CHAPTER X

ALPHA, BETA, AND ZERO COMPONENTS OF THREE-PHASE SYSTEMS

It is pointed out in Chapter II that a set of three voltage or current vectors pertaining to the phases of a three-phase system can be replaced by any one of a number of different systems of component vectors. The symmetrical component system is one of such systems. The positive-plus-negative, positive-minus-negative, and zero-sequence system of components discussed in Chapter V is another. The present chapter deals with a third system of components, here called α, β, and 0 components.

With phase a as reference phase in a three-phase system, the α, β, and 0 components of current and voltage are defined as follows:

α components in phases b and c are equal; they are opposite in sign and of half the magnitude of the α component of phase a.

β components in phases b and c are equal in magnitude and opposite in sign; in phase a they are zero.

0 components are equal in the three phases.

α components of current flow into a three-phase circuit in phase a and return one-half in phase b and one-half in phase c. β components of current are circulating currents in phases b and c. 0 components are zero-sequence components taken over from symmetrical components without change except in notation; they are here written 0 components for brevity and also to indicate that they are to be used with α and β components.

Relations between Phase Currents and Voltages and Their α, β, and 0 Components. Referring to Chapter II, equations [1]–[3], let $V_1 = V_\alpha$, $V_2 = V_\beta$, $V_3 = V_0$. The constant coefficients required to express a set of three vectors V_a, V_b, V_c of a three-phase system in terms of their $\alpha, \beta, 0$ components are:

$1, \quad -\frac{1}{2}, \quad -\frac{1}{2}$ for α components

$0, \quad \frac{\sqrt{3}}{2}, \quad -\frac{\sqrt{3}}{2}$ for β components

$1, \quad 1, \quad 1$ for 0 components

A set of three voltage vectors V_a, V_b, and V_c are expressed in terms of their α, β, and 0 components by the equations

$$V_a = V_\alpha + V_0 \qquad [1]$$

$$V_b = -\tfrac{1}{2}V_\alpha + \frac{\sqrt{3}}{2} V_\beta + V_0 \qquad [2]$$

$$V_c = -\tfrac{1}{2}V_\alpha - \frac{\sqrt{3}}{2} V_\beta + V_0 \qquad [3]$$

Equations [1]–[3] satisfy the required condition that the determinant[1] made up of the coefficients is not zero.

The three voltage vectors V_α, V_β, and V_0 are expressed in terms of the vectors V_a, V_b, and V_c by solving the simultaneous equations [1]–[3]:

Subtracting one-half the sum of [2] and [3] from [1] and solving for V_α,

$$V_\alpha = \tfrac{2}{3}\left(V_a - \frac{V_b + V_c}{2}\right) \qquad [4]$$

Subtracting [3] from [2] and solving for V_β,

$$V_\beta = \frac{1}{\sqrt{3}}(V_b - V_c) \qquad [5]$$

Adding the three equations and solving for V_0,

$$V_0 = \tfrac{1}{3}(V_a + V_b + V_c) \qquad [6]$$

The corresponding current equations are

$$I_a = I_\alpha + I_0 \qquad [7]$$

$$I_b = -\tfrac{1}{2}I_\alpha + \frac{\sqrt{3}}{2} I_\beta + I_0 \qquad [8]$$

$$I_c = -\tfrac{1}{2}I_\alpha - \frac{\sqrt{3}}{2} I_\beta + I_0 \qquad [9]$$

$$I_\alpha = \tfrac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) \qquad [10]$$

$$I_\beta = \frac{1}{\sqrt{3}}(I_b - I_c) \qquad [11]$$

$$I_0 = \tfrac{1}{3}(I_a + I_b + I_c) \qquad [12]$$

Equations [1]–[3] and [7]–[9] express any set of three voltage or current vectors, respectively, pertaining to the three phases of a three-phase system in terms of their α, β, and 0 components. Equations [4]–[6] and [10]–[12] express α, β, and 0 components in terms of phase voltages and currents, respectively. Equations [1]–[12] are analogous to the fundamental symmetrical component equations developed in Chapter II.

Line-to-Line Voltages. If V_a, V_b, and V_c in [1]–[3] represent the phase voltages to ground at a system point, the line-to-line voltages at the same point in terms of the α, β, 0 components of phase voltages to ground are

$$
\begin{aligned}
V_{ba} &= V_a - V_b = \tfrac{3}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta \\
V_{ac} &= V_c - V_a = -\tfrac{3}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta \qquad [13] \\
V_{cb} &= V_b - V_c = \sqrt{3}V_\beta
\end{aligned}
$$

If V_α and V_β are expressed in per unit of base line-to-neutral voltage, the line-to-line voltages will also be in per unit of base line-to-neutral voltage.

History of α, β, 0 Components. α, β, 0 components of current are not new. Components of current answering to the description of α, β, and 0, although not so named, were used in a method[2] developed by Dr. W. W. Lewis, and published in 1917, to determine system currents and voltages during line-to-ground faults. In Fig. 2 of the paper, which is similar to Fig. 1 of this chapter, phase currents are represented by arrows both in direction and magnitude, the number of arrows indicating relative magnitudes of currents in each circuit. Applying the definitions given above for α, β, and 0 currents to Fig. 1, it may be seen that all three components of current are present. Currents in the Y–Δ transformer bank and in the line to the right of the fault are 0 currents; currents in the transmission line to the left of the fault are α currents; currents in the second Y–Δ transformer bank and in the line at the generator terminals are β currents; currents in the generator are α currents. In the method as used before symmetrical components were applied to unsymmetrical short circuits, each component of phase current met its respective impedance, but calculations were made with phase voltages and currents, not with *component networks*, and therefore were time consuming if many circuits operated at different voltages had to be considered.

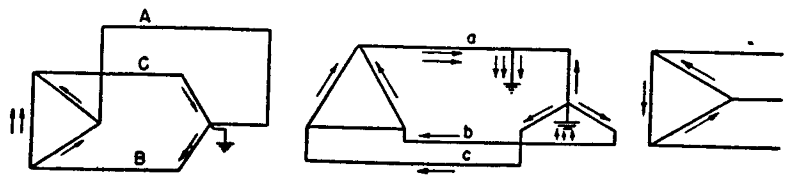

FIG. 1. Phase currents represented by arrows in direction and magnitude, number of arrows showing relative magnitudes of currents in any circuit.

In problems involving unsymmetrical three-phase circuits, and in particular circuits with two of the phases symmetrical with respect to the third phase, the use of components of current which flow in one phase and divide equally between the other two phases, and components of current which circulate in two phases, is a logical development. Such components, as yet unnamed, were used[3] in 1931; in 1938 they appeared under the names of α and β components in two papers,[4,5] both of which deal with transient conditions in rotating machines where the development is materially simplified by their use. Two papers have been devoted exclusively to these components. In one paper,[6] they are called α, β, and 0 components and the system *Modified Symmetrical Components*. In the other paper,[7] entitled "Two-Phase Co-ordinates of a Three-Phase System," by Dr. E. W. Kimbark, the components are called x, y, and z. Comparing these two sets: x and α components are identical; y and β components differ only in sign; z components of voltage are 0 (zero-sequence) components of voltage, z components of currents are twice 0 (zero-sequence) components of currents, and z impedances are one-half 0 (zero-sequence) impedances. At present, definitions and notation for the components (here called α, β, 0) are not definitely established[8] by usage. The choice of the sign for β or y components is arbitrary. The use of z components as defined in reference 7 has advantages which will be pointed out later. On the other hand, the familiar zero-sequence network, modified as required before interconnecting the component networks to satisfy unsymmetrical system conditions, is of advantage in analytic calculations. This is illustrated in Chapter V, Figs. 1 and 3, where two different modifications of the zero-sequence network are made. Dr. Kimbark's paper[7] and the discussions[8] by Messrs. Boyajian, Helwith, and Sligant in terms of matrix and tensor concepts should be read for a comprehensive view of these important components.

α, β, AND 0 ONE-LINE DIAGRAMS

When components of phase currents and voltages instead of phase quantities are used in calculations, each set of components is conveniently represented by a separate one-line diagram or component network from which the components of current and voltage in the three phases can be obtained. To draw component networks it is necessary to determine: (1) references for the components of voltage, (2) components of generated voltage, and (3) the impedances offered to the components of current, or the admittances associated with the components of voltage. Of interest also are the components of current present in a symmetrical system during normal operating conditions.

