EE 5240 - Lecture 13 Monday Feb 11, 2019

Topics for Today:

- Questions/Comments on Homework #4 ?
- LU Factorization (needed for each iteration)
- Loadflow Formulation: “NR Details” handout (Week 4)
- NR Algorithm implementation.
- More on Transformers, phase shifters

Coming up:

- More MatLab - build Jacobian, solve for $\Delta\delta$ and ΔV, iterate.
- Data structures
- Reordering to avoid zero divides and/or speed up solution.

LU Factorization (Crout's Method)

There are two approaches:

- "in situ" methods
- Sparse matrix storage.

- By rows (example shown here)
- By columns (your text's approach)

Basic Procedure:

1. Copy Column one
2. Divide Row 1 off-diagonal entries by diagonal term of Row 1.
3. For each element i,j where i > 1 and j >1, subtract from it the product of $a_{i1} \cdot a_{1j}$
4. If the resulting sub-matrix is of order 2 or greater, go back to step 1 and perform the same operations on that sub-matrix.

rank

Example:

$$[A][X]=[Y]$$

$$[L]\underbrace{[U][X]}_{*}=[Y]$$

$$\underbrace{[L][Z]}=[Y] \rightarrow \text{solve for } Z \quad ①$$

$$[U][X]=[Z] \rightarrow \text{solve for } [X]. \quad ②$$

step 2

$$\begin{bmatrix} \underline{\underline{2}} & 4 & 4 & 2 \\ 3 & 3 & 12 & 6 \\ 2 & 4 & -1 & 2 \\ 4 & 2 & 1 & 1 \end{bmatrix} \Longrightarrow [L][u]$$

②

①

$$\left[\begin{array}{c|ccc} 2 & 2 & 2 & 1 \\ \hline 3 & \boxed{-3} & 6 & 3 \\ 2 & 0 & -5 & 0 \\ 4 & -6 & -7 & -3 \end{array}\right]$$

③

$$u = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 2 & 2 & 2 & 1 \\ 3 & -3 & -2 & -1 \\ \hline 2 & 0 & -5 & 0 \\ 4 & -6 & -19 & -9 \end{array}\right] \Longrightarrow L = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & -3 & 0 & 0 \\ 2 & 0 & -5 & 0 \\ 4 & -6 & -19 & -9 \end{bmatrix}$$

CHECK!

$$[L][u] \overset{?}{=} [A]$$

$$\left[\begin{array}{c|ccc} 2 & 0 & \textcircled{0} & 6 \\ \hline \textcircled{0} & 4 & \underline{\underline{0}} & 8 \\ 0 & \underline{\underline{0}} & 2 & 4 \\ 4 & \underline{\underline{0}} & \underline{\underline{0}} & 6 \end{array}\right] \begin{bmatrix} x_1 \\ x_{\underline{\underline{2}}} \\ x_{\underline{\underline{3}}} \\ x_{\underline{\underline{4}}} \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 8 \\ 10 \end{bmatrix}$$

"A Fill" = changing a zero to nonzero entry.

Key: Re-order the equations to minimize fills.

Jacobian Structures

$$\left[\begin{array}{c:c} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \hdashline \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{array}\right] \left[\begin{array}{c} \Delta\delta \\ \hdashline \Delta V \end{array}\right] = \left[\begin{array}{c} \Delta P \\ \cdots \\ \Delta Q \end{array}\right]$$

8 Equation Forms

$nzr = \underline{find}(Y(i:0))$

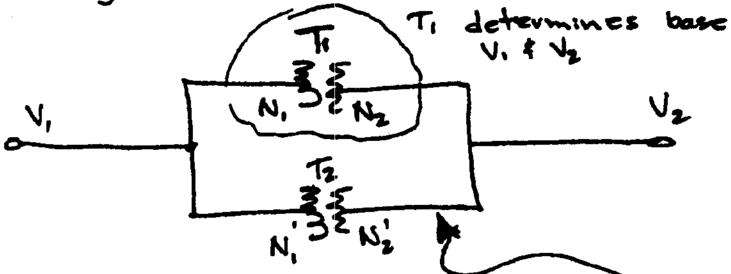
$nzr = find(Y(i:))$

$nzr = [1\ 2\ 3\ 4\ 5]$

```
for  n = 1:length(nzr)
          :
          Y(i, nzr(n))
          :
end

OR:   for  n = nzr
          :
      end
```

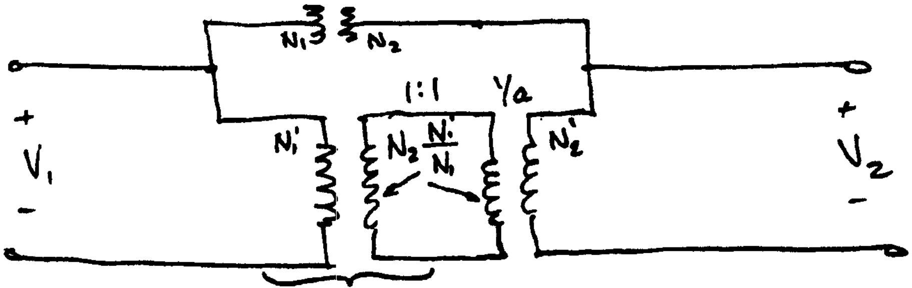
Paralleling Transformers of Unlike Turns Ratio



T1 determines base voltages V1 & V2

$$\frac{N_1}{N_2} \cong \frac{V_1}{V_2}$$

What happens for $\frac{N_1'}{N_2'} \neq \frac{N_1}{N_2}$?



