Chapter 9 Problem Solutions

9.1 In Example 9.3, suppose that the generator's maximum reactive power generation at bus ④ is limited to 125 Mvar. Re-compute the first-iteration value of the voltage at bus ④ using the Gauss-Seidel method.

Solution:

The net power injection found at bus ④ of Example 9.3 was

$$Q_4 \doteq 1.654151 \text{ per unit} = 165.4151 \text{ Mvar}$$

Considering the reactive load of 49.58 Mvar at the bus, the required reactive power generation is $165.4151 + 49.58 = 214.9951$ Mvar, which exceeds the 125 Mvar limit specified. The bus is now regarded as a load bus, with total reactive power generation of 125 Mvar. So the net injected reactive power in this case is

$$125 - 49.58 = 75.42 \text{ Mvar} = 0.7521 \text{ per unit}$$

V_4 is now calculated as

$$\begin{aligned}
V_4^{(1)} &= \frac{1}{Y_{44}}\left[\frac{P_{4,sch} - jQ_4^{(1)}}{V_4^{(0)*}} - \left(Y_{42}V_{2,acc}^{(1)} + Y_{43}V_{3,acc}^{(1)}\right)\right] \\
&= \frac{1}{8.193267 - j40.863838}\left[\frac{2.38 - j0.7542}{1.02} - (-5.573064 + j40.05939)\right] \\
&= 0.997117 - j0.006442 \text{ per unit}
\end{aligned}$$

and using an acceleration factor of 1.6 yields

$$V_{4,acc}^{(1)} = 1.02 + 1.6\,(0.997117 - j0.06442 - 1.02) = 0.983387 - j0.0103073 \text{ per unit}$$

(9.2) For the system of Fig. 9.2, complete the second iteration of the Gauss-Seidel procedure using the first iteration value of the bus voltages obtained in Examples 9.2 and 9.3. Assume an acceleration factor of 1.6.

Solution:

$$V_2^{(2)} = \frac{1}{Y_{22}}\left[\frac{P_{2,sch} - jQ_{2,sch}}{V_2^{(1)*}} - (Y_{21}V_1^{(1)} + Y_{24}V_{4,acc}^{(1)})\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-1.7 + j1.0535}{0.981113+j0.031518} - \{ -3.815629 + j19.078144\right.$$

$$\left. + (-5.169561 + j25.847809)(1.019922 + j0.012657)\} \right]$$

$$= \frac{7.718854 - j44.247184}{8.985190 - j44.835953}$$

$$= 0.9819338 - j0.0246233$$

$$V_{2,acc}^{(2)} = 0.981113 - j0.031518 + 1.6\ (\ 0.9819338 - j0.0246233$$
$$- 0.9819338 + j0.0246233)$$

$$= 0.982426 - j0.020486$$

$$V_3^{(2)} = \frac{1}{Y_{33}}\left[\frac{P_{3,\text{sch}} - jQ_{3,\text{sch}}}{V_3^{(1)*}} - (Y_{31}V_1^{(2)} + Y_{34}V_{4,\text{acc}}^{(1)})\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-2 + j1.2394}{0.966597+j0.040797} - \{ -5.16956 + j25.847809\right.$$

$$\left. + (-3.023705 + j15.118528)(1.019922 + j0.012657)\} \right].$$

$$= \frac{6.433447 - j39.862133}{8.193267 - j40.863838}$$

$$= 0.9681332 - j0.0366761$$

$$V_{3,\text{acc}}^{(2)} = 0.966597 - j0.00.040797 + 1.6\,(0.9681332 - j0.0366761$$
$$- 0.966597 + j0.00.040797)$$

$$= 0.969055 - j0.034195$$

$$Q_4^{(2)} = -\,\text{Im}\left\{ V_4^{(1)*}\left[Y_{42}V_2^{(2)} + Y_{43}V_3^{(2)} + Y_{44}V_4^{(1)}\right]\right\}$$

$$= -\,\text{Im}\{ (1.019922 - j0.012657)$$

$$\times [(-5.16956 + j\,25.847809)(0.982426 - j0.020486)$$
$$+ (-3.023705 + j\,15.118528)(0.969055 - j0.034195)$$
$$+ (8.193267 - j\,40.863837)(1.019922 + j0.012657)]\}$$

$$= -\,\text{Im}\{ 1.911362 - j1.320680\} = 1.320680$$

$$V_4^{(2)} = \frac{1}{Y_{44}}\left[\frac{P_{3,\text{sch}} - jQ_4^{(2)}}{V_4^{(1)*}} - (Y_{42}V_2^{(2)} + Y_{43}V_3^{(2)})\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{2.38 - j1.320680}{1.019922 - j0.012657}\right.$$

$$\left. - \{(-5.16956 + j25.847809)(0.982426 - j0.020486) + (-3.023705 + j15.118528)(0.969055 - j0.034195)\}\right]$$

$$= \frac{9.311570 - j41.519274}{8.193267 - j40.863838}$$

$$= 1.020695 + j0.023217$$

$$V_{4,\text{corr}}^{(2)} = \frac{1.02}{1.020959}(1.020695 + j0.023217)$$

$$= 1.019736 + j0.023195$$

(9.3) A synchronous condenser, whose reactive power capability is assumed to be unlimited, is installed at load bus 2 of the system of Example 9.2 to hold the bus-voltage magnitude at 0.99 per unit. Using the Gauss-Seidel method, find the voltage at buses 2 and 3 for the first iteration.

Solution:

$$Q_2 = -\operatorname{Im}\{ V_2^* [Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4]\}$$

$$= -\operatorname{Im}\{ (0.99) \times [(1.0)(-3.815629 + j19.078144) + (0.99)(8.985190 - j44.835953) + (1.02)(-5.169561 + j25.847809)]\}$$

$$= -1.0447626$$

Using the above value of Q_2, V_2 is computed:

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_{2,sch} - jQ_2}{V_2^{(0)*}} - (Y_{21}V_1 + Y_{24}V_4)\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-1.7 + j1.0447626}{0.99} - \{ (1.0)(-3.815629 + j19.078144) + (1.02)(-5.169561 + j25.847809)\}\right]$$

$$= \frac{7.3714095 - j44.387593}{8.985190 - j44.835953}$$

$$= 0.9834515 - j0.0326767$$

The magnitude of V_2 is now corrected to 0.99:

$$V_{2,\text{corr}} = \frac{0.99}{0.9839942}(0.9834515 - j0.0326767)$$

$$= 0.9894539 - j0.0328761$$

At bus 3, we have,

$$V_3 = \frac{1}{Y_{33}}\left[\frac{P_{3,\text{sch}} - jQ_{3,\text{sch}}}{V_3^*} - (Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4)\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-2 + j1.2394}{1.0}\right.$$

$$-\{(1.0)(-5.169561 + j25.847809)$$

$$\left. + (1.02)(-3.023705 + j15.118528)\}\right]$$

$$= \frac{6.2537401 - j40.029308}{8.193267 - j40.863838}$$

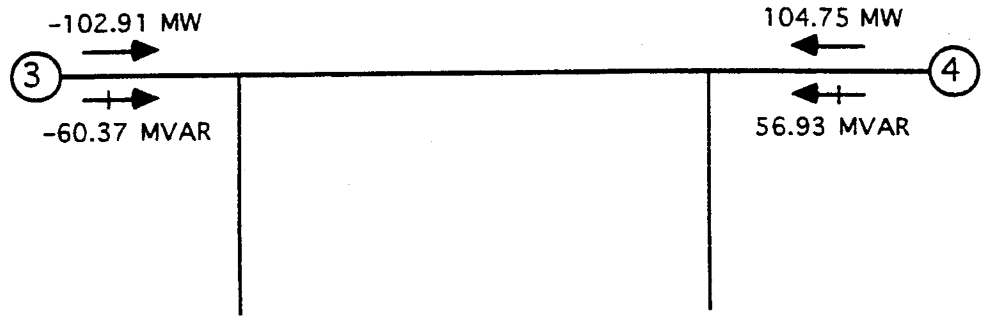
$$= 0.9712184 - j0.0416924$$

If desired, an acceleration factor may be used.

