EE 5240 - Lecture 3 Wed Jan 16, 2019

- Questions from last lecture?
- How are your matrix math skills?
- Questions on Hmwk #1 ? (Due 5pm Wed).
- Everybody all set w/text, matlab, etc?
- Web page ok? May want 3-ring binder - lots of papers!

TODAY:

- Kron Reduction (network reduction) – refer to Lecture 2.
- Modifications to [Y] according to system changes
- Switching things in and out: lines, shunt reactors/caps, gens.
- Augmenting the admittance matrix
  - Ungrounded Voltage Source
  - Short Circuit
  - Two-winding Transformer
- Next: in situ methods to solve $[Y_{bus}]\ [V_{bus}] = [I_{inj}]$

Admittance Equations

General Form:

$$\begin{bmatrix} Y_{Bus} \end{bmatrix} \begin{bmatrix} V_{NODE} \end{bmatrix} = \begin{bmatrix} I_{INJ} \end{bmatrix}$$

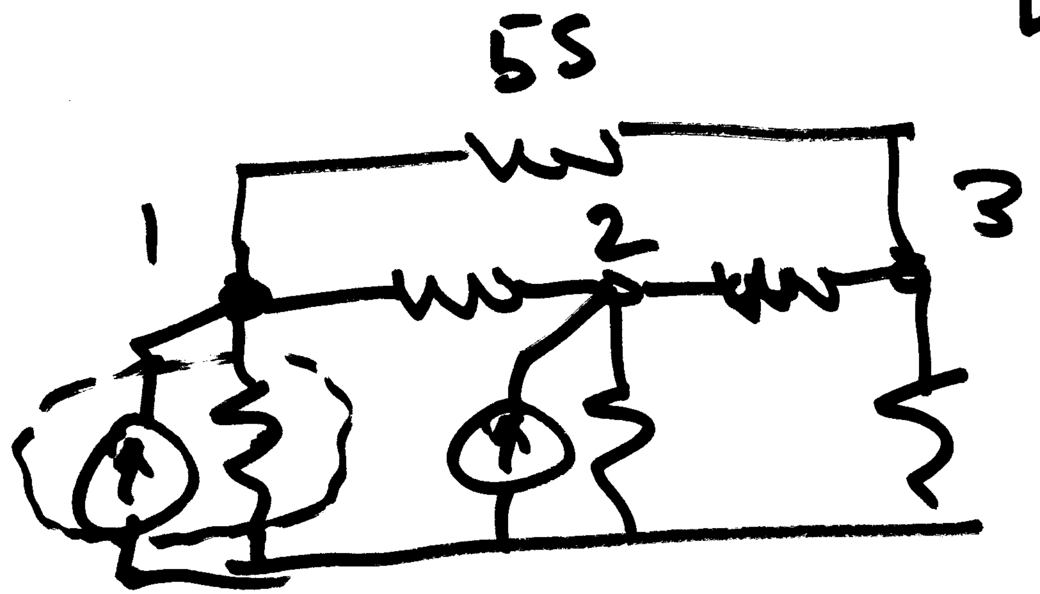
We can add constraints:

- V source Bus-Bus
- Short
- XFMR
- DEPENDENT SOURCES (OP-AMP)

SWITCHING LINES, ETC, in and out:

From Lecture 1:

$$[Y] = \begin{bmatrix} 8 & -2 & -5 \\ -2 & 9 & -4 \\ -5 & -4 & 15 \end{bmatrix}$$



Switch out Line 1-3:

$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 10 \end{bmatrix}$$

$\bar{y}_{1-3} = 5S$

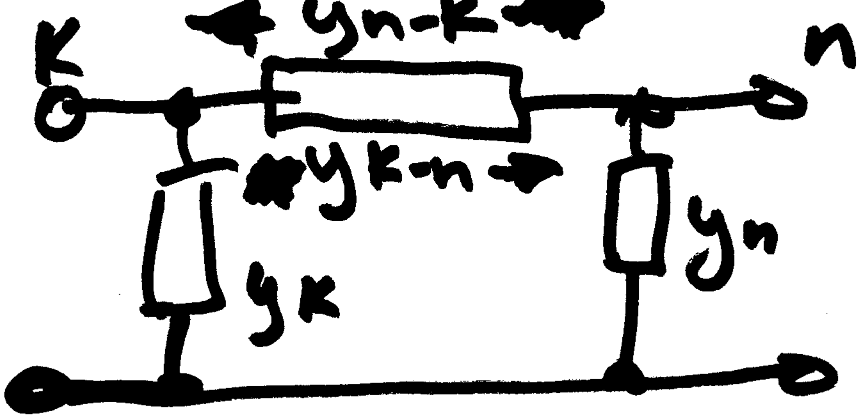
$\bar{y}_{11} = \bar{y}_{11} - \bar{y}_{1-3}$

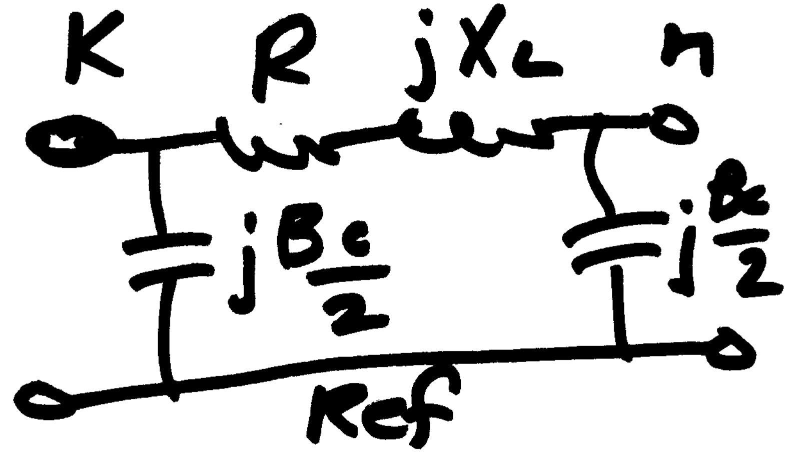
$\bar{y}_{33} = \bar{y}_{33} - \cancel{\bar{y}_{1-3}}\ \bar{y}_{3-1}$

$\bar{y}_{13} = \bar{y}_{13} + \bar{y}_{1-3}$

$\bar{y}_{31} = \bar{y}_{31} + \cancel{\bar{y}_{1-3}}\ \bar{y}_{3-1}$

How about a T-Line?





B_c is full-line Charging susc.

Modify [Y], switch out... ?