SAME RATIO AS T1

Replace T_2 with 2 XFMRS : - First is same ratio as T_1 $\frac{N_1}{N_2} = \frac{N_1'}{X}$

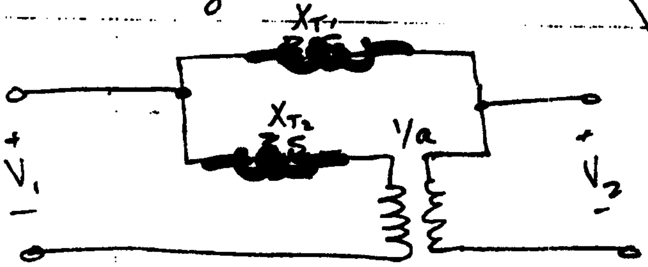
$$\boxed{X = N_2 \frac{N_1'}{N_1}}$$

Second XFMR has ratio of off-nominal turns

$$\frac{1}{a} = \frac{\left(N_2 \frac{N_1'}{N_1}\right)}{N_2'} = \frac{N_2 N_1'}{N_1 N_2'}$$

$$\therefore \boxed{a = \frac{N_1}{N_2}\frac{N_2'}{N_1'}} = \text{p.u. turns ratio}$$

Per unit equivalent:

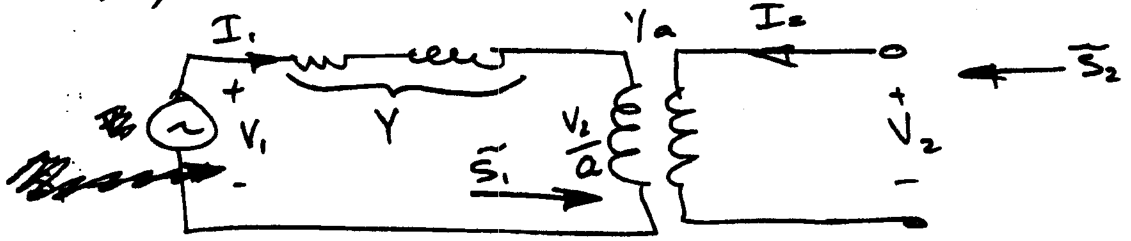


Three Methods to Analyze:

1) Admittance Method
2) Circuit Theory
3) Circulating Current Method (Approximate)

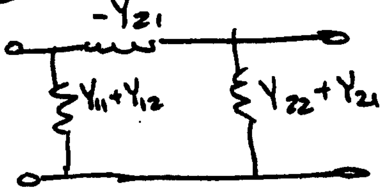
METHOD 1

So,



So we must find a way to model Y for Y_{12} as Y_a varies.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{12}V_1 + Y_{22}V_2$$