Reference for α, β, and 0 Voltages. The neutral of a Y-connected circuit is common to the three phases: therefore, in the limit as the neutral is approached, $V_a = V_b = V_c$. From [1]–[3], this condition is satisfied if $V_\alpha = 0$ and $V_\beta = 0$. All neutral points are therefore points of zero potential in the α and β networks, and α and β voltages are referred to neutral. As all neutrals are at zero potential in the α and β networks, the expressions "voltage to neutral" and "voltage to ground" can be used interchangeably for α and β voltages just as they are used interchangeably for positive- and negative-sequence voltages. 0 voltages at any point in a grounded system will be referred to ground at that point. In an ungrounded system with a neutral conductor, they will be referred to the neutral conductor.

Generated α, β, and 0 Voltages. In a synchronous machine with generated voltages E_a, E_b, and E_c, the generated α, β, and 0 voltages E_α, E_β, and E_0 obtained by substituting E_a, E_b, and E_c for V_a, V_b, and V_c, respectively, in [4]–[6] are

$$\begin{aligned} E_\alpha &= \tfrac{2}{3}\left(E_a - \frac{E_b + E_c}{2}\right) \\ E_\beta &= \frac{1}{\sqrt{3}}(E_b - E_c) \\ E_0 &= \tfrac{1}{3}(E_a + E_b + E_c) \end{aligned} \qquad [14]$$

If the generated voltages are balanced, $E_b = a^2E_a$, $E_c = aE_a$, and [14] becomes

$$\begin{aligned} E_\alpha &= E_a \\ E_\beta &= -jE_a \\ E_0 &= 0 \end{aligned} \qquad [15]$$

With balanced generated voltages in a synchronous machine, the generated voltage in the α network is E_a, the generated voltage of phase a. In the β network it is $-jE_a$, the generated voltage of phase a turned backward 90°. There is no generated voltage in the 0 network.

α and β Currents in a Balanced System. In a symmetrical system operating under balanced conditions, the currents in phases b and c at any point of the system are $I_b = a^2I_a$, $I_c = aI_a$. Substituting these values for I_b and I_c in [10]–[12],

$$\begin{aligned} I_\alpha &= I_a \\ I_\beta &= -jI_a \\ I_0 &= 0 \end{aligned} \qquad [16]$$

Equations [15] and [16] show that generated voltages and load currents are present in both the α and β networks of a symmetrical system during normal operation. Because two networks must be considered instead of one, α, β, and 0 components are not as convenient as symmetrical components for the study of symmetrical systems during normal operation or during three-phase faults.

α, β, and 0 Networks. Figure 2(*a*) shows a symmetrical three-phase system with balanced applied voltages and equal self-impedances Z in the three phases. I_α, flowing in phase a and returning one-half in each of phases b and c, flows in a loop circuit. The voltage applied to this loop, as shown in Fig. 2(*a*), is $E_a - (-E_a/2) = \tfrac{3}{2}E_a$. The α loop impedance for a symmetrical three-phase circuit of equal self-impedance Z in the three phases is $\tfrac{3}{2}Z$. The current I_α in phase a is

$$I_\alpha = \frac{\tfrac{3}{2}E_a}{\tfrac{3}{2}Z} = \frac{E_a}{Z}$$

The impedance met by I_α is Z. The equivalent circuit for phase a in the α system is shown in Fig. 2(*b*), with the applied voltage E_a and the self-impedance Z. In this equivalent circuit, voltages are referred to neutral, base voltage is line-to-neutral voltage, and base current is line current. Since the α currents and voltages in phases b and c at any point in the system are $-\tfrac{1}{2}$ those of phase a at the same point, it is unnecessary to have additional equivalent circuits for these phases. The equivalent circuit for phase a in the α system will be called the *α network*.

β currents, flowing in phase b and returning in phase c, flow in a loop circuit. The voltage applied to this loop, as shown in Fig. 2(*a*), is $-j\sqrt{3}E_a$. The β loop impedance for the symmetrical three-phase circuit of equal self-impedances Z in the three phases is $2Z$. The β current flowing in phase b in the direction indicated by arrow is $(\sqrt{3}/2)I_\beta$. Therefore

$$(\sqrt{3}/2)I_\beta = -j(\sqrt{3}E_a/2Z), \quad \text{and} \quad I_\beta = -j(E_a/Z)$$

The impedance met by I_β is Z. The equivalent circuit for the β system is shown in Fig. 2(*c*), with the applied voltage $-jE_a$ and the self-impedance Z. In this equivalent circuit, which will be called the *β network*, voltages are referred to neutral, base voltage is line-to-neutral voltage, and base current is line current. The β voltages and currents in phases b and c are the voltages and currents in the β network multiplied by $\sqrt{3}/2$ and $-\sqrt{3}/2$, respectively. *The β network does not give directly the β voltages and currents in either phase b or phase c.*

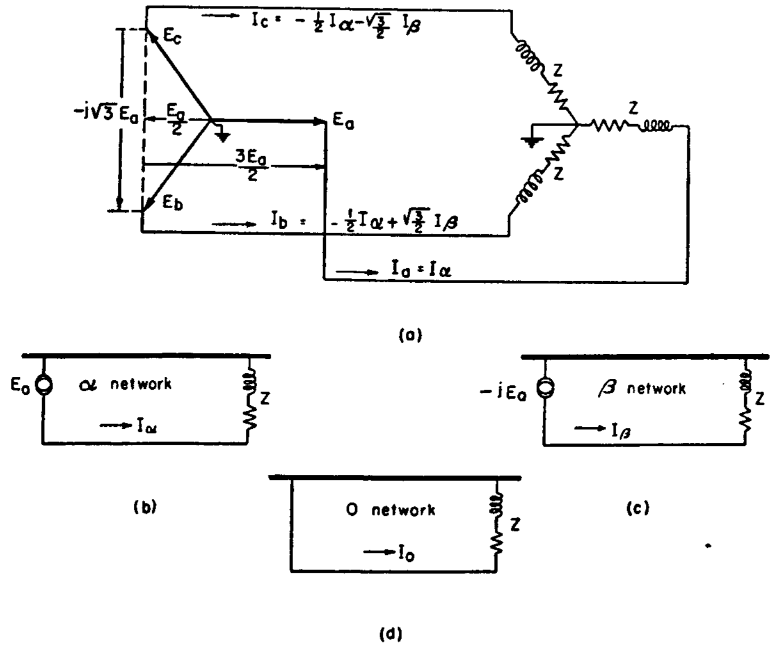

FIG. 2(*a*). Flow of α and β currents in balanced system with equal self-impedances in the three phases and balanced applied voltages. (*b*) α network for system shown in (*a*). (*c*) β network for system shown in (*a*). (*d*) 0 network for system shown in (*a*).

This slight disadvantage is more than offset by the convenience of having the same line-to-neutral voltage and line current as base quantities in the β as in the α and 0 networks.

With a path for 0 currents through the circuit of equal self-impedances Z in the three phases, the impedance met by I_0 is Z. The 0 network for the system of Fig. 2(*a*) is shown in Fig. 2(*d*).

α, β, and 0 equivalent circuits to replace the various equipment, machines, and transmission circuits of a three-phase power system in the α, β, and 0 networks can be determined when the α, β, and 0 self- and mutual impedances of the circuits are known. α, β, and 0 impedances, just as positive-, negative-, and zero-sequence impedances, can be obtained by calculation or test. Before developing equivalent circuits for use in the α, β, and 0 networks, relations between symmetrical components and α, β, and 0 components will be established.

α, β, and 0 Components of Voltage and Current in Terms of Symmetrical Components of Voltage and Current. From [1]–[3] and [7]–[9] of this chapter and [1]–[6] of Chapter V,

$$\begin{aligned} V_\alpha &= V_{a1} + V_{a2} \\ V_\beta &= -j(V_{a1} - V_{a2}) \\ V_0 &= V_{a0} \\ I_\alpha &= I_{a1} + I_{a2} \\ I_\beta &= -j(I_{a1} - I_{a2}) \\ I_0 &= I_{a0} \end{aligned} \qquad [17]$$

From [17], α components are positive-plus-negative components, β components are positive-minus-negative components turned backward 90°.