(9.4) Take Fig. 9.12 as the equivalent-π representation of the transmission line between bus 3 and 4 of the system of Fig. 9.2. Using the power-flow solution given in Fig. 9.4, determine and indicate on Fig. 9.12 the values of (*a*) P and Q leaving buses 3 and 4 on line 3-4, (*b*) charging megavars of the equivalent π of the line 3-4, and (*c*) P and Q at both ends of the series part of the equivalent π of the line 3-4.

Solution:

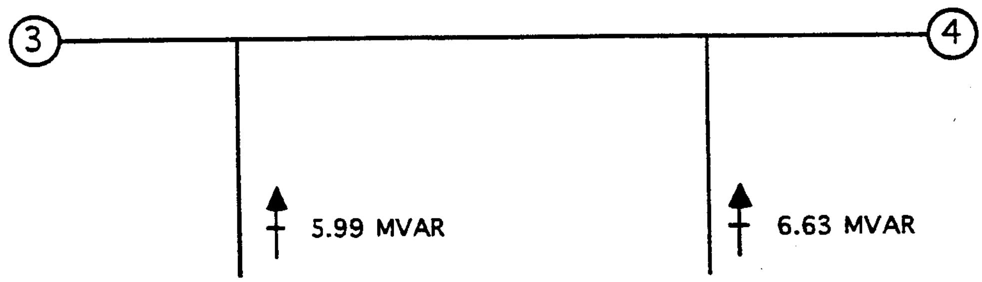
(*a*)



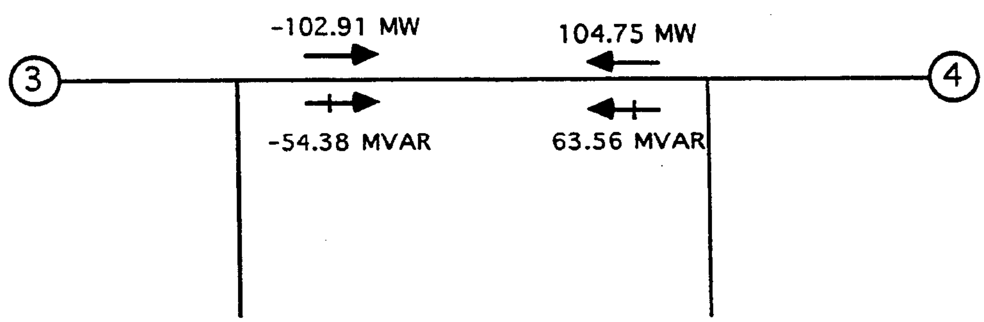
(*b*) Charging MVAR varies as the square of the voltage:

At bus 3: $\frac{12.75}{2}(0.969)^2 = 5.99 \text{ MVAR}$

At bus 4: $\frac{12.75}{2}(1.02)^2 = 6.63 \text{ MVAR}$



(*c*) These are the sums of (*a*) and (*b*):



(9.5) From the line-flow information of the power flow solution given in Fig. 9.4, determine I^2R loss in each of the four transmission lines, and verify that the sum of these line losses is equal to the total system loss of 4.81 MW.

Solution:

Line 1-2:

$$= 38.69 - 38.46 = 0.23 \text{ MW}$$

Line 1-3:

$$= 98.12 - 97.09 = 1.03 \text{ MW}$$

Line 2-4:

$$= 133.25 - 131.54 = 1.71 \text{ MW}$$

Line 3-4:

$$= 104.75 - 102.91 = 1.84 \text{ MW}$$

Summing up these four line loss components, we get the total loss is 4.8 MW.

(9.6) Suppose that a shunt capacitor bank rated 18 MVAR is connected between bus 3 and the reference node in the system of Example 9.5. Modify the $\mathbf{Y}_{BUS}$ given in Table 9.4 to account for this capacitor, and *estimate* the actual megavar reactive power injected into this system from this capacitor.

Solution:

Only the diagonal element corresponding to bus 3 needs to be modified:

$$Y_{33}^{new} = (8.193267 - j40.863838) + j0.18$$
$$= 8.193267 - j40.683838$$

Using the voltage at bus 3 given in Fig. 9.4, the approximate power injection is

$$18 \times (0.969)^2 = 16.9 \text{ MVAR}.$$

(9.7) For the system of Example 9.5 augmented with a synchronous condenser as described in Prob. 9.3, find the jacobian calculated at the initial estimates. *Hint:* It would be simpler to modify the jacobian matrix shown in Sec. 9.4 following Example 9.5 than to start calculations from the beginning.

Solution:
Since bus 2 is now a *PV* bus, the mismatch equations can be written in the following form:

$$
\begin{array}{cc}
 & \begin{array}{cccc} \quad ② & \quad ③ & \quad ④ & \qquad\qquad ③ \end{array} \\
\begin{array}{c} ② \\ ③ \\ ④ \\ ③ \end{array} &
\left[\begin{array}{ccc|c}
M_{22} & 0 & M_{24} & 0 \\
0 & M_{33} & M_{34} & N_{33} + 2|V_3|^2 G_{33} \\
M_{42} & M_{43} & M_{44} & -N_{43} \\
\hline
0 & N_{33} & N_{34} & -M_{33} - 2|V_3|^2 B_{33}
\end{array}\right]
\end{array}
\left[\begin{array}{c}
\Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \hline \Delta|V_3|/|V_3|
\end{array}\right]
=
\left[\begin{array}{c}
\Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \hline \Delta Q_3
\end{array}\right]
$$

Note that the values are given in Sec. 9.4. To determine the first iteration value of the above, we use the same initial voltages as before, except $|V_2| = 0.99$ p.u. Only the terms involving V_2 change. The M_{22}, M_{24} and M_{42} terms need only be corrected by 0.99:

$$M_{22} = 45.442909 * 0.99 = 44.98848$$
$$M_{24} = -26.364763 * 0.99 = -26.101115$$
$$M_{42} = -26.364763 * 0.99 = -26.101115$$

And M_{44} is recalculated as follows:

$$
\begin{aligned}
M_{44} &= \sum_{\substack{n=1 \\ n\neq 4}}^{4} |V_4 V_n Y_{4n}| \sin(\theta_{4n} + \delta_n - \delta_4) \\
&= 1.02 \times 0.99 \times 26.359696 \times \sin(101.30993^\circ) \\
&\quad + 1.02 \times 1 \times 15.41793 \times \sin(101.30993^\circ) \\
&= 26.101117 + 15.420899 \\
&= 41.522016
\end{aligned}
$$

Giving the jacobian:

$$
\begin{array}{c c}
 & \begin{array}{cccc} 2 & 3 & 4 & 3 \end{array} \\
\begin{array}{c} 2 \\ 3 \\ 4 \\ 3 \end{array} &
\left[\begin{array}{ccc|c}
44.98848 & 0 & -26.101115 & 0 \\
0 & 41.268707 & -15.420898 & 8.132792 \\
-26.101115 & -15.420898 & 41.522016 & -3.084180 \\
\hline
0 & -8.25374 & -3.084180 & 40.458969
\end{array}\right]
\end{array}
$$

(9.8) Suppose that in Fig. 9.7 the tap is on the side of node i so that the transformation ratio is $t:1$. Find the elements of $\mathbf{Y}_{BUS}$ similar to those in Eq. (9.74), and draw the equivalent-π representation similar to Fig. 9.8.