IN GENERAL

$$\tilde{S}_1 = -\tilde{S}_2$$

$$\tilde{S}_1 = \frac{V_2}{a} I_1^* \qquad S_2 = V_2 I_2^*$$

$$\frac{V_2}{a} I_1^* = -V_2 I_2^*$$

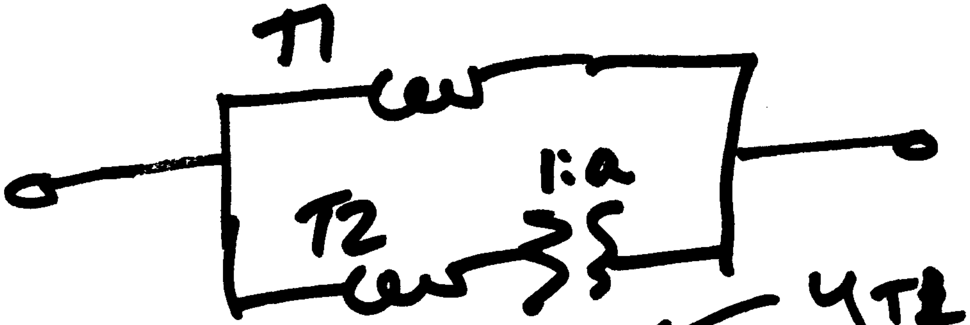
$$I_1^* = -a I_2^*$$

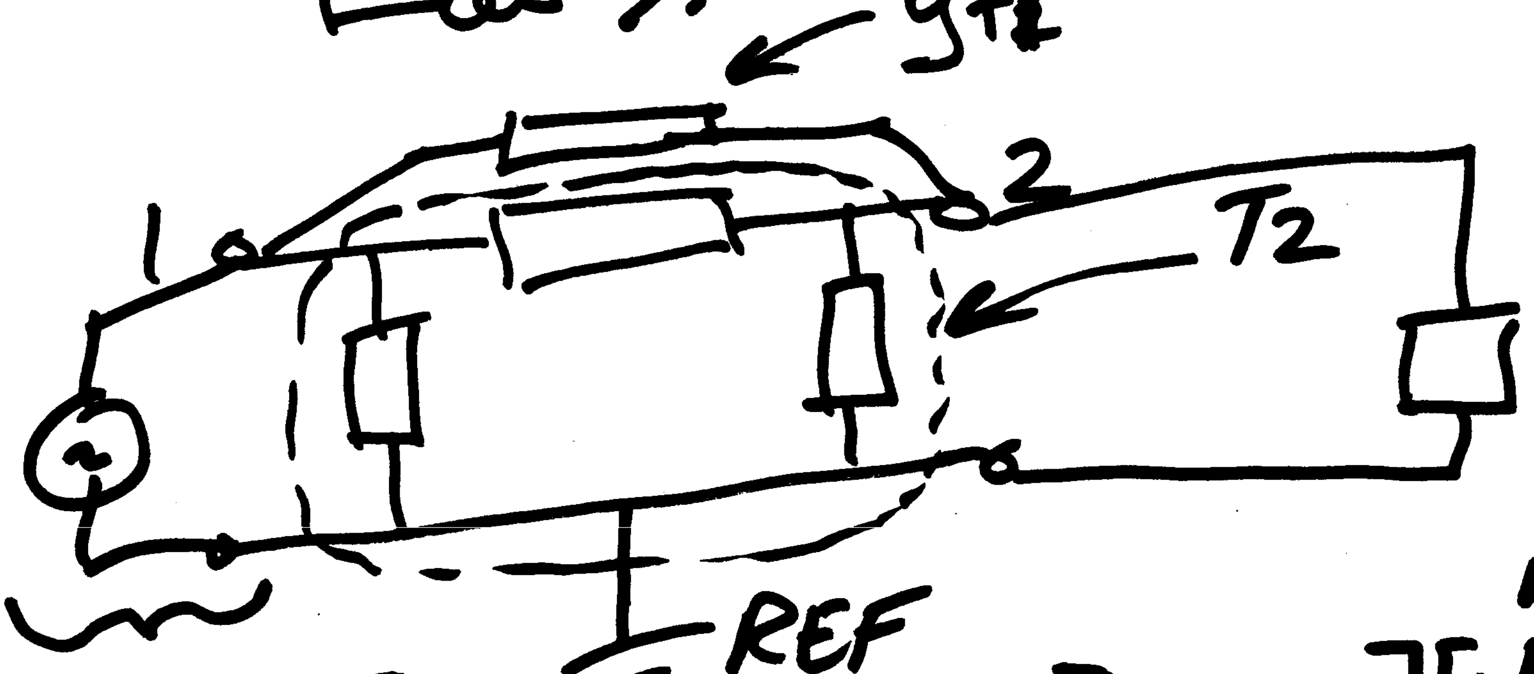
$$\boxed{I_1 = -a^* I_2}$$

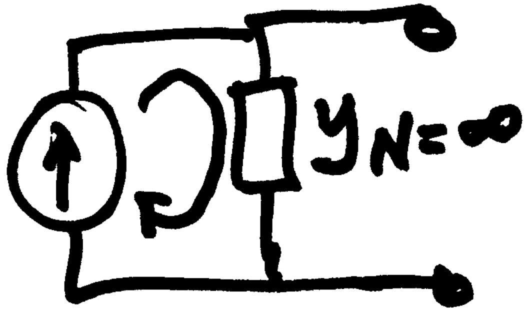
$$I_1 = \left(V_1 - \frac{V_2}{a}\right)Y = \boxed{Y}V_1 \boxed{-\frac{Y}{a}}V_2 = -a^* I_2$$

$$\therefore \quad I_2 = -\frac{I_1}{a^*} = \boxed{-\frac{YV_1}{a^*}} + \boxed{\frac{Y}{aa^*}}V_2$$

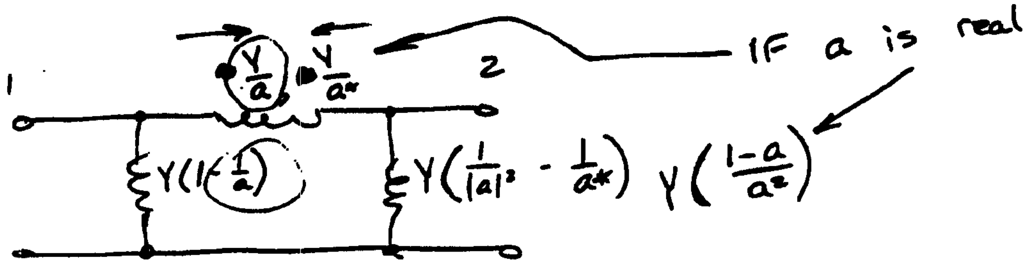
$$\therefore \quad \boxed{Y_{11} = Y \qquad Y_{12} = -\frac{Y}{a} \qquad Y_{21} = -\frac{Y}{a^*} \qquad Y_{22} = \frac{Y}{aa^*} = \frac{Y}{|a|^2}}$$



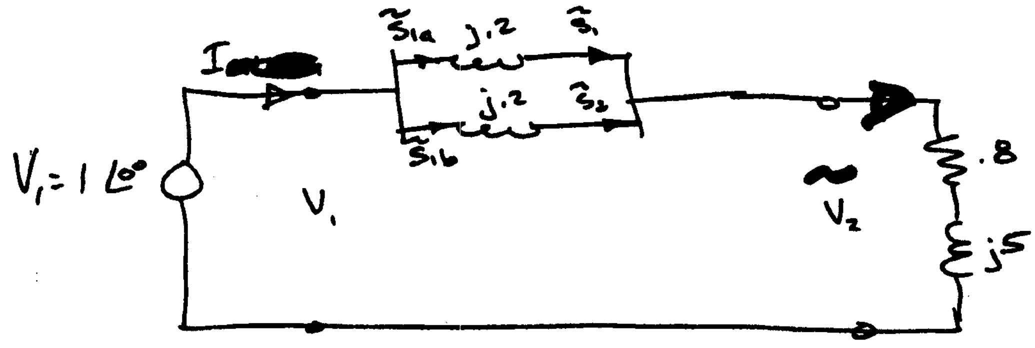


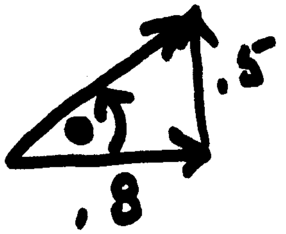


$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \overset{\text{known}}{V_1 \checkmark} \\ \hdashline V_2 \end{bmatrix} = \begin{bmatrix} \underset{0}{\cancel{I_1}} \\ \underset{0}{\cancel{I_2}} \end{bmatrix}$$



2) CIRCUIT THEORY APPROACH





$V_2 = \frac{(.8+j.5)(1\angle 0^\circ)}{.8+j.6} = .94 - j.08 = .9434\angle -4.86^\circ$

$I_2 = \frac{.94-j.08}{.8+j.5} = 1\angle -36.87^\circ$

$\tilde{S} = VI^* = .9434\angle -4.86^\circ (1\angle 36.87^\circ) = .9434\angle 32^\circ = .8+j.5$

$\left.\begin{matrix} S_1 = .4+j.25 \\ S_2 = .4+j.25 \end{matrix}\right\} = \frac{S_{TOTAL}}{2}$

