blocks are added:

$$\begin{matrix} & \textcircled{1} & \textcircled{3} \\ \textcircled{1} & 1 & -1 \\ \textcircled{3} & -1 & 1 \end{matrix}\;(-j2) \qquad \begin{matrix} & \textcircled{2} & \textcircled{5} \\ \textcircled{1} & 1 & -1 \\ \textcircled{5} & -1 & 1 \end{matrix}\;(-j5)$$

Giving:

$$
\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}
\begin{array}{c}
\begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\
\begin{bmatrix}
-j5.5 & j2.5 & j2 & 0 & 0 \\
j2.5 & -j7.5 & 0 & 0 & j5 \\
j2 & 0 & -j10 & j8 & 0 \\
0 & 0 & j8 & -j10 & j2 \\
0 & j5 & 0 & j2 & -j7.8
\end{bmatrix}
\end{array}
$$

(As in Prob. 7.6)

(7.13) Write nodal admittance equations for the circuit of Fig. 7.18 disregarding all mutual coupling. Solve the resultant equations for the bus voltages by the method of gaussian elimination.

Solution:
The nodal admittance equations are:

$$
\begin{bmatrix}
-j5.5 & j2.5 & j2 & 0 & 0 \\
j2.5 & -j11.5 & j4 & 0 & j5 \\
j2 & j4 & -j14 & j8 & 0 \\
0 & 0 & j8 & -j10 & j2 \\
0 & j5 & 0 & j2 & -j7.8
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}
=
\begin{bmatrix} 1.1\angle{-90^\circ} \\ 0 \\ 0 \\ 0 \\ 0.72\angle{-120^\circ} \end{bmatrix}
$$

Forward elimination gives:

$$
\left[\begin{array}{c:cccc}
1 & -0.45455 & -0.36364 & 0 & 0 \\ \hline
0 & -j10.36364 & j4.90909 & 0 & j5 \\
0 & j4.90909 & -j13.27273 & j8 & 0 \\
0 & 0 & j8 & -j10 & j2 \\
0 & j5 & 0 & j2 & -j7.8
\end{array}\right]
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}
=
\begin{bmatrix} 0.2 \\ -j0.5 \\ -j0.4 \\ 0 \\ -0.36-j0.62354 \end{bmatrix}
$$

$$\begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & -j10.94737 & j8 & j2.36842 \\ 0 & 0 & j8 & -j10 & j2 \\ 0 & 0 & j2.36842 & j2 & -j5.38772 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.04825 \\ -j0.63684 \\ 0 \\ -0.36-j0.62354 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & 1 & 0.73077 & -0.21635 \\ 0 & 0 & 0 & -j4.15385 & j3.73077 \\ 0 & 0 & 0 & j3.73077 & -j4.87532 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.04825 \\ 0.05817 \\ -j0.46538 \\ -0.36-j0.62354 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & 1 & -0.73077 & -0.21635 \\ 0 & 0 & 0 & 1 & -0.89815 \\ 0 & 0 & 0 & 0 & -j1.52454 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.04825 \\ 0.05817 \\ 0.11204 \\ -0.36-j0.62354 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & 1 & -0.73077 & -0.21635 \\ 0 & 0 & 0 & 1 & -0.89815 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.04825 \\ 0.05817 \\ 0.11204 \\ 0.93178-j0.23614 \end{bmatrix}$$

Back substitution yields the bus voltages:

$$V_5 = 0.93178 - j0.23614 = 0.96124\underline{/-14.2210^\circ}$$

$$V_4 = 0.94891 - j0.21209 = 0.97232\underline{/-12.5991^\circ}$$

$$V_3 = 0.95319 - j0.20607 = 0.9752\underline{/-12.1990^\circ}$$

$$V_2 = 0.94930 - j0.21154 = 0.97258\underline{/-12.5624^\circ}$$

$$V_1 = 0.97812 - j0.17109 = 0.99297\underline{/-9.9216^\circ}$$

(7.14) Prove Eq. (7.69) based on Eq. (7.68).

