- Optimization Problem in General
- Some terminologies
- Linear Programming
- Non-linear Programming (ELD)
- fmincon( )
- Linear Programming Example in power system

General Optimization Structure

$$\text{Minimize :} \qquad f(x)$$

$$\text{Subjected to :} \qquad g_k(x) = 0;\ k = 1, 2, ..., p$$

$$h_j(x) \leq 0;\ j = 1, 2, ..., m$$

Linear Programming

Minimize : $c^T x$

Subjected to : $A x \leq b$

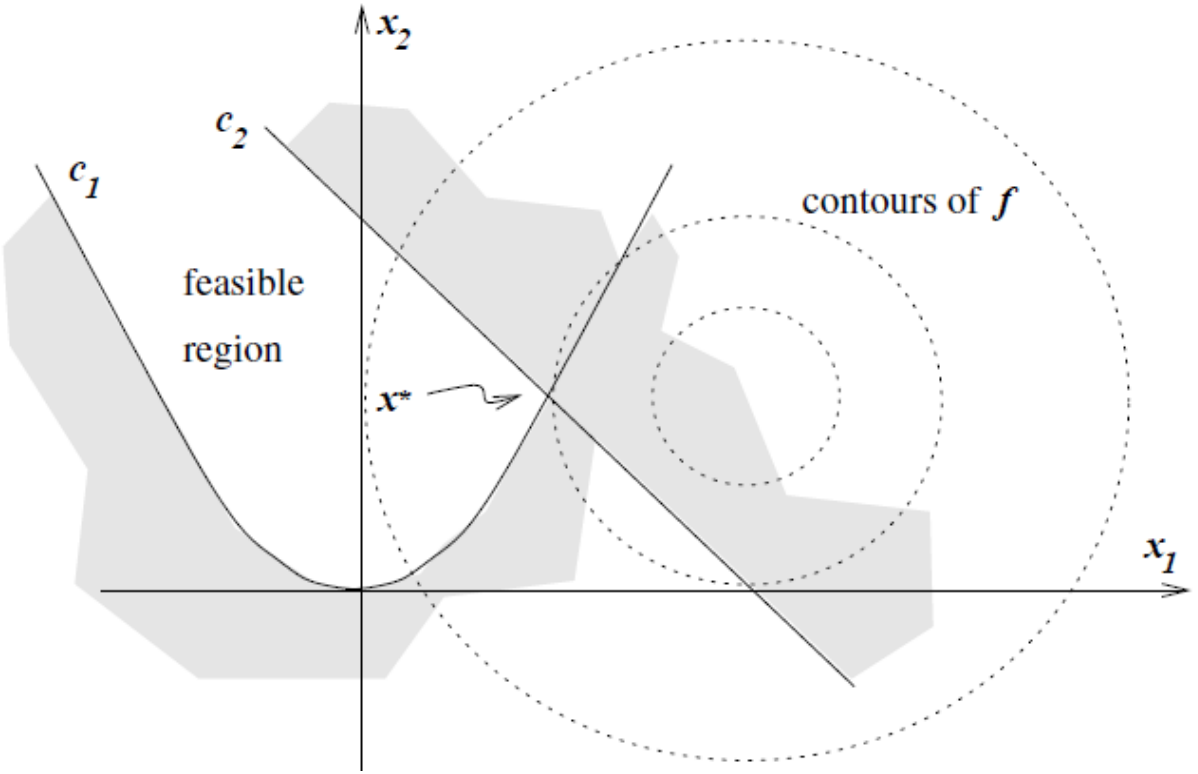
Mixed- Integer Linear Programming

Minimize : $c^T x + d^T y$

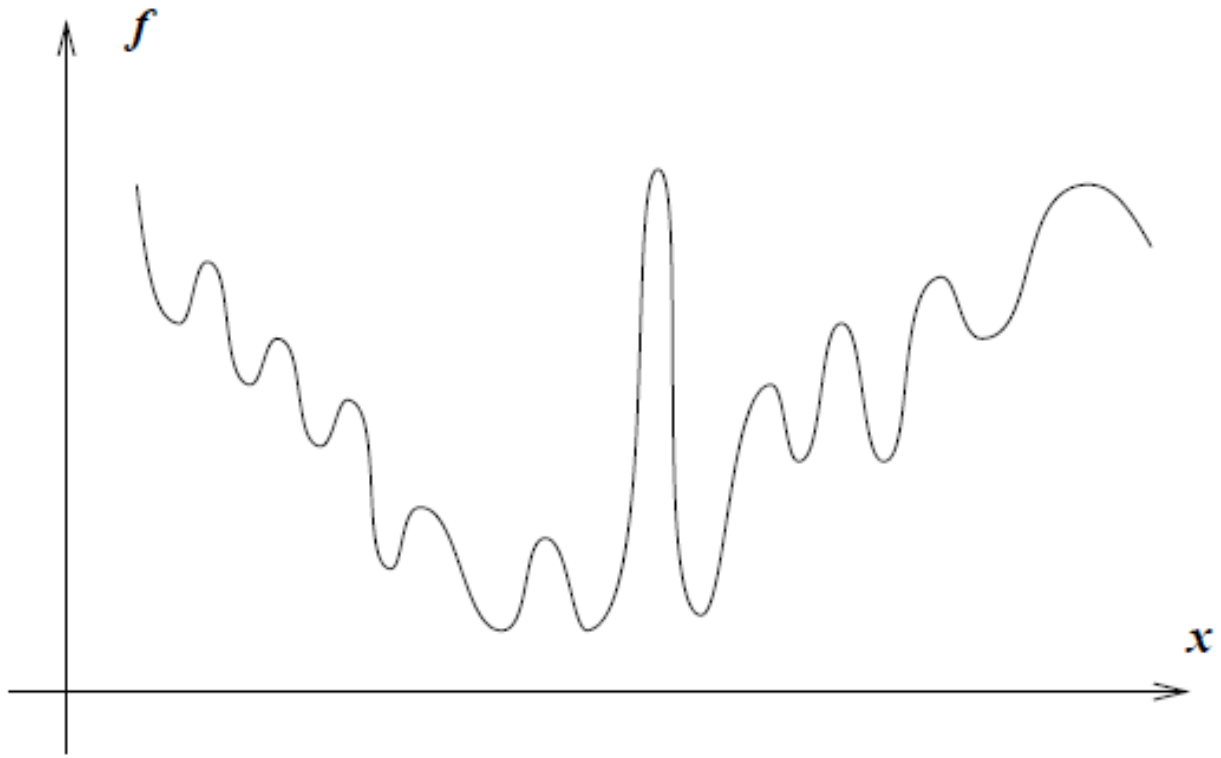
Subjected to : $A x + B y \leq b$

Feasible Solutions, Optimal Solution

$$\min\ (x_1 - 2)^2 + (x_2 - 1)^2 \qquad \text{subject to} \begin{cases} x_1^2 - x_2 & \leq 0, \\ x_1 + x_2 & \leq 2. \end{cases}$$

