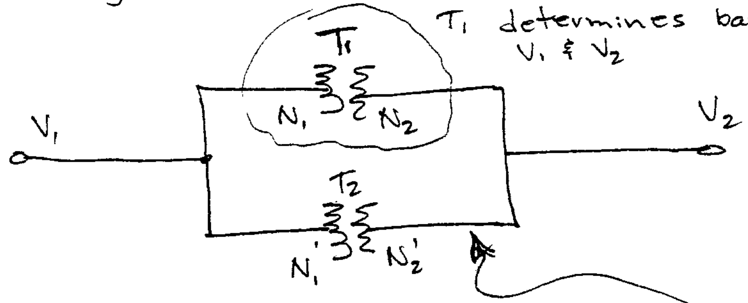
EE5200 - Study Prior to 9/28 Lecture 3

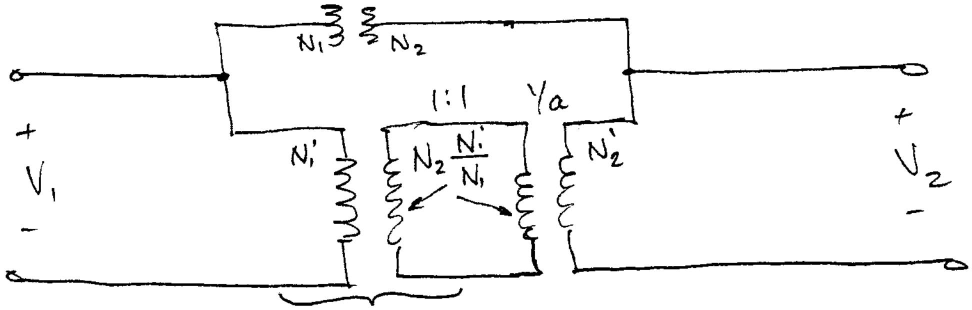
Paralleling Transformers of Unlike Turns Ratio



T1 determines base voltages V1 & V2

$$\frac{N_1}{N_2} \cong \frac{V_1}{V_2}$$

What happens for $\frac{N_1'}{N_2'} \neq \frac{N_1}{N_2}$?



SAME RATIO AS T1

Replace T_2 with 2 XFMRS : - First is same ratio as T_1 $\frac{N_1}{N_2} = \frac{N_1'}{X}$

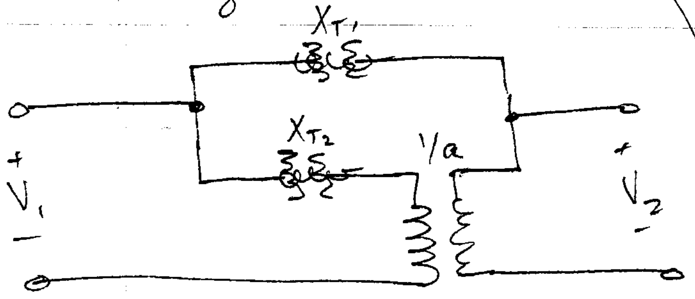
$$\boxed{X = N_2 \frac{N_1'}{N_1}}$$

Second XFMR has ratio of off-nominal turns

$$\frac{1}{a} = \frac{\left(N_2 \frac{N_1'}{N_1}\right)}{N_2'} = \frac{N_2 N_1'}{N_1 N_2'}$$

$$\therefore \boxed{a = \frac{N_1}{N_2} \frac{N_2'}{N_1'}} = \text{p.u. turns ratio}$$

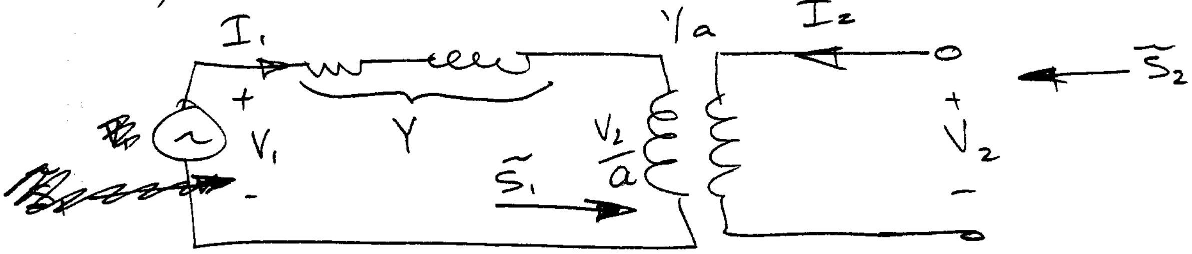
Per unit equivalent:



Three Methods to Analyze:
1) Admittance Method
2) Circuit Theory
3) Circulating Current Method (Approximate)

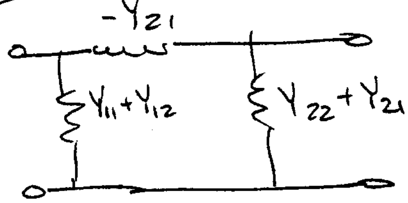
METHOD 1

So,



So we must find a way to model Y for Y_{12} as 1/a varies.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{12} V_1 + Y_{22} V_2$$



IN GENERAL

$$\tilde{S}_1 = -\tilde{S}_2$$

$$\tilde{S}_1 = \frac{V_2}{a} I_1^* \qquad S_2 = V_2 I_2^*$$

$$\frac{V_2}{a} I_1^* = -V_2 I_2^*$$

$$I_1^* = -a I_2^*$$

$$\boxed{I_1 = -a^* I_2}$$

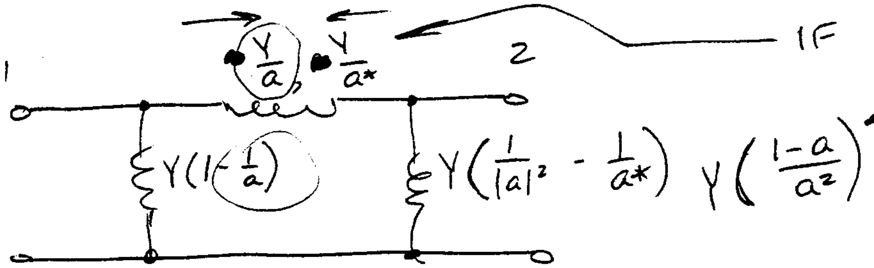
$$I_1 = \left(V_1 - \frac{V_2}{a}\right) Y = Y V_1 - \frac{Y}{a} V_2 = -a^* I_2$$

$\therefore$

$$I_2 = \frac{-I_1}{a^*} = \frac{-Y V_1}{a^*} + \frac{Y}{a a^*} V_2$$

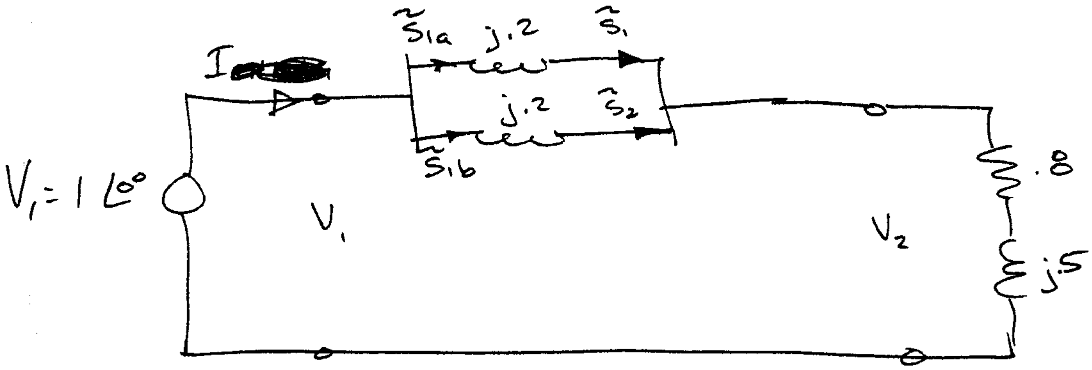
$\therefore$

$$\boxed{Y_{11} = Y \qquad Y_{12} = -\frac{Y}{a} \qquad Y_{21} = -\frac{Y}{a^*} \qquad Y_{22} = \frac{Y}{a a^*} = \frac{Y}{|a|^2}}$$



IF a is real

2) CIRCUIT THEORY APPROACH



$$V_2 = \frac{(.8+j.5)(1\angle 0°)}{.8+j.6} = .94-j.08 = .9434\angle -4.86°$$

$$I_2 = \frac{.94-j.08}{.8+j.5} = 1\angle -36.87°$$

$$\tilde{S} = VI^* = .9434\angle -4.86°(1\angle 36.87°) = .9434\angle 32° = .8+j.5$$

$$\left.\begin{array}{l} S_1 = .4+j.25 \\ S_2 = .4+j.25 \end{array}\right\} = \frac{S_{TOTAL}}{2}$$