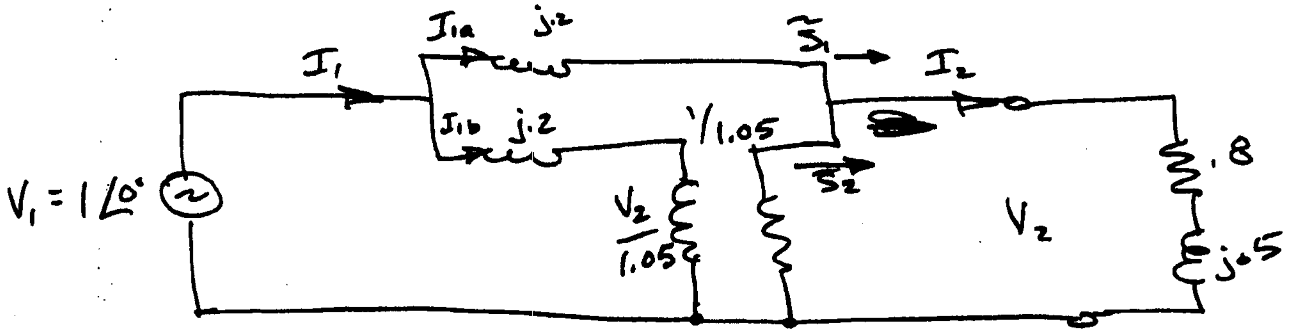
$I_1 = \frac{1}{.8+j6} = 1(.8-j.6) = .8-j.6$

$S_{1a} = \frac{V_1 I_1^*}{2} = 4+j.3$

$S_{1b} = \frac{V_1 I_1^*}{2} = .4+j.3$

Difference due to XFMR Inductance

Now add tap changer $a = 1.05$



$$I_1 = I_{1a} + I_{1b}$$

$$I_1 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}$$

$$I_2 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)$$

$$V_2 = I_2(.8 + j.5) = \left[\frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)\right](.8 + j.5)$$

$$V_2 = \left(1 - V_2 + \frac{1 - \frac{V_2}{1.05}}{1.05}\right)\left(\frac{.8 + j.5}{j.2}\right) = (1.952 - 1.907 V_2)\left(\overset{4.717 \quad -58^\circ}{\angle}\right)$$

[illegible]

$$\therefore V_2 = -1.907 V_2 (4.717 \angle -58^\circ) + (1.952)(4.717 \angle -58^\circ)$$

$$= -8.995 \angle -58^\circ V_2 + 9.207 \angle -58^\circ$$

$$V_2 = \frac{9.207 \angle -58^\circ}{1 + 8.995 \angle -58^\circ} = \frac{9.207 \angle -58^\circ}{9.563 \angle -52.9} = \boxed{.963 \angle -5.1^\circ}$$

$$I_{1a} = \frac{1 - V_2}{j.2} = .473 \angle -25.54^\circ \qquad I_{1b} = \frac{1 - \frac{V_2}{1.05}}{j.2} = .59 \angle -46.75^\circ$$

$$S_{1a} = V_1 I_{1a}^* = .427 + j.204$$

$$S_{1b} = V_1 I_{1b}^* = .4067 + j.4324$$

$$\tilde{S}_{1a} = V_2 I_{1a}^* = (.963 \angle -5.0°)(.4732 \angle 25.536)$$

$$= \underbrace{.427 + j.16} = S_1$$

$$\tilde{S}_2 = V_2 \frac{I_{1b}^*}{a} = .963 \angle -5.1° \left(\frac{.59 \angle -46.75}{1.05}\right)^* = \underbrace{.406 + j.3616}_{S_2}$$

$$\tilde{S} = V_2 I_2^* = (.963 \angle 5.1)\left(\frac{V_2}{Z}\right)^* = .983 \angle 32° = \tilde{S}_1 + \tilde{S}_2$$

		Before TC	After T.C.	
			Circ. Approx.	Ckt. Analysis
	V_1	1	1	1
	V_2	$.943 \angle -4.86°$	$.963 \angle -4.87°$	$.963 \angle -5.1$
at V_2	P_1	.4	.398	4.27
	P_2	.4	.418	.406
	Q_1	.25	.135	.159
	Q_2	.25	.375	.362

at V_2 at V_2

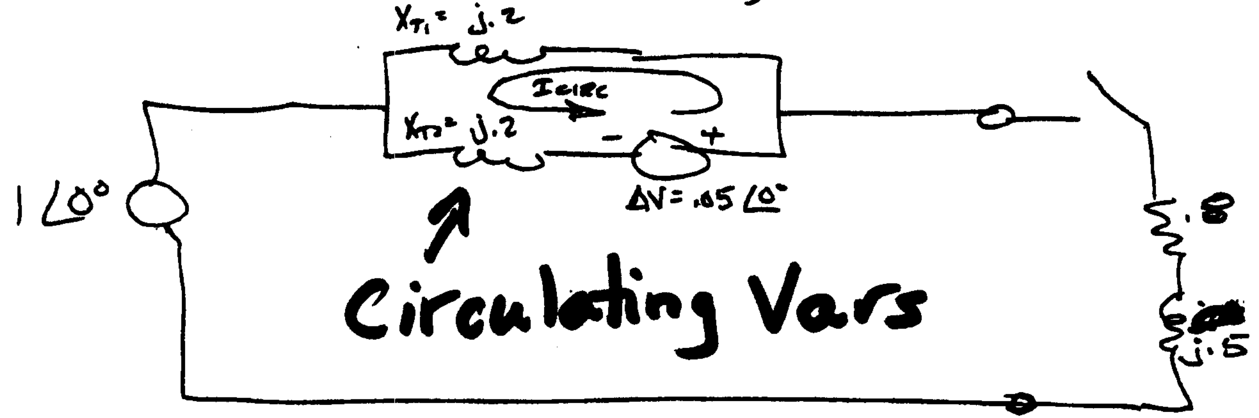
~~Tap Changing XFMR shifted Q to XFMR with tap ban to higher rated secondary voltage. higher tap setting~~

So:

Q is shifted to XFMR of higher tap setting.
P is still divided almost evenly.

