

SYSTEM STABILITY

Stability: Ability of machines in system to recover from system disturbances and still remain synchronized and ~~active~~ return to a steady-state operating point.

Types:

Steady-State Stability - Use loadflow to check for:

- 1 - Phase angle across T-line $\ll 90^\circ$
- 2 - $0.95 < V_{bus} < 1.05$
- 3 - Gens, T-Lines, XFMRs within P, Q, V, I limits.

Also - do incremental changes in operating points to check system sensitivities.

* Transient Stability - Check major disturbances

- Loss of generator
- Line switching
- Faults
- Load switching

Track frequency changes ($f_s = 60 \text{ Hz}$) and δ changes.

Objective: See if machines return to synch frequency with new s-s power angles.

Assumptions: ~~status not change~~

so-called "first swing" model, good for $\sim 1.0 \text{ sec}$

- P_M constant, include inertial dynamics.
- E_g doesn't change, use loadflow on electrical side.
- dc offsets & harmonics ignored
- Use symm components if unbalanced.

* This is what we'll focus on in this course.

Transient Stability (cont'd):

If $t > 1.0$ sec is desired ("multi-swing") then can include governor & exciter.

DYNAMIC STABILITY - out to several minutes.

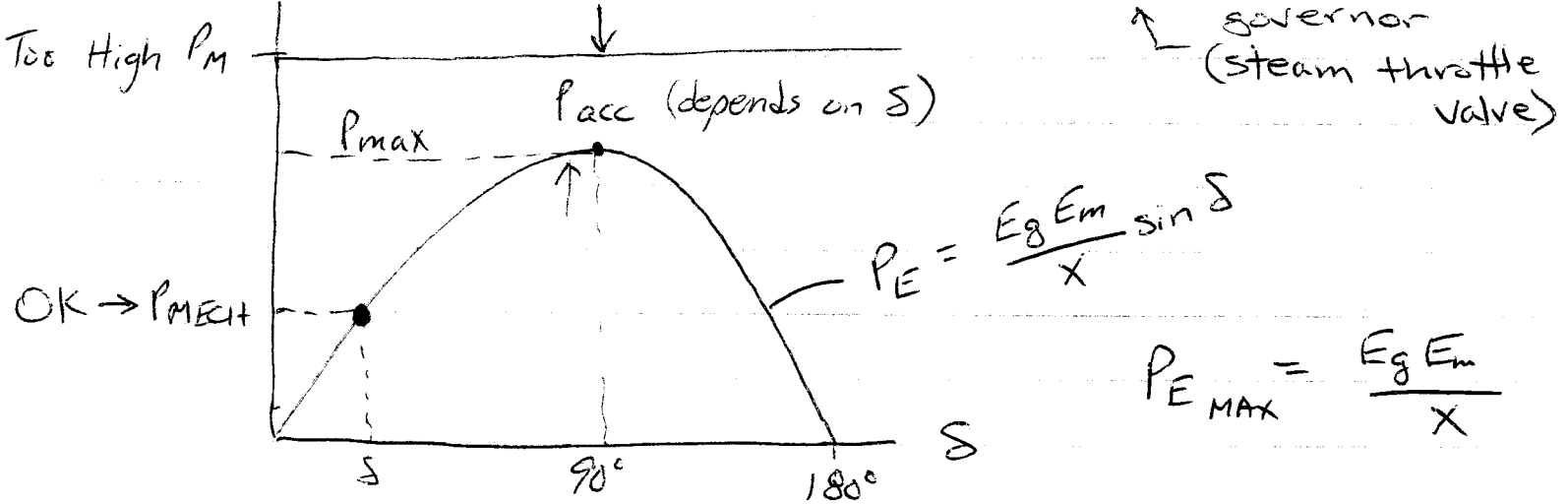
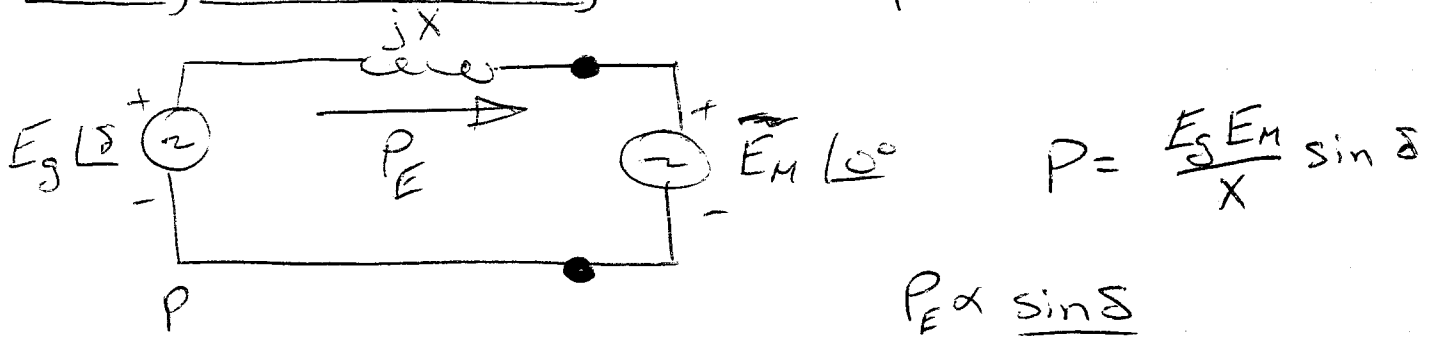
Effects Also included:

- Governors
- Exciters
- LTC XFMRS
- Dispatch/SCADA controls

Interactions can destabilize system even several minutes after disturbance occurs.
(Even when transient stability is maintained.)

Ex:

Steady-State Stability - Like power transfer



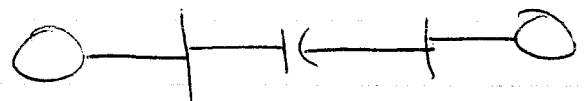
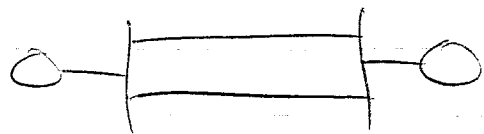
If $P_M > P_{E \text{ MAX}}$, lose synch so must disconnect from system
 Keep $P_{MECH} \leq P_{ELECT(MAX)}$ to assure stability.

To increase stability, we can:

- Increase E_g or E_m
- Decrease X by

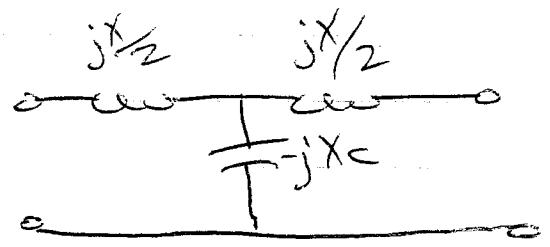
- a) Parallel lines:
- b) Series Cap:

$X = X_{LINE} - X_{CAP}$

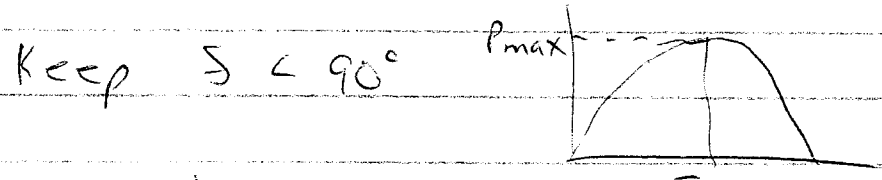


- c) Shunt Cap even helps:

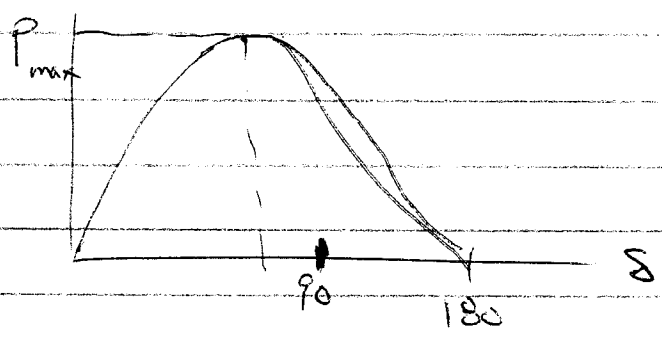
Transfer Impedance: $j \left(X - \frac{X^2}{4X_c} \right)$



For cylindrical rotor machines:

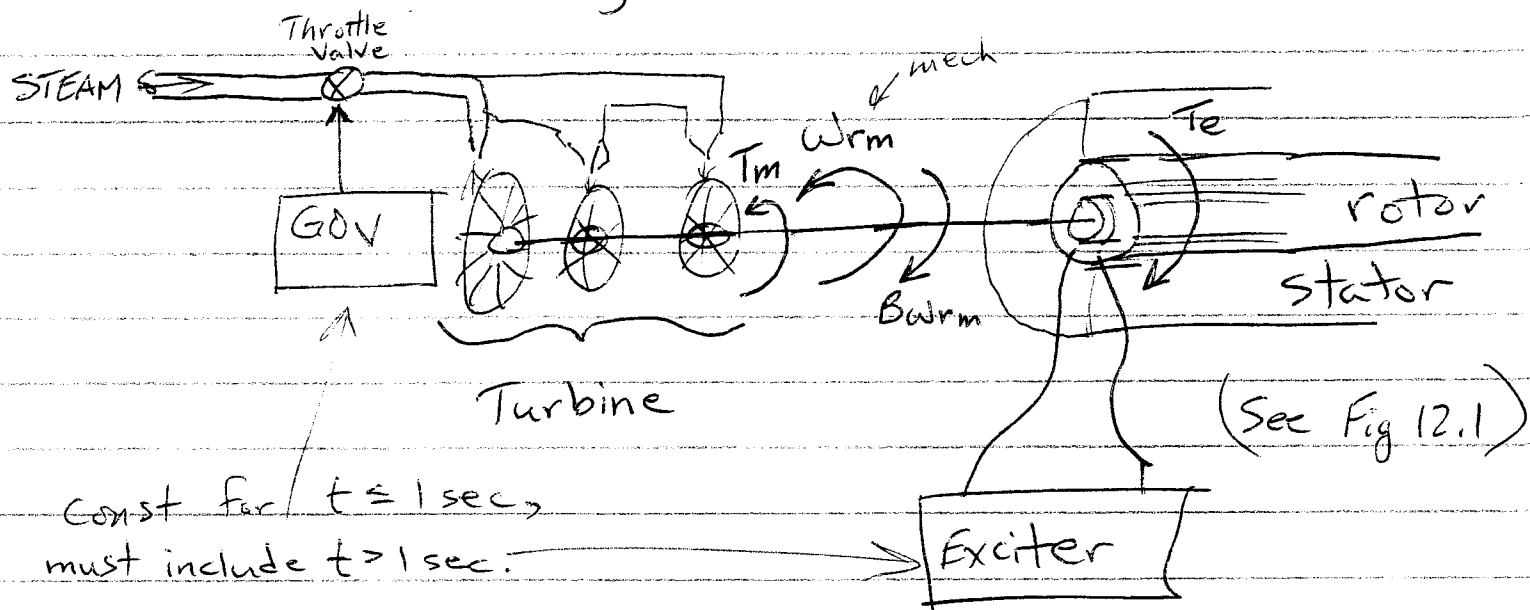


For salient machines, δ even less, since P_{max} occurs at



In reality, dangerous to operate close to P_{max} , since small increase in P_{mech} will put δ "over the top". Usually try to keep $\delta < 40^\circ$.