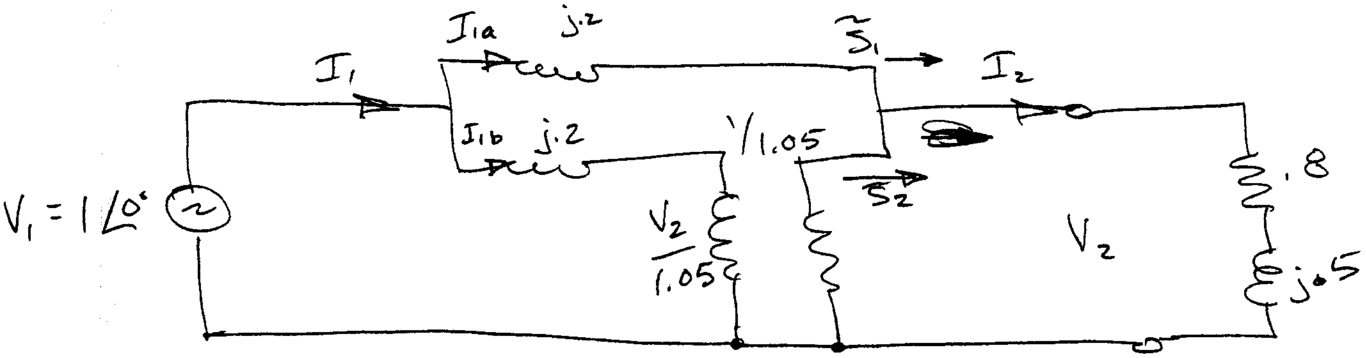
$$I_1 = \frac{1}{.8+j6} = 1(.8-j.6) = .8-j6$$

$$S_{1a} = \frac{V_1 I_1^*}{2} = 4+j.3$$

$$S_{1b} = \frac{V_1 I_1^*}{2} = .4+j.3$$

Difference due to XFMR Inductance

Now add tap changer $a = 1.05$



$$\begin{cases} I_1 = I_{1a} + I_{1b} \\ I_1 = \dfrac{1-V_2}{j.2} + \dfrac{1-\frac{V_2}{1.05}}{j.2} \\ I_2 = \dfrac{1-V_2}{j.2} + \dfrac{1-\frac{V_2}{1.05}}{j.2}\left(\dfrac{1}{1.05}\right) \\ V_2 = I_2(.8+j.5) = \left[\dfrac{1-V_2}{j.2} + \dfrac{1-\frac{V_2}{1.05}}{j.2}\left(\dfrac{1}{1.05}\right)\right](.8+j.5) \end{cases}$$

$$V_2 = \left(1-V_2+\frac{1-\frac{V_2}{1.05}}{1.05}\right)\left(\frac{.8+j.5}{j.2}\right) = (1.952 - 1.907V_2)\left(\overset{4.717}{}\angle\overset{-58°}{}\right)$$

~~[illegible] 32° = V_2([illegible] 32°)~~

$$V_2 = -1.907V_2(4.717\angle -58°) + (1.952)(4.717\angle -58°)$$
$$= -8.995\angle -58°\, V_2 + 9.207\angle -58°$$
$$V_2 = \frac{9.207\angle -58°}{1 + 8.995\angle -58°} = \frac{9.207\angle -58°}{9.563\angle -52.9} = \boxed{.963\angle -5.1°}$$

$$I_{1a} = \frac{1-V_2}{j.2} = \boxed{.4732\angle -25.54°} \qquad I_{1b} = \frac{1-\frac{V_2}{1.05}}{j.2} = \boxed{.59\angle -46.75°}$$

$$S_{1a} = V_1 I_{1a}^* = .427\cancel{3} + j.204$$
$$S_{1b} = V_1 I_{1b}^* = .4067 + j.4324$$

$$\tilde{S}_{1} = V_2 I_{1a}^* = (.963 \angle -5.0°)(.4732 \angle 25.536)$$

$$= .427 + j.16$$

$$\tilde{S}_2 = V_2 \frac{I_{1b}^*}{a} = .963 \angle -5.1° \left(\frac{.59 \angle -46.75}{1.05}\right)^* = .406 + j.3616$$

$$\tilde{S} = V_2 I_2^* = (.963 \angle 5.1)\left(\frac{V_2}{Z}\right)^* = .983 \angle 32° = \tilde{S}_1 + \tilde{S}_2$$

		Before TC		After T.C.		
				Circ. Approx.	Ckt. Analysis	
	V_1	1		1	1	
	V_2	$.943 \angle -4.86°$		$.963 \angle -4.87°$	$.963 \angle -5.1$	
at V_2	P_1	.4	at V_2	.398	‸4.27	at V_2
	r_2	.4		.418	.406	
	Q_1	.25		.135	.159	
	Q_2	.25		.375	.362	