Approximate solution:

Superposition of circulating current



Circulating Vars

First, look at XFMR before load applied.

$$I_{CIRC} = \frac{.05\angle 0°}{j.4} = .125 \angle -90° = \boxed{-j.125}$$

From first example,

$$I_{1a} = .4 - j.3 + \overbrace{j.125}^{I_{CIRC}} = \boxed{.4 - j.175}$$

$$I_{1b} = .4 - j.3 - j.125 = \boxed{.4 - j.425}$$

~~$V_2 = V_1 - (I_{1a})(j.2)$~~

~~$= 1 - (.4 - j.125)(j.2) =$~~

~~$= 1 - (j.08 + .025) = .98 \angle -4.6$~~

By Superposition,

$$V_2 = V_{2\,\text{BEFORE TAP CHANGE}} + \boxed{\underbrace{\frac{\Delta V (jX_{T1})}{jX_{T1} + jX_{T2}}}_{\Delta V_{oc}} \left[\frac{Z_{LOAD}}{Z_{TOTAL}} \right]}$$

Voltage at Load due to ΔV

$$\Delta V_{oc} = \frac{\Delta V (jX_{T1})}{(jX_{T1} + jX_{T2})}$$

$$Z_{th} = jX_{T1} \| jX_{T2}$$

$$= .943 \angle -4.86° + \frac{.05(j.2)}{j.4} \left[\frac{.8 + j.5}{\underbrace{.8 + j.6}_{j.5 + (jX_{T1} \| jX_{T2})}} \right]$$

$$\boxed{V_2 = .963 \angle -4.87°}$$

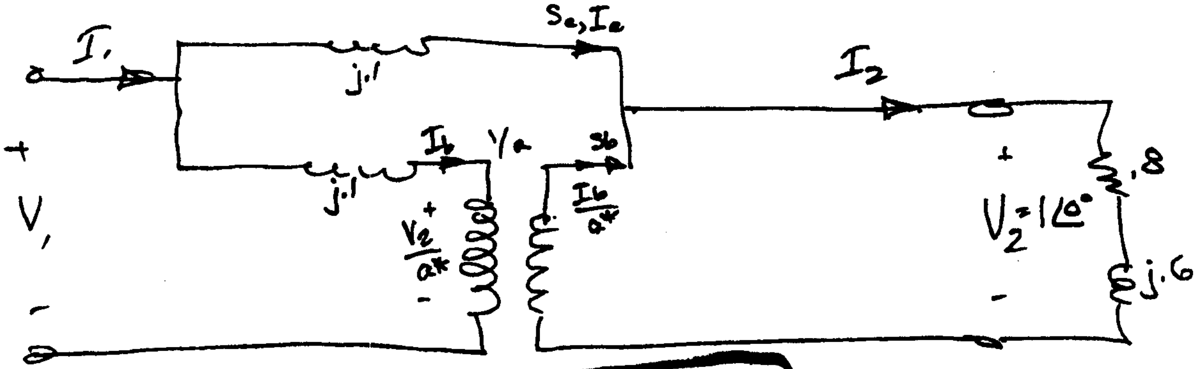
$S_{1a} = V_2 I_{1a}^* = (.963 \angle -4.87°)(.437 \angle -23.63)^* = .421 \angle$ ~~[illegible]~~ $18.76°$

$= .$ ~~[illegible] j~~ $= \boxed{.398 + j.135}$

$S_{2a} = V_2 I_{2a}^* = (.963 \angle -4.87°)(.584 \angle 46.73) = .562 \angle 41.86$

$= \boxed{.418 + j.375}$

Phase ~~[illegible]~~ Shifting XFMR



$$\boxed{a = 1 \angle 3° = e^{j\pi/60}}$$

Without phase shifter,

$$P_{LOAD} = \frac{V_2^2}{Z^*} = 1(.8+j.6) = .8+j.6$$

$S_a = .4 + j.3$

$S_b = .4 + j.3$

$I_a = .4 - j.3$

$I_b = .4 - j.3$

$V_1 = 1 \angle 0° + (.4+j.3)(j.1)$

$= 1.6307 \angle 2.2°$

With Phase Shifter,

$$\frac{V_1 - 1}{j.1} + \frac{V_1 - \frac{1}{1\angle 3°}}{j.1}\left(\frac{1}{1 \angle 3°}\right) = \frac{1}{.8+j.6} = I_2$$

$\frac{1}{a^*}$

$$V_1\left(1 + \frac{1}{1\angle 3°}\right) - 1 - \left(\frac{1}{1 \angle 3°}\right)^2 = j.1/(.8+j.6)$$

$V_1(1.999 \angle -1.5°)$ ~~$-1-(.9945-j.1045) = .1\angle 53.13 + 1.9945 - j.1045$~~