Transient Stability: The Problem



T_m - mechanical torque, N-m

T_e - electromagnetic counter torque, N-m

J - mass polar moment of inertia

B - Damping torque coefficient

- Bearing Friction
- Windage
- Magnetic ~~loss~~ Losses (leakage)
- Any Drag Torques in General
- Damper Winding Torque
- Exciter Torque

ω_{rm} - mechanical rotor speed, rad/sec

Steady-State: $P_m = P_e$, $T_m = T_e$. But:

* Loss of Line, * fault, * load, etc, reduces P_e

P_m will stay constant (gradually reduce, $t > 1 \text{ sec}$)

$P_A = P_m - P_e =$ Accelerating power, speeds up rotor

* Loss of generator in system makes $P_e > P_m$ and decelerates other generators in system.

Oscillations of machine δ 's w.r.t. each other is called SWING.

Adding Detail to Model

- 1) Governor - Measures f and
(each machine) a) increases P_m if $f < 60 \text{ Hz}$
b) decreases P_m if $f > 60 \text{ Hz}$
- 2) Add excitation -
Measure V_T (bus voltage)
a) Reduce I_f if $V_T > 1.0 \text{ p.u.}$
b) Increase I_f if $V_T < 1.0 \text{ p.u.}$
- 3) Fast valving
- 4) Power System Stabilizer
- 5) Single Pole Reclosing - Japan
- 6) Fast Reclosing
- 7) Load shedding
- 8) Switched Capacitors
- 9) Braking Resistors

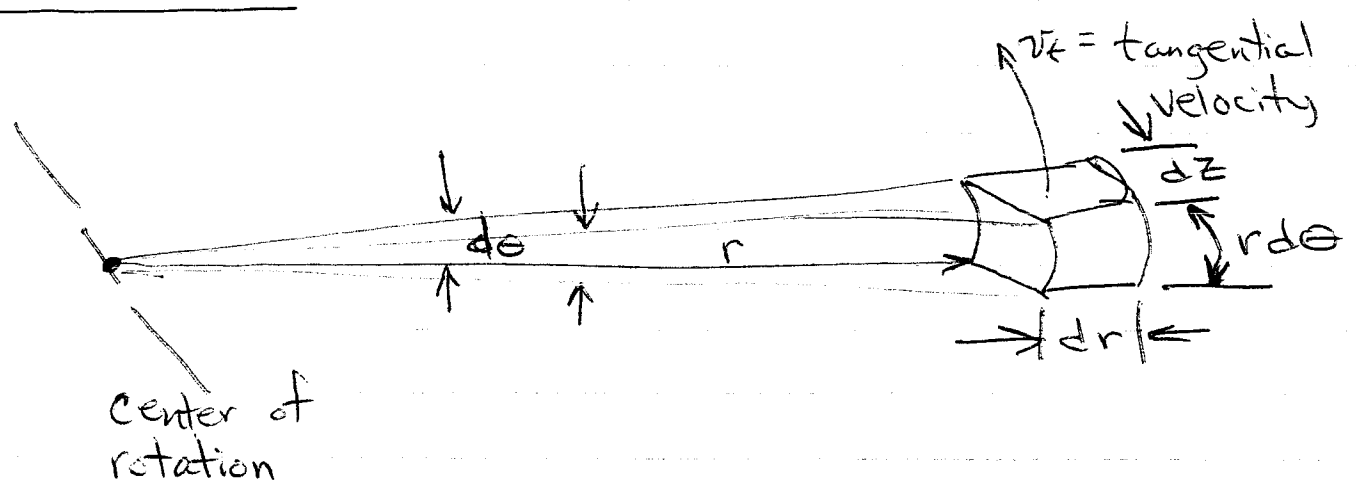
So computer simulation is a must.

To improve stability

- 1) Make H large
- 2) Reduce P_m during fault
 - a) Fast Valving
 - b) Gov - slow down during fault
but too slow to react

2.2 - Inertia

Newton's second Law: $F = ma$



Tangential Force on diff element: $\rightarrow F_t = (dm) a_t = dm \frac{dv_t}{dt}$

$$= r(dm) \frac{d\omega_{rm}}{dt} \quad \left| \quad v_t = r\omega_{rm} \right.$$

The accelerating Torque

$$T_a = r F_t = r^2(dm) \frac{d\omega_{rm}}{dt} = J \alpha \quad \text{N-m}$$

$dJ = r^2 dm =$ mass polar moment of inertia of element. (kg-m²)

If material has density ρ , $dm = \rho \underbrace{dv}_{\text{volume}}$

$$dv = (r d\theta)(dr)(dz)$$

$$J = \int_0^{2\pi} \int_0^z \int_0^R \rho r^3 dr dz d\theta$$

For actual rotor, field slots and use of different materials can make this complex problem.

$$\text{i.e. } R = f(\theta, z) \\ z = f(?)$$

If rotor is homogeneous,

$$J = \frac{\rho R^4 z 2\pi}{4} = \frac{\pi \rho R^4 z}{2}$$

Total Cylinder Mass is

$$M = \rho \int_0^{2\pi} \int_0^z \int_0^R r dr dz d\theta$$

$$= \rho \frac{R^2 z 2\pi}{2} = \boxed{\pi \rho R^2 z} \text{ Kg}$$

Substituting,

$$\boxed{J = \frac{MR^2}{2}} \text{ Kg-m}^2$$

If rotor not homogeneous, use numerical integration.

General Case:

$$\cancel{I} \tau_a = J \frac{d\omega_m}{dt}$$

Sometimes, radius of gyration is given:

K = radius from axis of rotation that a concentrated mass M could be placed to give identical M .

For diff mass or point mass, $J = MK^2$

$$K = \sqrt{\frac{J}{M}}$$

\therefore If K & M are given, J can be calculated.

~~Sometimes M is~~

Possible confusion:

$$W = Mg \quad \text{lbs}$$

	Mass	Weight	g accel. of grav
SI:	Kg	N	9.8 ft m/sec ²
Eng:	slugs	lb	32.2 ft ft/sec ²

also lbm vs. lbf

$$M' = Mg \quad \text{lbm}$$

$$W = Mg \quad \text{lbf}$$

Common Practice in US: (unfortunately):

J given as WK^2

W = mass in lbm
 K = rad gyration in feet

$$\therefore J = \text{WK}^2 \quad \text{lbm} \cdot \text{ft}^2 \left(\frac{0.3048 \text{ m}}{\text{ft}} \right)^2 \left(\frac{0.4536 \text{ Kg}}{\text{lbm}} \right)$$

$$= \underline{\hspace{2cm}} \text{ Kg} \cdot \text{m}^2$$

"If" several masses are on same shaft

$$\text{Then } J_{\text{TOT}} = J_1 + J_2 + J_3 + \dots + J_n$$

Then: Convert

$$H = \frac{1/2 J_{\text{TOT}} (2\omega_s/N_p)^2}{S_{3\phi, \text{BASE}}}$$

Conversion of H to different system base,

Ex: Gen base is 500 MVA, System base is 100 MVA

$$H_{\text{new base}} = H_{\text{old base}} \frac{S_{3\phi, \text{OLD BASE}}}{S_{3\phi, \text{NEW BASE}}}$$

$$H_{\text{new}} = 5 \cdot H_{\text{old}} \leftarrow$$

Typical H values:

2-pole thermal :	2.5 - 6.0	} ON MACHINE BASE
4-pole thermal :	4.0 - 10.0	
Hydro :	2.0 - 4.0	

Ex 1 Nuclear Generator, $WR^2 = 5.82 \times 10^6 \text{ lbm-ft}^2$
 MVA = 1333 MVA
 $N_s = 1800$

$J = WR^2 \times \text{conversion factors}$

$$= (5.82 \times 10^6 \text{ lbm-ft}^2) \left(\frac{\text{kg}}{2.2 \text{ lbm}} \right) \left(\frac{\text{m}}{3.28 \text{ ft}} \right)^2$$

$$= ~~2.459 \times 10^5 \text{ kg-m}^2~~ = \underline{2.459 \times 10^5 \text{ kg-m}^2}$$

$$H = \frac{1}{2} \frac{2.459 \times 10^5 \left[2 \left(\frac{2\pi \times 1800}{60} \right) / 2 \right]^2}{1333 \times 10^6} = \underline{3.27 \text{ sec}}$$

On 100 MVA system base,

$$H = 3.27 \left(\frac{1333}{100} \right) = \boxed{43.56 \text{ sec}}$$

Shortcut:

$$H = \frac{2.31 \times 10^{-10} WR^2 \times \text{RPM}^2}{\text{Machine base}}$$

in MVA

WR^2 in lbm-ft^2

Coherent Machines: 2 or more machines operating in parallel, whose rotors swing together ($\delta_1 = \delta_2$).

(Only possible if sharing equal p.u. loads, and running on same droop characteristic)

<u>Ex:</u>	Unit 1:	500 MVA	0.85 PF	} $H_1 = 4.8$
		20 KV	3600 RPM	
	Unit 2:	1333 MVA	0.9 PF	} $H_2 = 3.27$
		22 KV	1800 RPM	

~~4.8~~ SYSTEM BASE = 100 MVA

To combine, must convert to same base.

$$H_{TOT} = 4.8 \frac{500}{100} + 3.27 \frac{1333}{100} = \boxed{67.59 \text{ sec}}$$

Alternate Method: $H = \frac{\text{MJ stored}}{\text{MVA}}$

$$H = \frac{(4.8 \text{ MJ/MVA})(500 \text{ MVA}) + (3.27 \text{ MJ/MVA})(1333 \text{ MVA})}{100 \text{ MVA}}$$

$\delta \approx \omega$ must be in electrical units --
since one machine is 4-pole, other is 2-pole.

