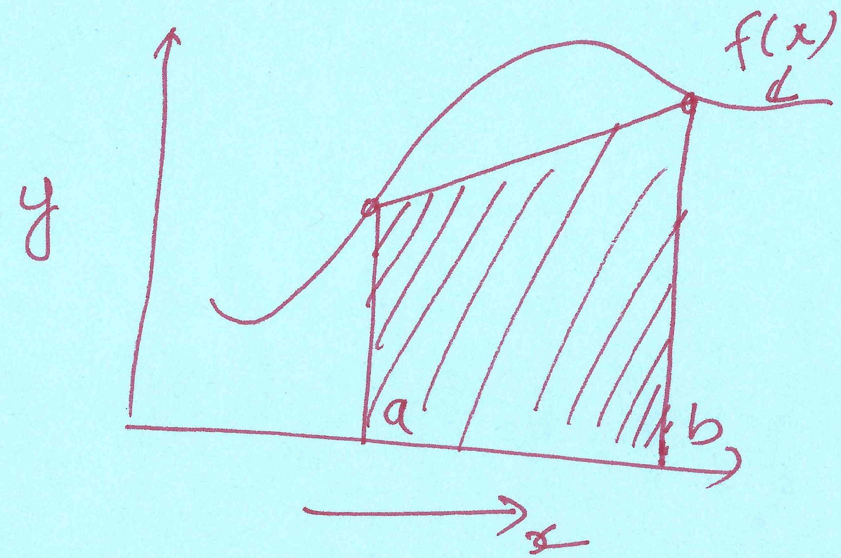
Trapezoidal Method

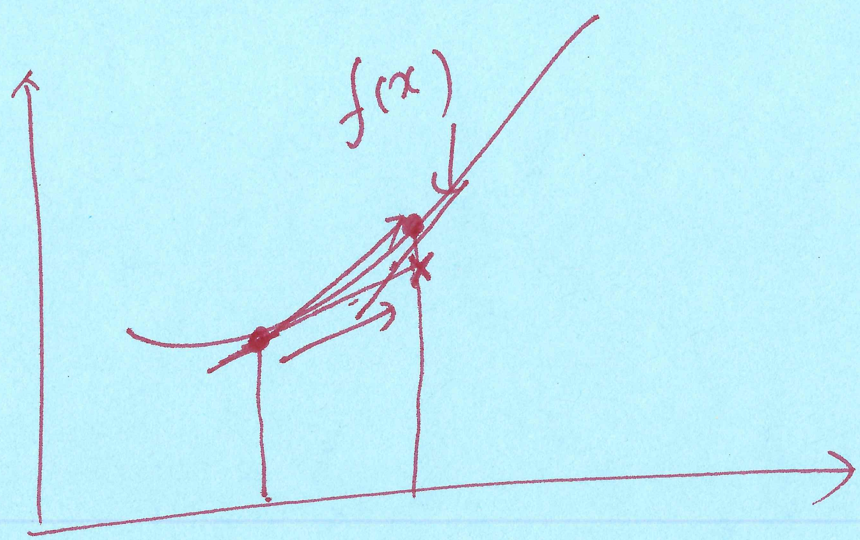


$$\int_a^b f(x) = \left\{ \frac{f(a) + f(b)}{2} (b-a) \right\}$$

Euler's method

$\dot{x}_1 = f(x)$

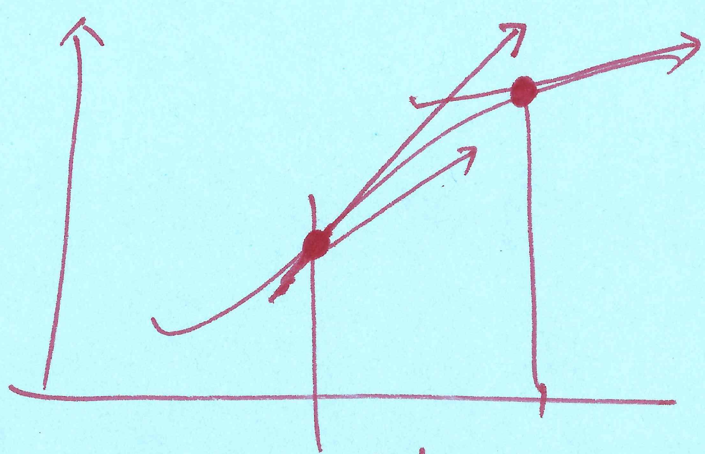
$$x_1 = \boxed{x_0 + \Delta t\,(\dot{x}_0)} + \frac{\Delta t^2}{2!}(\ddot{x}_0) + \cdots$$