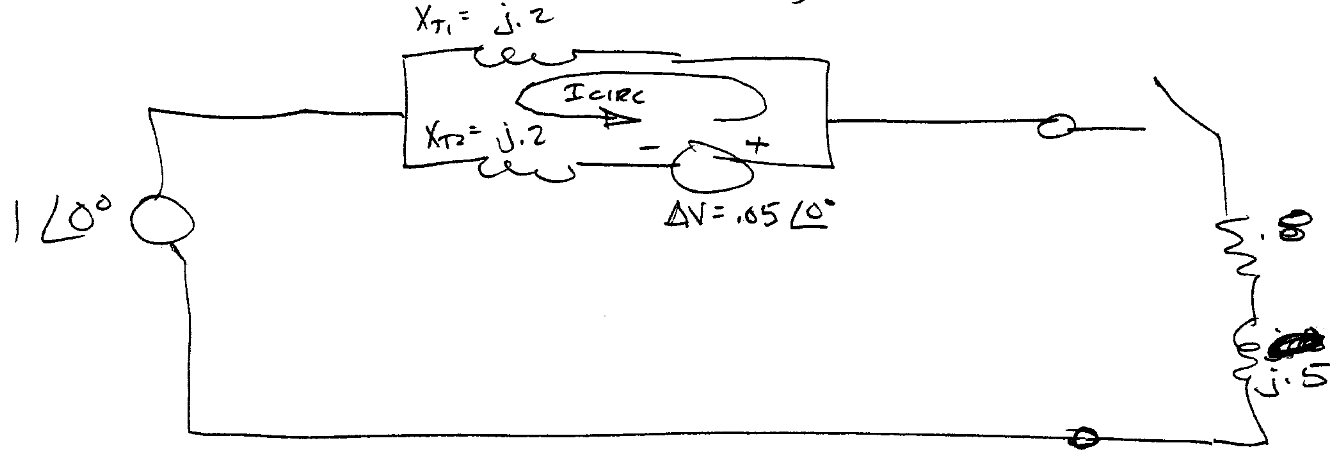
~~So Tap Changing XFMR shifted~~ ~~Q to XFMR with tap [illegible] to higher than rated secondary voltage.~~ ~~higher tap setting.~~

So:

Q is shifted to XFMR of higher tap setting.
P is still divided almost evenly.

Approximate solution:

Superposition of circulating current



First, look at XFMR before load applied.

$$I_{CIRC} = \frac{.05 \angle 0°}{j.4} = .125 \angle -90° = \boxed{-j.125}$$

From first example,

$$I_{1a} = .4 - j.3 \underbrace{+ j.125}_{I_{CIRC}} = \boxed{.4 - j.175}$$

$$I_{1b} = .4 - j.3 - j.125 = \boxed{.4 - j.425}$$

~~$S_{1a} = V_2 I_{1a}^*$~~ ~~$V_2 = V_1 - (I_{1a})(j.2)$~~

~~$= 1 - (.4 - j.125)(j.2) =$~~

~~$= 1 - (j.08 + .025) = .98 \angle -4.45$~~

By superposition,

$$V_2 = V_{2\ \text{BEFORE TAP CHANGE}} + \boxed{\underbrace{\frac{\Delta V (jX_{T_1})}{jX_{T_1} + jX_{T_2}}}_{\Delta V_{oc}} \left[\frac{Z_{LOAD}}{Z_{TOTAL}} \right]}$$

Voltage at Load due to ΔV

$$\Delta V_{oc} = \frac{\Delta V (jX_{T_1})}{(jX_{T_1} + jX_{T2})}$$

$$Z_{th} = jX_{T_1} \| jX_{T2}$$

$$= .943 \angle -4.86° + \frac{.05 (j.2)}{j.4} \left[\frac{.8 + j.5}{.8 + j.6} \right]$$