12.1 The Swing Equation:

$$\bar{T}_A = T_{acc} = T_m - T_e = \underbrace{J\alpha}_{\substack{\uparrow \\ (T=J\alpha \Leftrightarrow F=ma)}} = J \frac{d\omega_{rm}}{dt}$$

In terms of Power,

$$P_m - P_e = \omega_{rm} J \frac{d\omega_{rm}}{dt}$$

$$P_m = \omega_{rm} T_m$$

$$P_e = \omega_{rm} T_e$$

$$= \sqrt{E_f} - \sqrt{V_T}$$

Review: $\delta' = \text{electrical torque angle} = \sqrt{B_r} - \sqrt{B_s}_e$

Here,

δ taken as angle between ω_{ref} and rotor

$$\delta = \sqrt{B_r} - \sqrt{\omega_{ref}} \quad \text{see fig 12.2}$$

ω_{ref} can be synch, or set to match some other machine in system. Usually, it is both, i.e. matches ω of ~~static~~ infinite bus. If there is no infinite bus, then probably set to ω of machine having largest "H".

For an N-Pole machine,

$$\omega_{re} = \omega_{rm} \left(\frac{N_p}{2} \right)$$

~~The~~ The electrical angular velocity:

$$\omega_{re} = \omega_{ref} + \frac{d\delta}{dt}$$

$$\omega_{rm} = \frac{\omega_{re}}{N_p/2}$$

$$P_m - P_e = \omega_{rm} J \frac{d\omega_{rm}}{dt}$$

Substituting, $P_m - P_e = \left(\frac{\omega_{re}}{N_p/2} \right) (J) \frac{d}{dt} \left(\frac{\omega_{re}}{N_p/2} \right)$

$$= \frac{\omega_{re}}{N_p^2/4} J \frac{d}{dt} \left(\omega_{ref} + \frac{d\delta}{dt} \right)$$

$$P_m - P_e = \frac{\omega_{re} J}{N_p^2/4} \left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2} \right)$$

Converting to per unit,

$$P_{m,pu} - P_{e,pu} = \frac{\omega_{re} J}{N_p^2/4 \cdot S_{3\phi \text{ BASE}}} \left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2} \right)$$

At this point we define

$$H = \frac{\text{Kinetic Energy of all rotating parts at } \omega_s}{S_{3\phi \text{ rated}}} = \frac{\frac{1}{2} J \omega_s^2}{S_{3\phi \text{ rated}}}$$

mechanical speed \downarrow

$$\uparrow \text{ units} = \frac{\text{Joules}}{\text{watt}} = \frac{\text{Watt}\cdot\text{s}}{\text{watt}} = \text{s}$$

$$H = \frac{\frac{1}{2} J \left(\frac{\omega_{se}}{Np/2} \right)^2}{S_{3\phi \text{ BASE}}} \quad \omega_{s, \text{mech}}$$

$$\text{Substituting, } J = \frac{2H S_{3\phi \text{ BASE}} Np^2}{4\omega_{se}^2}$$

$$J = \frac{H Np^2 S_{3\phi \text{ BASE}}}{2\omega_{se}^2}$$

if $\omega_{se} \stackrel{\text{synch}}{=} \omega_{re} \stackrel{\text{actual}}{=} \omega_{re}$ (not too far wrong if stability maintained)

$$\text{Substituting, } P_{m, pu} - P_{e, pu} = \left[\frac{\omega_{re}}{Np/2} \right] \left[\frac{H Np^2 S_{3\phi \text{ BASE}}}{2 \omega_{re}^2} \right] \left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2} \right)$$

$$P_m - P_e = \frac{H}{\pi f} \left(\frac{d\omega_{ref}}{dt} + \frac{d^2\delta}{dt^2} \right)$$

Again,

Term = 0 if infinite bus $\omega_{ref} = \omega_s$ is used.

$P_m =$ turbine input power (mech) } per unit
 $P_e =$ generator output

$H =$ Inertia Constant in seconds.

$\delta =$ $\angle \vec{B}_r - \angle \omega_{ref}$, radians

$\omega_{ref} =$ ang velocity of reference machine, rad/sec

$t =$ time in seconds

12.6 - Defining the Transient Stability Problem

- Assumptions:
- Ignore magnetic saturation
 - Assume machine is lossless.
 - Assume balanced 3 ϕ operation.
 - Assume P_m constant
 - Assume $|E_f|$ constant

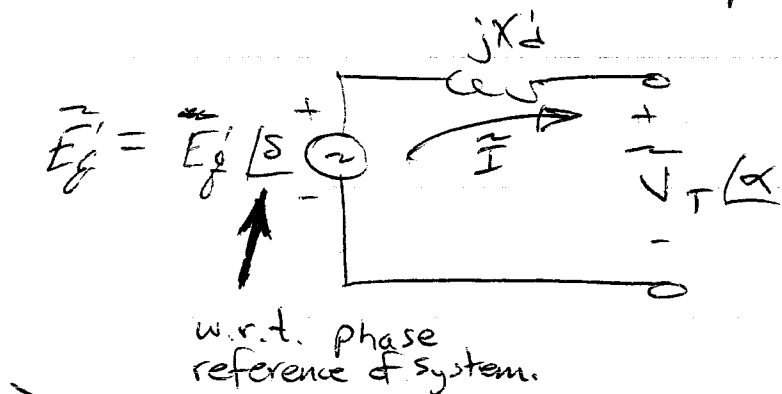
For salient pole machine:

$$E_f = \tilde{V}_r + jX_f \tilde{I}_f + jX'_d \tilde{I}_d = \tilde{E}_f + j(X_d - X_f) \tilde{I}_d$$

Recall $\tilde{I} = \tilde{I}_d + \tilde{I}_q$ (see fig. p. 515 in Gross)

For stability calculations, use X'_d .

Illustration on p. 515 shows that if $X'_d \approx X_f$ then machine can be simplified to



$$P_e = \frac{E'_f V}{X_d} \sin \delta \quad | \quad X'_d = X_f$$

If $X'_d \neq X_f$,

$$P_e = \frac{E'_f V}{X_d} \sin \delta + \frac{V^2}{2} \left(\frac{1}{X_f} + \frac{1}{X'_d} \right) \sin 2\delta$$

at $t = 0^-$, prior to disturbance.

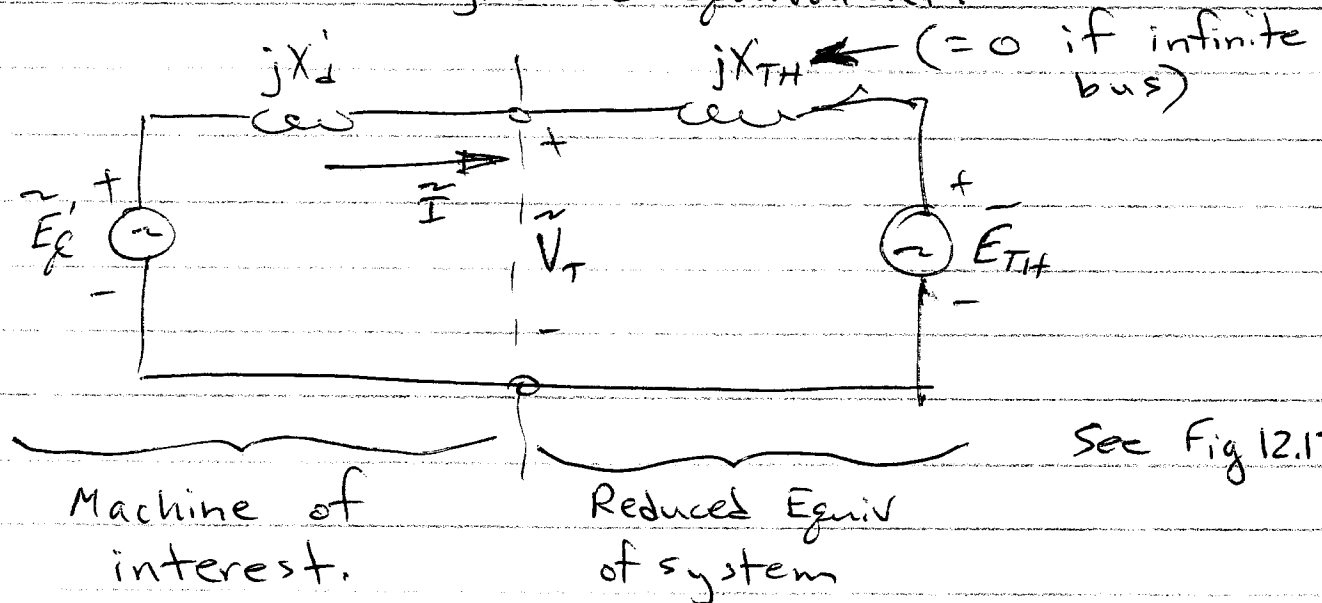
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Infinite Bus - Ideal voltage source of constant frequency

In terms of a generator, this means:

- ⇒ - Constant Internal voltage
- Zero impedance
- Infinite Inertia ($\omega_{ref} = \omega_s$)

When considering how a disturbance affects one particular machine, it is possible to approximate the remainder of the system as a reduced dynamic equivalent!



If response does not involve governor or exciter, the relationship

$$P_m - P_e = \frac{H}{\pi f} \left(\frac{d^2\delta}{dt^2} + \frac{d\omega_{ref}}{dt} \right)$$

if infinite bus

can be used. This assumes that $\omega_{ref} = \omega_s$.

To evaluate,

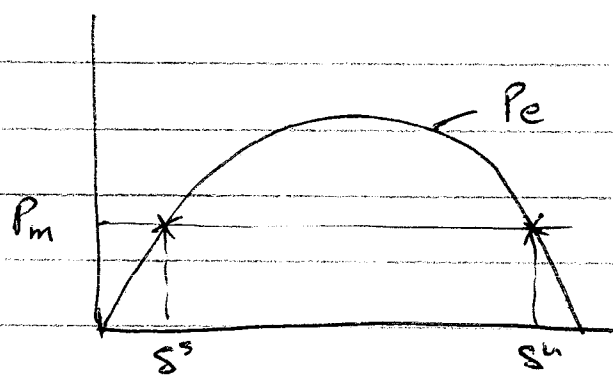
$P_m = \text{constant}$

$$P_e = \frac{E'_q E_{TH}}{X'_d + X_{TH}} \sin \delta$$

Equilibria Points:

when $P_e = P_m$

$$P_m - P_e = \frac{H}{\pi f} \frac{d^2\delta}{dt^2}$$



Perturbations: δ^s is stable

- From δ^s , if δ increases, $\frac{d^2\delta}{dt^2} < 0$ ($P_m < P_e$) $\Rightarrow \delta$ decreases
- if δ decreases, $\alpha > 0$ ($P_m > P_e$) $\Rightarrow \delta$ increases.