Modified Euler's

predicter step

$$x_1^p = x_0 + \dot{x}_0 \Delta t$$



Correcter step

$$x_1^c = x_0 + \frac{1}{2}(\dot{x}_0 + \dot{x}_p)\Delta t$$

Runge - Kutta (RK 2)

(RK4)

RK 2

$$x_1 = x_0 + \Delta x$$

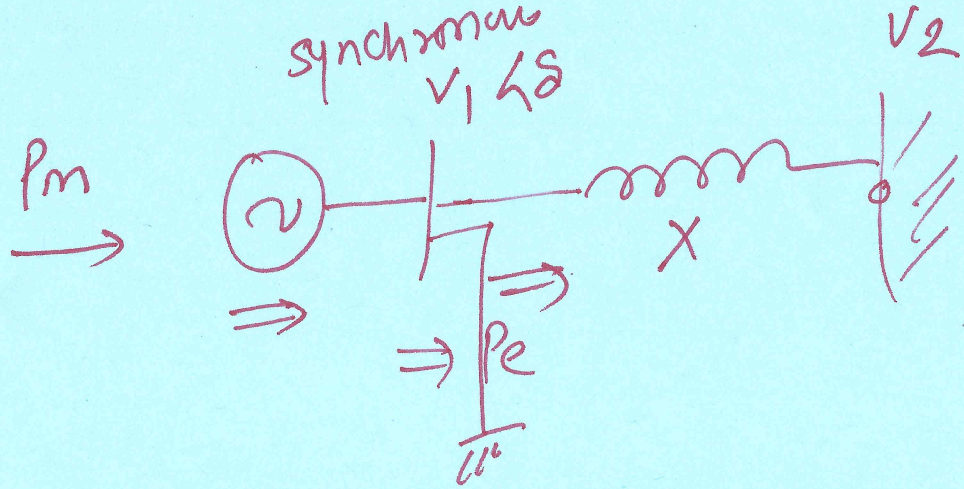
$$= x_0 + \frac{k_1 + k_2}{2}$$

$$k_1 = f(x_0, t_0)\,\Delta t$$

$$k_2 = f(x_0 + k_1, t_0 + \Delta t)\,\Delta t$$

Swing Equation

$t=0 \quad t=t_1$



grid

stiff voltage (ideal

$$\left[\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \right]$$

acceleration power

$P_m - P_e$

$\frac{V_1 V_2}{X} \sin\delta$

$\delta(t)$

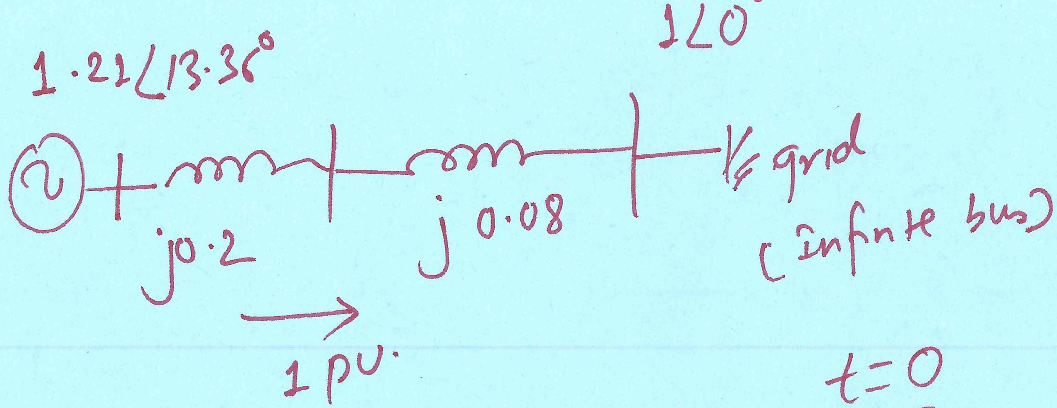
$\omega(t)$

$$\frac{d\delta}{dt} = \omega - \omega_{syn}$$

$$\frac{d\omega}{dt} = \frac{P_m - P_{max}\sin\delta}{M}$$

$P_e =$

$\delta(t)$
$\omega(t)$



$\underline{\underline{t=0}}$

$$P_e = P_m \sin\delta$$

$$= \frac{V_1 V_2}{X} \sin\delta$$

$$= 4.321 \sin\delta$$

$$\delta_0 = 13.36^\circ$$

$$\frac{1}{M} = 47.123$$

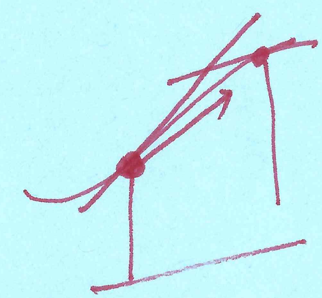
$$\frac{\pi f_0}{H} = 4$$

$\delta_0 = 13.365^\circ$ $\quad\quad \omega_0 = 2\pi f_0 = 376.99 \text{ rad s}^{-1}$