Solution:

$$S_i = V_i I_i^* \qquad ; \qquad S_j = \frac{1}{t} V_j I_j^*$$

Since $S_i + S_j = 0$,

$$I_i = \frac{-1}{t^*} I_j$$

$$I_j = \left(V_j - \frac{V_i}{t} \right) Y = - \frac{Y}{t} V_i + Y V_j$$

and

$$-t^* I_i = - \frac{Y}{t} V_i + Y V_j$$

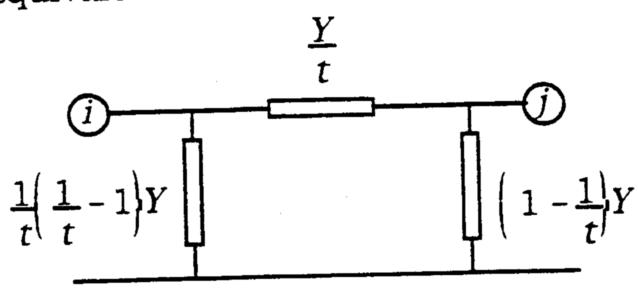
from which:

$$I_i = - \frac{Y}{tt^*} V_i + \frac{Y}{t^*} V_j$$

giving:

$$\begin{bmatrix} I_i \\ \\ I_j \end{bmatrix} = \begin{bmatrix} \dfrac{Y}{|t|^2} & \dfrac{-Y}{t^*} \\ \\ \dfrac{-Y}{t} & Y \end{bmatrix} \begin{bmatrix} V_i \\ \\ V_j \end{bmatrix}$$

For real t, the equivalent network is:



(9.9) In the four-bus system of Example 9.5, suppose that a magnitude-regulating transformer with 0.2 per-unit reactance is inserted between the load and the bus at bus 3, as shown in Fig. 9.10. The variable tap is on the load side of the transformer. If the voltage magnitude at the new load bus 5 is prespecified, and therefore is not a state variable, the tap t of the transformer should be regarded as a state variable. The Newton-Raphson method is to be applied to the solution of the power-flow equations.

(*a*) Write mismatch equations for this problem in symbolic form similar to Eq. (9.45)

(*b*) Write equations of the jacobian elements of the column corresponding to the variable t (that is, partial derivatives with respect to t), and evaluate them using the initial voltage estimates shown in Table 9.3 and assuming that the voltage magnitude at bus 5 is specified to be 0.97. The initial estimate of δ_5 is 0.

(*c*) Write equations of P and Q mismatches at bus 5 and evaluate them for the first iteration. Assume the initial estimate of variable t is 1.0.

Note to instructor: In future printings, this value of 0.2 per unit for the transformer reactance should be changed to a value of 0.02 per unit.

Solution:

(*a*)

$$
\left[\begin{array}{cccc|cc|c}
\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial \delta_5} & |V_2|\frac{\partial P_2}{\partial |V_2|} & |V_3|\frac{\partial P_2}{\partial |V_3|} & \frac{\partial P_2}{\partial t} \\
\frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & & \cdots & |V_3|\frac{\partial P_3}{\partial |V_3|} & \frac{\partial P_3}{\partial t} \\
\frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \ddots & & & & \frac{\partial P_4}{\partial t} \\
\frac{\partial P_5}{\partial \delta_2} & \frac{\partial P_5}{\partial \delta_3} & \frac{\partial P_5}{\partial \delta_4} & & & & \frac{\partial P_5}{\partial t} \\
\hline
\frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_2} & \cdots & & \cdots & |V_3|\frac{\partial Q_2}{\partial |V_3|} & \frac{\partial Q_2}{\partial t} \\
\frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_2} & \ddots & & & \cdots & \frac{\partial Q_3}{\partial t} \\
\frac{\partial Q_5}{\partial \delta_2} & \cdots & & & & \cdots & \frac{\partial Q_5}{\partial t}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \hline \frac{\Delta|V_2|}{|V_2|} \\ \frac{\Delta|V_3|}{|V_3|} \\ \hline t
\end{array}\right]
=
\left[\begin{array}{c}
\Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_5
\end{array}\right]
$$

(b) For the last column:

$$\frac{\partial P_2}{\partial t} = 0$$

$$\frac{\partial P_3}{\partial t} = \frac{\partial}{\partial t}\left\{ |V_3|^2 G_{33} + \sum_{\substack{n=1 \\ n\neq 3}}^{5} |V_3 V_n Y_{3n}| \cos(\theta_{3n} + \delta_n - \delta_3) \right\}$$

$$= \frac{\partial}{\partial t}\{ |V_3 V_5(-tY)| \cos(\theta_{35} + \delta_5 - \delta_3)\}$$

$$= -|V_3 V_5 Y| \cos(90°) = 0$$

$$\frac{\partial P_4}{\partial t} = 0$$

$$\frac{\partial P_5}{\partial t} = \frac{\partial}{\partial t}\left\{ |V_5|^2 G_{55} + \sum_{\substack{n=1 \\ n\neq 5}}^{5} |V_5 V_n Y_{5n}| \cos(\theta_{5n} + \delta_n - \delta_5) \right\}$$

$$= \frac{\partial}{\partial t}\{ |V_5 V_3(-tY)| \cos(\theta_{53} + \delta_3 - \delta_5)\}$$

$$= -|V_5 V_3 Y| \cos(90°) = 0$$

$$\frac{\partial Q_2}{\partial t} = 0$$

$$\frac{\partial Q_3}{\partial t} = \frac{\partial}{\partial t}\left\{ -|V_3|^2 B_{33} - \sum_{\substack{n=1 \\ n\neq 3}}^{5} |V_3 V_n Y_{3n}| \sin(\theta_{3n} + \delta_n - \delta_3) \right\}$$

$$= \frac{\partial}{\partial t}\left\{ -|V_3|^2 (B_{33,\text{fixed}} - t^2 |Y|) - |V_5 V_3 (-tY)| \sin(\theta_{53} + \delta_3 - \delta_5) \right\}$$