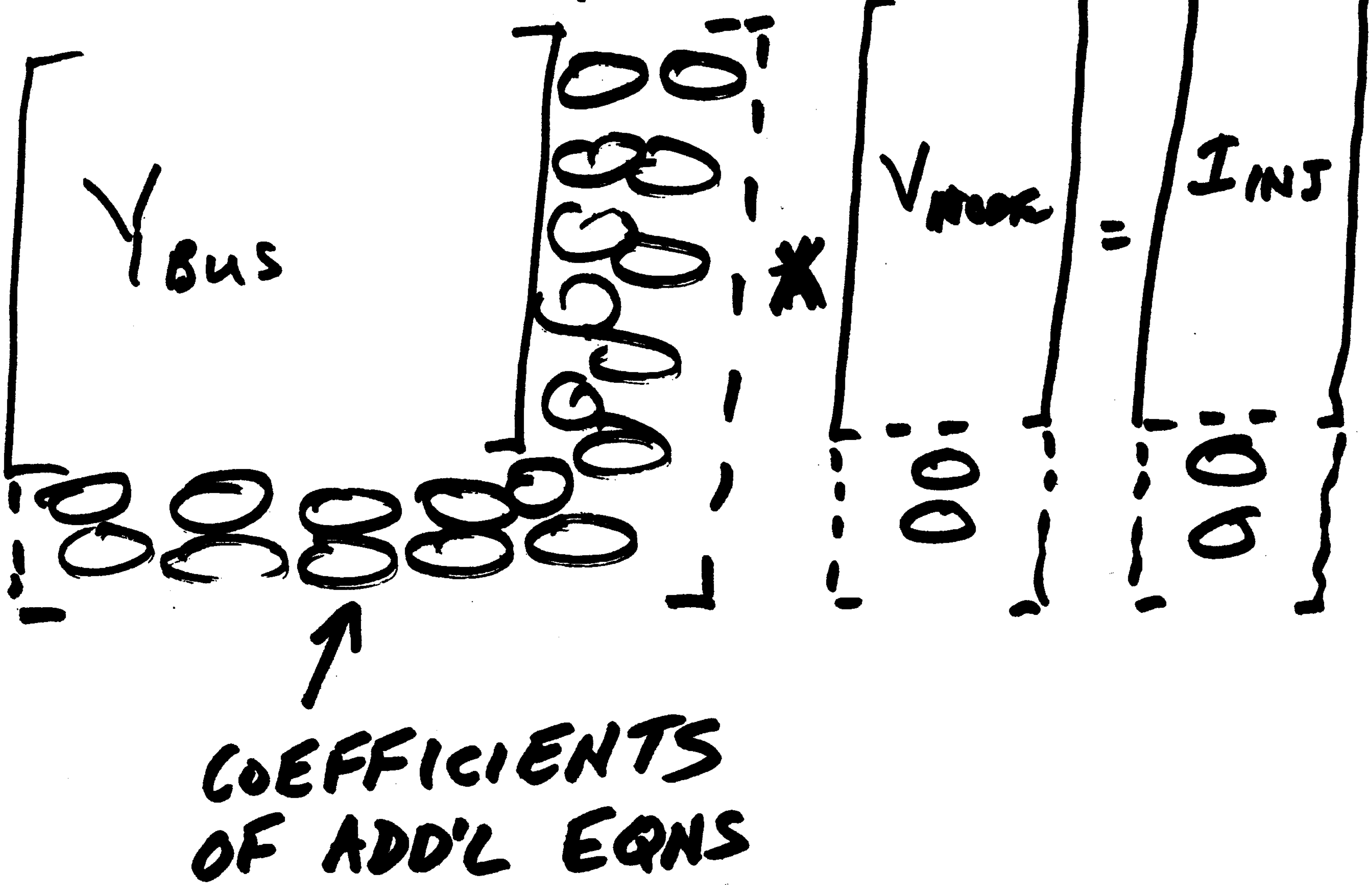
$\bar{y}_{KK} = \bar{y}_{KK} - y_K - y_{K-n}$