$$j.5 + (jX_{T_1} \| jX_{T_2})$$

$$\boxed{V_2 = .963 \angle -4.87°}$$

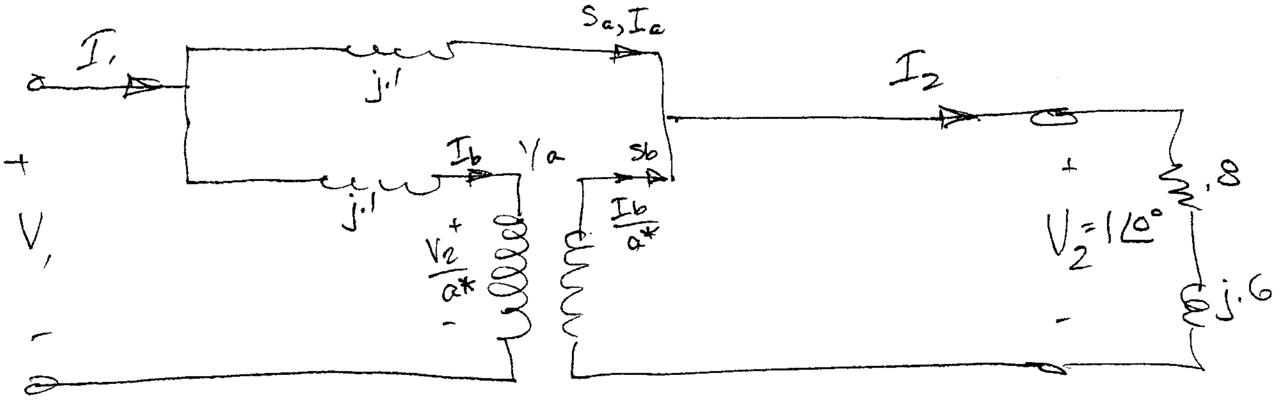
$S_{1a} = V_2 I_{1a}^* = (.963 \angle -4.87^\circ)(.437 \angle -23.63)^* = .421 \angle$ ~~-[illegible]~~ 18.76°

$= .$ ~~[illegible] + j~~ $= \boxed{.398 + j.135}$

$S_{2a} = V_2 I_{2a}^* = (.963 \angle -4.87^\circ)(.584 \angle 46.73) = .562 \angle 41.86$

$= \boxed{.418 + j.375}$

Phase ~~Changing~~ Shifting XFMR



$$a = 1 \angle 3^\circ = e^{j\pi/60}$$

Without phase shifter,

$$P_{LOAD} = \frac{V_2^2}{Z^*} = 1(.8+j.6) = .8+j.6$$

$S_a = .4 + j.3$

$S_b = .4 + j.3$

$I_a = .4 - j.3$

$I_b = .4 - j.3$

$V_1 = 1\angle 0^\circ + (.4+j.3)(j.1)$

$= 1.6307 \angle 2.2^\circ$

With Phase Shifter,

$$\frac{V_1 - 1}{j.1} + \frac{V_1 - \frac{1}{1\angle -3^\circ}}{j.1}\left(\frac{1}{1\angle -3^\circ}\right) = \frac{1}{.8+j.6} = I_2$$

$\frac{1}{a^*}$

$$V_1\left(1 + \frac{1}{1\angle -3^\circ}\right) - 1 - \left(\frac{1}{1\angle -3^\circ}\right)^2 = j.1/(.8+j.6)$$

$V_1(1.999 \angle 1.5^\circ)$ ~~$- 1 - (.9945 - j.1045)$~~ $=$ ~~$.1 \angle 53.13 + 1.9945 - j.1045$~~ .06 ~~+.08~~

~~$V_1 = 2.054 \angle -.8^\circ / 1.999 \angle 1.5^\circ = 1.028 \angle .82^\circ$~~

~~$= 1.028 + j.016$~~

$$\boxed{V_1 = 1.031 \angle .73^\circ = 1.031 + j.013}$$

~~$I_a = \frac{1.028 + j.016 - 1}{j.1} = .322 \angle -60.3^\circ = .159 - j.28$~~

$$I_a = \frac{1.031 + j.013 - 1}{j.1} = .131 - j.311$$

$$\frac{I_b}{a^*} = I_2 - I_a = .8 - j.6 - .131 + j.311 = \boxed{.669 - j.289}$$

$$S_a = .131 + j.311$$

$$S_b = .669 + j.289$$

↑ NOTE SHIFT IN POWER FLOW THRU XFMRS

	BEFORE	AFTER
I_a	$.4 + j.3$	$.131 - j.311$
I_b/a^*	$.4 - j.3$	$.669 - j.289$
P_a	$.4$	$.131$
P_b	$.4$	$.669$
Q_a	$+.3$	$.311$
Q_b	$+.3$	$.289$
V_1	$1.031 \angle 2.2^\circ$	$1.031 \angle .73^\circ$
V_2	$1.0 \angle 0^\circ$	$1.0 \angle 0^\circ$

if a is at positive angle, power flow through phase shifting XFMR increases