Symmetrical Components of Voltage and Current in Terms of α, β, and 0 Components of Voltage and Current. Solving the simultaneous voltage and current equations of [17],

$$\begin{aligned} V_{a1} &= \tfrac{1}{2}(V_\alpha + jV_\beta) \\ V_{a2} &= \tfrac{1}{2}(V_\alpha - jV_\beta) \\ V_{a0} &= V_0 \\ I_{a1} &= \tfrac{1}{2}(I_\alpha + jI_\beta) \\ I_{a2} &= \tfrac{1}{2}(I_\alpha - jI_\beta) \\ I_{a0} &= I_0 \end{aligned} \qquad [18]$$

α, β, and 0 Self- and Mutual Impedances

In [4]–[6] of Chapter VIII, the symmetrical components of voltage drop in an unsymmetrical three-phase series circuit without internal voltages are expressed in terms of the symmetrical components of current flowing in the circuit and the self- and mutual impedances of the sequence networks. Equations for the α, β, and 0 voltage drops in terms of the α, β, and 0 currents flowing in the circuit and the α, β, and 0 self- and mutual impedances of the circuit likewise will be written. In these equations, as in the corresponding symmetrical component equations, the effects of saturation are neglected and linear relations between currents and voltages assumed.

Finite 0 Self-Impedance. Let V_α, V_β, V_0 and V'_α, V'_β, V'_0 be the α, β, 0 components of voltage to ground at P and Q, respectively, and I_α, I_β, and I_0 the components of line current flowing from P to Q. Then

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + I_0 Z_{\alpha 0} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + I_0 Z_{\beta 0} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + I_0 Z_{00} \end{aligned} \qquad [19]$$

where $Z_{\alpha\alpha}$, $Z_{\beta\beta}$, and Z_{00} are the α, β, and 0 self-impedances, respectively, of the circuit. The Z's with two unlike subscripts represent mutual impedances, the first subscript referring to the voltage and the second to the current associated with the impedance. $Z_{\alpha\alpha}$ is the ratio of the voltage drop in the α network produced by I_α to I_α; $Z_{\alpha\beta}$ is the ratio of the voltage drop in the α network produced by I_β to I_β; $Z_{\beta\alpha}$ is the ratio of the voltage drop in the β network produced by I_α to I_α, etc. If $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha 0} = Z_{0\alpha}$, and $Z_{\beta 0} = Z_{0\beta}$, the mutual impedances between the component networks are reciprocal. If the mutual impedances between any two networks are zero, there is no mutual coupling between these networks.

Infinite 0 Self-Impedance. If there is no path for zero-sequence currents through the circuit, $I_0 = 0$, and no voltages are induced in the α and β networks by I_0. $I_0 Z_{\alpha 0}$ and $I_0 Z_{\beta 0}$ are zero, $Z_{00} = \infty$, and [19] becomes

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + 0 \cdot \infty \text{ (indeterminate)} \end{aligned} \qquad [20]$$

The voltage drop v_0 between P and Q is indeterminate from [20] but can be evaluated when the 0 impedance diagram and operating conditions are known. (See [36]–[39] for an evaluation of v_0.)

Equations [19] and [20], written for a series circuit between P and Q, can be applied to a circuit connected at one point of the system. If currents flow *out of the circuit*, the components of *series voltage drop* in the circuit between ground (or neutral) and terminals in the direction of current flow are given by [19] or [20]. If currents flow *into a circuit* without internal voltages, the components of voltage to ground or to neutral *at the circuit terminals* are given by the right-hand sides of equations [19] or [20].

The α, β, and 0 self- and mutual impedances in [19] and [20] will be expressed in terms of the self- and mutual impedances of the sequence networks, and also in terms of the self- and mutual impedances of the phases.

α, β, and 0 Self- and Mutual Impedances in Terms of the Sequence Impedances. Replacing I_{a1}, I_{a2}, and I_{a0} in [4]–[6] of Chapter VIII by their values in terms of I_α, I_β, and I_0 given by [18], v_{a1}, v_{a2}, and v_{a0} are expressed in terms of I_α, I_β, and I_0. Substituting these equations for v_{a1}, v_{a2}, and v_{a0} in [17], v_α, v_β, and v_0 are expressed in terms of I_α, I_β, I_0. Equating the coefficients of I_α, I_β, and I_0 in the resultant equations for v_α, v_β, and v_0 to the corresponding coefficients in [19], the α, β, and 0 self- and mutual impedances are

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{1}{2}(Z_{11} + Z_{22} + Z_{21} + Z_{12}) \\ Z_{\beta\beta} &= \tfrac{1}{2}(Z_{11} + Z_{22} - Z_{12} - Z_{21}) \\ Z_{00} &= Z_{00} \\ Z_{\beta\alpha} &= -j\tfrac{1}{2}(Z_{11} - Z_{22} + Z_{12} - Z_{21}) \\ Z_{\alpha\beta} &= j\tfrac{1}{2}(Z_{11} - Z_{22} + Z_{21} - Z_{12}) \\ Z_{0\alpha} &= \tfrac{1}{2}(Z_{01} + Z_{02}) \\ Z_{\alpha 0} &= (Z_{10} + Z_{20}) \\ Z_{0\beta} &= j\tfrac{1}{2}(Z_{01} - Z_{02}) \\ Z_{\beta 0} &= -j(Z_{10} - Z_{20}) \end{aligned} \qquad [21]$$

Self- and Mutual Impedances of the Sequence Networks in Terms of α, β, 0 Self- and Mutual Impedances. Proceeding in a manner analogous to that used to determine [21] or by solving [21] for Z_{11}, Z_{22}, etc.:

$$\begin{aligned} Z_{11} &= \tfrac{1}{2}[Z_{\alpha\alpha} + Z_{\beta\beta} - j(Z_{\alpha\beta} - Z_{\beta\alpha})] \\ Z_{22} &= \tfrac{1}{2}[Z_{\alpha\alpha} + Z_{\beta\beta} + j(Z_{\alpha\beta} - Z_{\beta\alpha})] \\ Z_{00} &= Z_{00} \\ Z_{12} &= \tfrac{1}{2}[Z_{\alpha\alpha} - Z_{\beta\beta} + j(Z_{\alpha\beta} + Z_{\beta\alpha})] \\ Z_{21} &= \tfrac{1}{2}[Z_{\alpha\alpha} - Z_{\beta\beta} - j(Z_{\alpha\beta} + Z_{\beta\alpha})] \\ Z_{10} &= \tfrac{1}{2}(Z_{\alpha 0} + jZ_{\beta 0}) \\ Z_{01} &= (Z_{0\alpha} - jZ_{0\beta}) \\ Z_{20} &= \tfrac{1}{2}(Z_{\alpha 0} - jZ_{\beta 0}) \\ Z_{02} &= (Z_{0\alpha} + jZ_{0\beta}) \end{aligned} \qquad [22]$$

Equations [17] and [21] can be used to pass from symmetrical components to α, β, 0 components. Equations [18] and [22] give symmetrical components in terms of α, β, 0 components. If a problem is to be solved by α, β, 0 components and the sequence self- and mutual impedances are known, the α, β, 0 self- and mutual impedances are obtained by substituting the sequence impedances in [21]. If the problem is to be solved by symmetrical components, but the α, β, 0 self- and mutual impedances of an unsymmetrical circuit are more readily obtained than the sequence self- and mutual impedances, [22] will be found useful.

Symmetrical Circuit with Equal Positive- and Negative-Sequence Impedances. In a circuit with equal positive- and negative-sequence self-impedances and no mutual impedances between the sequence networks, the α, β, and 0 self- and mutual impedances obtained from [21] are

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = Z_{11} = Z_1 \\ Z_{00} &= Z_{00} = Z_0 \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \end{aligned} \quad [23]$$

When there are no mutual impedances between the sequence networks, the positive-, negative-, and zero-sequence self-impedances are customarily indicated by Z_1, Z_2, and Z_0, respectively, instead of Z_{11}, Z_{22}, and Z_{00}.