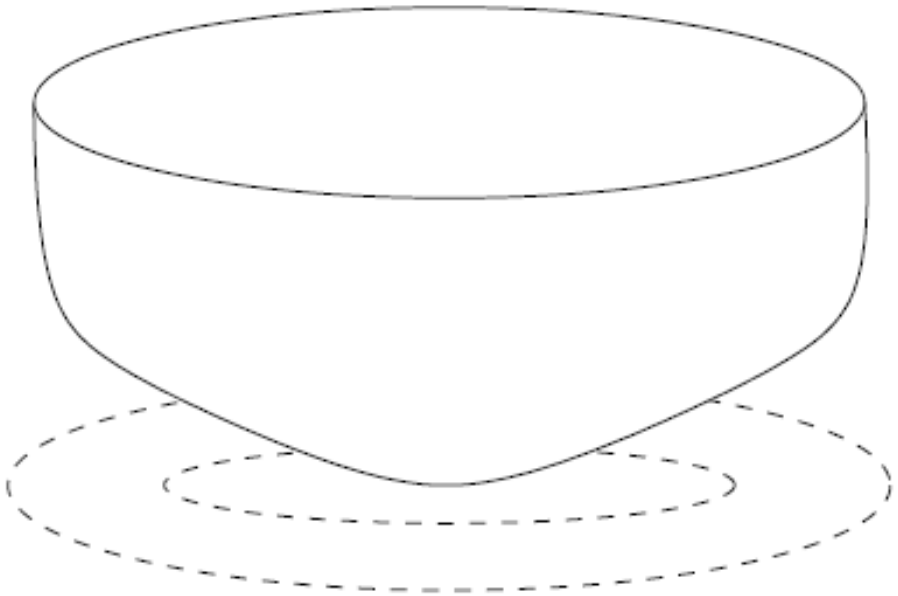


Local Optimal, Global Optimal



Convex Function

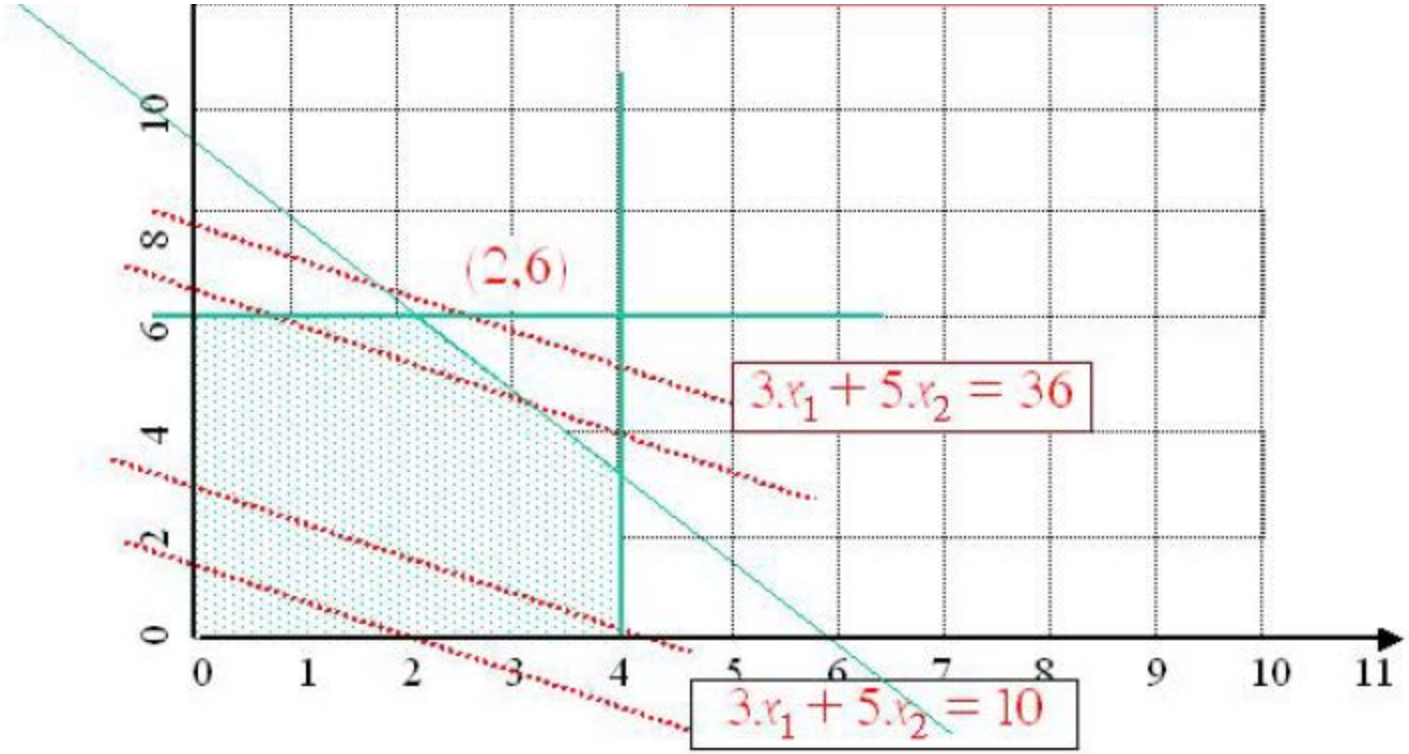
$(x_1-6)^2+\frac{1}{25}(x_2-4.5)^4$.