δ^u is unstable

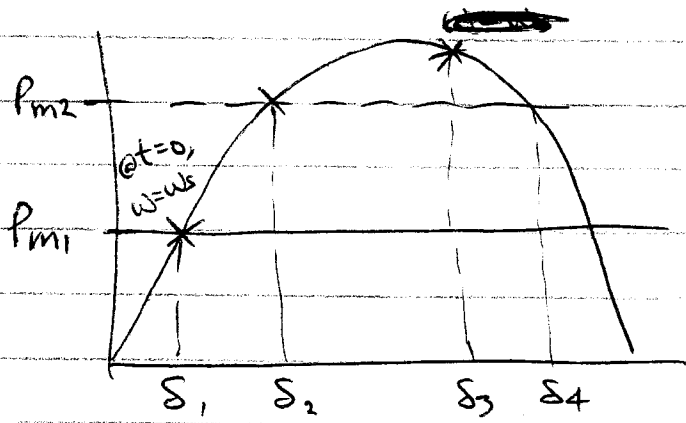
- From δ^u , if δ increases, $\alpha > 0$ ($P_m > P_e$) $\Rightarrow \delta$ increases further.
- if δ decreases, $\alpha < 0$ ($P_m < P_e$) $\Rightarrow \delta$ decreases further.

P_e proportional to $\sin \delta$ \uparrow acceleration

\therefore Better keep $\delta < 90^\circ$ for stable equilibrium.

Conceptual example:

If P_m is initially P_{m1} , then suddenly increases to P_{m2} . What happens?

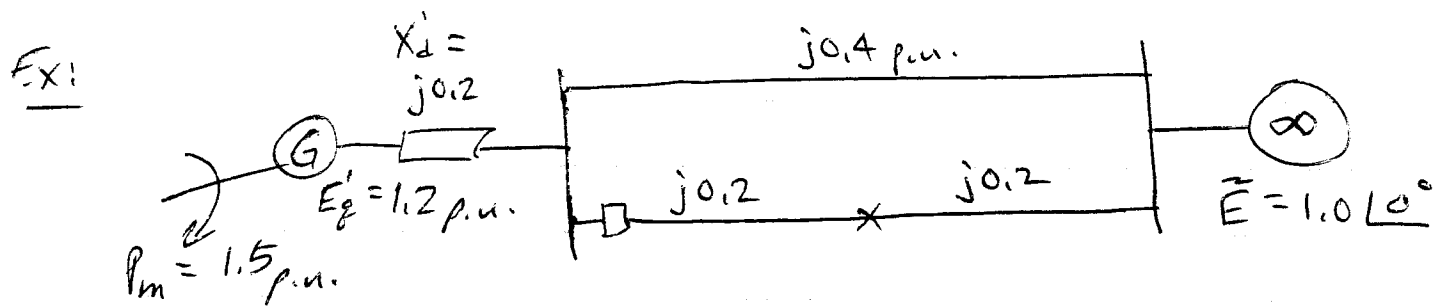


For $t \geq 0^+$, rotor accelerates, until δ reaches δ_2 , where it begins to decelerate.

Oscillation in δ (about δ_2) continues, damping slowly.

No chance to maintain stability if δ goes past δ_4 .

- $\delta < \delta_2 \Rightarrow P_m > P_e \Rightarrow$ accel
- $\delta > \delta_2 \Rightarrow P_m < P_e \Rightarrow$ decel
- $\delta > \delta_4 \Rightarrow P_m > P_e \Rightarrow$ accel

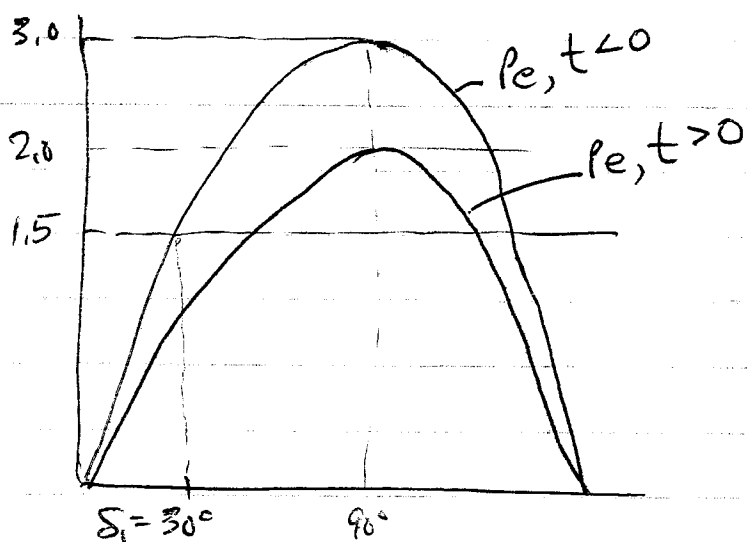


@ $t=0^-$ $X = j0.2$ p.u. (X refers to T-Line tie)

$$P_m = P_e = \frac{(1.2)(1.0)}{0.2 + 0.2} \sin \delta = 1.5 \text{ p.u.} \Rightarrow \delta_1 = 30^\circ$$

$= 3.0 = P_{e \max}$

~~broken~~



Breaker opens at $t=0$, ~~⊗~~

$$P_{\max} = \frac{(1.2)(1.0)}{0.2 + 0.4} = 2.0 \text{ p.u.}$$

New equilibrium point is at

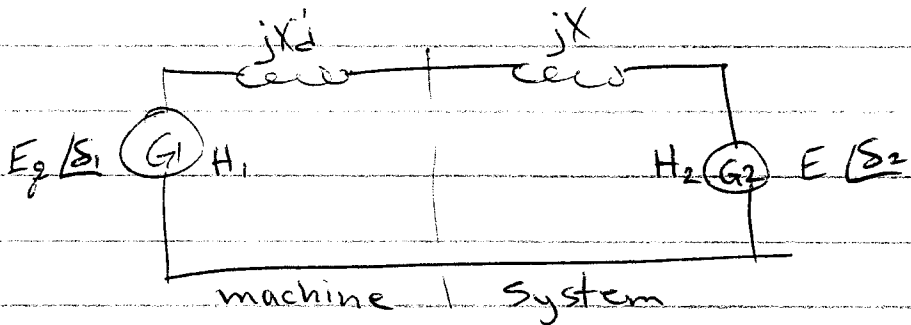
$$1.5 = P_{\max} \sin \delta = 2.0 \sin \delta$$

$$\sin \delta = 0.75 \Rightarrow \delta = \underline{\underline{48.6^\circ}} \text{ or } \cancel{131.4^\circ} \text{ unstable}$$

Considerations / Questions

1) What if system equivalent cannot be assumed to be infinite bus (i.e. infinite inertia)?

This is a case where machines are noncoherent.



See pp. 705-706 in Grainger/Stevenson © 1994

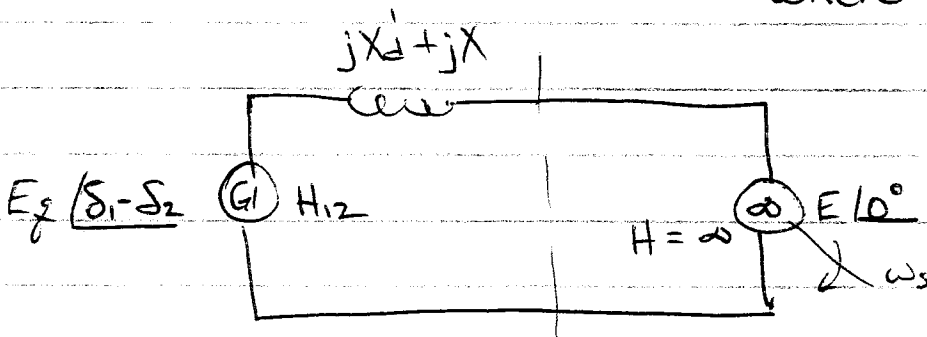
$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_m - P_e = \frac{H_{12}}{\pi f} \frac{d^2 \delta_{12}}{dt^2}$$

where $P_m = P_{m1} = -P_{m2}$

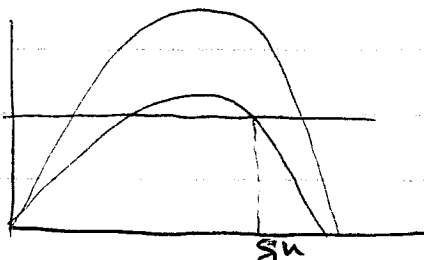
$P_e = P_{e1} = -P_{e2}$

$\delta_{12} = \delta_1 - \delta_2$



∴ Convert system equiv to ∞ bus.

2) How do we know if δ swings past δ^u in new P_{max} curve? All we know so far is how to find new s-s equilibrium point.



Focus on this for remainder of Ch. 12.

12.7 - EQUAL AREA CRITERION

Back to swing equation:

$$P_m - P_e = \frac{H}{\pi f} \frac{d^2 \delta}{dt^2}$$

Rearranging, $\frac{d^2 \delta}{dt^2} = \frac{\pi f}{H} (P_m - P_e) = \frac{\pi f}{H} \left[P_m - \frac{E_p V}{X} \sin \delta \right]$

Because of $\sin \delta$ term in P_e , this is nonlinear differential equation. How to solve?

One "trick" that works:

$$\frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = 2 \frac{d\delta}{dt} \left(\frac{d^2 \delta}{dt^2} \right)$$

We can substitute:

$$\frac{d^2 \delta}{dt^2} = \frac{1}{2} \frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 \frac{dt}{d\delta}$$

$$\frac{1}{2} \frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 \frac{dt}{d\delta} = \frac{\pi f}{H} (P_m - P_e)$$

$$\frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = \frac{2\pi f}{H} (P_m - P_e) \frac{d\delta}{dt}$$

Integrating both sides ~~with respect to t~~,

$$\left(\frac{d\delta}{dt} \right)^2 = \frac{2\pi f}{H} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta = (\omega - \omega_s)^2$$

($\omega_{ref} = \omega_s$)

\Rightarrow Area between P_m & P_e ($\Delta P \cdot d\delta$) is proportional to relative velocity squared, $(\omega - \omega_{ref})^2$

$$A_1 = \int_{0.524}^{0.848} (1.5 - 2 \sin \delta) d\delta = 1.5\delta + 2 \cos \delta \Big|_{0.524}^{0.848} = \underline{\underline{0.0773}}$$

Since equilibrium is (hopefully) about δ_2 , A_2 will be equal to A_1 .

$$\therefore \int_{\delta_3}^{\delta_2} (P_m - P_e) d\delta = \int_{\delta_3}^{0.848} (1.5 - 2 \sin \delta) d\delta = 0.0773$$

$$1.5\delta + 2 \cos \delta \Big|_{\delta_3}^{0.848} = 0.0773$$

$$(1.5)(0.848) + 2 \cos(0.848) - 1.5(\delta_3) - 2 \cos(\delta_3) = 0.0773$$

$$2 \cos \delta_3 + 1.5 \delta_3 = 2.518$$

Solve? Use Newton Raphson, iteration, etc.