~~$V_1 = 2.0$ [illegible] $/ 1.999 \angle -1.5°$~~ ~~$= 1.028 + j.016$~~

$$\boxed{V_1 = 1.031 \angle .73° = 1.031 + j.013}$$

~~$I_a = \frac{1.028 + j.016 - 1}{j.1} = .322 \angle -60.3 = .159 - j.28$~~

$$I_a = \frac{1.031 + j.013 - 1}{j.1} = .131 - j.311$$

$$\frac{I_b}{a^*} = I_2 - I_a = .8 - j.6 - .131 + j.311 = \boxed{.669 - j.289}$$

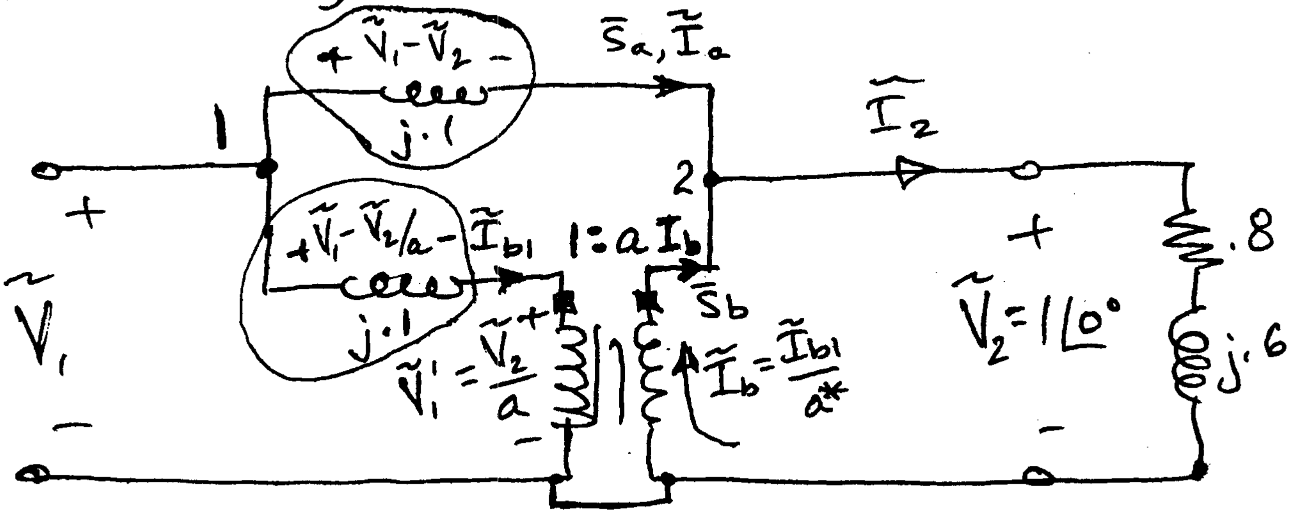
$$\boxed{\begin{aligned} S_a &= .131 + j.311 \\ S_b &= .669 + j.289 \end{aligned}}$$

NOTE SHIFT IN POWER FLOW THRU XFMRS

	BEFORE	AFTER
I_a	$.4 + j.3$	$.131 - j.311$
I_b/a^*	$.4 - j.3$	$.669 - j.289$
P_a	$.4$	$.131$
P_b	$.4$	$.669$
Q_a	$+.3$	$.311$
Q_b	$+.3$	$.289$
V_1	$1.031 \angle 2.2^\circ$	$1.031 \angle .73^\circ$
V_2	$1.0 \angle 0^\circ$	$1.0 \angle 0^\circ$

if a is at positive angle, power flow through phase shifting XFMR increases

Phase-Shifting XFMR:



Solve:

KCL at Node 2

$$\tilde I_2 = \underbrace{\frac{\tilde V_2}{.8+j.6}}_{\tilde I_2} = \underbrace{\frac{\tilde V_1 - \tilde V_2}{j.1}}_{\tilde I_a} + \underbrace{\underbrace{\frac{\tilde V_1 - \frac{\tilde V_2}{a}}{j.1}}_{\tilde I_{b1}}\left(\frac{1}{a^*}\right)}_{\tilde I_b = \tilde I_{b1} \div a^*}$$

Since $\tilde V_2$ is known (it's typical to state desired load voltage and then solve for required load voltage $\tilde V_1$) we can solve for $\tilde V_1$.

Since $\tilde V_2 = 1\angle 0°$,

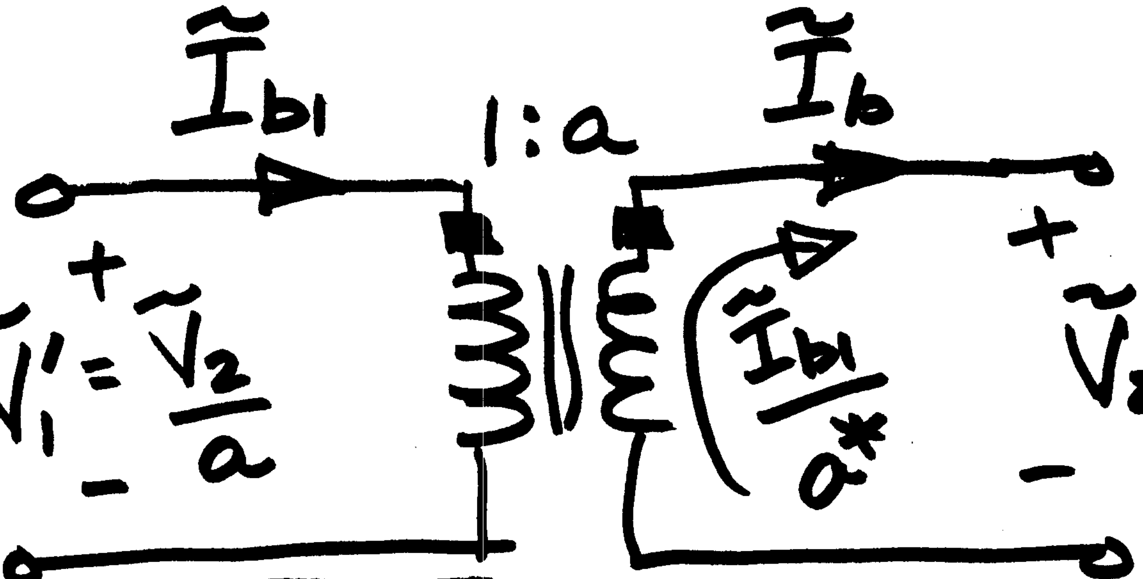
$$\frac{1}{.8+j.6} = \frac{\tilde V_1 - 1}{j.1} + \left(\frac{\tilde V_1 - 1/a}{(j.1)}\right)\frac{1}{a^*}$$

Multiplying through by j0.1,

$$\frac{j.1}{(8+j6)} = \tilde V_1 - 1 + \left(V_1 - \frac{1}{a}\right)\frac{1}{a^*}$$

Grouping, $\tilde V_1\left(1+\frac{1}{a^*}\right) - 1 - \frac{1}{|a|^2} = j\frac{0.1}{(.8+j.6)}$