$$= 2t |V_3|^2 |Y| - |V_5 V_3 Y| \sin(90°) =$$

$$= \frac{2}{0.2} - \frac{0.97}{0.2} = 5.15$$

$$\frac{\partial Q_4}{\partial t} = 0$$

$$\frac{\partial Q_5}{\partial t} = \frac{\partial}{\partial t}\left\{ -|V_5|^2 B_{55} - \sum_{\substack{n=1 \\ n\neq 5}}^{5} |V_5 V_n Y_{5n}| \sin(\theta_{5n} + \delta_n - \delta_5) \right\}$$

$$= \frac{\partial}{\partial t}\left\{ -|V_5 V_3 (-tY)| \sin(\theta_{53} + \delta_3 - \delta_5) \right\}$$

$$= -|V_5 V_3 Y| \sin(90°) = -\frac{0.97}{0.2} = -4.85$$

(c)

$$P_{5,\text{calc}}^{(0)} = |V_5|^2 G_{55} + \sum_{\substack{n=1 \\ n\neq 5}}^{5} |V_5 V_n Y_{5n}| \cos(\theta_{5n} + \delta_n - \delta_5)$$

$$= 0 + |V_5 V_3 Y_{53}| \cos(\theta_{53} + \delta_3 - \delta_5)$$

$$= 0 + 0.97 \times 5 \times \cos(90^\circ) = 0$$

$$\Delta P_5^{(0)} = P_{5,\text{sch}} - P_{5,\text{calc}}^{(0)}$$

$$= -2 - 0 = -2$$

$$Q_{5,\text{calc}}^{(0)} = - |V_5|^2 B_{55} - \sum_{\substack{n=1 \\ n\neq 5}}^{5} |V_5 V_n Y_{5n}| \sin(\theta_{5n} + \delta_n - \delta_5)$$

$$= - |V_5|^2 B_{55} - |V_5 V_3 Y_{53}| \sin(\theta_{53} + \delta_3 - \delta_5)$$

$$= -(0.97)^2(-5) - 0.97 \times 5 \times \sin(90^\circ) =$$

$$= 4.7045 - 4.85$$

$$= -0.1455$$

$$\Delta Q_5^{(0)} = Q_{5,\text{sch}} - Q_{5,\text{calc}}^{(0)}$$

$$= -1.2394 - (-0.1455) = -1.0939$$

(9.10) If the tap setting of the transformer of Prob. 9.9 is prespecified instead of the voltage magnitude at bus 5, then V_5 should be regarded as a state variable. Suppose that the tap setting t is specified to be 1.05.

(*a*) In this case write mismatch equations in symbolic form similar to Eq. (9.45)

(*b*) Write equations of the jacobian elements which are partial derivatives with respect to $|V_5|$, and evaluate them using the initial estimates. The initial estimate of V_5 is $1.0\underline{/0^\circ}$.

(*c*) Write equations of P and Q mismatches at bus 5 and evaluate them for the first iteration.

Solution:

(*a*)

$$\left[\begin{array}{cccc|ccc}
\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial \delta_5} & |V_2|\frac{\partial P_2}{\partial |V_2|} & |V_3|\frac{\partial P_2}{\partial |V_3|} & |V_5|\frac{\partial P_2}{\partial |V_5|} \\
\frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & & \cdots & |V_3|\frac{\partial P_3}{\partial |V_3|} & |V_5|\frac{\partial P_3}{\partial |V_5|} \\
\frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \ddots & & & & |V_5|\frac{\partial P_4}{\partial |V_5|} \\
\frac{\partial P_5}{\partial \delta_2} & \frac{\partial P_5}{\partial \delta_3} & \frac{\partial P_5}{\partial \delta_4} & & & & |V_5|\frac{\partial P_5}{\partial |V_5|} \\
\hline
\frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_2} & \cdots & & \cdots & |V_3|\frac{\partial Q_2}{\partial |V_3|} & |V_5|\frac{\partial Q_2}{\partial |V_5|} \\
\frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_2} & \ddots & & & \cdots & |V_5|\frac{\partial Q_3}{\partial |V_5|} \\
\frac{\partial Q_5}{\partial \delta_2} & \cdots & & & & \cdots & |V_5|\frac{\partial Q_5}{\partial |V_5|}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \hline \frac{\Delta|V_2|}{|V_2|} \\ \frac{\Delta|V_3|}{|V_3|} \\ \frac{\Delta|V_5|}{|V_5|}
\end{array}\right]
=
\left[\begin{array}{c}
\Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_5
\end{array}\right]$$

(*b*)

$$|V_5|\,\frac{\partial P_2}{\partial |V_5|} = |V_5|\,|V_2 Y_{25}|\cos(\,\theta_{52} + \delta_5 - \delta_2\,) = 0$$

$$\begin{aligned} |V_5|\,\frac{\partial P_3}{\partial |V_5|} &= |V_5|\,|V_3 Y_{35}|\cos(\,\theta_{53} + \delta_5 - \delta_3\,) \\ &= |V_5|\,|V_3(-tY)|\cos(90^\circ\,) = 0 \end{aligned}$$

$$|V_5|\,\frac{\partial P_4}{\partial |V_5|} = |V_5|\,|V_4 Y_{45}|\cos(\,\theta_{54} + \delta_5 - \delta_4\,) = 0$$

$$\begin{aligned} |V_5|\,\frac{\partial P_5}{\partial |V_5|} &= |V_5|\left\{ 2|V_5|G_{55} + \sum_{\substack{n=1 \\ n\neq 5}}^{5} |V_n Y_{5n}|\cos(\,\theta_{5n} + \delta_n - \delta_5\,) \right\} \\ &= |V_5|\,|V_3(-tY)|\cos(90^\circ\,) = 0 \end{aligned}$$

$$|V_5|\,\frac{\partial Q_2}{\partial |V_5|} = -\,|V_5|\,|V_2 Y_{52}|\sin(\,\theta_{52} + \delta_5 - \delta_2\,) = 0$$

$$\begin{aligned} |V_5|\,\frac{\partial Q_3}{\partial |V_5|} &= -\,|V_5|\,|V_3 Y_{53}|\sin(\,\theta_{53} + \delta_5 - \delta_3\,) \\ &= -\,|V_5|\,|V_3(-tY)|\sin(90^\circ\,) \\ &= -\,1 \times \frac{1.05}{0.2} = -\,5.25 \end{aligned}$$

$$|V_5|\,\frac{\partial Q_5}{\partial |V_5|} = -\,2|V_5|^2 B_{55} - \sum_{\substack{n=1\\ n\neq 5}}^{5} |V_5 V_n Y_{5n}|\ \sin(\ \theta_{5n} + \delta_n - \delta_5\ )$$

$$= -\,2|V_5|^2 B_{55} - |V_5|\ |V_3(-tY)|\ \sin(90^\circ\ )$$

$$= -\,2\left(\frac{-1}{0.2}\right) - \frac{1.05}{0.2} = 4.75$$

(c)

$$P_{5,\text{calc}}^{(0)} = |V_5|^2 G_{55} + \sum_{\substack{n=1\\ n\neq 5}}^{5} |V_5 V_n Y_{5n}|\ \cos(\ \theta_{5n} + \delta_n - \delta_5\ )$$