$\bar{y}_{Kn} = \bar{y}_{Kn} + \bar{y}_{K-n}$

$\bar{y}_{nn} = \bar{y}_{nn} - y_n - \bar{y}_{n-k}$

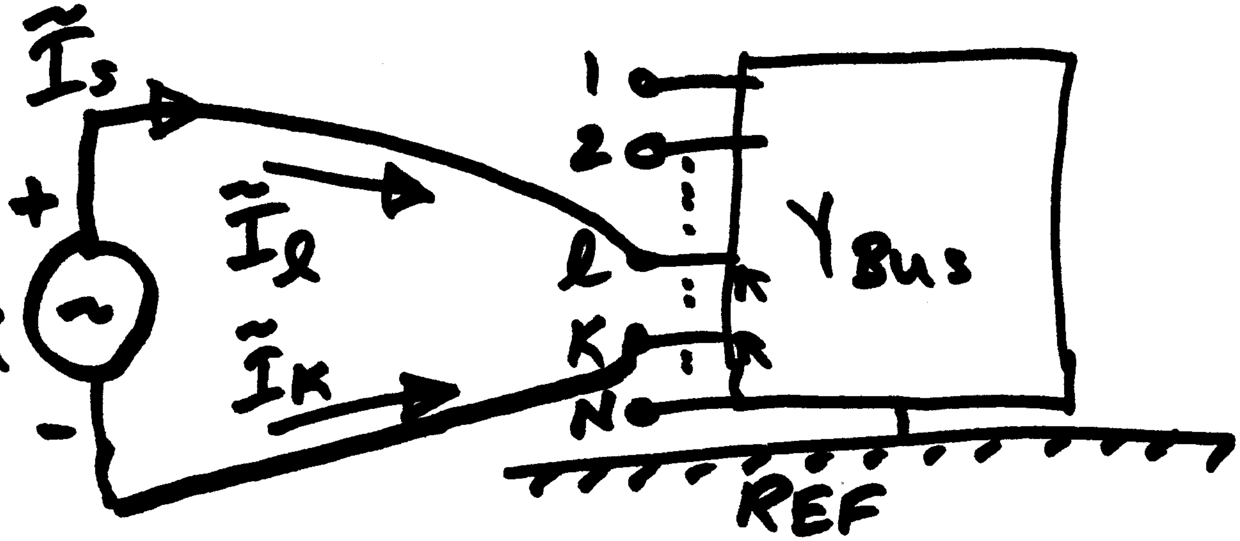
$\bar{y}_{nk} = \bar{y}_{nk} + y_{n-k}$

AUGMENTING [Y_{BUS}]



NODE-NODE VOLTAGE SOURCE

$\tilde{V}_S = \tilde{V}_{\ell K}$



New Constraints being added:

① $\tilde{V}_S = \tilde{V}_\ell - \tilde{V}_K$ = Known (forced voltage)

② $\tilde{I}_\ell = +\tilde{I}_S$

③ $\tilde{I}_K = -\tilde{I}_S$

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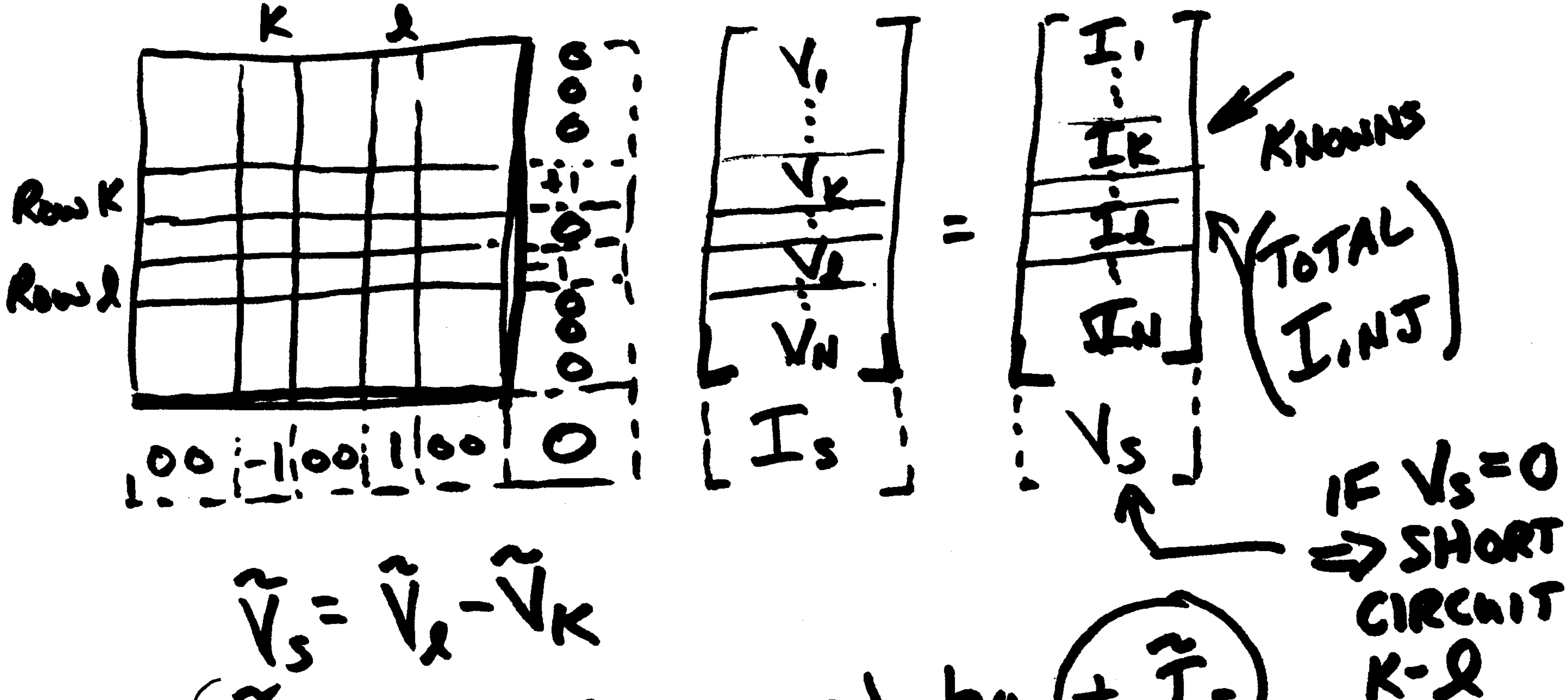