$$\delta_3 = 1.218 \text{ rad } (69.8^\circ)$$

Note: Max A_2 available to "counter" A_1 is

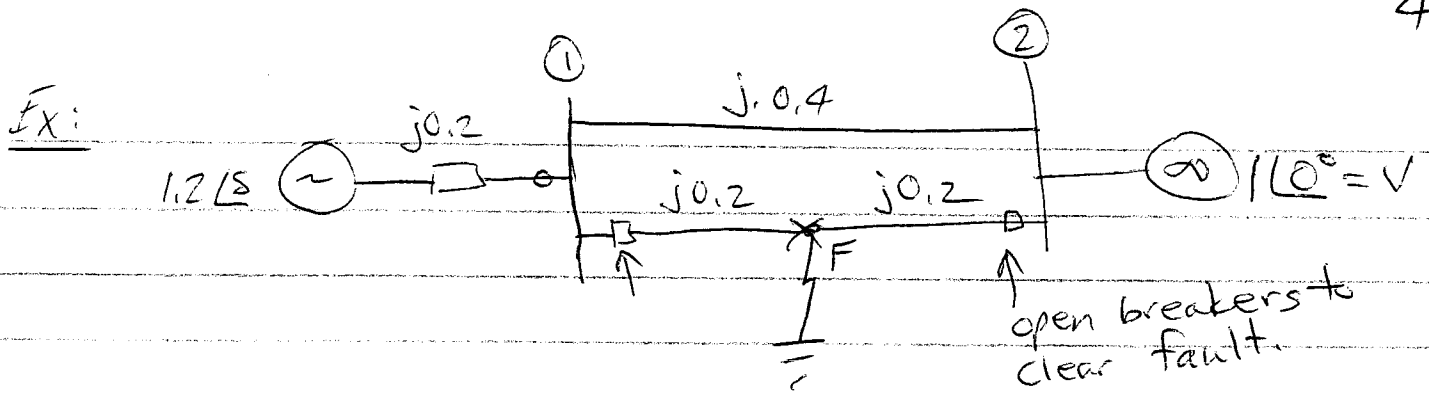
$$A_2 = \int_{\delta_4}^{\delta_2} (P_m - P_e) d\delta = \int_{2.293}^{0.848} (1.5 - 2 \sin \delta) d\delta = \underline{\underline{0.478}}$$

$$\text{If } \delta_1 \text{ had been } 0^\circ, \quad A_1 = \int_0^{0.524} (1.5 - 2 \sin \delta) d\delta$$

$$= 1.5(0.524) + 2 \cos 0.524 - 2$$

$$= \underline{\underline{0.5176}}$$

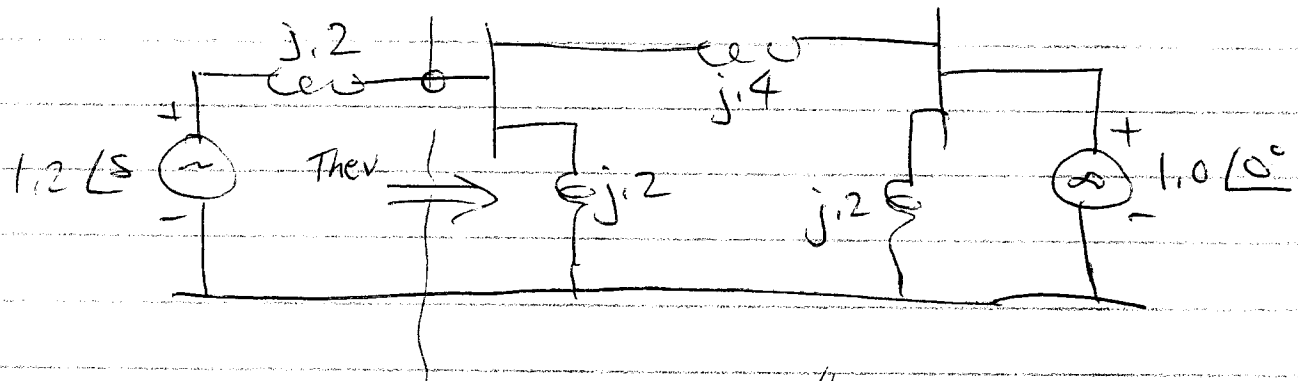
\therefore Stability would not have been possible!



What if fault occurs at $t=0$?

$$P_{\text{max}}(t=0^-) = \frac{(1.2)(1)}{j0.2 + j0.4 // j0.4} = \frac{1.2}{0.4} = 3.0 \text{ p.u.}$$

For fault, Thevenize at bus 1 look back to inf. bus.



$$Z_{TH} = j0.4 // j0.2 = \cancel{j0.2667} j0.1333$$

$$V_{TH} = 1 \left(\frac{j0.2}{j0.6} \right) = 0.333$$

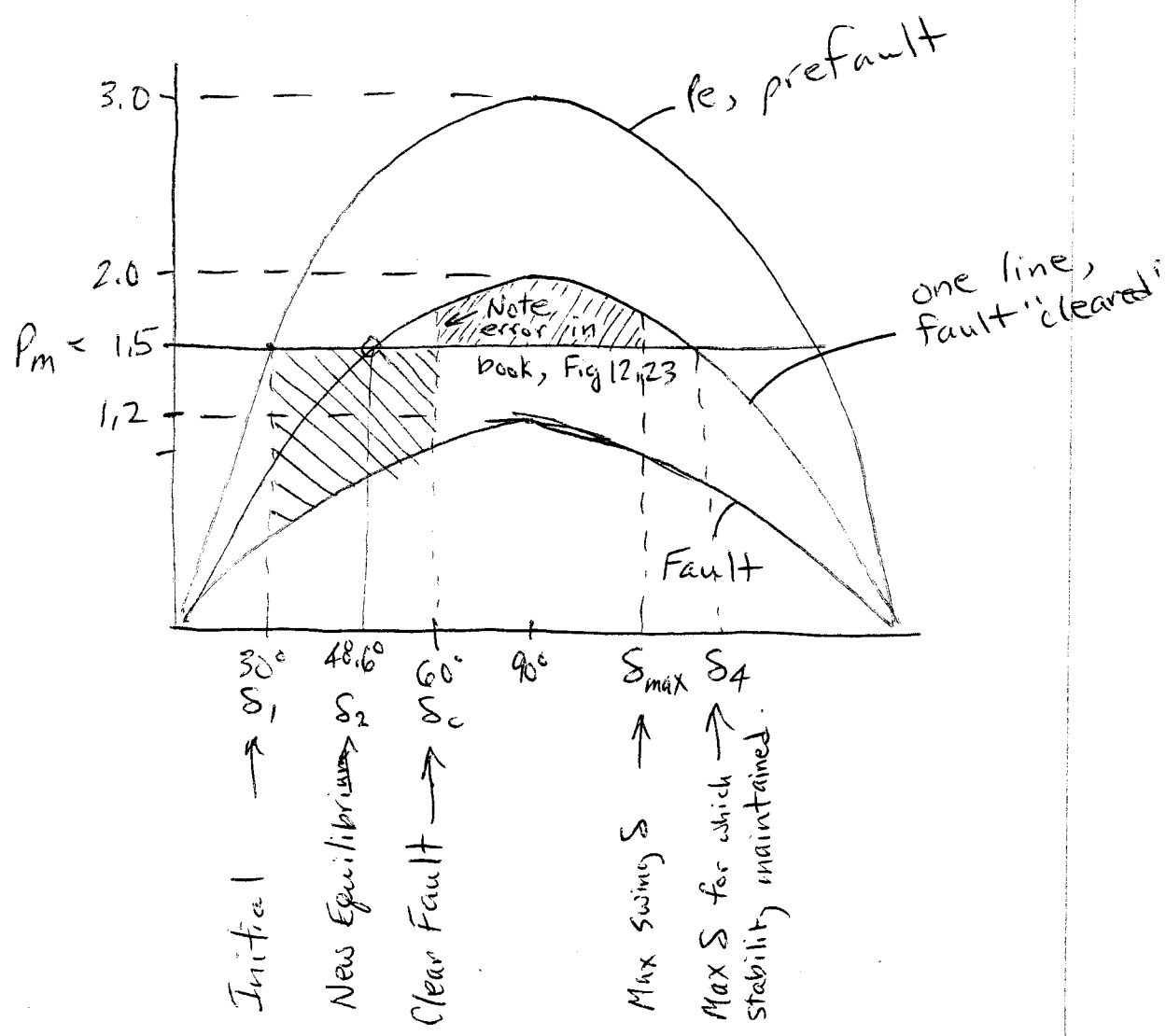
$$\therefore P_{\text{Fault Max}} = \frac{(1.2)(0.333)}{0.2 + 0.1333} = 1.2$$

Note: $P_m = 1.5$ Conclusion?

Impossible to maintain stability, if fault persists.

If fault can be cleared,

$$P_{e \max} = \frac{(1.2)(1)}{12+1.4} = 2.0 \quad (\text{from before, one line})$$



The accelerating area, A_1 is

$$\int_{\delta_1}^{\delta_c} (P_m - P_e) d\delta = \int_{.524}^{1.047} (1.5 - 1.2 \sin \delta) d\delta$$

$$= 1.5 \delta + 1.2 \cos \delta \Big|_{.524}^{1.047} = 0.346$$

How far will δ increase after fault is cleared?

$$|A_2| = |A_1| \therefore \int_{\delta_{\max}}^{1.047} (1.5 - 2.0 \sin \delta) d\delta = 0.346$$

$$1.5\delta + 2.0 \cos \delta \Big|_{\delta_{\max}}^{1.047} = 0.346$$

$$1.5\delta_{\max} + 2 \cos \delta_{\max} = 0.346$$

$$\Rightarrow \delta_{\max} = 1.848 \text{ rad} \\ = 105.9^\circ$$

Max it could "swing" is out to $131.4^\circ = \delta_4$

Question: What is critical clearing angle?

This is largest angle that fault could be cleared and stability still maintained.

Call this δ_{cc} .

Ex 12.5

$$\int_{0.524}^{\delta_{cc}} \overbrace{(1.5 - 1.2 \sin \delta)}^{\text{faulted}} d\delta = \int_{2.293}^{\delta_{cc}} (1.5 - 2 \sin \delta) d\delta$$

Solving,

$$1.5\delta + 1.2 \cos \delta \Big|_{0.524}^{\delta_{cc}} = 1.5\delta + 2 \cos \delta \Big|_{2.293}^{\delta_{cc}}$$

$$\underline{\underline{\delta_{cc} = 1.196 \text{ rad} = 68.6^\circ}}$$

How long do we have to clear?