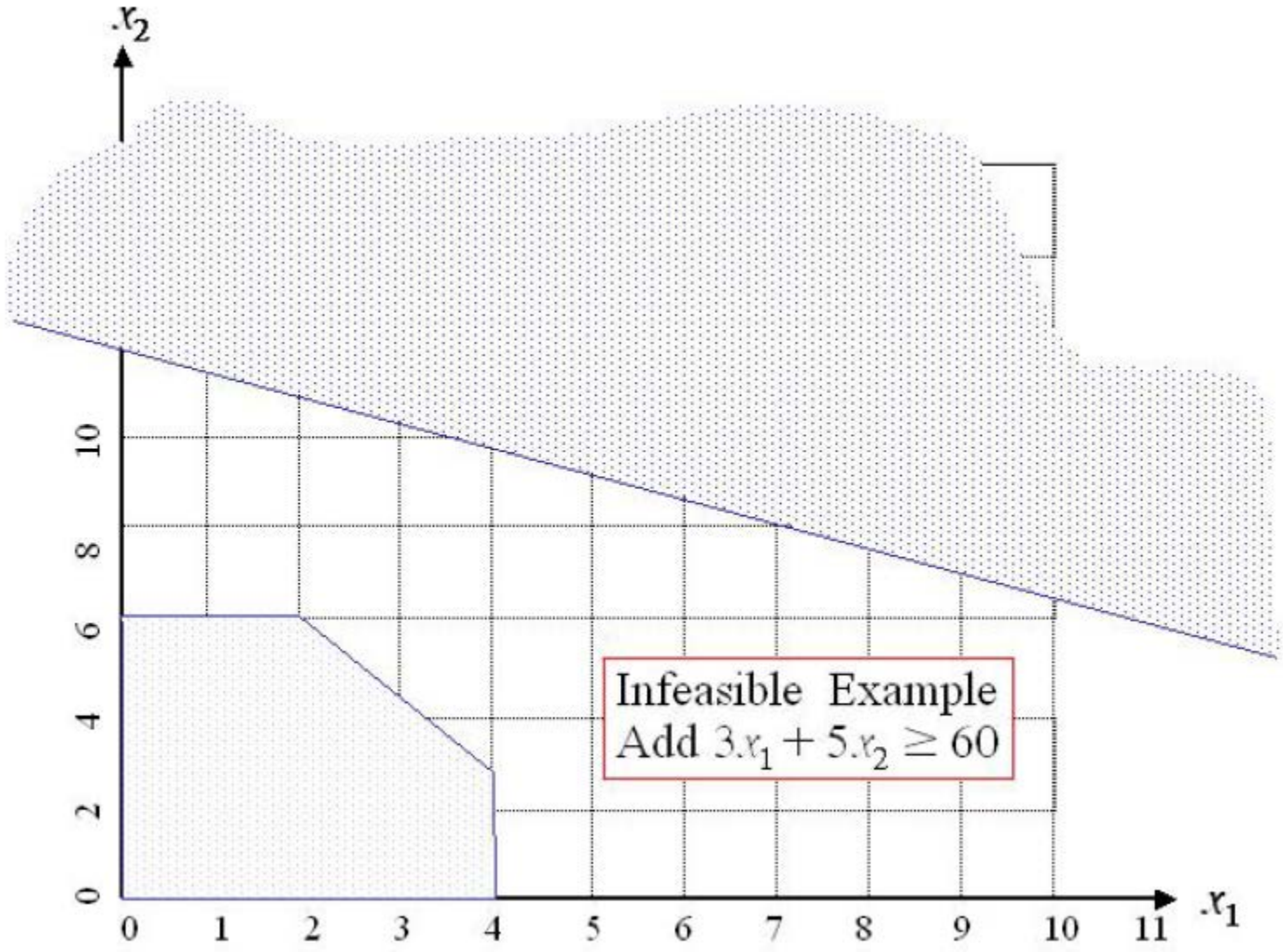


Linear Programming:

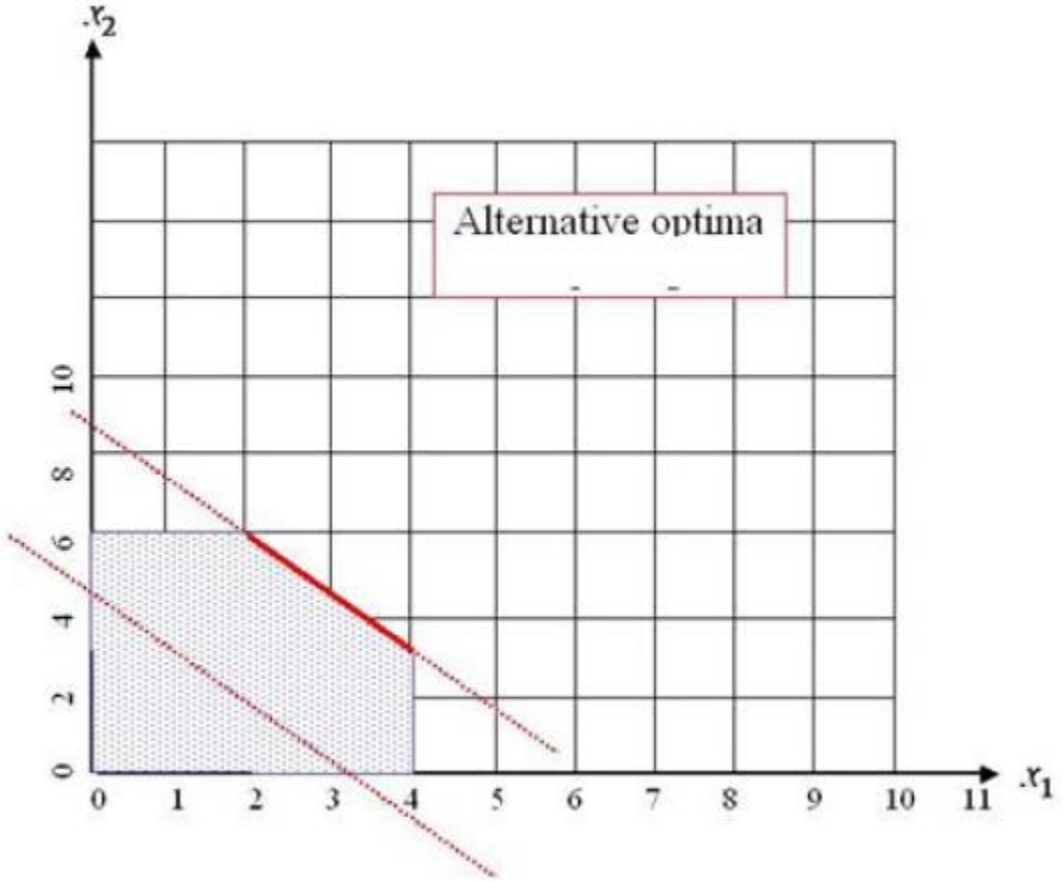
- **Graphical Solution Approach**

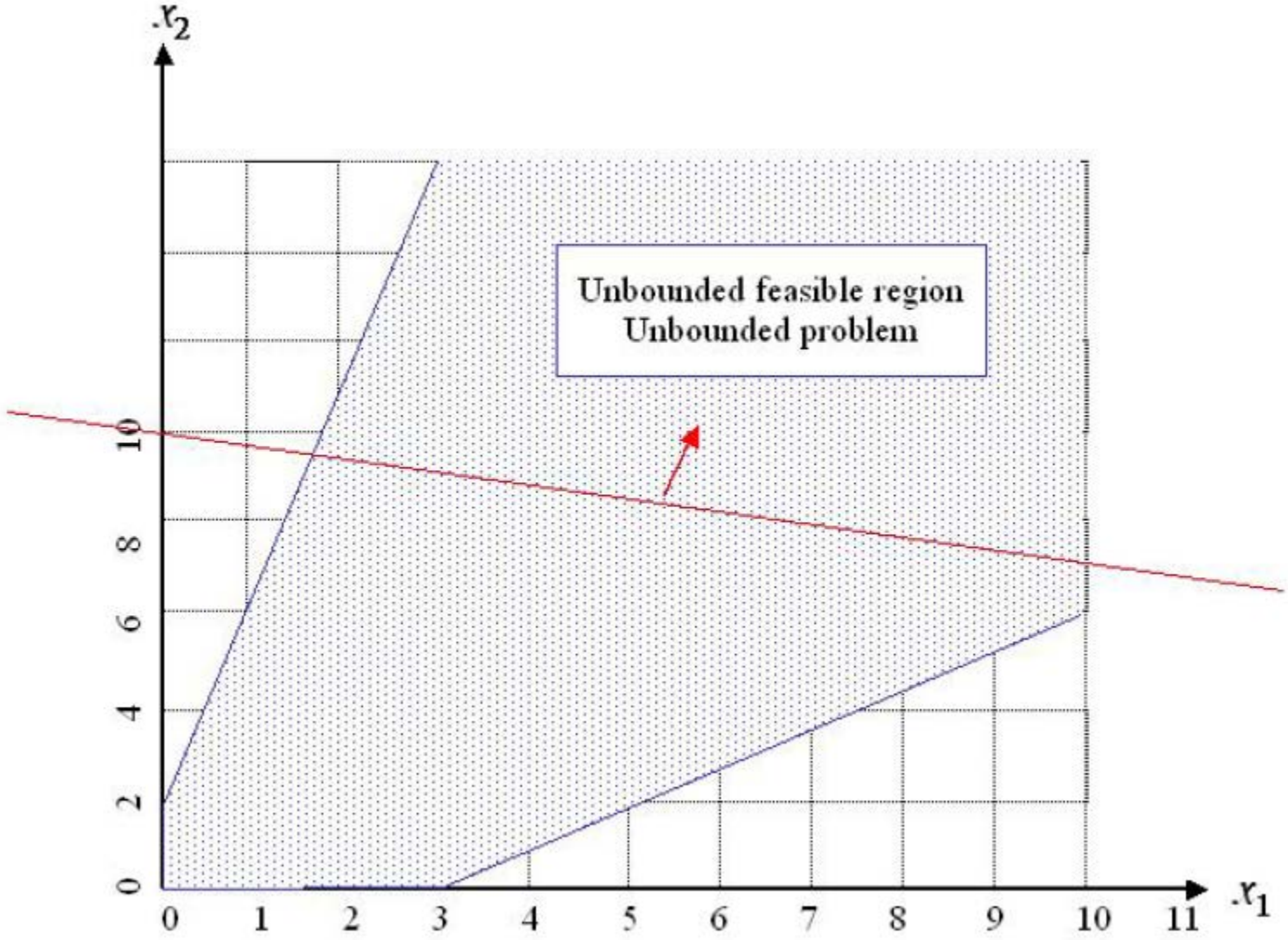
$$
\begin{array}{lrrl}
\max & 3x_1 & +5x_2 & \\
s.t. & & & \\
& x_1 & & \leq 4 \\
& & 2x_2 & \leq 12 \\
& 3x_1 & +2x_2 & \leq 18 \\
& x_1, & x_2 & \geq 0
\end{array}
$$