$$= 0 + |V_5 V_3 Y_{53}|\ \cos(\ \theta_{53} + \delta_3 - \delta_5\ )$$

$$= \frac{1.05}{0.2}\ \cos(90^\circ) = 0$$

Therefore,

$$\Delta P_5^{(0)} = P_{5,\text{sch}} - P_{5,\text{calc}}^{(0)}$$

$$= -2 - 0 = -2$$

and,

$$Q_{5,\text{calc}}^{(0)} = -\ |V_5|^2 B_{55} - \sum_{\substack{n=1 \\ n\neq 5}}^{5} |\ V_5 V_n Y_{5n}|\sin(\ \theta_{5n} + \delta_n - \delta_5)$$

$$= -\ |V_5|^2 B_{55} - |\ V_5 V_3 Y_{53}|\sin(\ \theta_{53} + \delta_3 - \delta_5)$$

$$= -\ 1 \times (-5) - \frac{1.05}{0.2} \times \sin(90°) =$$

$$= 5 - 5.25$$

$$= -0.25$$

$$\Delta Q_5^{(0)} = Q_{5,\text{sch}} - Q_{5,\text{calc}}^{(0)}$$

$$= -1.2394 - (\ -0.25) = -0.9894$$

(9.12) The generator at bus 4 of the system of Example 9.5 is to be represented by a generator connected to bus 4 through a generator step-up transformer as shown in Fig. 9.13. The reactance of this transformer is 0.02 per unit; the tap is on the high-voltage side of the transformer with the off-nominal turns ratio of 1.05. Evaluate the jacobian elements of the rows corresponding to buses 4 and 5.

Solution:
Note to instructor: In this problem it is assumed that the new bus 5 becomes the regulated bus with $|V_5| = 1.02$ p.u., and that bus 4 is a load bus with an initial voltage value of $|V_4| = 1.0$ p.u.

The transformer contribution to Y_{BUS} is:

$$\begin{array}{c} \begin{array}{cc} \text{④} & \text{⑤} \end{array} \\ \begin{bmatrix} Y & -tY \\ -tY & t^2 Y \end{bmatrix} \end{array}$$

The augmented Y_{BUS} is:

$$\begin{array}{c|c} & \begin{array}{ccccc} \text{①} & \text{②} & \text{③} & \text{④} & \text{⑤} \end{array} \\ \begin{array}{c} \text{①} \\ \text{②} \\ \text{③} \\ \text{④} \\ \text{⑤} \end{array} & \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & 0 & 0 \\ Y_{21} & Y_{22} & 0 & Y_{24} & 0 \\ Y_{31} & 0 & Y_{33} & Y_{34} & 0 \\ 0 & Y_{42} & Y_{43} & Y_{44} + Y & -tY \\ 0 & 0 & 0 & -tY & t^2 Y \end{bmatrix} \end{array}$$

where $Y = -j50$ and $t = 1.05$. Some of the desired jacobian entries corresponding to bus 4 are:

$$\frac{\partial P_4}{\partial \delta_4} = \sum_{\substack{n=1 \\ n \neq 4}}^{5} |V_4 V_n Y_{4n}| \sin(\theta_{4n} + \delta_n - \delta_4)$$

$$= |V_4 V_2 Y_{42}| \sin(\theta_{42} + \delta_2 - \delta_4)$$
$$+ |V_4 V_3 Y_{43}| \sin(\theta_{43} + \delta_3 - \delta_4)$$
$$+ |V_4 V_5 Y_{45}| \sin(\theta_{45} + \delta_5 - \delta_4)$$

$$= |1.0 \times 1.0 \times 26.359696| \sin(101.30993°)$$
$$+ |1.0 \times 1.0 \times 15.417934| \sin(101.30993°)$$
$$+ |1.0 \times 1.02 \times 1.05 \times 50| \sin(90°)$$

$$= 94.516337$$

$$\frac{\partial P_4}{\partial \delta_5} = -|V_4 V_5 Y_{45}| \sin(\ \theta_{45} + \delta_5 - \delta_4)$$

$$= -|1 \times 1.02 \times 1.05 \times 50| \sin(90°)$$

$$= \ -53.55$$

$$\frac{\partial Q_4}{\partial \delta_2} = -|V_4 V_2 Y_{42}| \cos(\ \theta_{42} + \delta_2 - \delta_4)$$

$$= -|1 \times 1 \times 26.359696| \cos(101.30993°)$$

$$= \ 5.1695606$$

$$|V_2| \frac{\partial Q_4}{\partial |V_2|} = -|V_2 V_4 Y_{42}| \sin(\ \theta_{42} + \delta_2 - \delta_4)$$

$$= -|\ 1.0 \times 1.0 \times 26.359696| \sin(101.30993°)$$

$$= -25.847809$$

The remaining terms corresponding to bus 4 are calculated in a similar manner. The terms corresponding to bus 5 are:

$$\frac{\partial P_5}{\partial \delta_4} = -\ |V_5 V_4 Y_{54}| \sin(\ \theta_{54} + \delta_4 + \delta_5)$$

$$= -\ |\ 1.02 \times 1.0 \times 1.05 \times 50| \sin(90°)$$

$$= -53.55$$

$$\frac{\partial P_5}{\partial \delta_5} = \sum_{\substack{n=1 \\ n \neq 5}}^{5} |V_5 V_n Y_{5n}| \sin(\theta_{5n} + \delta_n - \delta_5)$$

$$= |1.02 \times 1.0 \times 1.05 \times 50| \sin(90^\circ)$$

$$= 53.55$$

$$|V_4| \frac{\partial P_5}{\partial |V_4|} = |V_5 V_4 Y_{54}| \cos(\theta_{54} + \delta_4 - \delta_5)$$

$$= |1.02 \times 1.0 \times 1.05 \times 50| \cos(90^\circ) = 0$$

Since bus 5 is a regulated bus, there are no terms involving Q_5 or $|V_5|$.

(9.13) For the system of Prob. 9.12, find the matrices **B'** and **B''** for use in the decoupled power-flow method.

Solution:

$\mathbf{Y}_{BUS}$ is:

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array}\begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & 0 & 0 \\ Y_{21} & Y_{22} & 0 & Y_{24} & 0 \\ Y_{31} & 0 & Y_{33} & Y_{34} & 0 \\ 0 & Y_{42} & Y_{43} & Y_{44}+Y & -tY \\ 0 & 0 & 0 & -tY & t^2Y \end{bmatrix} \end{array}$$

where $Y = -j50$ and the remaining entries are given in Table 9.4. The imaginary part of $\mathbf{Y}_{BUS}$ is called **B** and is given by:

$$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & 0 & 0 \\ B_{21} & B_{22} & 0 & B_{24} & 0 \\ B_{31} & 0 & B_{33} & B_{34} & 0 \\ 0 & B_{42} & B_{43} & B_{44} & B_{45} \\ 0 & 0 & 0 & B_{54} & B_{55} \end{bmatrix}$$

with:

$$B_{44} = -40.863838 - 50$$
$$B_{45} = 50t$$
$$B_{54} = 50t$$
$$B_{55} = -50t^2$$

and the remaining B_{ij}'s are obtained from Table 9.4.