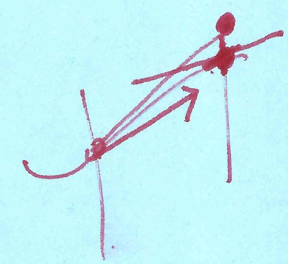
$\Delta t = 0.02$ seconds

Euler's $\quad\quad$ predictor $\quad\quad$ corrector

$$\delta^{p}_{0.02} = \delta_0 + \left.\frac{\partial \delta}{\partial t}\right|_0 \Delta t$$



$$\omega^{p}_{0.02} = \omega_0 + \left.\frac{\partial \omega}{\partial t}\right|_0 \Delta t$$



$$\left.\frac{\delta \delta}{\delta t}\right|_{t=0} = ? \quad 0$$

$$\delta^{p}_{0.02} = 13.365^\circ$$

$$\omega'_{0.02} = 376.99 + \left(\frac{1 - 4.321 \sin 13.365^{\circ}}{M} \right) \Bigg|_{\delta = 0}$$

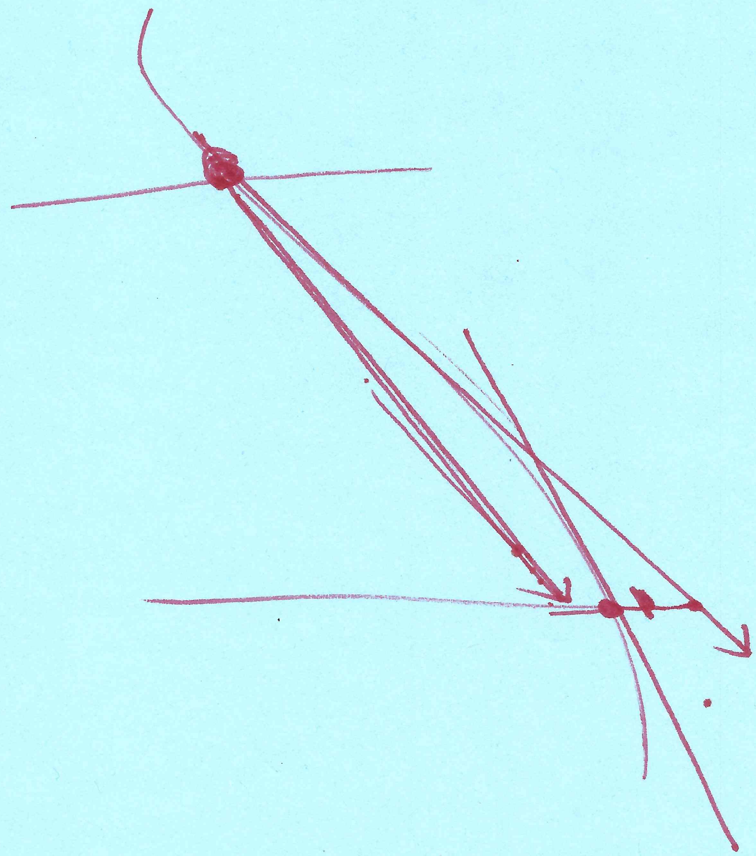
$$= 377.935 \text{ rad/second.}$$

Corrector step

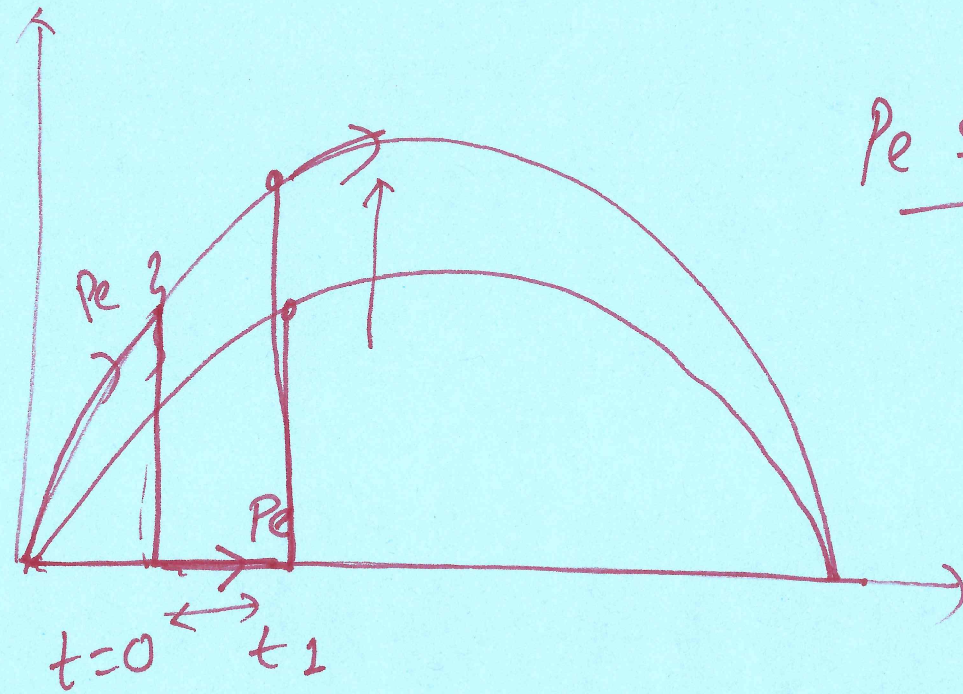
$$\left.\frac{d\delta}{dt}\right|_{0.02} = \omega - \underbrace{2\pi f_0}_{2\pi \times 60} \Big|_{0.02}$$

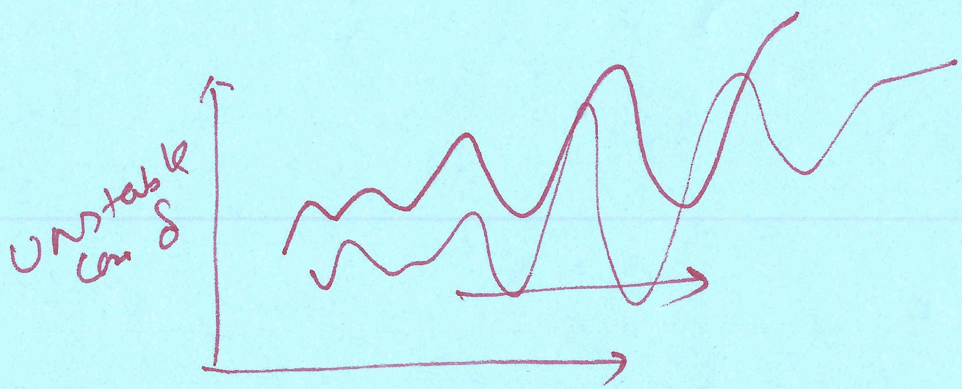
$$= \underline{\underline{0.95}}$$

$$\left.\frac{d\omega}{dt}\right|_{0.02} = \left(\frac{P_m - P_e}{M} \right) \Bigg|_{0.02} = -0.0558$$