Not so simple, since the accelerating torque varies ~~is~~ with δ . Some key relationships:

$$P_{acc} = P_m - P_e = P_m - 1.2 \sin \delta$$

P_{acc} gradually decreases as δ increases to δ_{cc} .

$$T_{acc} = (P_m - 1.2 \sin \delta) \omega_{\text{mech}}(\delta)$$

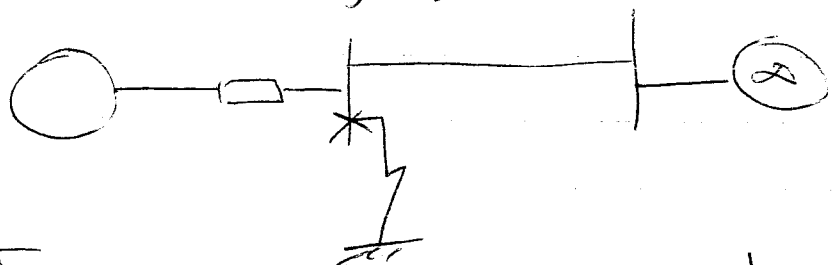
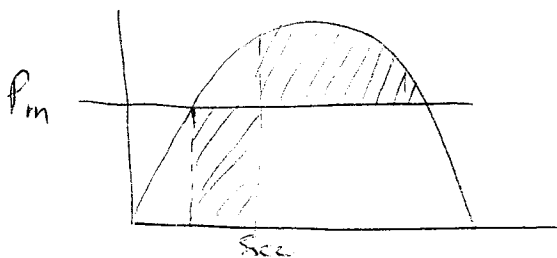
$$\omega = \omega_0 + \int_0^{t_{cc}} \alpha(\delta) dt \quad \leftarrow \quad \omega(t) = \omega_0 + \alpha t$$

From swing equation,

~~$$\omega_{cc}^2 - \omega_1^2 = \frac{2\pi f}{H} \int_{\delta_1}^{\delta_{cc}} (P_m - P_e) d\delta$$~~
$$\omega_{cc}^2 - \omega_1^2 = \frac{2\pi f}{H} \int_{\delta_1}^{\delta_{cc}} (P_m - P_e) d\delta ?$$

Should be able to get closed-form expression for $t_{critical}$. Usually this is done trial & error, in available texts anyway.

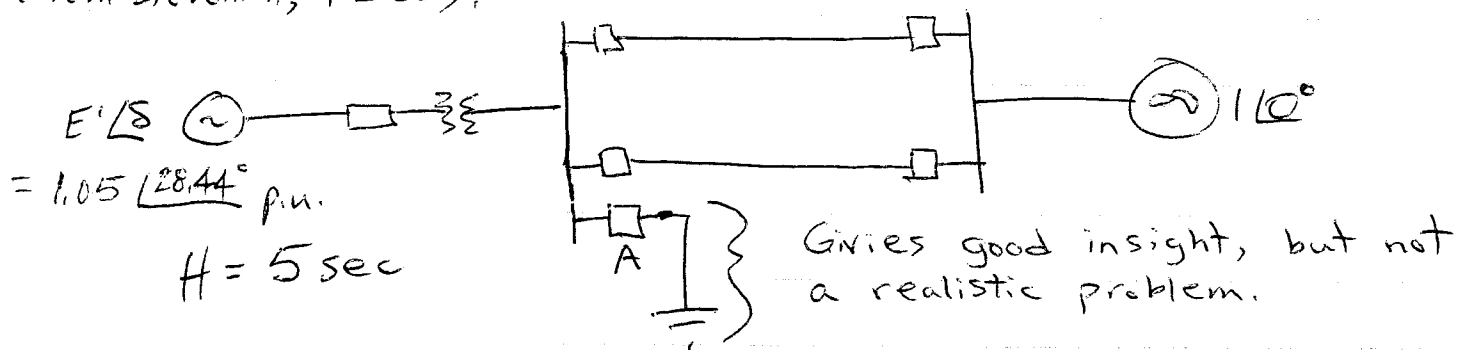
Easier case:



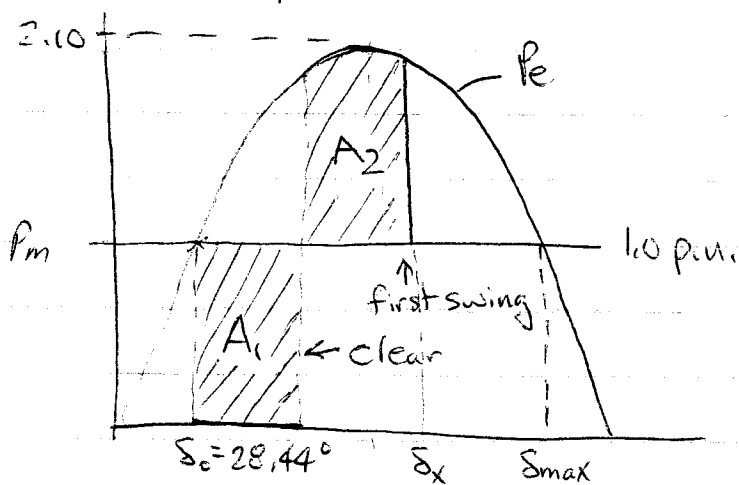
$P_e = 0$ during fault!

More on Equal Area Criterion: Fault on Gen Bus

(From Stevenson, 4th Ed).



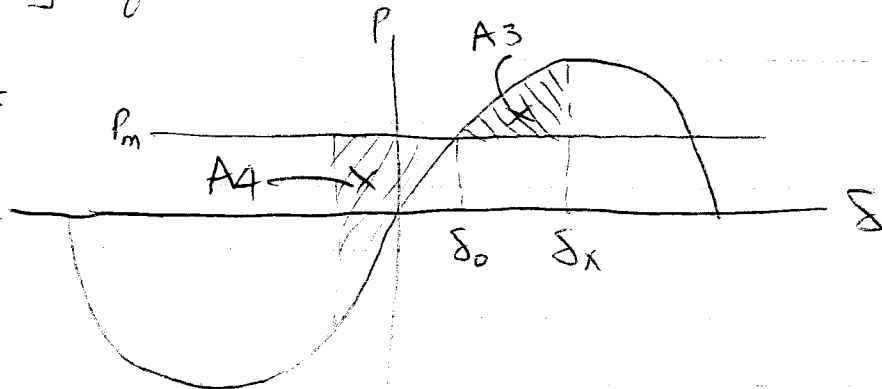
Assume a bus fault occurs (Breaker A is closed).
 Prefault P_{max} is 2.10 pu.



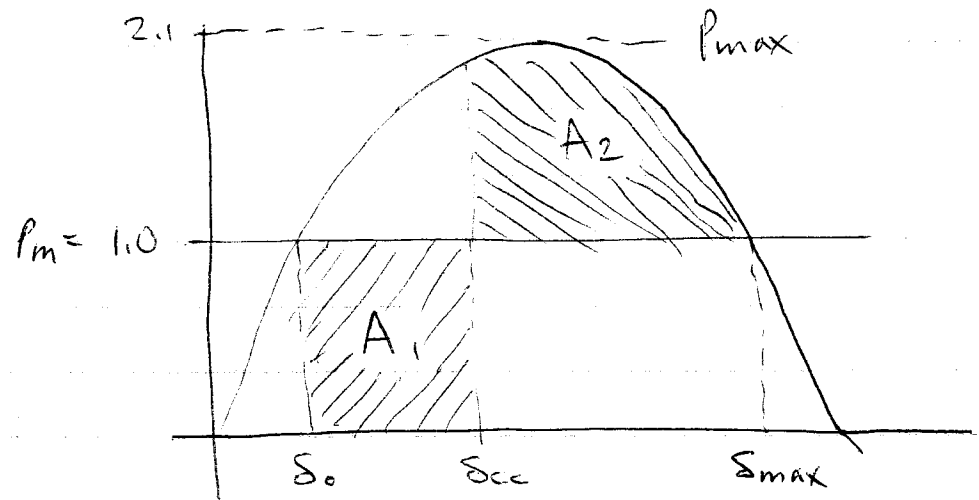
When fault occurs in this case, $P_e = 0$. $(P_m - P_e) = P_m$
 P_e remains at zero until fault is cleared (open A).
 S will continue to increase until $A_2 = A_1$.

After clearing, equilibrium is still at δ_0 .

Now, S swings about δ_0 , with
 $A_3 = A_4$



What is maximum δ_x for which stability can be maintained?



$$\underbrace{\int_{\delta_0}^{\delta_{cc}} (P_m - P_e) d\delta}_{A_1} = \underbrace{\int_{\delta_{max}}^{\delta_{cc}} (P_m - P_e) d\delta}_{A_2}$$

$$P_m \delta \Big|_{\delta_0}^{\delta_{cc}} = P_m \delta + P_{max} \cos \delta \Big|_{\delta_{max}}^{\delta_{cc}}$$

$$\cancel{P_m \delta_{cc}} - P_m \delta_0 = \cancel{P_m \delta_{cc}} - P_m \delta_{max} + P_{max} \cos \delta_{cc} - P_{max} \cos \delta_{max}$$

$$P_m (\delta_{max} - \delta_0) = P_{max} (\cos \delta_{cc} - \cos \delta_{max})$$

$$\cos \delta_{cc} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}$$

but $\delta_{max} = \pi - \delta_0$, $P_m = P_{max} \sin \delta_0$

$$\delta_{cc} = \cos^{-1} \left[(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0 \right] \text{ only for bus fault at gen!}$$

For this case, it is possible to directly calculate t_{cc} , the time before which the fault must be removed.

$P_A = P_m$, since P_e is zero in this case. Therefore, the accelerating Torque and the acceleration are roughly constant.