Using $t = 1$, changing the signs, and deleting the first row and column, we get **B'**:

$$\mathbf{B}' = -\begin{bmatrix} B_{22} & 0 & B_{24} & 0 \\ 0 & B_{33} & B_{34} & 0 \\ B_{42} & B_{43} & B_{44} & B_{45} \\ 0 & 0 & B_{54} & B_{55} \end{bmatrix}$$

or

$$\begin{array}{c} \qquad\quad ② \qquad\qquad\qquad ③ \qquad\qquad\qquad ④ \qquad\qquad\qquad ⑤ \\ \mathbf{B}' = \begin{bmatrix} 44.835953 & 0 & -25.847809 & 0 \\ 0 & 40.863838 & -15.118528 & 0 \\ -25.847809 & -15.118528 & 90.863838 & -50 \\ 0 & 0 & -50 & 50 \end{bmatrix} \end{array}$$

B'' is obtained from **B** by deleting the rows and columns corresponding to the slack bus (bus 1) and the *PV* bus (bus 5), while t is set to 1.05 as specified:

$$\begin{array}{c} \qquad\quad ② \qquad\qquad\qquad ③ \qquad\qquad\qquad ④ \\ \mathbf{B}'' = \begin{bmatrix} 44.835953 & 0 & -25.847809 \\ 0 & 40.863838 & -15.118528 \\ -25.847809 & -15.118528 & 90.863838 \end{bmatrix} \end{array}$$

(9.14) A five-bus power system is shown in Fig. 9.14. The line, bus, transformer, and capacitor data are given in Tables 9.6, 9.7, 9.8, and 9.9, respectively. Use the Gauss-Seidel method to find the bus voltages for the first iteration.

Solution:
For t = 0.975:

$$-j25t = -j24.375$$
$$t(t-1)(-j25) = j0.609375$$
$$(1-t)(-j25) = -j0.625$$

And $\mathbf{Y}_{BUS}$ is:

$$\begin{bmatrix} 5-j24.947 & -2.5+j15 & 0 & -2.5+j10 & 0 \\ -2.5+j15 & 7.5-j59.932 & j24.375 & 0 & -5+j20 \\ 0 & j24.375 & 4-j39.546 & 0 & -4+j16 \\ -2.5+j10 & 0 & 0 & 6.5-j27.8 & -4+j18 \\ 0 & -5+j20 & -4+j16 & -4+j18 & 13-53.895 \end{bmatrix}$$

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_{2,sch} - jQ_{2,sch}}{V_2^{(0)*}} - (Y_{21}V_1^{(0)} + Y_{23}V_3^{(0)} + Y_{25}V_5^{(0)})\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6+j0.35}{1.0} - \{(-2.5+j15)(1.01) + (j24.375)(1.0) + (-5+j20)(1.0))\}\right]$$

$$= \frac{6.925 - j59.175}{7.5 - j59.932}$$

$$= 0.986382 - j0.007890$$

$$V_{2,acc}^{(1)} = 1 + 1.6\,(\,0.986382 - j0.007890 - 1)$$

$$= 0.978211 - j0.012624$$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_{3,sch} - jQ_{3,sch}}{V_3^{(0)*}} - (Y_{32}V_2^{(1)} + Y_{35}V_5^{(0)}\ )\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-0.7 + j0.42}{1.0} - \left\{\ (j24.375)(0.978211 - j0.012624)\right.\right.$$

$$\left.\left. + (-4 + j16)(1.0)\right\}\ \right]$$

$$= \frac{2.99229 - j39.423893}{4 - j39.546}$$

$$= 0.994392 - j0.024915$$

$$V_{3,acc}^{(1)} = 1 + 1.6\,(\,0.994392 - j0.024915 - 1)$$

$$= 0.991028 - j0.039864$$

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_{4,sch} - jQ_{4,sch}}{V_4^{(0)*}} - (Y_{41}V_1^{(1)} + Y_{45}V_5^{(0)}\ )\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.8 + j0.5}{1.0} - \{\ (-2.5 + j10)(1.01)\right.$$

$$\left. + (-4 + j18)(1.0)\}\ \right]$$

$$= \frac{5.725 - j27.6}{6.5 - j27.8}$$

$$= \ 0.9869984 - j0.0248378$$

$$V_{4,acc}^{(1)} = 1 + 1.6\ (\ 0.9869984 - j0.0248378 - 1)$$

$$= \ 0.979197 - j0.039740$$

$$Q_5^{(1)} = -\operatorname{Im}\left\{\ V_5^{(0)*}\left[Y_{52}V_2^{(1)} + Y_{53}V_3^{(1)} + Y_{54}V_4^{(1)} + Y_{55}V_5^{(0)}\right]\right\}$$

$$\begin{aligned} = -\operatorname{Im}\{\ (1.0) \times [\ (-5 + j\,20)(\ 0.978211 - j0.012624) \\ + (-4 + j\,16)(\ 0.991028 - j0.039864) \\ + (-4 + j\,18)(\ 0.979197 - j0.039740) \\ + (13 - j\,53.895)(\ 1.0)]\} \end{aligned}$$

$$= -\operatorname{Im}\{\ 1.833669 - j0.46725\} = 0.46725$$

$$V_5^{(1)} = \frac{1}{Y_{55}}\left[\frac{P_{5,\text{sch}} - jQ_5^{(1)}}{V_4^{(0)*}} - (Y_{52}V_2^{(1)} + Y_{53}V_3^{(1)} + Y_{54}V_4^{(1)}\ )\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{1.25 - j0.46725}{1.0} - \left\{\ (-11.166331 + j53.427750)(1.01)\right\}\ \right]$$

$$= \frac{12.527994 - j54.429278}{13 - j53.895}$$

$$= \ 1.007372 - j0.010536$$

and,

$$V_{5,\mathrm{corr}}^{(1)} = \frac{1.0}{1.007427}(1.007372 - j0.010536)$$
$$= 0.999903 - j0.010458$$

(9.15) To apply the Newton-Raphson method to the power-flow solution of the system of Fig. 9.14, determine (*a*) Y_{BUS} of the system, (*b*) the mismatch equation at bus 5 evaluated at the initial voltage estimates of Table 9.7 for the first iteration, and (*c*) write mismatch equations in a form similar to Eq. (9.45).

Solution:
(*a*) For t = 0.975:

$$-j25t = -j24.375$$
$$t(t-1)(-j25) = j0.609375$$
$$(1-t)(-j25) = -j0.625$$

And Y_{BUS} is:

$$\begin{bmatrix} 5-j24.947 & -2.5+j15 & 0 & -2.5+j10 & 0 \\ -2.5+j15 & 7.5-j59.932 & j24.375 & 0 & -5+j20 \\ 0 & j24.375 & 4-j39.546 & 0 & -4+j16 \\ -2.5+j10 & 0 & 0 & 6.5-j27.8 & -4+j18 \\ 0 & -5+j20 & -4+j16 & -4+j18 & 13-53.895 \end{bmatrix}$$