Pe sin δ



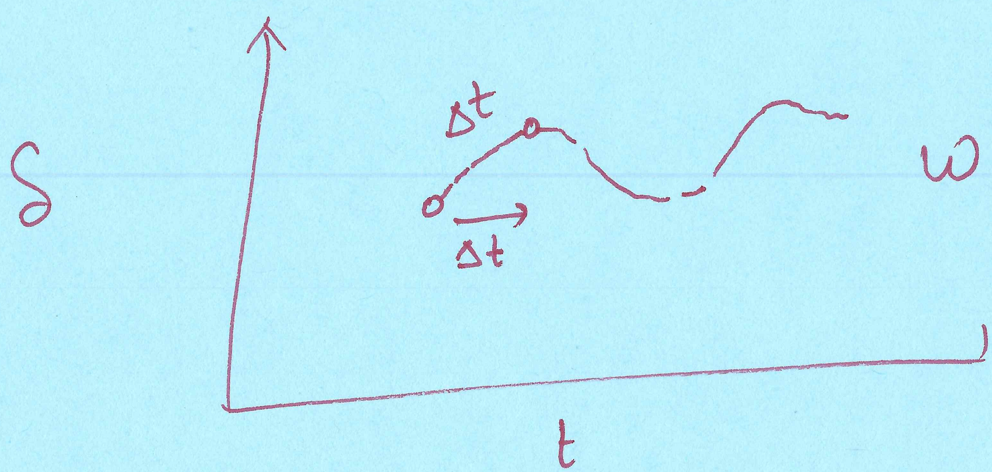


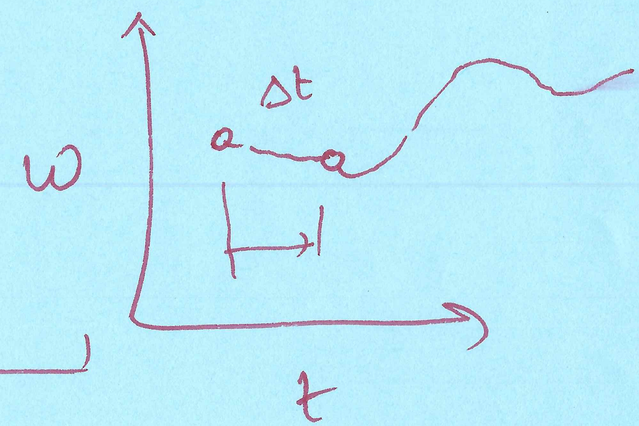
$$\delta^{c}_{0.02} = \delta_0 + \frac{\Delta t}{2}\left[\left.\frac{d\delta}{dt}\right|_0 + \left.\frac{\delta\delta}{dt}\right|_{0.02} \right]$$

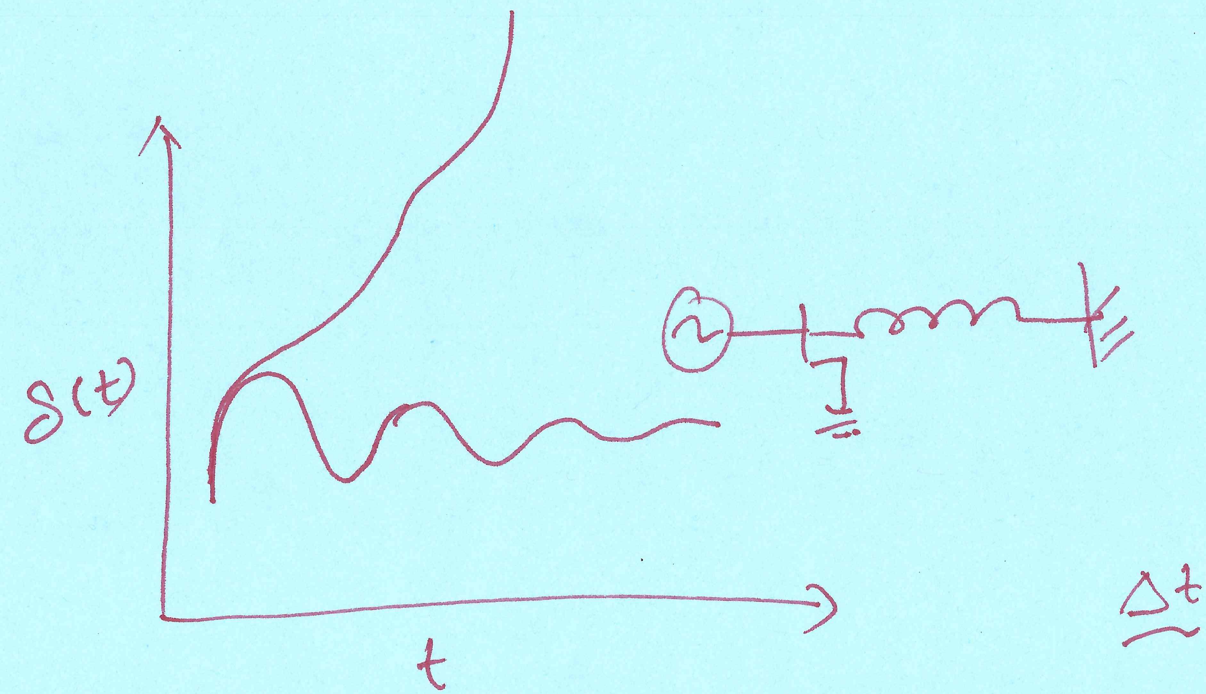
$$= 13.815^{\circ}$$

$$\omega^{c}_{0.02} = \omega_0 + \frac{\Delta t}{2}\left[\left.\frac{d\omega}{dt}\right|_0 + \left.\frac{d\omega}{dt}\right|_{0.02} \right]$$