Unsymmetrical Static Circuits. If $Z_{11} = Z_{22}$, $Z_{10} = Z_{02}$, and $Z_{20} = Z_{01}$, [21] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{11} + \tfrac{1}{2}(Z_{21} + Z_{12}) \\ Z_{\beta\beta} &= Z_{11} - \tfrac{1}{2}(Z_{21} + Z_{12}) \\ Z_{00} &= Z_{00} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = j\tfrac{1}{2}(Z_{21} - Z_{12}) \\ Z_{\alpha 0} &= 2Z_{0\alpha} = (Z_{10} + Z_{20}) \\ Z_{\beta 0} &= 2Z_{0\beta} = -j(Z_{10} - Z_{20}) \end{aligned} \quad [24]$$

If $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$, and $Z_{10} = Z_{02} = Z_{20} = Z_{01}$, [21] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{11} + Z_{12} \\ Z_{\beta\beta} &= Z_{11} - Z_{12} \\ Z_{00} &= Z_{00} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = 2Z_{10} \end{aligned} \quad [25]$$

If the sequence self- and mutual impedances of the unsymmetrical static circuits developed in Chapter VIII are substituted in [21], [24], or [25], their α, β, and 0 self- and mutual impedances will be obtained.

Modified 0 Network. In circuits in which $Z_{0\alpha} = \tfrac{1}{2}Z_{\alpha 0}$ and $Z_{0\beta} = \tfrac{1}{2}Z_{\beta 0}$, equations [19] are conveniently expressed in terms of a *modified* 0 network in which the voltage is 0 voltage, the current is $2I_0$, and the impedance is one-half the 0 impedance. Rewriting [19] in terms of $2I_0$,

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + (2I_0)\frac{Z_{\alpha 0}}{2} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + (2I_0)\frac{Z_{\beta 0}}{2} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + (2I_0)\frac{Z_{00}}{2} \end{aligned} \quad [26]$$

Equations [26] can be used instead of [19] in cases where $\frac{Z_{\alpha 0}}{2} = Z_{0\alpha}$ and $\frac{Z_{\beta 0}}{2} = Z_{0\beta}$, thereby giving reciprocal mutual coupling between the α (or β) network and a modified 0 network in which the current is $2I_0$, the impedances are one-half 0 impedances, and the voltages are 0 voltages. This is the **z** network used in reference 7.

An alternate modification of the 0 network is obtained by rewriting [19] in terms of $2v_0$. Retaining the equations for v_α and v_β in [19] and multiplying the equation for v_0 by 2,

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + I_0 Z_{\alpha 0} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + I_0 Z_{\beta 0} \\ 2v_0 &= 2(V_0 - V'_0) = I_\alpha(2Z_{0\alpha}) + I_\beta(2Z_{0\beta}) + I_0(2Z_{00}) \end{aligned} \quad [27]$$

Equations [27] apply to a modified 0 network in which currents are 0 currents, voltages are twice 0 voltages, and impedances are twice 0 impedances.

In unsymmetrical circuits in which $2Z_{0\alpha} = Z_{\alpha 0}$ and $2Z_{0\beta} = Z_{\beta 0}$, either of the modified 0 networks defined in [26] and [27] is mutually coupled with the α and β networks. Where either of the modified networks can be used equally well, the one corresponding to the **z** network[7] will be chosen in the work which follows.

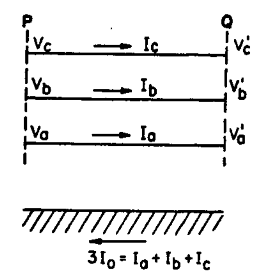

FIG. 3. Unsymmetrical series circuit between P and Q.

α, β, 0 Self- and Mutual Impedances in Terms of the Phase Impedances. Let Fig. 3 represent a general three-phase static circuit composed of bilateral circuit elements without internal voltages between points P and Q, with a return path for 0 currents. With phase voltages at P and Q referred to ground or to a neutral conductor at P and Q, respectively, the voltage drops

v_a, v_b, and v_c in phases a, b, and c in the direction of current flow are

$$\begin{aligned} v_a &= V_a - V_a' = I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} \\ v_b &= V_b - V_b' = I_aZ_{ab} + I_bZ_{bb} + I_cZ_{bc} \\ v_c &= V_c - V_c' = I_aZ_{ac} + I_bZ_{bc} + I_cZ_{cc} \end{aligned} \qquad [28]$$

Equations [28] are general equations expressing phase voltage drops in terms of phase currents after all other currents in the circuit have been eliminated. For example, in a three-phase transmission circuit with a neutral conductor or ground wires, Z_{aa}, $Z_{ab} = Z_{ba}$, etc., may include the effects of neutral conductor or ground wires.

Replacing I_a, I_b, and I_c in [28] by their α, β, 0 components given by [7]–[9], v_a, v_b, and v_c are expressed in terms of I_α, I_β, and I_0. Substituting these equations for v_a, v_b, and v_c in [4]–[6], v_α, v_β, and v_0 are expressed in terms of I_α, I_β, I_0. Equating the coefficients of I_α, I_β, and I_0 in these resultant equations for v_α, v_β, and v_0 to the corresponding coefficients in [19], the α, β, 0 self- and mutual impedances in terms of phase impedances are

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left[Z_{aa} + \frac{Z_{bb} + Z_{cc}}{4} - \left(Z_{ab} + Z_{ac} - \frac{Z_{bc}}{2}\right)\right] \\ Z_{\beta\beta} &= \tfrac{1}{2}(Z_{bb} + Z_{cc} - 2Z_{bc}) \\ Z_{00} &= \tfrac{1}{3}[Z_{aa} + Z_{bb} + Z_{cc} + 2(Z_{ab} + Z_{ac} + Z_{bc})] \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = \frac{1}{2\sqrt{3}}[Z_{cc} - Z_{bb} + 2(Z_{ab} - Z_{ac})] \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{1}{3}[2Z_{aa} - Z_{bb} - Z_{cc} + (Z_{ab} + Z_{ac} - 2Z_{bc})] \\ Z_{\beta 0} &= 2Z_{0\beta} = \frac{1}{\sqrt{3}}(Z_{bb} - Z_{cc} + Z_{ab} - Z_{ac}) \end{aligned} \qquad [29]$$

Two Phases with Equal Self-Impedances and Equal Mutual Impedances with the Third Phase. Let $Z_{bb} = Z_{cc}$ and $Z_{ac} = Z_{ab}$. Equations [29] then become

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left[Z_{aa} + \frac{Z_{bb}}{2} - \left(2Z_{ab} - \frac{Z_{bc}}{2}\right)\right] \\ Z_{\beta\beta} &= \tfrac{1}{2}(Z_{bb} + Z_{cc} - 2Z_{bc}) \\ Z_{00} &= \tfrac{1}{3}[Z_{aa} + 2Z_{bb} + 2(2Z_{ab} + Z_{bc})] \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{2}{3}[Z_{aa} - Z_{bb} + (Z_{ab} - Z_{bc})] \end{aligned} \qquad [30]$$

Symmetrical Circuit. With all self-impedances equal to Z_{aa} and all mutual impedances equal to Z_{ab}, [29] or [30] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = Z_{aa} - Z_{ab} \\ Z_{00} &= Z_{aa} + 2Z_{ab} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = Z_{\alpha 0} = Z_{0\alpha} = 0 \end{aligned} \qquad [31]$$

Unsymmetrical Three-Phase Self-Impedance Circuit with Finite 0 Self-Impedance. In a three-phase series circuit between P and Q, let the self-impedances of phases a, b, and c be Z_a, Z_b, and Z_c, respectively, with no mutual impedance between phases. The α, β, and 0 self- and mutual impedances in terms of the phase impedances can be obtained by replacing Z_{aa}, Z_{bb}, and Z_{cc} in [29] by Z_a, Z_b, and Z_c, respectively, and equating all mutual impedances between phases to zero. Then,

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left(Z_a + \frac{Z_b + Z_c}{4}\right) \\ Z_{\beta\beta} &= \frac{Z_b + Z_c}{2} \\ Z_{00} &= \frac{Z_a + Z_b + Z_c}{3} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = \frac{Z_c - Z_b}{2\sqrt{3}} \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \frac{2Z_a - Z_b - Z_c}{3} = 2(Z_a - Z_{\alpha\alpha}) \\ Z_{\beta 0} &= 2Z_{0\beta} = \frac{Z_b - Z_c}{\sqrt{3}} \end{aligned} \qquad [32]$$

Two Phases with Equal Self-Impedances. Let $Z_b = Z_c$, then [32] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left(Z_a + \frac{Z_b}{2}\right) \\ Z_{\beta\beta} &= Z_b \\ Z_{00} &= \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{2}{3}(Z_a - Z_b) = 2(Z_a - Z_{\alpha\alpha}) \end{aligned} \qquad [33]$$

Symmetrical Self-Impedance Circuit. Let $Z_a = Z_b = Z_c = Z$. From [32] or [33],

$$Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{00} = Z \quad [34]$$

and there are no mutual couplings between the α, β, and 0 networks. Figure 2(*a*), used to illustrate the flow of α and β currents and to determine α and β impedances from the α and β loop circuits, is a symmetrical self-impedance circuit.