(*b*)

$$\frac{\partial P_5}{\partial \delta_2} = -|V_5 V_2 Y_{52}| \sin(\theta_{52} + \delta_2 - \delta_5)$$

$$= -20.616 \sin(104.04°) = -20$$

$$\frac{\partial P_5}{\partial \delta_3} = -|V_5 V_3 Y_{53}| \sin(\theta_{53} + \delta_3 - \delta_5)$$

$$= -16.492 \sin(104.04°) = -16$$

$$\frac{\partial P_5}{\partial \delta_4} = -|V_5 V_4 Y_{54}| \sin(\theta_{54} + \delta_4 - \delta_5)$$

$$= -18.439 \sin(102.53°) = -18$$

$$\frac{\partial P_5}{\partial \delta_5} = -\left(\frac{\partial P_5}{\partial \delta_2} + \frac{\partial P_5}{\partial \delta_3} + \frac{\partial P_5}{\partial \delta_4}\right)$$

$$= -\ (-20 - 16 - 18) = 54.0$$

$$|V_2| \frac{\partial P_5}{\partial |V_2|} = |V_5 V_2 Y_{52}| \cos(\theta_{52} + \delta_2 - \delta_5)$$

$$= 20.616 \cos(104.04°) = -5.0$$

$$|V_3| \frac{\partial P_5}{\partial |V_3|} = |V_5 V_3 Y_{53}| \cos(\theta_{53} + \delta_3 - \delta_5)$$

$$= 16.492 \cos(104.04°) = -4.0$$

$$|V_4|\frac{\partial P_5}{\partial |V_4|} = |V_5 V_4 Y_{54}| \cos(\theta_{54} + \delta_4 - \delta_5)$$

$$= 18.439 \cos(102.53°) = -4.0$$

$$P_{5,\text{calc}}^{(0)} = 1.0^2 \times 13 + 20.616 \cos(104.04) + 16.492 \cos(104.04) + 18.439 \cos(104.04)$$

$$= 13 - 5 - 4 - 4 = 0$$

$$\Delta P_5^{(0)} = P_{5,\text{sch}} - P_{5,\text{calc}}^{(0)}$$

$$= (1.9 - 0.65) - 0 = 1.25$$

The mismatch equation at bus 5 is:

$$-20\Delta\delta_2 - 16\Delta\delta_3 - 18\Delta\delta_4 + 54\Delta\delta_5 - 5\Delta|V_2| - 4\Delta|V_3| - 4\Delta|V_4| = 0$$

(c) For this system, we have:

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial \delta_5} & |V_2|\frac{\partial P_2}{\partial |V_2|} & |V_3|\frac{\partial P_2}{\partial |V_3|} & |V_4|\frac{\partial P_2}{\partial |V_4|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & & \cdots & |V_3|\frac{\partial P_3}{\partial |V_3|} & |V_4|\frac{\partial P_3}{\partial |V_4|} \\ \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \ddots & & & & |V_4|\frac{\partial P_4}{\partial |V_4|} \\ \frac{\partial P_5}{\partial \delta_2} & \frac{\partial P_5}{\partial \delta_3} & \frac{\partial P_5}{\partial \delta_4} & & & & |V_4|\frac{\partial P_5}{\partial |V_4|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_2} & \cdots & & \cdots & |V_3|\frac{\partial Q_2}{\partial |V_3|} & |V_4|\frac{\partial Q_2}{\partial |V_4|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_2} & \ddots & & & \cdots & |V_4|\frac{\partial Q_3}{\partial |V_4|} \\ \frac{\partial Q_4}{\partial \delta_2} & \cdots & & & & \cdots & |V_4|\frac{\partial Q_4}{\partial |V_4|} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \frac{\Delta|V_2|}{|V_2|} \\ \frac{\Delta|V_3|}{|V_3|} \\ \frac{\Delta|V_4|}{|V_4|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix}$$

(9.16) For the system of Fig. 9.14, find matrices **B'** and **B''** for use in the decoupled power-flow method. Also, determine the first iteration P and Q mismatch equations at bus 4, and find the voltage magnitude at bus 4 at the end of the first iteration.

Solution:

To form **B'**, the capacitors are neglected and transformer tap t is set equal to 1.

$$\begin{array}{c} \begin{array}{cccc} \quad (2) \quad & \quad (3) \quad & \quad (4) \quad & \quad (5) \quad \end{array} \\ \begin{bmatrix} 59.932 & -25 & 0 & -20 \\ -25 & 40.96 & 0 & -16 \\ 0 & 0 & 27.95 & -18 \\ -20 & -16 & -18 & 53.895 \end{bmatrix} \end{array}$$

where:

$$\begin{bmatrix} & & \\ & \mathbf{B}' & \\ & & \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \end{bmatrix} = \begin{bmatrix} \Delta P_2/|V_2| \\ \Delta P_3/|V_3| \\ \Delta P_4/|V_4| \\ \Delta P_5/|V_5| \end{bmatrix}$$

In the formation of **B"**, the capacitors and off-nominal tap settings are considered. Bus 5 is deleted since it is a regulated bus.

$$\begin{array}{c} \begin{array}{ccc} \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\ \begin{bmatrix} 59.932 & -24.375 & 0 \\ -24.375 & 39.546 & 0 \\ 0 & 0 & 27.8 \end{bmatrix} \end{array}$$

where:

$$\begin{bmatrix} & & \\ & \mathbf{B}'' & \\ & & \end{bmatrix} \begin{bmatrix} \Delta|V_2| \\ \Delta|V_3| \\ \Delta|V_4| \end{bmatrix} = \begin{bmatrix} \Delta Q_2/|V_2| \\ \Delta Q_3/|V_3| \\ \Delta Q_4/|V_4| \end{bmatrix}$$

$$P_{4,calc}^{(0)} = |V_4|^2 G_{44} + \sum_{\substack{n=1 \\ n\neq 4}}^{5} |V_4 V_n Y_{4n}| \cos(\theta_{4n} + \delta_n - \delta_4)$$

$$= 6.5 + 1 \times 1.01 \times 10.308 \cos(104.04°) + 18.439 \cos(102.53°)$$

$$= 6.5 - 1.01 \times 2.5 - 4 = -0.025$$

$$\Delta P_4^{(0)} = P_{4,sch} - P_{4,calc}^{(0)} = -0.8 - (-0.025)$$

$$= -0.775 \text{ p.u.}$$

$$Q_{4,\text{calc}}^{(0)} = -|V_4|^2 B_{44} - \sum_{\substack{n=1 \\ n\neq 4}}^{5} |V_4 V_n Y_{4n}| \sin(\theta_{4n} + \delta_n - \delta_4)$$

$$= -(-27.8) - 1 \times 1.01 \times 10.308 \sin(104.04°) - 18.439 \sin(102.53°)$$

$$= 27.8 - 1.01 \times 10 - 18 = -0.3$$

$$\Delta Q_4^{(0)} = Q_{4,\text{sch}} - Q_{4,\text{calc}}^{(0)} = -0.5 - (-0.3)$$

$$= -0.2 \text{ p.u.}$$

The P- and Q-equations at bus 4 are:

$$27.95\ \Delta\delta_4 - 18\Delta\delta_5 = -0.775/|V_4| = -0.775$$

$$27.8\ \Delta|V_4| = -0.2/|V_4| = -0.2$$

Therefore, the voltage magnitude at bus 4 after the first iteration is:

$$|V_4^{(1)}| = |V_4^{(0)}| + \Delta|V_4| = 1 + \frac{-0.2}{27.8} = 0.9928$$

(9.17) Suppose that in Fig. 9.14 the transformer between buses 2 and 3 is a phase shifter where t is now the complex variable and is $1.0\angle{-2°}$. (a) Find $\mathbf{Y}_{\text{BUS}}$ of this system, (b) When compared with the power-flow solution of Prob. 9.15, will the real power in the line from bus 5 to bus 3 increase or decrease? What about the reactive power flow? Explain why qualitatively.