Solution:
Consider the nodal admittance equations in the form:

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1p} & \cdots & Y_{1n} \\ \vdots & & & & \\ Y_{p1} & \cdots & Y_{pp} & \cdots & Y_{pn} \\ \vdots & & & & \\ Y_{n1} & \cdots & Y_{np} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix}$$

The p^{th} equation is written as

$$\sum_{k=1}^{n} Y_{pk} V_k = I_p$$

If $I_p = 0$, V_p is given by:

$$V_p = \frac{-1}{Y_{pp}} \left[\sum_{\substack{k=1 \\ k \neq p}}^{n} Y_{pk} V_k \right]$$

By substitution of the above V_p into the j^{th} equation, the j^{th} equation becomes:

$$\sum_{k=1}^{n} Y_{jk}V_k = \sum_{\substack{k=1\\k\neq p}}^{n} \left(Y_{jk} - \frac{Y_{jp}Y_{pk}}{Y_{pp}}\right)V_k$$
$$= \sum_{\substack{k=1\\k\neq p}}^{n} Y_{jk,\text{new}}\, V_k$$

In this reduced set of equations, the new coefficient is as defined in Eq. (7.69).

(7.15) Using the gaussian-elimination calculations of Prob. 7.13, find the triangular factors of Y_{BUS} for the circuit of Fig. 7.18.

Solution:
Elements of two matrices **L** and **U** are (refer to Prob. 7.14):

$$\mathbf{L} = \begin{bmatrix} -j5.5 & 0 & 0 & 0 & 0 \\ j2.5 & -j10.36364 & 0 & 0 & 0 \\ j2 & j4.90909 & -j10.94737 & 0 & 0 \\ 0 & 0 & j8 & -j4.15385 & 0 \\ 0 & j5 & j2.36842 & j3.73077 & -j1.52454 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & 1 & -0.73077 & -0.21635 \\ 0 & 0 & 0 & 1 & -0.89815 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(7.16) Use the triangular factors obtained in Prob. 7.15 to calculate new bus voltages for Fig. 7.18 when the voltage source at bus 5 is changed to $1.0\underline{/-45^\circ}$ per unit. Follow the procedure of Example 7.9.

Solution:
The new voltage source at bus 5 is converted to a current source, $1.0\underline{/-45^\circ} \div j1.25 = 0.8\underline{/-125^\circ}$ per unit. Using the **L** and **U** of Prob. 7.15, the equation to solve is

$$\mathbf{LUV} = \mathbf{I} = \begin{bmatrix} 1.1\underline{/-90^\circ} \\ 0 \\ 0 \\ 0 \\ 0.8\underline{/-125^\circ} \end{bmatrix}$$

We first let **UV** = **V'** and solve **LV'** = **I** for **V'** as follows:

$$\begin{bmatrix} -j5.5 & 0 & 0 & 0 & 0 \\ j2.5 & -j10.36364 & 0 & 0 & 0 \\ j2 & j4.90909 & -j10.94737 & 0 & 0 \\ 0 & 0 & j8 & -j4.15385 & 0 \\ 0 & j5 & j2.36842 & j3.73077 & -j1.52454 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_5' \end{bmatrix} = \begin{bmatrix} 1.1\underline{/-90^\circ} \\ 0 \\ 0 \\ 0 \\ 0.8\underline{/-125^\circ} \end{bmatrix}$$

Solving by back substitution:

$$\mathbf{V'} = \begin{bmatrix} 0.2 \\ 0.04825 \\ 0.05817 \\ 0.11204 \\ 0.89383 - j0.37105 \end{bmatrix}$$

V is then determined from solution of **UV** = **V'**:

$$\begin{bmatrix} 1 & -0.45455 & -0.36364 & 0 & 0 \\ 0 & 1 & -0.47368 & 0 & -0.48246 \\ 0 & 0 & 1 & -0.73077 & -0.21635 \\ 0 & 0 & 0 & 1 & -0.89815 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.04825 \\ 0.05817 \\ 0.11204 \\ 0.89383 - j0.37105 \end{bmatrix}$$