Z_{00} *Infinite.* If there is no path for 0 currents in the unsymmetrical circuit, the α and β components of voltage drop in the circuit are given by [20]. The α and β self-impedances and the mutual impedances between the α and β networks are not affected by the presence or absence of 0 currents, nor are the mutual impedances $Z_{0\alpha}$ and $Z_{0\beta}$. The 0 voltage drops $I_\alpha Z_{0\alpha}$ and $I_\beta Z_{0\beta}$ caused by I_α and I_β flowing through an unsymmetrical circuit can be determined by calculation, if required, after I_α and I_β are known.

With $Z_{00} = \infty$, if two of the phases have equal self-impedances and equal mutual impedances (including no mutual impedance) with the other phase, substituting [30] or [33] in [20],

$$V_\alpha = I_\alpha Z_{\alpha\alpha}$$
$$V_\beta = I_\beta Z_{\beta\beta} \quad [35]$$

This case is of special interest as there is no mutual coupling between the α, β, and 0 networks because of the unsymmetrical circuit.

Unsymmetrical Y-Connected Static Circuits. The α, β, and 0 self- and mutual impedances given in terms of the phase impedances by [29]–[34] apply to an unsymmetrical Y-connected circuit with grounded neutral, if I_α, I_β, and I_0 in [19] are the components of currents flowing into the circuit and v_α, v_β, and v_0 the components of voltages to neutral at the circuit terminals. Z_{00} in these equations is the 0 self-impedance between circuit terminals and neutral. If the neutral is grounded through Z_n, and Z_{00} is replaced by $Z_{00} + 3Z_n$, v_0 will be referred to ground.

If the neutral is ungrounded, equations [20] apply. The voltage at the neutral of the ungrounded Y may be evaluated as follows: α and β currents flowing in the unsymmetrical circuit produce a voltage drop v_0 between circuit terminals T and neutral N, where

$$v_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} \quad [36]$$

If the 0 voltage at T is $V_{0(T)}$, the voltage V_N at N is

$$V_N = V_{0(T)} - v_0 = V_{0(T)} - I_\alpha Z_{0\alpha} - I_\beta Z_{0\beta} \quad [37]$$

$V_{0(T)}$ can be evaluated when the 0 impedance diagram and operating conditions are given. If the 0 voltage at T is zero, the voltage at the neutral is

$$V_N = -I_\alpha Z_{0\alpha} - I_\beta Z_{0\beta} \quad [38]$$

where $Z_{0\alpha}$ and $Z_{0\beta}$ are defined in [29]–[33].

If $Z_b = Z_c$; $Z_{0\beta} = 0$. Then, if $V_{0(T)} = 0$,

$$V_N = -I_\alpha Z_{0\alpha} \quad [39]$$

α and β Self-Impedances from α and β Loop Impedances. I_α flows in phase a and one-half I_α flows in phase b and one-half in phase c. The one-line α impedance diagram for determining α currents and voltages in phase a of the system is also the impedance diagram for phases b or c because $-\frac{1}{2}I_\alpha$ flows in phases b and c and the applied α voltage in these phases is $-\frac{1}{2}E_\alpha$. The α self-impedance $Z_{\alpha\alpha}$ is therefore two-thirds the α loop impedance, regardless of the type of circuit. I_β, flowing in phases b and c in series, meets an impedance which is twice the β impedance. The β self-impedance is therefore one-half the β loop impedance. In certain unsymmetrical circuits, $Z_{\alpha\alpha}$ and $Z_{\beta\beta}$ are more readily determined from the α and β loop impedances than from the phase impedances. In a circuit in which the self-impedances of phases b and c are equal and α currents induce no voltages in the β loop and β currents induce no voltages in the α loop, the α loop impedance may be determined by connecting the three phases at one terminal and applying a voltage at the other terminal between phase a and phases b and c connected to a common point; and the β loop impedance by applying a voltage between phases b and c with phase a open. In either case, the loop impedances are the ratios of the applied voltages to the resultant α and β currents, respectively, flowing in the α and β loops. Then

$$Z_{\alpha\alpha} = \tfrac{2}{3}(\alpha \text{ loop impedance}) \quad [40]$$

$$Z_{\beta\beta} = \tfrac{1}{2}(\beta \text{ loop impedance}) \quad [41]$$

In determining the α loop impedance for an unsymmetrical self-impedance static circuit in which the self-impedances are unequal in phases b and c, it should be noted (see [32]) that the α loop impedance is the sum of Z_a and $(Z_b + Z_c)/4$. Z_b and Z_c are *not* paralleled.

α and β Applied Voltages Determined from α and β Loop Voltages. Equations [14] give α, β, 0 voltages in terms of the applied phase voltages. In unsymmetrical circuits involving transformers, the voltages applied to the α and β loops may be more readily obtained than the phase voltages. In such cases, the applied α voltage is two-thirds the voltage applied to the α loop; the applied β voltage is $1/\sqrt{3}$ times the voltage applied to the β loop.

EQUIVALENT CIRCUITS TO REPLACE AN ACTUAL CIRCUIT IN THE α, β, AND 0 NETWORKS

Synchronous Machine with Equal Positive- and Negative-Sequence Impedances. From [23], the α and β self-impedances are equal to Z_1, and the 0 self-impedance to Z_0. There are no mutual impedances between the α, β, and 0 networks. With balanced generated voltages in the machine, the generated voltage in the α network from [15] is E_a; in the β network it is $-jE_a$. The α, β, and 0 equivalent circuits for a synchronous machine with balanced generated voltages and equal positive- and negative-sequence impedances are shown in Fig. 4.

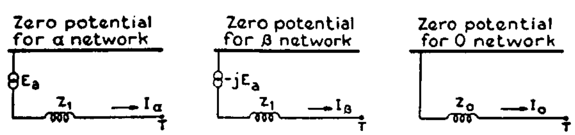

FIG. 4. α, β, and 0 equivalent circuits for a synchronous machine with balanced generated voltages and equal positive- and negative-sequence impedances.

Points T are the terminals of the machine to which the equivalent α, β, and 0 circuits for the rest of the system are to be connected.

In a three-phase power system consisting of symmetrical circuits with equal positive- and negative-sequence impedances, the one-line impedance diagrams for the α and β systems are the same as the positive-sequence impedance diagram. Generated α voltages are positive-sequence generated voltages; the α network is the same as the positive-sequence network. The β network differs from the positive-sequence network only in its generated voltages, which are positive-sequence voltages multiplied by $-j$.

Symmetrical Circuit with Unequal Positive- and Negative-Sequence Impedances. From [21] with $Z_1 \neq Z_2$ and all sequence mutual impedances zero,

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = \tfrac{1}{2}(Z_1 + Z_2) \\ Z_{00} &= Z_0 \\ Z_{\alpha\beta} &= -Z_{\beta\alpha} = j\tfrac{1}{2}(Z_1 - Z_2) \\ Z_{\alpha 0} &= Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \end{aligned} \qquad [42]$$

When the positive- and negative-sequence self-impedances of a circuit are unequal and there are no sequence mutual impedances, the α and β self-impedances are the average of the positive- and negative-sequence impedances. There is no mutual coupling with the 0 network; but the α and β networks are coupled through *non-reciprocal* mutual impedances. Because of this non-reciprocal coupling between the α and β networks in rotating machines in which $Z_1 \neq Z_2$, the α, β, 0 components are not convenient for determining fundamental-frequency currents and voltages in systems in which the positive- and negative-sequence impedances cannot be assumed equal. However, if there is but one machine or group of machines in which $Z_1 \neq Z_2$, [19] can be rewritten to give a reciprocal mutual coupling between the α network and a *modified* β network, from which an equivalent circuit can be obtained.