$$= 377.46 \text{ rad/seconds}$$

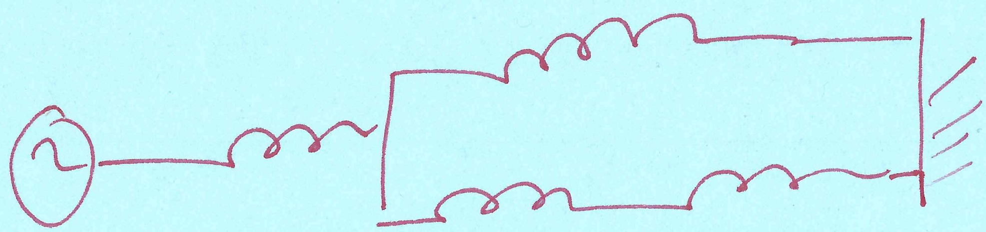


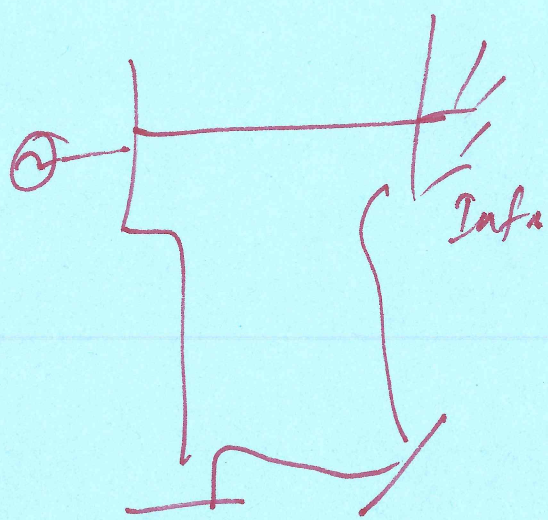



δ(t)
t
Δt

$$\dot{\omega}_i = \frac{1}{M_i}\left(P_{m_i} - \underbrace{E_i^2 G_{ii} - E_i \sum_{j \neq i} E_j \left(B_{ij} \sin \delta_{ij} + G_{ij} \cos \delta_{ij} \right)} \right)$$

