EE 5240 - Lecture 2 Wed Jan 14/16, 2018

Topics for Today:

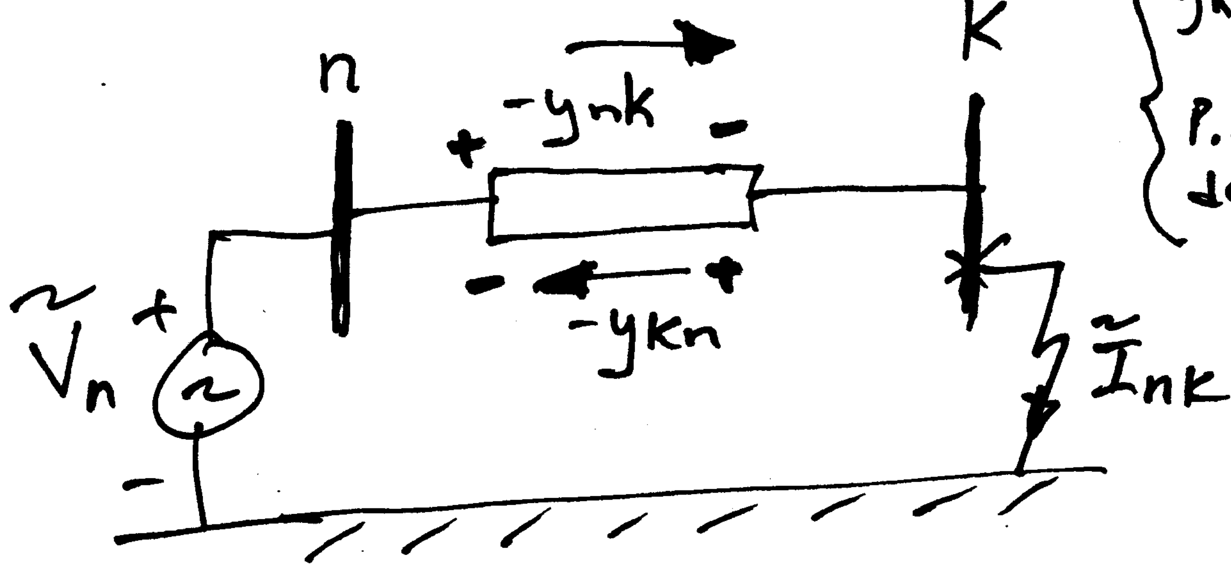
- Any Remaining Startup Questions?
- Recap of Mesh and NODE equations from Lecture 1:
  - Symmetric about main diagonal
  - [Y_{BUS}] is invertible, usually
- More on node equation formulations, sparse storage, etc.
- Possible Solution Methods
  - Brute Force Inversion and pre-multiplication
  - in situ methods:
    - Gauss Elimination
    - Gauss-Jordan Elimination
    - LU Factorization
- Matrix "manipulations"
  - Kron Reduction
  - Augmentation
    - Adding constraints (add'l variables) to system of equations
    - Adding a source, short circuit, ideal transformer, etc.

- Homework #1 -- To get started:
  - 1) Go thru videotaped EE5200 MatLab tutorials, refer to Matlab online help for matrix operations and take basic notes for your future reference.
  - 2) Use MatLab to solve the matrix equations for the mesh and the node problems in Lecture 1.
  - 3) Go thru the terminology listed in assignment, look up in text or other references. Find corresponding MatLab functions or capabilities. Take notes on MatLab syntax and application “in’s and out’s”
  - 4) Find out how to enter a sparse matrix into MatLab and document the procedure. Learn how to view network topology via the Matlab spy function.
  - Don’t hesitate to send e-mail our group e-mail forum,
  - it can be helpful for everyone to contribute questions and comments.

Implications of symmetry:

i.e. $\boxed{\text{if } y_{nk} = y_{kn}}$?

$\boxed{\text{Bilateral vs. non-Bilateral}}$



$y_{kn} \neq y_{nk}$ if P.S. xfmr or dependent source

"Transfer admittances"

$$\boxed{-y_{nk} = \frac{I_{nk}}{V_n}} \qquad \boxed{-y_{kn} = \frac{I_{kn}}{V_k}}$$

- If symmetric about main diagonal, then <u>might</u> get by with storing only lower half of off-diagonal terms.
- Careful! a) in situ methods will produce "fills" Can't look <u>statically</u> at storage requirements!
  b) Produce errors in solution if non-bilateral.