Modified β Network. Substituting $Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$ from [42] in [19], the α, β, and 0 components of voltage drop in the circuit in the direction of current flow are

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} = I_\alpha \tfrac{1}{2}(Z_1 + Z_2) + jI_\beta \tfrac{1}{2}(Z_1 - Z_2) \\ v_\beta &= I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} = -jI_\alpha \tfrac{1}{2}(Z_1 - Z_2) + I_\beta \tfrac{1}{2}(Z_1 + Z_2) \\ v_0 &= I_0 Z_0 \end{aligned} \qquad [43]$$

Rewriting v_α and v_β in [43] in terms of $(-I_\beta)$, with $-Z_{\alpha\beta}$ replaced by $Z_{\beta\alpha}$,

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + (-I_\beta)(-Z_{\alpha\beta}) = I_\alpha Z_{\alpha\alpha} + (-I_\beta)Z_{\beta\alpha} \\ &= I_\alpha(Z_{\alpha\alpha} - Z_{\beta\alpha}) + (I_\alpha - I_\beta)Z_{\beta\alpha} \\ v_\beta &= I_\alpha Z_{\beta\alpha} + (-I_\beta)(-Z_{\beta\beta}) \\ &= (-I_\beta)(-Z_{\beta\beta} - Z_{\beta\alpha}) + (I_\alpha - I_\beta)Z_{\beta\alpha} \end{aligned} \qquad [44]$$

Retaining the equation for v_α in [43] but rewriting that for v_β,

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} = I_\alpha(Z_{\alpha\alpha} - Z_{\alpha\beta}) + (I_\alpha + I_\beta)Z_{\alpha\beta} \\ -v_\beta &= I_\alpha(-Z_{\beta\alpha}) + I_\beta(-Z_{\beta\beta}) = I_\alpha Z_{\alpha\beta} + I_\beta(-Z_{\beta\beta}) \\ &= I_\beta(-Z_{\beta\beta} - Z_{\alpha\beta}) + (I_\alpha + I_\beta)Z_{\alpha\beta} \end{aligned} \qquad [45]$$

In [44] and [45] the mutual impedances between the α network and a modified β network are reciprocal. In [44], v_β has been retained, but I_β has been replaced by $(-I_\beta)$ flowing in the direction assumed as positive for I_β. In [45], I_β has been retained but v_β has been replaced by $-v_\beta$ measured in the direction of v_β.

Equivalent Circuits for a Synchronous Machine with $Z_1 \neq Z_2$. Figures 5(a) and (b) give equivalent circuits to replace a synchronous machine with balanced generated voltages in the α and β networks when $Z_1 \neq Z_2$. The α and β components of balanced generated voltage are given by [15]. Figures 5(a) and (b) satisfy the equations for the α and β components of voltage drop in the direction of current flow given by [44] and [45], respectively, where $Z_{\alpha\alpha}$, $Z_{\beta\beta}$, $Z_{\alpha\beta}$, and $Z_{\beta\alpha}$ are defined in [42]. In both equivalent circuits, currents and voltages in

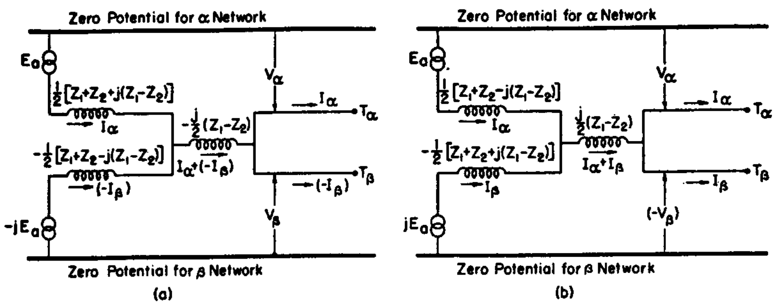

FIG. 5. Equivalent circuits to replace a synchronous machine with $Z_1 \neq Z_2$ in the α and modified β network. T_α and T_β are the machine terminals to which the α and β networks for the rest of the system are to be connected after all β impedances have been multiplied by -1. (a) Currents in the β network are negative β currents; voltages are β voltages. (b) Voltages in the β network are negative β voltages; currents are β currents.

the α network are correctly represented. In Fig. 5(a) and equations [44], current in the β network is $-I_\beta$, giving a modified β network in which voltages are β voltages. In Fig. 5(b) and equations [45], voltages in the β network are negative β voltages, giving a modified β network in which currents are β currents. The generated voltage in the modified β network of Fig. 5(b) becomes $-(-jE_a) = jE_a$, as indicated. The points T_α and T_β are the terminals of the synchronous machine to which the α and β networks, respectively, for the system exclusive of the synchronous machine are to be connected, after all impedances in the β network have been multiplied by -1. If the impedances in the β network include resistances, negative resistances will be present in the network; capacitive reactances will become inductive reactances, and vice versa. The modification of the β network presents no difficulties in an analytic solution.

The equivalent circuit for the synchronous machine with $Z_1 \neq Z_2$ in the 0 network is the same as that given in Fig. 4.

As α, β, and 0 components will be used instead of symmetrical components only if calculations are simplified by their use, the equivalent synchronous machine circuits of Figs. 5(a) and (b), which are much less simple than the symmetrical component equivalent circuits, have but limited application. They can sometimes be used to advantage in solutions of special problems involving unsymmetrical circuits and unsymmetrical faults, where the conditions of the problem do not require that the β network be coupled with the 0 network, or have a second coupling with the α network.

In approximate solutions, where the mutual impedance $\pm j(Z_1 - Z_2)$ is omitted, $\frac{1}{2}(Z_1 + Z_2)$ should be used as the self-impedance in both the α and β networks.

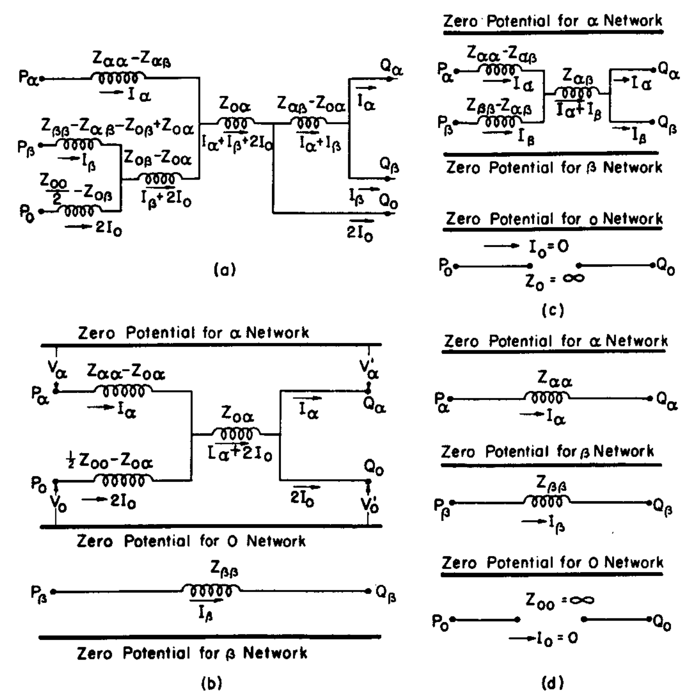

FIG. 6. Equivalent circuits for unsymmetrical series circuit in which (a) $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha 0} = 2Z_{0\alpha}$, $Z_{\beta 0} = 2Z_{0\beta}$. ($b$) $Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$, and $Z_{\alpha 0} = 2Z_{0\alpha}$. ($c$) $Z_{00} = \infty$, $Z_{\alpha\beta} = Z_{\beta\alpha}$. ($d$) $Z_{00} = \infty$, $Z_{\alpha\beta} = Z_{\beta\alpha} = 0$.