$$\dot{\delta}_i = \omega_i - \omega_s \qquad i = 1 \ldots n$$





$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

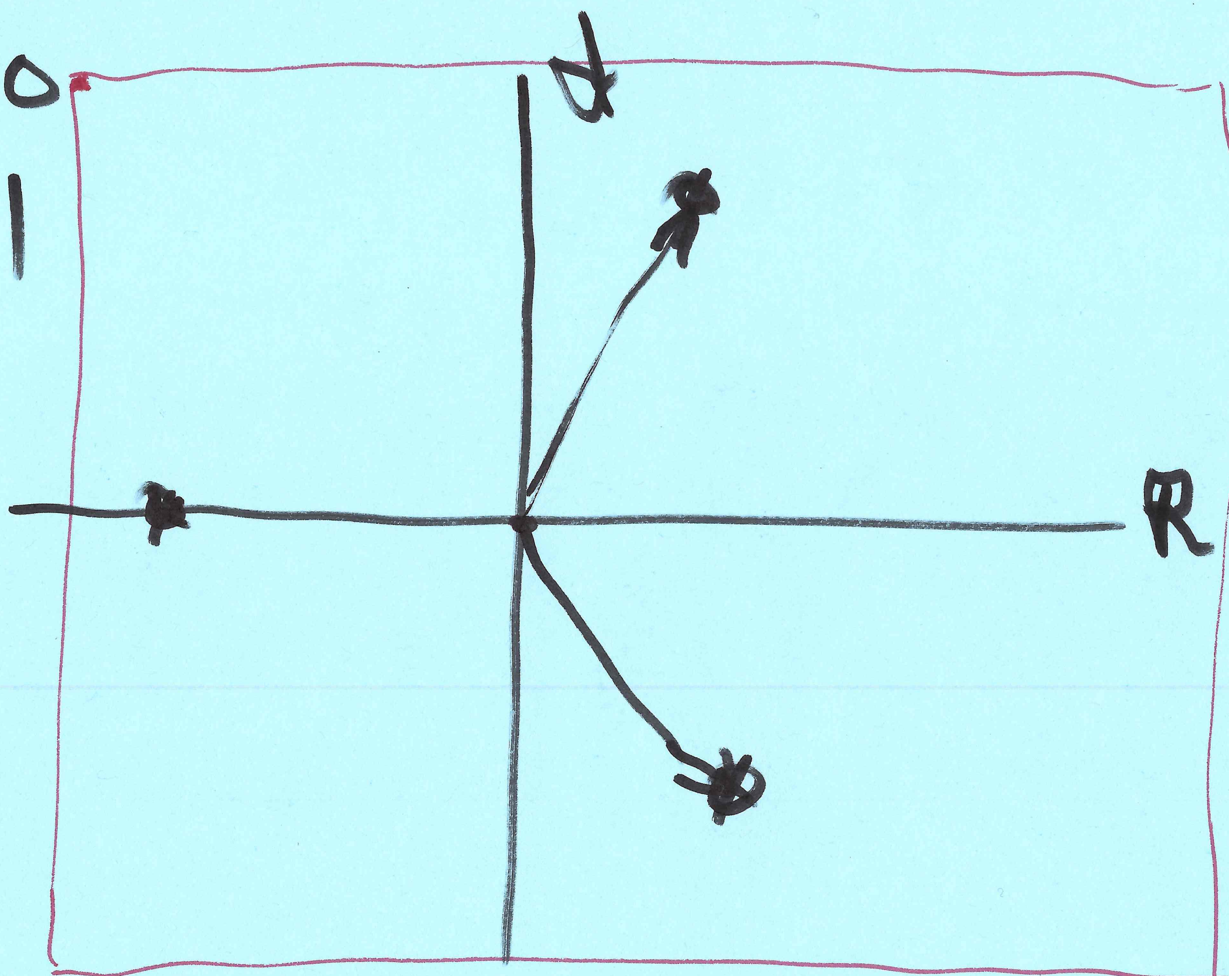
NEWTON-RAPHSON

- Convergence
- Initial value

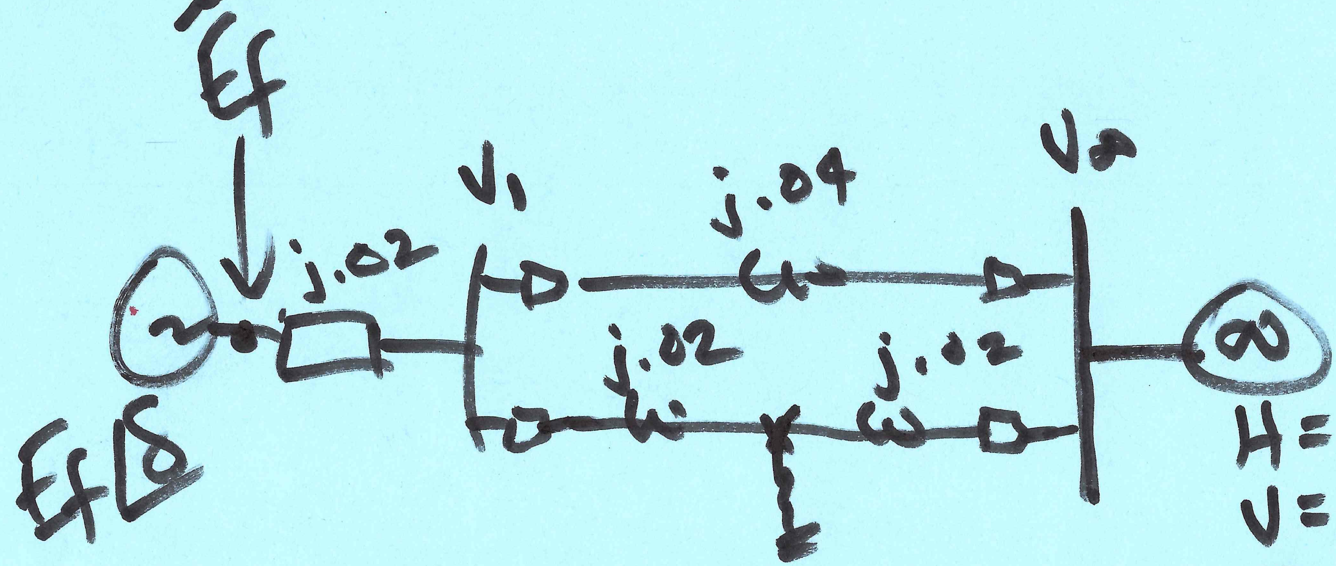
$[x]^0$

$z^3+1=0$