$$\tilde{V}_s = \tilde{V}_\ell - \tilde{V}_K$$

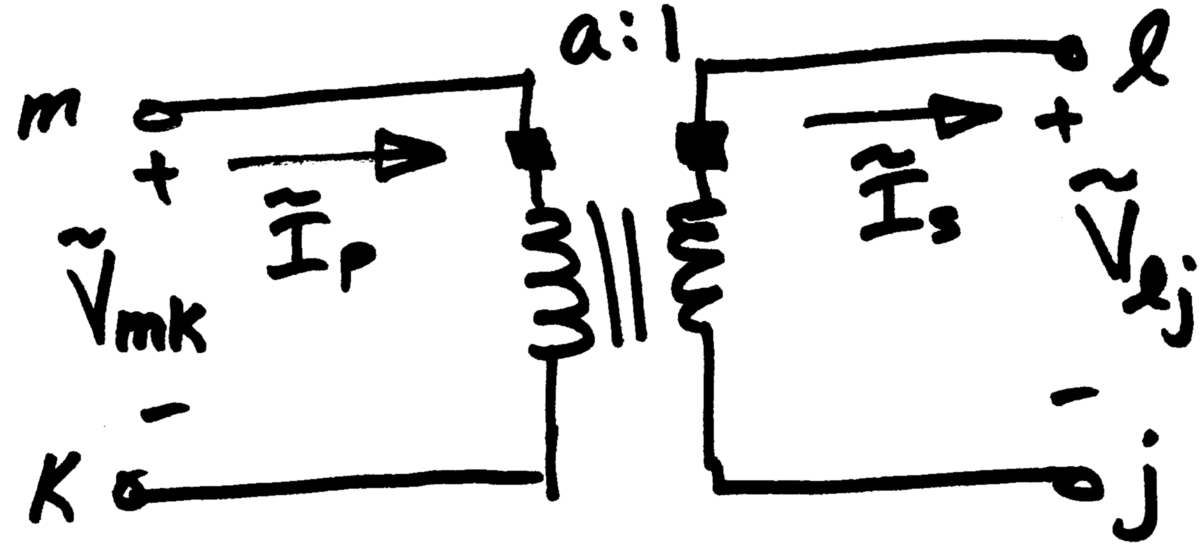
Note: This effect is on <u>left</u> side of eqn!

$\tilde{I}_\ell (INJ)$ is increased by $+\tilde{I}_s$

$\tilde{I}_K (INJ)$ is "increased" by $-\tilde{I}_s$

∴ change sign when entering in column N+1 of [Y]!

XFMR - IDEAL 2-WINDING



Polarity Marks: $\angle \tilde{I}_p = \angle \tilde{I}_s$

$$\angle \tilde{V}_{mk} = \angle \tilde{V}_{\ell j}$$

The turns ratio "a" is in general, complex.