Remaining topics:

- Linked list storage
- Thev → Norton for gen's & $[Y_{Bus}]$
- Augmenting $[Y_{Bus}]$
- **Partitioning (Kron Reduction)**

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Ex: Coefficient matrix

$$A = \begin{bmatrix} 3 & 0 & 0 & 2 & 0 \\ 0 & 5 & 1 & 0 & 0 \\ 0 & 1 & 2 & 7 & 9 \\ 2 & 0 & 7 & 1 & 0 \\ 0 & 0 & 9 & 0 & 4 \end{bmatrix}$$

Full storage:

25 numbers.

Single precision:

Real: 4 bytes

Complx: 8 bytes $\Rightarrow$ $8 \times 25 = \underline{200 \text{ bytes}}$

10,000-bus

$\Rightarrow [Y]$: 100,000,000 entries

Linked List: Storage

A ← actual Values

	A	ICOL	NEXT	NBEG
1	3	1	2	1
2	2	4	0	3
3	5	2	4	5
4	1	3	0	9
5	1	2	6	12
6	2	3	7	
7	7	4	8	
8	9	5	0	
9	2	1	10	
10	7	3	11	
11	1	4	0	
12	9	3	13	
13	4	5	0	

A: COMPLEX 13x8

ICOL: INT 13x2

NEXT: INT 13x2

NBEG: INT 5x2

NBEG: pointers to start of each row

If NBUS < 32,000, then we can use 2-Byte Int.

104
52
10

166 Bytes

KEY: Vital to understand data structure.