Take a close look at the PS transformer:



$$\boxed{\begin{gathered}\tilde{V}_1'\tilde{I}_{b1}^* = \tilde{V}_2\tilde{I}_b^* \\ \bar{S}_1 = \bar{S}_2\end{gathered}}$$

$$a = \frac{\tilde{I}_{b1}^*}{\tilde{I}_b^*}$$

"a" is defined as the voltage ratio:

$$a = \frac{\tilde{V}_2}{\tilde{V}_1'} \Longrightarrow \boxed{\tilde{V}_1' = \frac{\tilde{V}_2}{a}}$$

$$a^* = \frac{\tilde{I}_{b1}}{\tilde{I}_b}$$

$$\boxed{\tilde{I}_{b1} = a^*\tilde{I}_b}$$

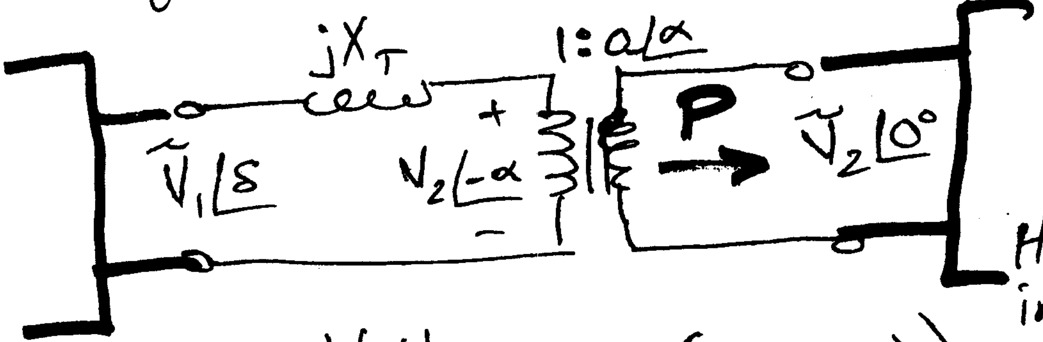
Or, $$\tilde{V}_1 = \frac{\frac{j0.1}{(.8+j.6)} + 1 + \frac{1}{|a|^2}}{\left(1+\frac{1}{a^*}\right)}$$

If Load Impedance & transformer reactance are expressed as variables,

$$\tilde{V}_1 = \left.\frac{\frac{jX_T}{Z_{LOAD}} + 1 + \frac{1}{|a|^2}}{1+\frac{1}{a^*}}\right| \tilde{V}_2 = 1\angle 0^\circ$$

After $\tilde{V}_1$ is known, all currents, P_S, Q_S can be solved for. (see next page).

We note that as the angle of a increases, so does the power flow thru XFMR #2. This is easy to explain using the power flow equation for a short lossless line:



$$P = \frac{V_1 V_2}{X}\underbrace{\sin(\delta-(-\alpha))}_{}$$

Hence, as α increases, so does P.

PS-3

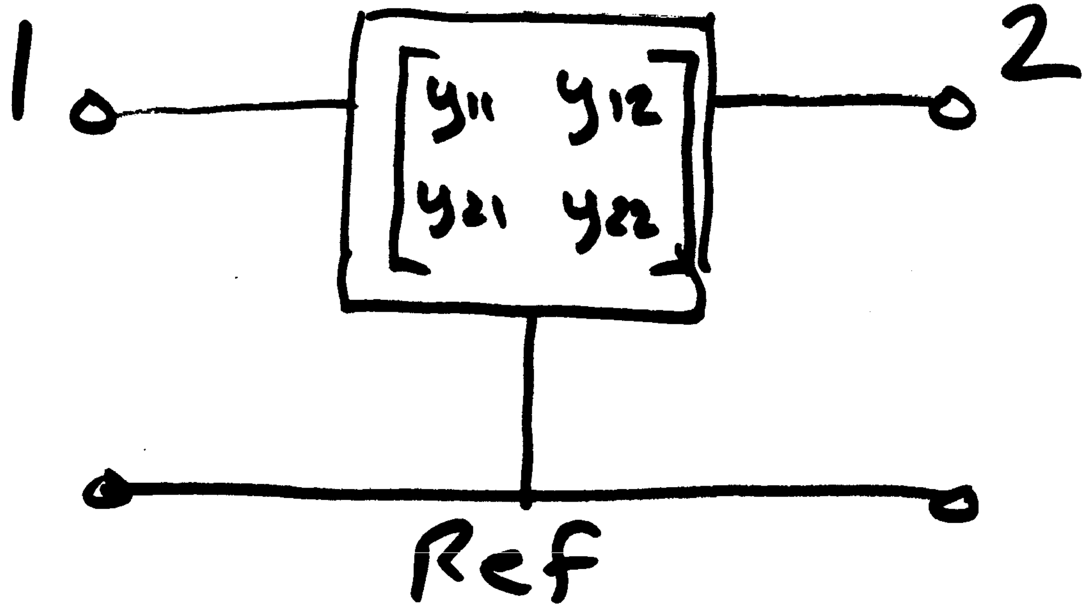
Summary of Effect of Phase Shift in Paralleled Transformers:

Quantity	Case #1	Case #2	Case #3
Turns Ratio "a"	1/-3°	1/0°	1/+3°
V_1	1.031 /3.72°	1.031/2.22°	1.031/0.72°
V_2	1/0°	1/0°	1/0°
I_1	1.009/-35.99°	1.0/-36.87°= I_2	1.009/-38.99°
I_2	1.0/-36.87°	1.0/-36.87°	1.0/-36.87°
I_{b1}	0.337 /-64.23°	0.5/-36.87°	0.730/-26.38°
I_b	0.337 /-67.24°	0.5/-36.87°	0.730/-23.38°
$I_{a1} = I_a$	0.730 /-23.38°	0.5/-36.87°	0.337/-67.24°
P_1	0.8000	0.8000	0.8000
P_2	0.8000	0.8000	0.8000
P_{a1}	0.6697	0.4000	0.1303
P_a	0.6697	0.4000	0.1303
P_{b1}	0.1303	0.4000	0.6697
P_b	0.1303	0.4000	0.6697
Q_1	0.6646	0.6500	0.6646
Q_2	0.6000	0.6000	0.6000
Q_{a1}	0.3428	0.3250	0.3218
Q_a	0.2895	0.3000	0.3105
Q_{b1}	0.3218	0.3250	0.3428
Q_b	0.3105	0.3000	0.2895