Solution:

(a) The following is used in construction of Y_{BUS} where $t = 1.0\angle{-2^\circ}$ and $Y = 1/j0.04$:

$$\begin{array}{cc} \text{②} & \text{③} \\ \left[\begin{array}{cc} Y & -tY \\ -t^*Y & |t|^2 Y \end{array}\right] \end{array} = \begin{array}{cc} \text{②} & \text{③} \\ \left[\begin{array}{cc} -j25 & j25\angle{-2^\circ} \\ j25\angle{+2^\circ} & -j25 \end{array}\right] \end{array} = \begin{array}{cc} \text{②} & \text{③} \\ \left[\begin{array}{cc} -j25 & 0.873+j24.985 \\ -0.873+j24.985 & -j25 \end{array}\right] \end{array}$$

Y_{BUS} is then given by:

$$\left[\begin{array}{ccccc} 5-j24.947 & -2.5+j15 & 0 & -2.5+j10 & 0 \\ -2.5+j15 & 7.5-j59.932 & 0.873+j24.985 & 0 & -5+j20 \\ 0 & -0.873+j24.985 & 4-j40.78 & 0 & -4+j16 \\ -2.5+j10 & 0 & 0 & 6.5-j27.8 & -4+j18 \\ 0 & -5+j20 & -4+j16 & -4+j18 & 13-53.895 \end{array}\right]$$

Compare this with that obtained in Prob. 9.15.

(b) Because of the phase angle of -2°, more real power will flow from bus 2 to 3. Consequently, less real power will flow from bus 5 to bus 3. Since the magnitude of t is 1.0, the voltage magnitude at bus 3 will drop compared to that of Prob. 9.15. As a result, more reactive power will flow from bus 5 to bus 3.

(9.18) To apply the decoupled power-flow method to the system of Prob. 9.17, find matrices **B'** and **B"**.

Solution:
To form **B'**, only capacitors are to be neglected. Using the result of Y_{BUS} determined in Prob. 9.17, **B'** is:

$$\begin{array}{c} \begin{array}{cccc} \quad ② & \quad ③ & \quad ④ & \quad ⑤ \end{array} \\ \begin{bmatrix} 59.932 & -24.985 & 0 & -20 \\ -24.985 & 40.96 & 0 & -16 \\ 0 & 0 & 27.95 & -18 \\ -20 & -16 & -18 & 53.895 \end{bmatrix} \end{array}$$

In determining **B"**, the phase shifter angle is neglected and the rows and columns corresponding to regulated bus 5 are deleted. Thus, **B"** is given by:

$$\begin{array}{c} \begin{array}{ccc} \quad ② & \quad ③ & \quad ④ \end{array} \\ \begin{bmatrix} 59.932 & -25 & 0 \\ -25 & 40.78 & 0 \\ 0 & 0 & 27.8 \end{bmatrix} \end{array}$$

(9.19) Redo Example 9.10 when an 18-MVAR shunt capacitor bank is added to bus 3.

Solution:
The **B** matrix (the imaginary part of Y_{BUS}) is modified to account for the 18 MVAR capacitor. This is done by adding a 0.18 p.u. value to B_{33}. All other elements are unchanged.

Thus:

$$\begin{array}{cccc} & 2 & 3 & 4 \\ \mathbf{B} = & \left[\begin{array}{c} -44.835953 \\ 0 \\ 25.847809 \end{array}\right. & \begin{array}{c} 0 \\ -40.863838+0.18 \\ 15.118528 \end{array} & \left.\begin{array}{c} 25.847809 \\ 15.118528 \\ -40.863838 \end{array}\right] \end{array}$$

The capacitor is ignored in the P-equations, and so

$$\mathbf{B}' = \begin{bmatrix} 44.835953 & 0 & -25.847809 \\ 0 & 40.863838 & -15.118528 \\ -25.847809 & -15.118528 & 40.863838 \end{bmatrix}$$

Which is the same as in the example. Thus, the initial angle corrections for the first iteration will be the same as calculated in the example.

The matrix **B''** is:

$$\begin{array}{cc} & \begin{array}{cc} 2 & \quad 3 \end{array} \\ \mathbf{B}'' = & \begin{bmatrix} 44.835953 & 0 \\ 0 & 40.683838 \end{bmatrix} \end{array}$$

The Q-equations to be solved are:

$$\begin{bmatrix} 44.835953 & 0 \\ 0 & 40.683838 \end{bmatrix} \begin{bmatrix} \Delta|V_2| \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} \Delta Q_2/\Delta|V_2| \\ \Delta Q_3/\Delta|V_3| \end{bmatrix}$$

The value calculated for $\Delta Q_3/|V_3|$ must be modified to reflect the change in B_{33}. To do this, we add an additional term as follows:

$$\frac{\Delta Q_3}{\Delta|V_3|} = -1.27684 + \frac{1}{|1.0|} 1.0^2(0.18) = -1.09684$$

Solution of the Q-equations gives:

$\Delta|V_2| = -0.01793$ and $\Delta|V_3| = -0.02696$

And the new voltage magnitudes at buses 2 and 3 are:

$|V_2| = 0.98207$ and $|V_3| = 0.97304$

(9.20) In applying the Newton-Raphson method, if the amount of reactive power required to maintain the specified voltage at a *PV* bus exceeds the maximum limit of its reactive power generation capacity, the reactive power at that bus is set to that limit and the type of bus becomes a load bus. Suppose the maximum reactive power generation at bus 4 is limited to 150 MVAR in the system of Example 9.5. Using the first-iteration result given in Sec. 9.4 following Example 9.5, determine whether or not the type of bus 4 should be converted to a load bus at the start of the second iteration. If so, calculate the reactive power mismatch at bus 4 that should be used in the second-iteration mismatch equation.

Solution:

$$Q_{4,\mathrm{calc}}^{(0)} = -|V_4|^2 B_{44} - \sum_{\substack{n=1 \\ n\neq 4}}^{4} |V_4 V_n Y_{4n}| \sin(\theta_{4n} + \delta_n - \delta_4)$$

$$\begin{aligned}
&= -1.02^2(-40.863838) - 0 \\
&\quad - 1.02 \times 0.98335 \times 26.359696 \sin(101.30993 - 0.93094 - 1.54383°) \\
&\quad - 1.02 \times 0.97095 \times 15.417934 \sin(101.30993 - 1.78790 - 1.54383°) \\
&= 42.514737 - 26.125504 - 15.12165 \\
&= 1.2675831
\end{aligned}$$

When the reactive load of 49.58 MVAR is added to this reactive power requirement of 126.76 MVAR, the actual reactive power generation is 176.34 MVAR which exceeds the limit of 150 MVAR. Therefore, bus 4 should be considered to be a load bus. The net reactive power becomes:

150 MVAR − 49.58 = 100.42 MVAR = 1.0042 p.u.

Thus the scheduled reactive power at this bus is 1.0042 p.u. Since the calculated reactive power is 1.2676 p.u., the reactive power mismatch at bus 4 is 1.0042 - 1.2676 = -0.2634 p.u.