Equivalent Circuits for Unsymmetrical Three-Phase Static Circuits. In [29] and [32], $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha 0} = 2Z_{0\alpha}$, and $Z_{\beta 0} = 2Z_{0\beta}$. An equivalent circuit which satisfies equations [26] for the general case of unequal self- and mutual impedances (including no mutual impedances) is shown in Fig. 6(a). P and Q with subscripts α, β, and 0 indicate the terminals of the equivalent circuit in the α, β, and 0 networks, respectively, to which the equivalent α, β, 0 networks for the rest of the system are to be connected after all 0 impedances have been divided by 2. Figure 6(*a*) is tested for correct self-impedances in each network by opening both of the other networks at P or Q. It is tested for correct mutual impedance between any two networks by opening the third network at P or Q. When an a-c network analyzer is available, mutual coupling between the networks can be obtained by either mutual coupling transformers or direct connections as in Fig. 6(*a*).

For the special case of two phases symmetrical with respect to the third phase, the α, β, 0 self- and mutual impedances are given by [30] and [33]. In these equations there is no mutual coupling between the β network and either the α or 0 networks. The equivalent circuits for this case are shown in Fig. 6(*b*).

With $Z_{00} = \infty$, the zero network is open between P and Q so that no 0 current flows into or out of the circuit at either P or Q. The equivalent circuits for this case can be determined from Figs. 6(*a*) and (*b*) by opening the 0 network at P or Q, giving the equivalent circuits shown in Figs. 6(*c*) and (*d*), respectively.

Comparing equations [29], [30], [32], and [33], with equations [13], [17], [18], and [19], respectively, of Chapter VIII, it may be seen that non-reciprocal mutual impedances between the symmetrical component networks because of unsymmetrical static circuits become reciprocal mutual impedances between the α, β, 0 networks; while reciprocal mutual impedances between symmetrical component networks become zero between the β network and both the α and 0 networks. This may also be seen from equations [24] and [25].

Δ-CONNECTED CIRCUITS

Line Currents and Line-to-Neutral Voltages on Opposite Sides of a Δ–Y Transformer Bank. The difference in phase of positive-sequence line-to-neutral voltages on opposite sides of the bank at no load can be determined by inspection when the connection diagram is given. (See Chapter III, Fig. 19.) If positive-sequence components of line current and voltage to neutral are turned forward 90° and negative-sequence components of current and voltage backward 90° in passing through the bank,

$$\begin{aligned} V_\alpha &= (V_{a1} + V_{a2}) \text{ becomes } j(V_{a1} - V_{a2}) = -V'_\beta \\ V_\beta &= -j(V_{a1} - V_{a2}) \text{ becomes } (V_{a1} + V_{a2}) = V'_\alpha \end{aligned} \qquad [46]$$

If the shift in phase of positive-sequence components is backward 90°

and that of negative-sequence components forward 90°,

$$\begin{aligned} V_\alpha &= (V_{a1} + V_{a2}) \text{ becomes } -j(V_{a1} - V_{a2}) = V'_\beta \\ V_\beta &= -j(V_{a1} - V_{a2}) \text{ becomes } -(V_{a1} + V_{a2}) = -V'_\alpha \end{aligned} \qquad [47]$$

Replacing V by I in [46] and [47] the corresponding current equations are obtained. When the connection diagram is not given, it is immaterial whether [46] or [47] is used when the *relative phases* of currents and voltages on the two sides of the bank are not required. (See Chapter III, Problem 6.) In system studies in which the positive- and negative-sequence impedances of rotating machines can be assumed equal, the α and β impedance diagrams of the system, exclusive of unsymmetrical circuits, are the same as the positive-sequence impedance diagram. The shift of components from β to α and α to β presents no difficulty where the *same impedances* are met by α and β currents. Dissymmetries on opposite sides of a Δ–Y transformer bank will be discussed later.

Voltages and Currents of Δ-Connected Circuits. As the fundamental-frequency currents or voltages in the three phases of a Δ constitute a set of three vectors, they can be replaced by their α, β, 0 components. There will be no 0 components of line-to-line voltage, but there may be 0 components of Δ current. The three phases of the Δ will be indicated by A, B, and C, with A, B, and C opposite terminals a, b, and c, respectively, as in Fig. 7. Base Δ voltage is $\sqrt{3}$ times base line-to-neutral voltage, base Δ current is $1/\sqrt{3}$ times base line current, and base Δ impedance is 3 times base line-to-neutral impedance. In per unit of base line-to-line voltage, the Δ phase voltages in terms of their components are

$$\begin{aligned} V_A &= V'_\alpha \\ V_B &= -\tfrac{1}{2}V'_\alpha + \frac{\sqrt{3}}{2}V'_\beta \\ V_C &= -\tfrac{1}{2}V'_\alpha - \frac{\sqrt{3}}{2}V'_\beta \end{aligned} \qquad [48]$$

where V'_α and V'_β indicate components of line-to-line voltage in per unit of base line-to-line voltage. Using a similar notation for components of Δ current, the current equations are of the form of [7]–[9].

Relations between per Unit α and β Components of Current and Voltage in the Line and in the Δ. Line-to-line voltages are given by [13] in terms of α and β components of line-to-neutral voltage. Dividing equations [13] by $\sqrt{3}$ and replacing V_{cb} by V_A, V_{ac} by V_B, and

V_{ba} by V_C, in per unit of base line-to-line voltage,

$$\begin{aligned} V_A &= V_{cb} = V_\beta \\ V_B &= V_{ac} = -\frac{\sqrt{3}}{2} V_\alpha - \tfrac{1}{2} V_\beta \\ V_C &= V_{ba} = \frac{\sqrt{3}}{2} V_\alpha - \tfrac{1}{2} V_\beta \end{aligned} \qquad [49]$$

Comparing α and β components of line-to-neutral and line-to-line voltages, by eliminating V_A, V_B, and V_C in [48] and [49],

$$\begin{aligned} V'_\alpha &= V_\beta \\ V'_\beta &= -V_\alpha \end{aligned} \qquad [50]$$

where V'_α and V'_β are in per unit of base line-to-line voltage, and V_α and V_β in per unit of base line-to-neutral voltage.

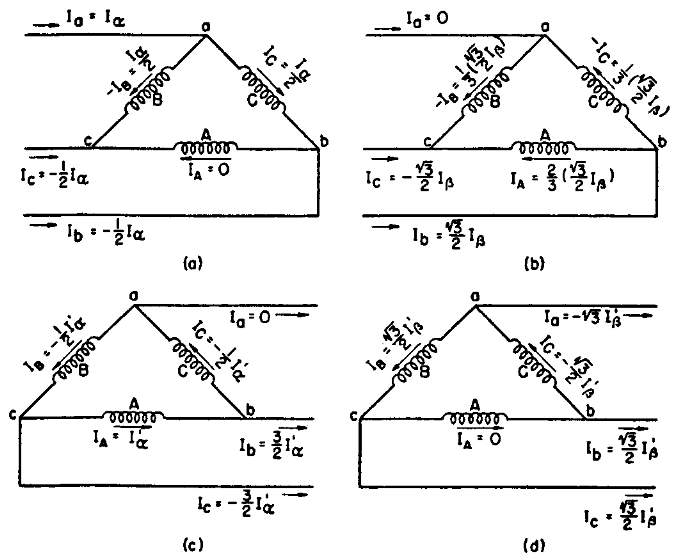

FIG. 7. (a) α components of line currents flowing into Δ circuit. (b) β components of line currents flowing into Δ circuit. (c) α components of Δ currents flowing into the line. (d) β components of Δ currents flowing into the line. In (a) and (b), base current is line current; in (c) and (d), base current is Δ current.

In Figs. 7(a) and (b), currents in per unit of *base line current* are shown in the line and in the Δ. In Fig. 7(a) there are only α components (I_α), in Fig. 7(b) only β components (I_β). In Figs. 7(c) and (d), currents in per unit of *base Δ current* are shown in the Δ and in the line.