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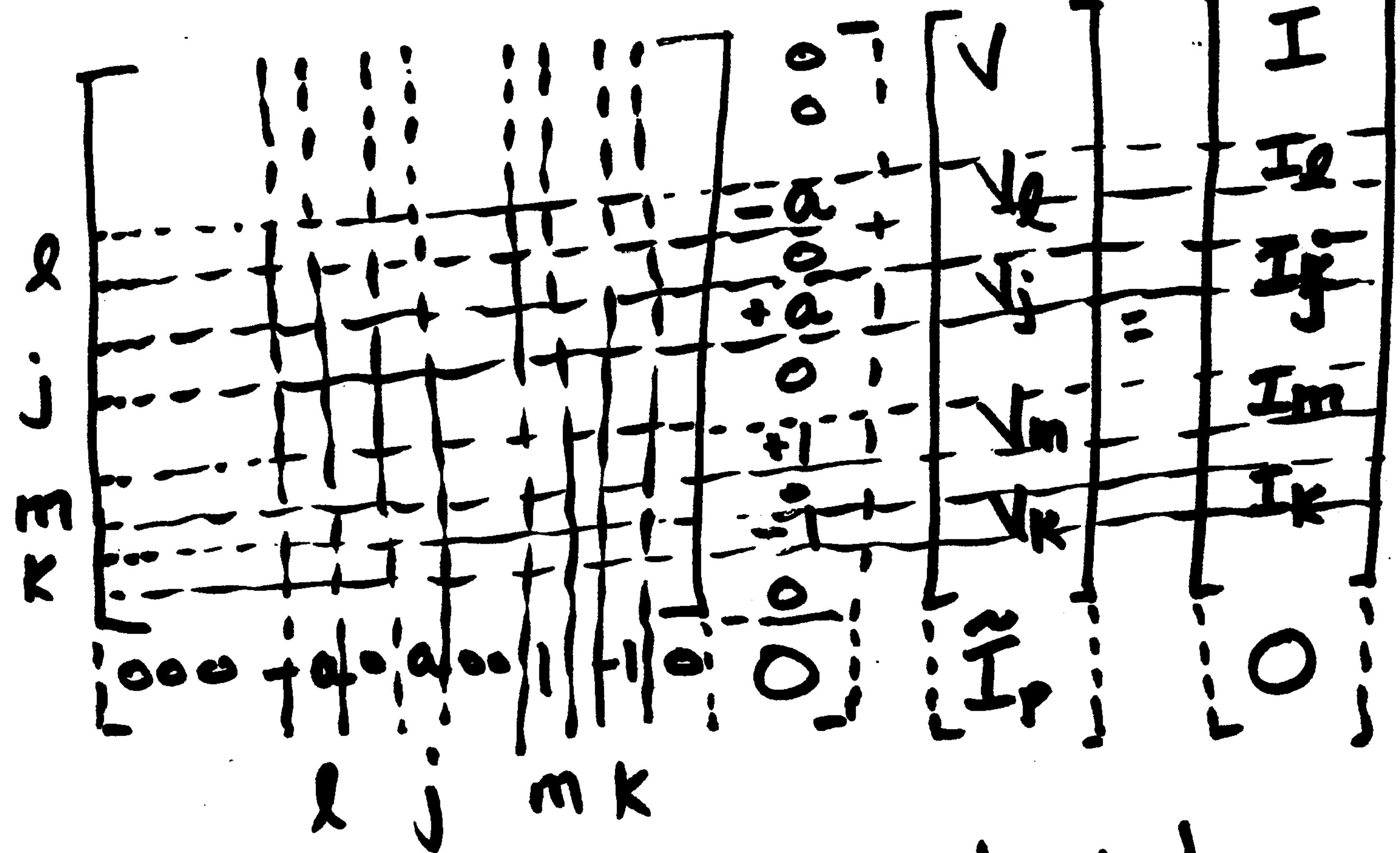

Aug Row → voltage constraints

Aug Col → current constraints

TYPICAL: INTRODUCE ZERO IN MAIN DIAG!