From which we get:

$$\mathbf{V} = \begin{bmatrix} 0.95063 - j0.26884 \\ 0.91531 - j0.33240 \\ 0.92008 - j0.32381 \\ 0.91483 - j0.33326 \\ 0.89383 - j0.37105 \end{bmatrix} = \begin{bmatrix} 0.98791\angle -15.791^\circ \\ 0.97380\angle -19.9588^\circ \\ 0.97540\angle -19.3888^\circ \\ 0.97364\angle -20.016^\circ \\ 0.96779\angle -22.5445^\circ \end{bmatrix}$$

(7.17) Using the triangular factors obtained in Example 7.9, find the voltage at bus 3 of the circuit of Fig. 7.11 when an *additional* current of $0.2\angle -120^\circ$ per unit is injected at bus 2. All other conditions of Fig. 7.11 are unchanged.

Solution:
The equations to be solved are:

$$\mathbf{LUV} = \mathbf{I} = \begin{bmatrix} 0 \\ 0.2\angle -120^\circ \\ 1.0\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

L and **U** are given in Example 7.9. Let **UV** = **V'** ; solve **LV'** = **I** for **V'**:

$$\begin{bmatrix} -j16.75 & 0 & 0 & 0 \\ j11.75 & -j11.00746 & 0 & 0 \\ j2.5 & j4.25373 & -j3.78305 & 0 \\ j2.5 & j6.75373 & j2.98305 & -j1.43082 \end{bmatrix} \begin{bmatrix} \\ \mathbf{V'} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0.2\angle{-120^\circ} \\ 1.0\angle{-90^\circ} \\ 0.68\angle{-135^\circ} \end{bmatrix}$$

Solving by back substitution:

$$\begin{bmatrix} \\ \mathbf{V'} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01574 - j0.00908 \\ 0.28203 - j0.01022 \\ 0.99832 - j0.40023 \end{bmatrix}$$

V is next determined from solution of **UV** = **V'**:

$$\begin{bmatrix} 1 & -0.70149 & -0.14925 & -0.14925 \\ 0 & 1 & -0.38644 & -0.61356 \\ 0 & 0 & 1 & -0.78853 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \mathbf{V} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01574 - j0.00908 \\ 0.28203 - j0.01022 \\ 0.99832 - j0.40023 \end{bmatrix}$$

From which we get:

$$\begin{bmatrix} \\ \mathbf{V} \\ \\ \end{bmatrix} = \begin{bmatrix} 1.03916 - j0.37532 \\ 1.04146 - j0.38055 \\ 1.06924 - j0.32581 \\ 0.99832 - j0.40023 \end{bmatrix} = \begin{bmatrix} 1.10486\angle{-19.8585^\circ} \\ 1.10881\angle{-20.0723^\circ} \\ 1.11778\angle{-16.9466^\circ} \\ 1.07556\angle{-21.8460^\circ} \end{bmatrix}$$

(7.18) *(a)* Kron reduce Y_{BUS} of the circuit of Fig. 7.18 to reflect elimination of node 2. *(b)* Use the Y-Δ transformation of Table 1.2 to eliminate note 2 from the circuit of Fig. 7.18 and find Y_{BUS} for the resulting reduced network. Compare results of parts *(a)* and *(b)*.

Solution:

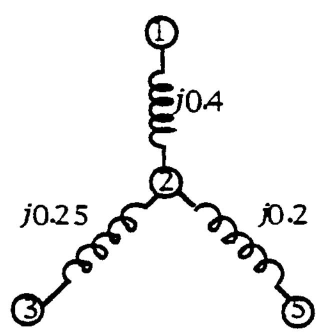
Y_{BUS} of the circuit of Fig. 7.18 is given by:

$$Y_{BUS} = j\begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} \end{matrix} \\ \begin{bmatrix} -5.5 & 2.5 & 2 & & \\ 2.5 & -11.5 & 4 & & 5 \\ 2 & 4 & -14 & 8 & \\ & & 8 & -10 & 2 \\ & 5 & & 2 & -7.8 \end{bmatrix} \end{array}$$