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Data Type

[A] – Single-Prec Complex A

– " " Int. ICOL

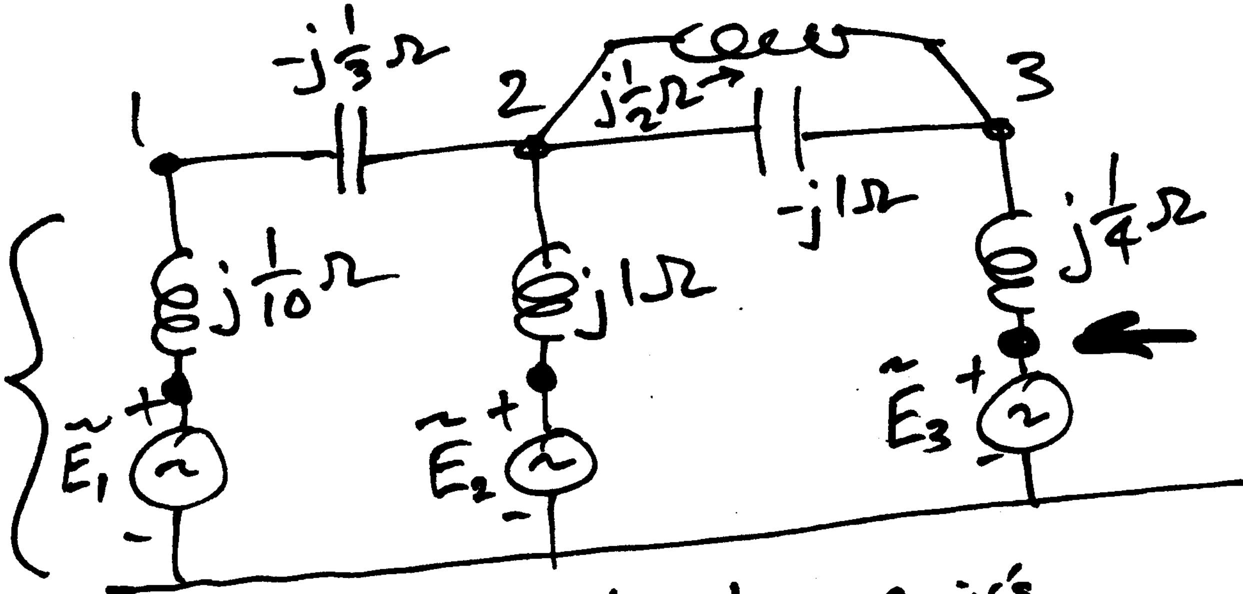
– " " Int. NEXT

A
ICOL
NEXT

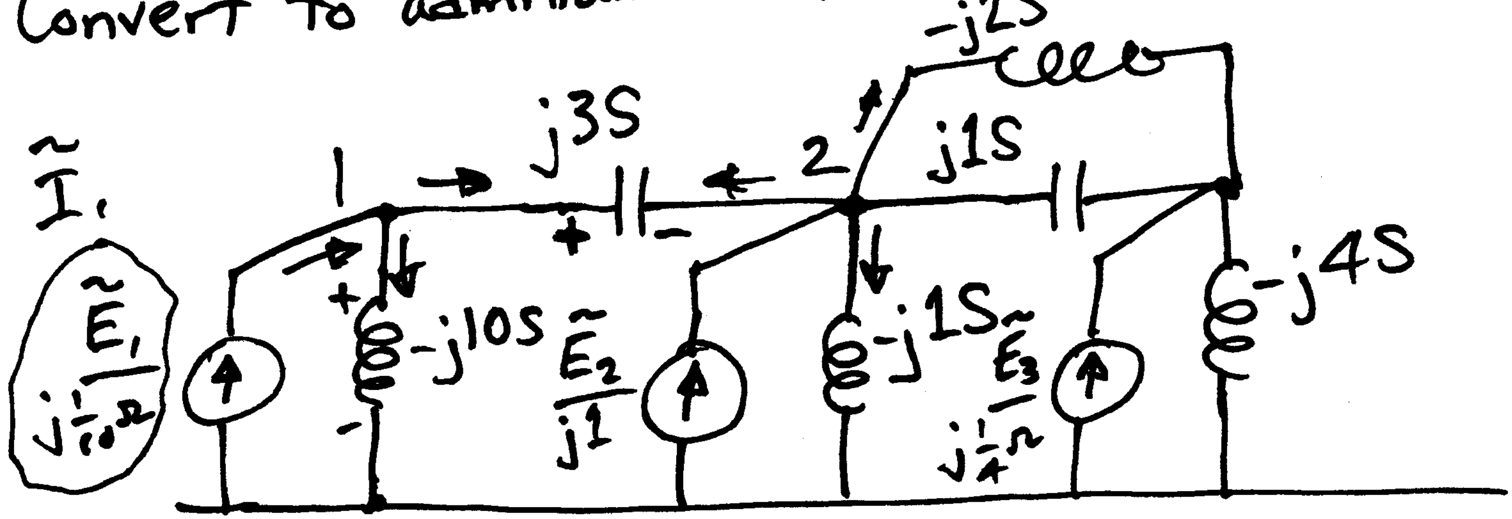
A(1) =

3
1
2

Ex:



Convert to admittances w/Norton equiv's



Node 1

$$\tilde{I}_1 = \underline{(\tilde{V}_1 - \tilde{V}_2)}\,j3 + (\tilde{V}_1 - 0)(-j105)$$

$$\tilde{I}_2 = (\tilde{V}_2 - \tilde{V}_1)\,j3 + (\tilde{V}_2 - \tilde{V}_3)\,j1 + (\tilde{V}_2 - 0)(-j1) + (\tilde{V}_2 - \tilde{V}_3)(-j2)$$

$$\tilde{I}_3 = (\tilde{V}_3 - V_2)(j1 - j2) + (\tilde{V}_3 - 0)(-j4)$$

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = \begin{bmatrix} \text{or, build} \\ [Y] \text{ by} \\ \text{inspection.} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \end{bmatrix}$$

Continue, see homework.