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Constraints

NODE VOLTAGES

$$\tilde{V}_m - \tilde{V}_k = a(\tilde{V}_\ell - V_j)$$

i.e. $\tilde{V}_{mk} = a\tilde{V}_{\ell j}$

$$\boxed{\tilde{V}_m - \tilde{V}_k + aV_j - a\tilde{V}_\ell = 0}$$

Injected Currents: (ADD'L)

$$\left.\begin{aligned} \ell &: +\tilde{I}_s = a\tilde{I}_p \\ j &: -\tilde{I}_s = -aI_p \\ m &: -\tilde{I}_p \\ k &: +\tilde{I}_p \end{aligned}\right\}$$

Again: effect is on left side of eqn, ∴ change signs in Column N+1 of [Y].

General Reference for this is on next page. (From Vlach - refer to syllabus reference list).

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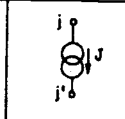
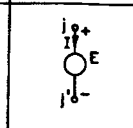
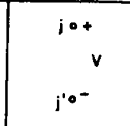
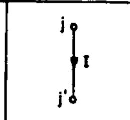
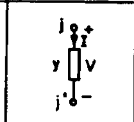
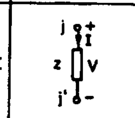
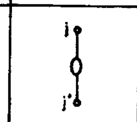
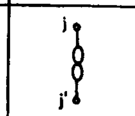
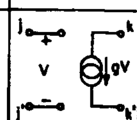
ELEMENT	SYMBOL	MATRIX	EQUATIONS
CURRENT SOURCE		$\begin{array}{c} \\ j \\ j' \end{array}\begin{bmatrix} -J \\ J \end{bmatrix}$ SOURCE VECTOR	$I_j = J$ $I_{j'} = -J$
VOLTAGE SOURCE		$\begin{array}{c\|ccc} & V_j & V_{j'} & I \\ \hline j & & & 1 \\ j' & & & -1 \\ m+1 & 1 & -1 & \end{array}$ SOURCE VECTOR $\begin{bmatrix} \\ \\ E \end{bmatrix}$	$V_j - V_{j'} = E$ $I_j = I$ $I_{j'} = -I$
OPEN CIRCUIT		—	$V = V_j - V_{j'}$
SHORT CIRCUIT		$\begin{array}{c\|ccc} & V_j & V_{j'} & I \\ \hline j & & & 1 \\ j' & & & -1 \\ m+1 & 1 & -1 & \end{array}$	$V_j - V_{j'} = 0$ $I_j = I$ $I_{j'} = -I$
ADMITTANCE		$\begin{array}{c\|cc} & V_j & V_{j'} \\ \hline j & y & -y \\ j' & -y & y \end{array}$	$I_j = y(V_j - V_{j'})$ $I_{j'} = -y(V_j - V_{j'})$
IMPEDANCE		$\begin{array}{c\|ccc} & V_j & V_{j'} & I \\ \hline j & & & 1 \\ j' & & & -1 \\ m+1 & 1 & -1 & -z \end{array}$	$V_j - V_{j'} - zI = 0$ $I_j = -I_{j'} = I$
NULLATOR		$\begin{array}{c\|cc} & V_j & V_{j'} \\ \hline m+1 & 1 & -1 \end{array}$	$V_j - V_{j'} = 0$ $I_j = I_{j'} = 0$
NORATOR		$\begin{array}{c\|c} & I \\ \hline j & 1 \\ j' & -1 \end{array}$	V, I ARE ARBITRARY
VCT		$\begin{array}{c\|cc} & V_j & V_{j'} \\ \hline k & g & -g \\ k' & -g & g \end{array}$	$I_j = 0$ $I_{j'} = 0$ $I_k = g(V_j - V_{j'})$ $I_{k'} = -g(V_j - V_{j'})$

Fig. 4.4.1. Ideal elements in the modified nodal formulation without graphs.