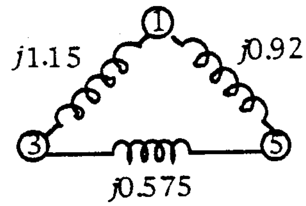
After Kron reduction of row 2 and column 2:

$$Y_{BUS}^{reduced} = j\begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{3} & \textcircled{4} & \textcircled{5} \end{matrix} \\ \begin{bmatrix} -4.95652 & 2.86957 & 0 & 1.08696 \\ 2.86957 & -12.60870 & 8 & 1.73913 \\ 0 & 8 & -10 & 2 \\ 1.08696 & 1.73913 & 2 & -5.62609 \end{bmatrix} \end{array}$$

(*b*) Node 2 is connected to nodes 1,3 and 5 as shown, all impedances are in per-unit.



The Δ-equivalent circuit is:



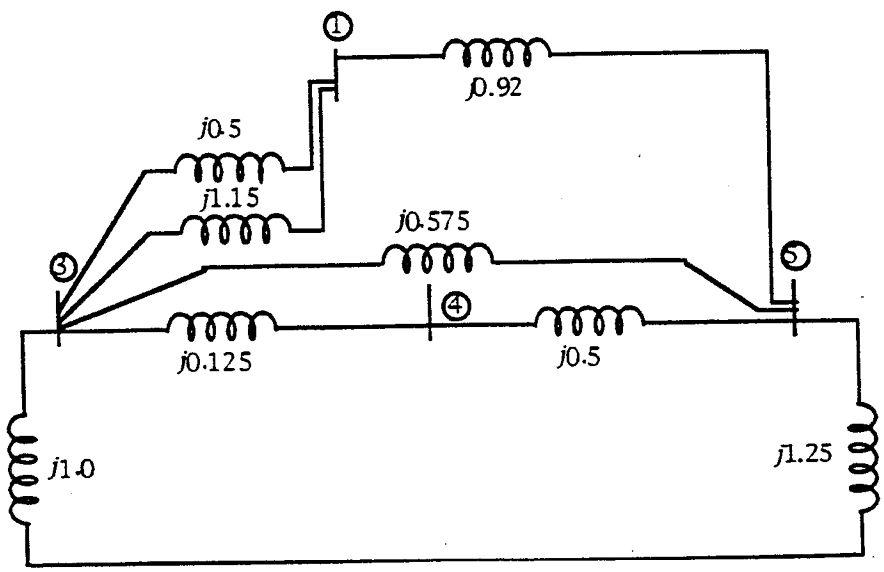
where:

$$Z_{15} = (j0.4)(j0.2)\frac{11.5}{j} = j0.92$$

$$Z_{13} = (j0.4)(j0.25)\frac{11.5}{j} = j1.15$$

$$Z_{35} = (j0.25)(j0.2)\frac{11.5}{j} = j0.575$$

When the Δ-equivalent circuit replaces the original star, the following results:



Giving :

$$Y_{BUS} = j \begin{array}{c} \begin{array}{cccc} ① & ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} -4.95652 & 2.86957 & 0 & 1.08696 \\ 2.86957 & -12.60870 & 8 & 1.73913 \\ 0 & 8 & -10 & 2 \\ 1.08696 & 1.73913 & 2 & -5.62609 \end{bmatrix} \end{array}$$

confirming the earlier result.

(7.19) Find the L and U triangular factors of the symmetric matrix

$$M = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

Verify the result using Eq. (7.75).

Solution:
Forward elimination yields the following:

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 9/2 & 5/2 \\ 0 & 5/2 & 5/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 5/9 \\ 0 & 0 & 10/9 \end{bmatrix}$$

L and U are:

$$\mathbf{L} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 9/2 & 0 \\ 3 & 5/2 & 10/9 \end{bmatrix} ; \ \mathbf{U} = \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 5/9 \\ 0 & 0 & 1 \end{bmatrix}$$

And

$$\mathrm{U^TD} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & 5/9 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 9/2 & 0 \\ 0 & 0 & 10/9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 1 & 9/2 & 0 \\ 3 & 5/2 & 10/9 \end{bmatrix} = \mathbf{L}$$

Which verifies Eq. (7.75).