Admittance Equations

General Form:

$$\begin{bmatrix} Y_{Bus} \end{bmatrix} \begin{bmatrix} V_{NODE} \end{bmatrix} = \begin{bmatrix} I_{INJ} \end{bmatrix}$$

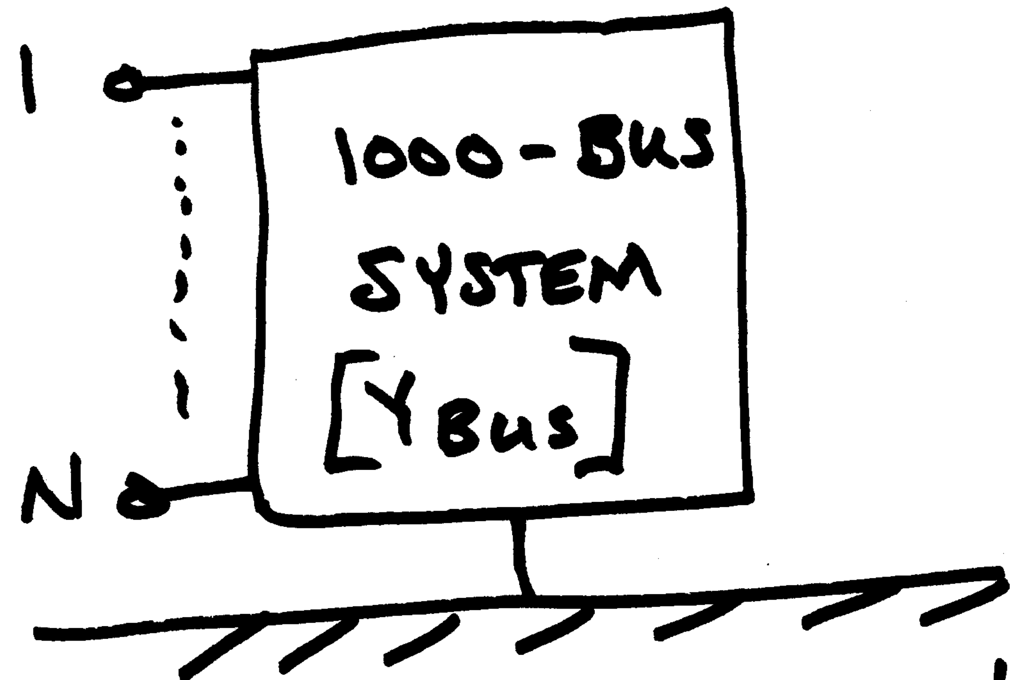
We can add constraints:

- V source Bus-Bus
- Short
- XFMR
- DEPENDENT SOURCES (OP-AMP)

Augmentation.

Kron Reduction – System Reduction
– Kron Elimination

Reference: Stevenson, 4th Ed., pp. 174-178.



Possible to reduce to equiv system of fewer nodes.

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Goal: Only buses of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

STEPS:

1) Reorder system – move buses to be kept to top, i.e. 1....K
Remaining L...Z nodes are absorbed into system.

2) Perform Kron Reduction.

$$\underbrace{\left[\begin{array}{c|c} [K] & [L] \\ \hline [L^T] & [M] \end{array}\right]}_{Y_{BUS}} \underbrace{\left[\begin{array}{c} V_A \\ \hline V_B \end{array}\right]}_{V} = \underbrace{\left[\begin{array}{c} I_A \\ \hline I_X \end{array}\right]}_{I}$$

$$\left.\begin{array}{ll} ① & I_A = K V_A + L V_B \\ ② & I_X = L^T V_A + M V_B \end{array}\right\}$$

$$\text{Since } I_X = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

3 $-L^T V_A = M V_B$ ← From Eqn. 2 for $I_X = 0$.

4 $-M^{-1} L^T V_A = V_B$ ← premultiply both sides by M^{-1}.

Substituting V_B into Eqn. 1,

$$I_A = K V_A - L M^{-1} L^T V_A$$

$$[I_A] = \underbrace{[K - L M^{-1} L^T]} [V_A]$$

The $[Y_{Bus}]$ for this reduced system is thus implied to be $[K - L M^{-1} L^T]$.

Derivation assumes bilateral system (note L, L^T)

Reduced $[Y_{Bus}]$ is

$$\boxed{\left[Y_{\substack{Bus\\Reduced}}\right] = K - LM^{-1}L^T}$$

IMPORTANT OBSERVATION:

If L & L^T are off-diagonals, then this eqn. only valid for bilateral System!