From swing equation, $\alpha = \frac{d^2\delta}{dt^2} = \frac{\pi f}{H} (P_m - P_e)$

$$\therefore \alpha = \frac{\pi f}{H} P_m \frac{\text{rad}}{\text{sec}^2}$$

$$\omega(t) = \omega_0 + \alpha t = \frac{\pi f P_m t}{H}$$

(w ref = ω_s)

$$\delta(t) = \delta_0 + \int_0^t \omega(t) dt = \delta_0 + \frac{\pi f}{2H} t^2$$

$$\therefore \delta_{cc} = \delta_0 + \frac{\pi f}{2H} t_{cc}^2$$

$$\Rightarrow t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\pi f P_m}} \quad \begin{array}{l} \text{only for bus} \\ \text{fault at gen!} \\ (P_e = 0) \end{array}$$

For previous problem, $\delta_0 = 28.44^\circ = 0.496 \text{ rad}$

$$\delta_{max} = \pi - \delta_0$$

$$\delta_{cc} = \cos^{-1} \left[(\pi - 2 \times 0.496) \sin 0.496 - \cos 0.496 \right] = 1.426 = \underline{\underline{81.70^\circ}}$$

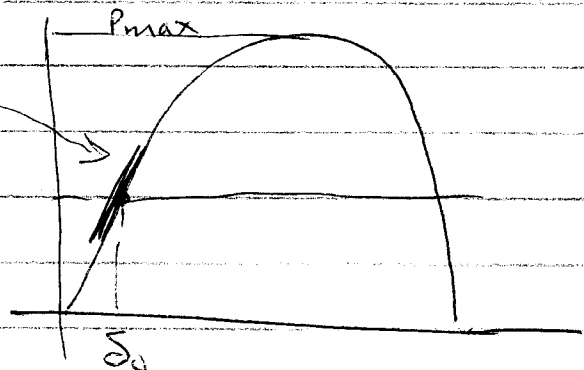
$$t_{cc} = \sqrt{\frac{2(5)(1.426 - 0.496)}{(60 \times \pi)(110)}} = \underline{\underline{0.222 \text{ sec}}}$$

Natural Frequency of Oscillation -

$$\omega_n = \sqrt{\frac{\omega_s P_{\max} \cos \delta}{2H}} \quad (\text{For small disturbances.})$$

$S_p = P_{\max} \cos \delta$ is the "synchronizing power coefficient". It is the slope of $P_e = P_{\max} \sin \delta$ at $\delta = \delta_0$.

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{\omega_s P_{\max} \cos \delta}{2H}} \quad P_m$$



Ex: $\delta_0 = 28.44^\circ$, $H = 5 \text{ sec}$, $P_{\max} = 2.10 \text{ p.u.}$

$$\omega_n = \sqrt{\frac{(377)(2.10 \cos 28.44^\circ)}{2(5)}} = 8.343 \frac{\text{elec rad}}{\text{sec}}$$

$$f_n = \underline{\underline{1.33 \text{ Hz}}}$$

∴ Small perturbations in load (normal occurrence throughout day) will cause low-freq machine oscillations. Usually on order of $\sim 1 \text{ Hz}$.

They are quickly damped by prime mover, generator and system load. Note that R of transmission system has virtually no damping effect.

12.8

Solution of Swing Equation:

For practical cases, critical clearing time cannot be calculated with equal area criterion. Engineers must know how much time they have to clear the fault - the fault must be detected, a trip signal sent to the CB, and then breaker must operate. t_{cc} must be known before the CB & protective relaying can be specified and before coordinating the relaying.

If we can solve for $\delta(t)$, then we can determine t_{cc} . The approach is typically to use trial & error, clearing fault at different times.

An analog computer implementation is given in book.

$\frac{\pi f}{H} (P_m - P_e)$ is integrated. (Note inverting integrator).
gain

basically:

$$\alpha(t) = \frac{d^2\delta(t)}{dt^2} = \frac{\pi f}{H} (P_m - \underbrace{P_{\max} \sin(\delta(t))}_{P_e})$$
$$\omega(t) = \underbrace{\omega_0}_0 + \int \alpha(t) dt$$
$$\delta(t) = \delta_0 + \int \omega(t) dt$$

Note that P_{\max} changes with fault, clear, reclose, ~~etc~~ line switching.

The book sets up a problem using Runge-Kutta numerical integration. This is more typical of the modern solution approach on computer.

Results for the previous example, where $\delta_{cc} = 68.6^\circ$, are given in Figs 12.25 & 12.26.

→ In these programs, t_{cc} is specified and simulation is run to see if stability is maintained.

** Best guess/judgement is that if ~~first pos/neg~~ first negative fluctuation in δ is \leq to first pos fluctuation then system will be stable.
i.e. if $(\delta_{\max} - \delta_0) \geq (\delta_c - \delta_{\min}) \Rightarrow$ stable.

For this simple system (two machines), δ_{cc} could be found by equal-area. t_{cc} could be found simply by running simulation and noting the value of t when $\delta = \delta_{cc}$.

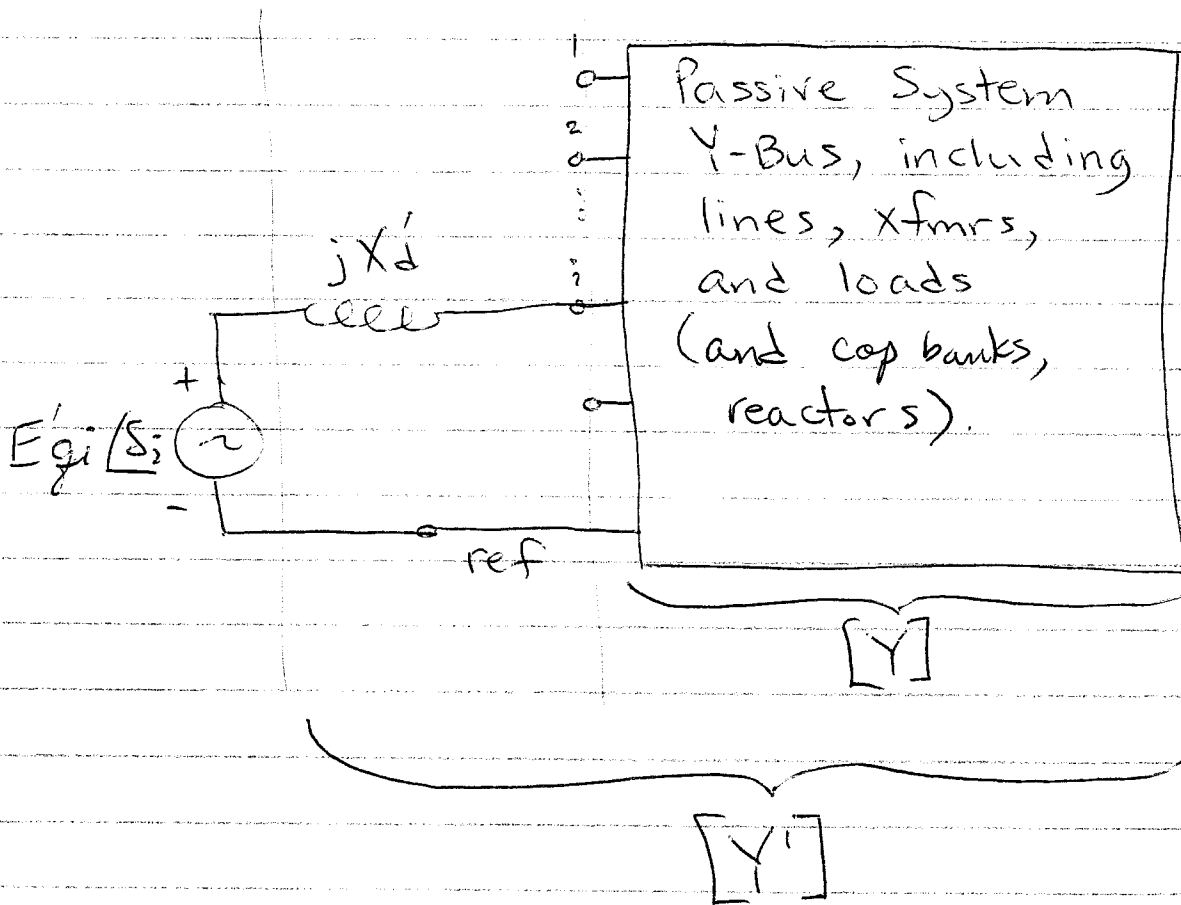
What if there are more than 2 machines?

Before going on,

- Do you understand swing equation?
- Do you understand [Y] system description?
- Do you have equal area method figured out?

12.9 - Multimachine Systems

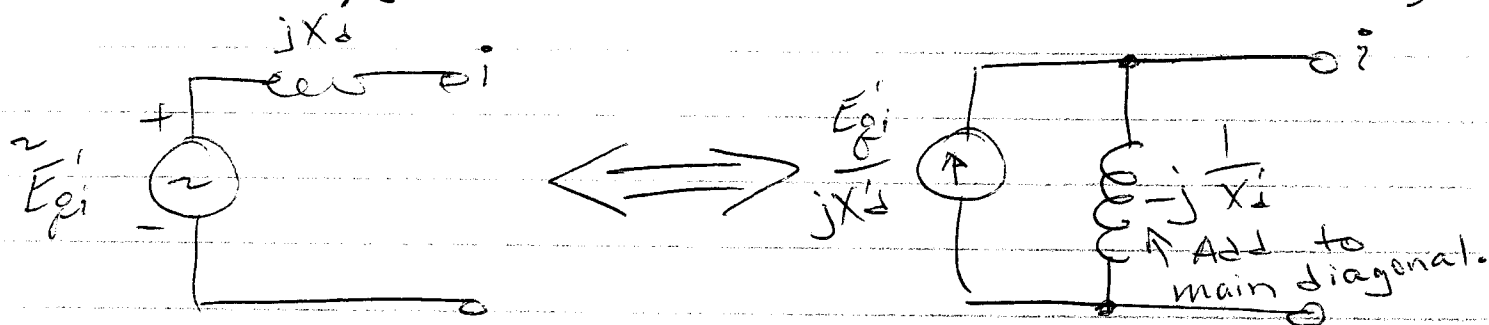
A network approach must be used where more than 2 machines are to be simulated.



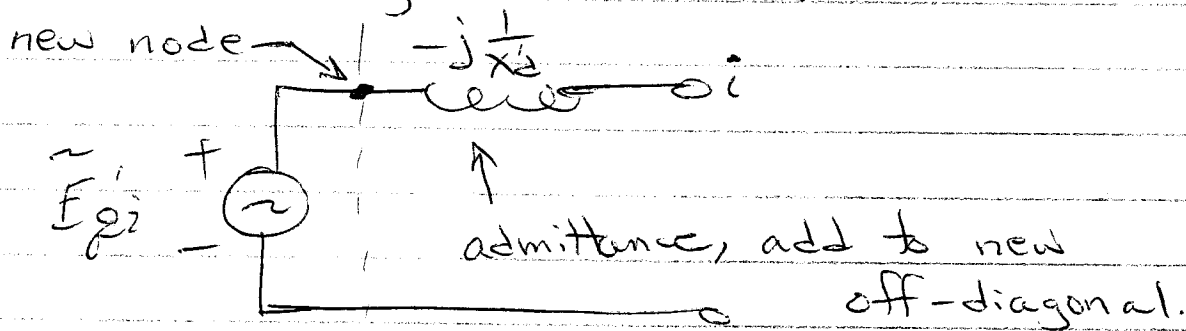
- Pre-~~transient~~ transient bus voltages typically obtained from load flow program.
- All buses not connected to a generator ~~are~~ have no current injected.

$$\begin{matrix}
 \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_{n+1} \\ \vdots \\ I_{n+m} \end{bmatrix} \\
 \left. \begin{matrix} \text{All} \\ \text{zero} \end{matrix} \right\}
 \end{matrix}
 = [Y']
 \begin{matrix}
 \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \\ V_{n+1} \\ \vdots \\ V_{n+m} \end{bmatrix} \\
 \left. \begin{matrix} n \text{ generators} \\ m \text{ buses w/o generators,} \\ \text{Voltage due to } I_1 \rightarrow I_n \end{matrix} \right\}
 \end{matrix}$$

$[Y']$ is formed by adding Norton admittance of generators to main diagonal of $[Y]$. (No extra nodes added to network).