(2,6)
$3.x_1 + 5.x_2 = 36$
$3.x_1 + 5.x_2 = 10$
0
1
2
3
4
5
6
7
8
9
10
11
0
2
4
6
8
10


x_2
0
2
4
6
8
10
0
1
2
3
4
5
6
7
8
9
10
11
x_1
Infeasible Example
Add $3x_1 + 5x_2 \geq 60$

$$
\begin{array}{lrll}
\max & 3x_1 & +2x_2 & \\
s.t. & & & \\
 & x_1 & & \leq 4 \\
 & & 2x_2 & \leq 12 \\
 & 3x_1 & +2x_2 & \leq 18 \\
 & x_1, & x_2 & \geq 0
\end{array}
$$




x_2
Unbounded feasible region
Unbounded problem
0
2
4
6
8
10
0
1
2
3
4
5
6
7
8
9
10
11
x_1

ELD

$$C_1 = 0.00128\,P_1^2 + 6.48\,P_1 + 459$$

$$C_2 = 0.00194\,P_2^2 + 7.85\,P_2 + 310$$

$$C_3 = 0.00482\,P_3^2 + 7.97\,P_3 + 78$$

$$P_1 + P_2 + P_3 = P_D \Downarrow 850$$

MATLAB:

Min $f(x)$

nonlcon $\Leftarrow$ [$c(x) \leq 0$ → nonlinear inequality

$Ceq(x) = 0$ → " equality

$Ax \leq b$ → linear inequality

$Aeq\,x = beq$ → " equality

$lb \leq x \leq ub$ ⇒ Bounds

$[x_{opt}, y_{opt}] =$ fmin(@function, X0, A, b, Aeq, beq, lb, ub, @nonlcon)

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{A_{eq}} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 850 \end{bmatrix}}_{B_{eq}}$$

With limits:

$$lb = [150 \quad 100 \quad 50]$$
$$ub = [600 \quad 400 \quad 200]$$

DCOPF:

Same Cost functions

C_1
C_2
C_3

$PD1 = 250$
$PD2 = 400$
$PD3 = 200$

```
% Linear inequalities
A=[];
b=[];
% Linear equalities
Aeq=[1 1 1];
beq=[850];
x0=[500 200 10];
% lower and upper bound
lb=[];
ub=[];
lb=[150 100 50];
ub=[600 400 200];
[xopt, fopt] = fmincon(@ELD,x0,A,b,Aeq,beq,lb,ub);
%[xopt,fopt,flag,somethinguseful,lagrange] =
fmincon(@ELD,x0,A,b,Aeq,beq,lb,ub);
```

```
function [f]=ELD(x)
c1=0.00128*x(1)*x(1)+6.48*x(1)+459;
c2=0.00194*x(2)*x(2)+7.85*x(2)+310;
c3=0.00482*x(3)*x(3)+7.97*x(3)+78;
f=c1+c2+c3;
```

```
xopt =
```

```
  600.0000  187.1301   62.8699

>> fopt

fopt =

   7.2528e+03
```

B3LP Revisited