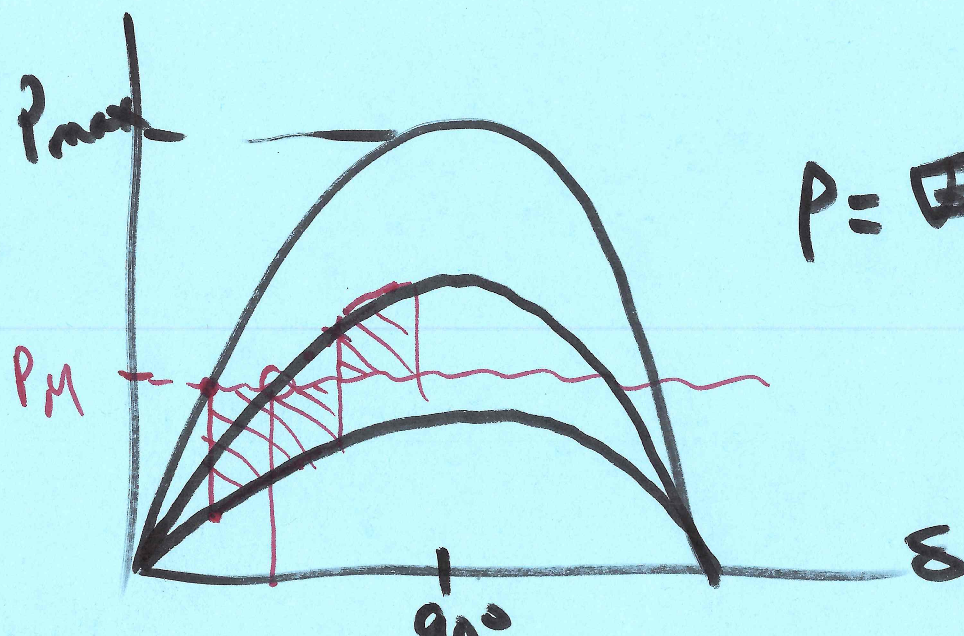
$z^3=-1$



STAB



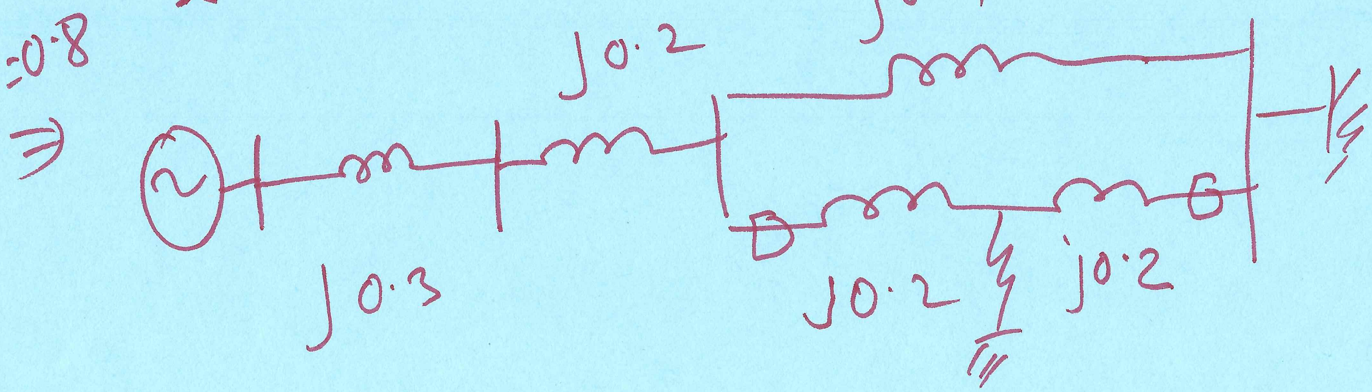
$H = \infty$
$V = \text{Const}$
$Z = 0$

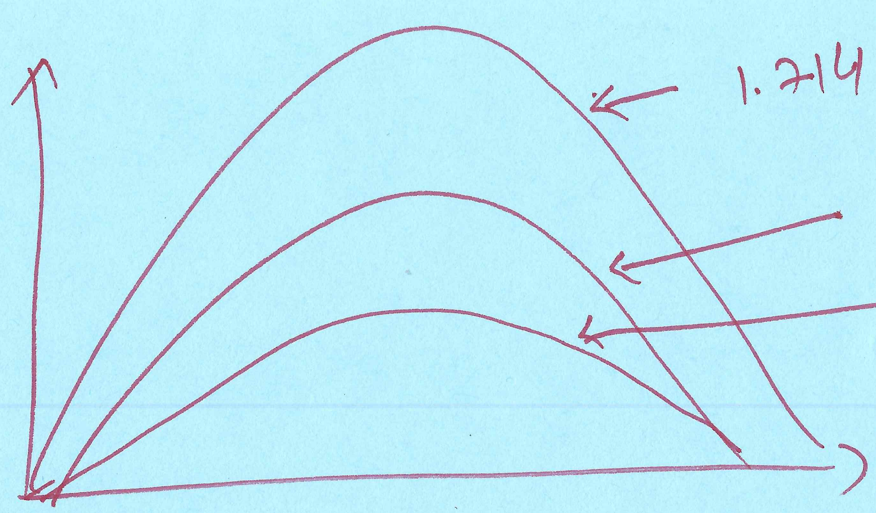


$$P = \cancel{E_f}\,\frac{E_f V_\infty}{X_{XFR}} \sin\delta$$

$P_m = 0.8$

$1.0\angle\delta$





$$M \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$M \frac{d^2\delta}{dt^2} + D(\delta) \frac{d\delta}{dt} = P_m - P_e.$$

Multi-machine swing

$$\overset{\circ}{\omega}_i = \frac{1}{M_i}\left(P_{mi} - \quad \right)$$

$$\overset{\circ}{\delta}_i = \omega_i - \omega_s \qquad i = 1 \ldots n.$$

Energy function