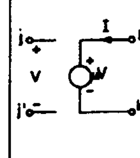
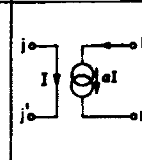
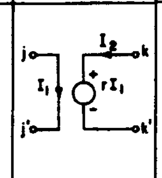
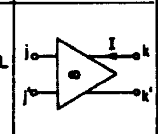
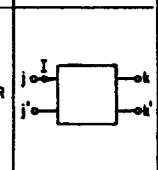
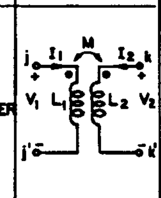
ELEMENT	SYMBOL	MATRIX	EQUATIONS
VVT		$\begin{array}{c\|ccccc} & V_j & V_{j'} & V_k & V_{k'} & I \\ \hline j & & & & & \\ j' & & & & & \\ k & & & & & 1 \\ k' & & & & & -1 \\ m+1 & -\mu & \mu & 1 & -1 & \end{array}$	$-\mu V_j + \mu V_{j'} + V_k$ $-V_{k'} = 0$ $I_k = I$ $I_{k'} = -I$
CCT		$\begin{array}{c\|ccccc} & V_j & V_{j'} & V_k & V_{k'} & I \\ \hline j & & & & & 1 \\ j' & & & & & -1 \\ k & & & & & \alpha \\ k' & & & & & -\alpha \\ m+1 & 1 & -1 & & & \end{array}$	$V_j - V_{j'} = 0$ $I_j = -I_{j'} = I$ $I_k = -I_{k'} = \alpha I$
CVT		$\begin{array}{c\|cccccc} & V_j & V_{j'} & V_k & V_{k'} & I_1 & I_2 \\ \hline j & & & & & 1 & \\ j' & & & & & -1 & \\ k & & & & & & 1 \\ k' & & & & & & -1 \\ m+1 & 1 & -1 & & & & \\ m+2 & & & 1 & -1 & -r & \end{array}$	$V_j - V_{j'} = 0$ $V_k - V_{k'} - rI_1 = 0$ $I_j = -I_{j'} = I_1$ $I_k = -I_{k'} = I_2$
OPERATIONAL AMPLIFIER		$\begin{array}{c\|ccccc} & V_j & V_{j'} & V_k & V_{k'} & I \\ \hline j & & & & & \\ j' & & & & & \\ k & & & & & 1 \\ k' & & & & & -1 \\ m+1 & 1 & -1 & & & \end{array}$	$V_j - V_{j'} = 0$ $I_k = -I_{k'} = I$
CONVERTOR		$\begin{array}{c\|ccccc} & V_j & V_{j'} & V_k & V_{k'} & I \\ \hline j & & & & & 1 \\ j' & & & & & -1 \\ k & & & & & -K_2 \\ k' & & & & & K_2 \\ m+1 & 1 & -1 & -K_1 & K_1 & \end{array}$	$V_j - V_{j'} - K_1V_k + K_1V_{k'} = 0$ $I_j = -I_{j'} = I$ $I_k = -I_{k'} = -K_2I$ FOR IDEAL TRANSFORMER $K_1 = K_2 = n$
TRANSFORMER		$\begin{array}{c\|cccccc} & V_j & V_{j'} & V_k & V_{k'} & I_1 & I_2 \\ \hline j & & & & & 1 & \\ j' & & & & & -1 & \\ k & & & & & & 1 \\ k' & & & & & & -1 \\ m+1 & 1 & -1 & & & -sL_1 & -sM \\ m+2 & & & 1 & -1 & -sM & -sL_2 \end{array}$	$V_j - V_{j'} - sL_1I_1 - sMI_2 = 0$ $V_k - V_{k'} - sMI_1 - sL_2I_2 = 0$ $I_j = -I_{j'} = I_1$ $I_k = -I_{k'} = I_2$

Fig. 4.4.1. *(Continued)*