In Fig. 7(c) there are only α components (I'_α); in Fig. 7(d) only β components (I'_β). Let line-to-line voltages in the Δ measured in the directions *cb*, *ba*, and *ac* represent voltage rises, the direction *cba* around the Δ corresponding to increasing potentials. Following the convention that positive direction for currents in a circuit is in the direction of increasing potential when currents flow *from the circuit*, and in the direction of decreasing potential when currents flow *towards a circuit*, a minus sign before a phase current in the Δ in Figs. 7(a) or (b) means that the direction indicated by arrow is negative for the convention used.

Multiplying Δ currents in per unit of base line-to-neutral current by $\sqrt{3}$ to express them in per unit of base Δ current, I'_β in the Δ of Fig. 7(a) is

$$I'_\beta = \frac{1}{\sqrt{3}}(I_B - I_C) = \frac{1}{\sqrt{3}}\left[-\frac{\sqrt{3}}{2} I_\alpha - \frac{\sqrt{3}}{2} I_\alpha\right] = -I_\alpha$$

Similarly, in Fig. 7(b),

$$I'_\alpha = \tfrac{2}{3}\left(I_A - \frac{I_B + I_C}{2}\right) = \tfrac{2}{3}\sqrt{3}\left(\frac{\sqrt{3}}{2} I_\beta\right) = I_\beta$$

Multiplying line currents given in per unit of base line-to-line current in Figs. 7(c) and (d) by $1/\sqrt{3}$ to express them in per unit of base line current,

$$I_\beta = \frac{1}{\sqrt{3}}(I_b - I_c) = \tfrac{1}{3}(3I'_\alpha) = I'_\alpha$$

$$I_\alpha = \tfrac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) = \tfrac{2}{3} \times \frac{1}{\sqrt{3}}(-\tfrac{3}{2}\sqrt{3}I'_\beta) = -I'_\beta$$

Expressed in per unit each on their respective base voltages and currents, α and β components of Δ voltages and currents are equal to β and $-\alpha$ components of line-to-neutral voltages and line currents, respectively.

α and β Impedances of Δ-Connected Circuits. In a symmetrical static Δ, let the impedance of each phase be Z in per unit, based on system kva per phase and system line-to-line voltage; then, based on line-to-neutral voltage and system kva per phase, the Δ impedances are $3Z$. The α loop impedance in Fig. 7(a) is $\frac{1}{2}(3Z) = \frac{3}{2}Z$. The self-impedance $Z_{\alpha\alpha}$ from [40] is $\frac{2}{3}(\frac{3}{2}Z) = Z$. The β loop impedance in Fig. 7(b) is $3Z(6Z)/9Z = 2Z$. The self-impedance $Z_{\beta\beta}$ from [41] is $\frac{1}{2}(2Z) = Z$. This also follows directly from [23] in which $Z_{\alpha\alpha} = Z_{\beta\beta} = Z_1$, for the symmetrical circuit in which $Z_1 = Z_2$.

An *unsymmetrical Δ-connected self-impedance circuit* can be replaced by its equivalent Y with per unit impedances based on line-to-neutral voltage for determining voltages and currents at its terminals. After the α and β voltages at the terminals of the equivalent Y have been calculated, the line-to-line voltages in per unit of base line-to-neutral voltage can be obtained from [13]. These line-to-line voltages divided by the corresponding Δ impedances, expressed in per unit based on line-to-neutral voltage, give the Δ currents in per unit of base line current.

In considering unsymmetrical Δ-connected circuits in which two phases have equal self-impedances and equal mutual impedances (including no mutual impedance) with the other phase, the loop impedances offered to α and β line currents viewed from the circuit terminals are conveniently calculated in terms of base impedance in the line-to-neutral circuit. From [40] and [41], the α and β self-impedances can then be determined from the loop impedances. This is illustrated below.

Open-Δ Transformer Bank. Assume that exciting currents can be neglected and that the open phase is opposite terminal a, as in Fig. 12(a), Chapter VIII. Referring to Fig. 7(a), no α current flows in phase A opposite terminal a. The α impedance therefore is not changed because of the open phase. In Fig. 7(b) with phase A open, the β impedance is increased; the β self-impedance is one-half the impedance of phases C and B in series.

Bank of Two Identical Transformers. Let the per unit leakage impedance of the transformers be Z_t, based on system kva per phase and base line-to-line voltage. The α and β self-impedance in per unit, based on system kva per phase and base line-to-neutral voltage, are

$$Z_{\alpha\alpha} = \tfrac{2}{3}\left(\frac{3Z_t}{2}\right) = Z_t \qquad [51]$$

$$Z_{\beta\beta} = \tfrac{1}{2}(3Z_t + 3Z_t) = 3Z_t$$

There is no mutual impedance between the α and β networks.

To illustrate the usefulness of [22] in passing from α and β components to symmetrical components, [51] will be substituted in [22], to obtain the positive- and negative-sequence self- and mutual impedances of the open-Δ transformer banks.

$$Z_{11} = Z_{22} = 2Z_t \qquad [52]$$

$$Z_{12} = Z_{21} = -Z_t$$

Equations [52] check equations [61] of Chapter VIII.

Short Circuits on Systems Containing an Unsymmetrical Circuit

With phases a, b, and c specified in defining the unsymmetrical circuit, the connections of the component networks for faults of a given type will depend upon the phase or phases faulted. Table I gives relations between V_α, V_β, and V_0, the α, β, and 0 components of phase voltages to ground (or to the neutral conductor) at the fault, and between I_α, I_β, and I_0, the components of line currents flowing from the system into the fault for faults involving any phases. The following solution illustrates the procedure followed in determining the equations of Table I.

Double Line-to-Ground Fault on Phases a and b. Conditions at the Fault: $V_a = V_b = 0$; $I_c = 0$. Substituting $V_a = V_b = 0$ in [4]–[6] and $I_c = 0$ in [9],

$$V_\alpha = -V_0 = \frac{V_\beta}{\sqrt{3}} \qquad [53]$$

$$I_\alpha = -\sqrt{3}I_\beta + 2I_0$$

This is Case IV(b) of Table I.

TABLE I

SHORT CIRCUIT ON THREE-PHASE SYSTEM

Relations between α, β, and 0 Components of Voltages and Currents at the Fault

Case	Type of Fault	Phases Involved	Equations for Components of Voltage at Fault	Equations for Components of Currents Flowing into Fault
I(a)	Three-phase	a, b, c	$V_\alpha = 0;\ V_\beta = 0$	$I_0 = 0$
(b)		a, b, c and ground	$V_\alpha = 0;\ V_\beta = 0,\ V_0 = 0$	
II(a)	Line-to-ground	a and ground	$V_\alpha = -V_0$	$I_\beta = 0;\ I_\alpha = 2I_0$
(b)	Line-to-ground	b and ground	$V_\alpha = \sqrt{3}V_\beta + 2V_0$	$I_\alpha = -\frac{I_\beta}{\sqrt{3}};\ I_\alpha = -I_0$
(c)	Line-to-ground	c and ground	$V_\alpha = -\sqrt{3}V_\beta + 2V_0$	$I_\alpha = \frac{I_\beta}{\sqrt{3}};\ I_\alpha = -I_0$
III(a)	Line-to-line	b and c	$V_\beta = 0$	$I_\alpha = 0;\ I_0 = 0$
(b)	Line-to-line	a and b	$V_\alpha = \frac{V_\beta}{\sqrt{3}}$	$I_\alpha = -\sqrt{3}I_\beta;\ I_0 = 0$
(c)	Line-to-line	a and c	$V_\alpha = -\frac{V_\beta}{\sqrt{3}}$	$I_\alpha = \sqrt{3}I_\beta;\ I_0 = 0$
IV(a)	Two line-to-ground	$b, c,$ and ground	$V_\beta = 0;\ V_\alpha = 2V_0$	$I_\alpha = -I_0$
(b)	Two line-to-ground	$a, b,$ and ground	$V_\alpha = \frac{V_\beta}{\sqrt{3}};\ V_\alpha = -V_0$	$I_\alpha = -\sqrt{3}I_\beta + 2I_0$
(c)	Two line-to-ground	$a, c,$ and ground	$V_\alpha = -\frac{V_\beta}{\sqrt{3}};\ V_\alpha = -V_0$	$I_\alpha = \sqrt{3}I_\beta + 2I_0$