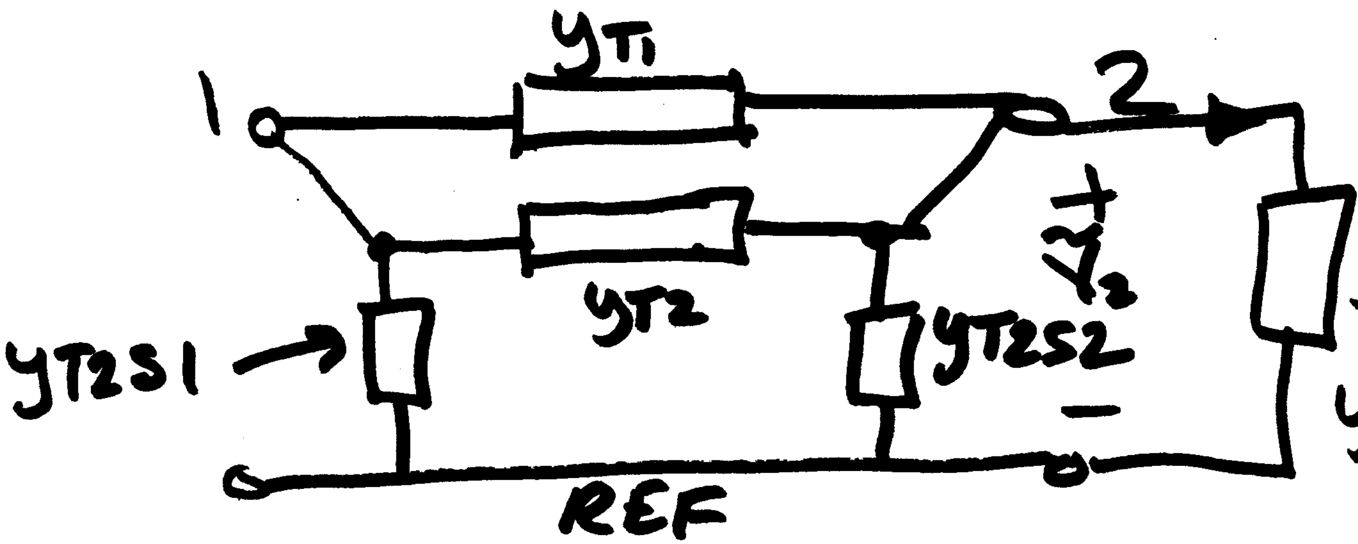
$Q_{T1} =$

$|I_a|^2 X_{T1}$

$0.5^2(.1) = \underline{.025}$



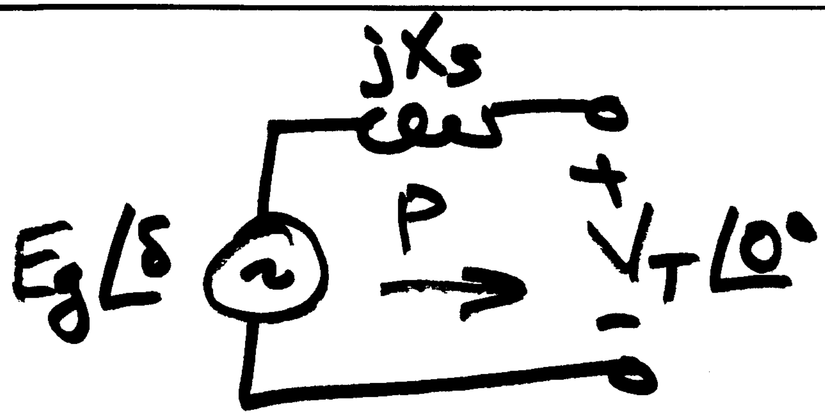
$$y_{11} = y_{11} + y_{T1}$$
$$y_{22} = y_{22} + y_{T1}$$
$$y_{12} = y_{12} - y_{T1}$$
$$y_{21} = y_{21} - y_{T1}$$

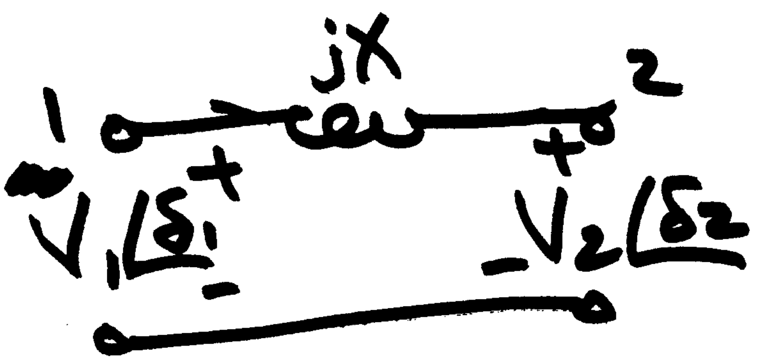


$$y_L = \frac{1}{R+jX}$$

$$y_{22} = y_{22} + y_L$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ 1\angle 0^\circ \end{bmatrix} = \begin{bmatrix} \tilde{I}_1 \\ \underset{0}{\cancel{\tilde{I}_2}} \end{bmatrix}$$





$$P = \frac{E_g V_T}{X_s}\sin(\delta)$$

$$P_{1\to 2} = \frac{V_1 V_2}{X_s}\sin(\delta_1-\delta$$