In this book, author adds an extra node at each generator bus:



$\Rightarrow [Y']$

Therefore, end up with $2n+m$ nodes in system. We will follow author's method, even though not the best in terms of computer application.

Nodes not connected to a generator are typically eliminated, using matrix partitioning.

$[Y]$ is partitioned:

$$\begin{array}{c|c} \begin{array}{c} 1 \rightarrow n \\ \downarrow \\ n \end{array} & \begin{array}{c} [Y_{AA}] \quad [Y_{AB}] \\ \hline [Y_{BA}] \quad [Y_{BB}] \end{array} \\ \begin{array}{c} n+1 \\ \downarrow \\ n+m \end{array} & \end{array}$$

$$[Y_{red}] = [Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}] = n \times n$$

See derivation, p. 533

Ex: Go thru Ex. 12.7

Effect of loads: Loads are added to main diagonal term of $[Y]$

Ex: see example 12.8

Calculating P_{max} for various ~~conditions~~ conditions is done using $[Y]$ for each of the operating states.

Ex: If $[Y] = \begin{bmatrix} -j2.5 & j2.5 \\ j2.5 & -j2.5 \end{bmatrix}$ Pre-fault

$$\begin{aligned} E_1 = E_{f1} = 1.2 \text{ p.u.} & \quad \delta_1 = \delta & \quad g_1 \angle \delta_1 = 2.5 \angle -90^\circ \\ E_2 = 1.0 \text{ p.u.} & \quad \delta_2 = 0 \text{ (infinite bus)} & \quad g_2 \angle \delta_2 = 2.5 \angle +90^\circ \end{aligned}$$

$$\begin{aligned} P_{e1} &= \sum_{j=1}^2 E_i E_j y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \\ &= (1.2)^2 (2.5) \cos(\delta_1 - \delta_1 - (-90^\circ)) + (1.2)(1.0)(2.5) \cos(\delta_1 - 0 - 90^\circ) \\ &= 3.0 \sin \delta \Rightarrow P_{max} = 3.0 \end{aligned}$$

Using same method,

$$P_{\max} = 1.2 \quad \text{during fault.}$$

$$P_{\max} = 2.0 \quad \text{when cleared.}$$

Each machine's swing equation is solved separately, although δ depends on other machines.

$$P_{m_i} - P_{e_i} = \frac{H_i}{\pi f} \left(\frac{d^2 \delta_i}{dt^2} + \frac{d\omega_{\text{ref}}}{dt} \right) \quad i = 1, 2, \dots, n$$

Caution: use ~~same~~ system base for all H and P values.

Usually pick infinite bus machine for ω_{ref}

$$P_{m_i} - P_{e_i} = \frac{H_i}{\pi f} \left(\frac{d^2 \delta_i}{dt^2} \right) \quad i = 1, 2, \dots, n$$

Reformulate as

$$\frac{d\delta_i}{dt} = \omega_i$$

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i} \left(P_{m_i} - \underbrace{\sum_{j=1}^n E_i E_j y_{ij} \cos(\delta_i - \delta_j - \alpha_j)}_{P_{e_i}} \right)$$

Initial conditions: $\omega_i = 0$ for all machines $\omega_i(0)$
 $\delta_{0i} = \delta_i(0)$

$$\frac{dS(t)}{dt} = f = \omega(t)$$

$$\frac{d\omega(t)}{dt} = g = \alpha(t) = \frac{\pi f}{H} (P_{im} - P_e)$$

$$S(0) = S_0 = \sin^{-1} \left(\frac{P_m}{P_{max}} \right)$$

$$\omega(0) = 0$$

INITIATE

CALCULATE

$$f(s,t) = \omega_{avg} = \omega(t)$$

$$g(s,t) \text{ (alpha)} = \frac{\pi f}{H} [P_m - P_{max} \sin S(t)]$$

~~K1 = OMEGA * dt~~

~~L1 = ALPHA * dt~~

~~K2 = OMEGA * dt~~

~~L2 = ALPHA (S + K1/2) * dt~~

~~K3 = OMEGA * dt~~

~~L3 = ALPHA (S + L2/2) * dt~~

~~K4 = OMEGA * dt~~

~~L4 = ALPHA (S + L3/2) * dt~~

$$K1 = OMEGA * dt = ds$$

$$L1 = \text{ALPHA}^{(s)} * dt$$

$$K2 = OMEGA * dt$$

$$L2 = \text{ALPHA} (S + \frac{K1}{2}) * dt$$

$$K3 = OMEGA * dt$$

$$L3 = \text{ALPHA} (S + \frac{K2}{2}) * dt$$

$$K4 = OMEGA * dt$$

$$L4 = \text{ALPHA} (S + \frac{K3}{2}) * dt$$

$$D\text{OMEGA} = OMEGA * DT$$

$$D\text{DELTA} = (L1 + 2 * L2 + 2 * L3 + L4) / 6.0$$

$$OMEGA = OMEGA + D\text{OMEGA}$$

$$DELTA = DELTA + D\text{DELTA}$$

~~FUNCTION OMEGA (X, T)~~

~~FUNCTION ALPHA (DELTA, Pm, Pmax)~~

OMEGA (0:1000)

DELTA (0:1000)

ALPHA (1:1000)

LECTURE 22

23

MULTI-MACHINE STUDIES

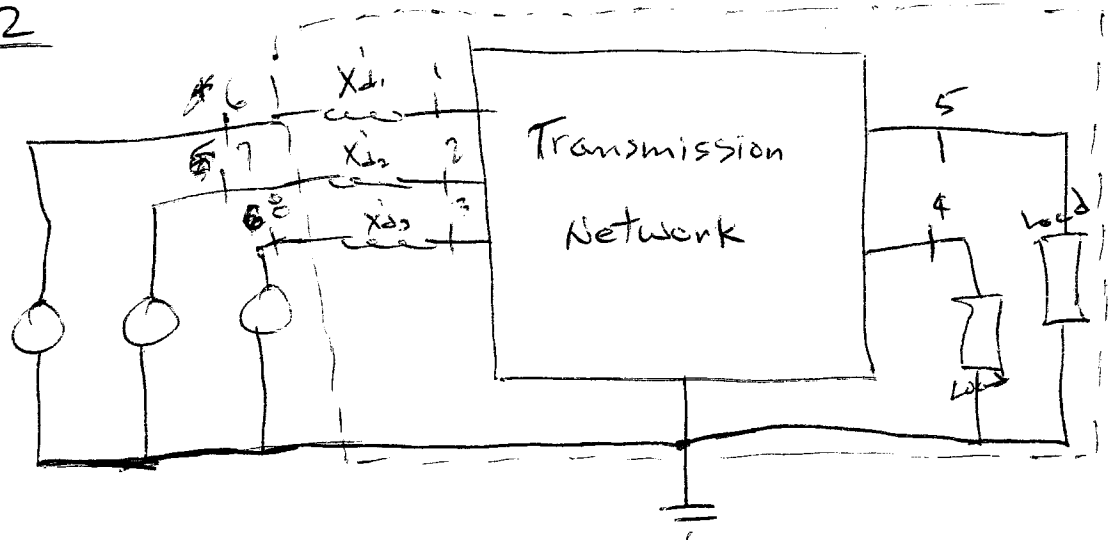
(Can't use equal-area if ≥ 2 machines)

- Use load flow to establish initial conditions.
- Convert loads to Admittances
(Can be constant admittance, or voltage dependent.)

ex:

$$Y_{LF} = \frac{P_L - jQ_L}{|V_L|^2} \quad \text{then all to } Y_{SS}$$

Fig 14.2



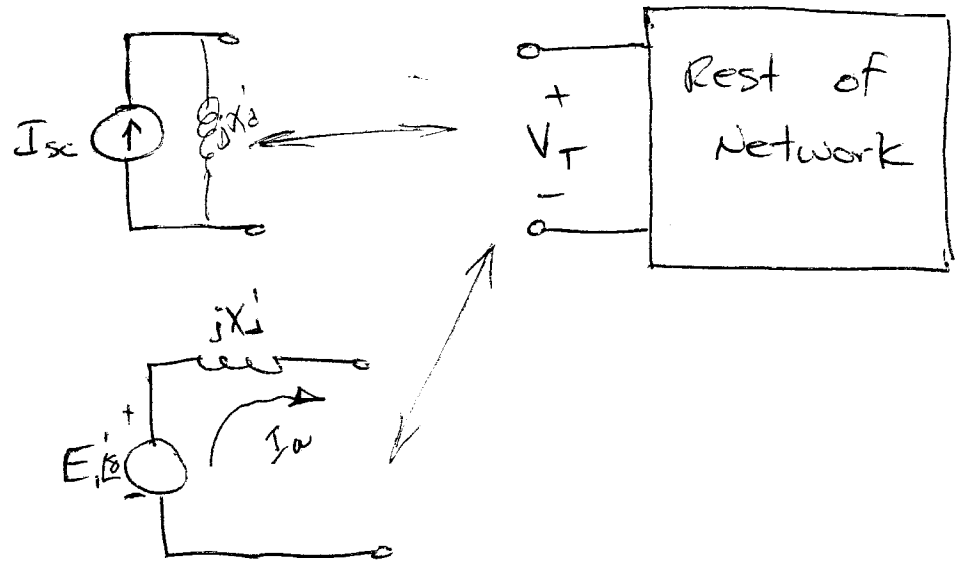
- Two ways to absorb X_d'
- Make new bus
 - Use Norton Equiv

Electrical power:
Add buses:

$$P_{e1} = |E_1'|^2 G_{11} + |E_1'| |E_2'| |Y_{12}| \cos(\delta_{12} - \theta_{12}) + |E_1'| |E_3'| |Y_{13}| \cos(\delta_{12} - \theta_{12})$$

So do summation

OR: use Norton Equiv
 For each Mach:



~~By you know~~

and power flow

1) Use V_T to find $|E'| = \text{constant}$.
 (From load flow) and δ_0 & I_0

2) Switch system (Fault, etc)

$$P_e = E_1' I_a^* = \frac{E_1' [(E_1' - V_T)]^*}{jX_d}$$

Modified Euler Method - ^{Adapted} from Stagg & El-Abiad 3

- 1) Use V_T and power flow from Load flow to find $|E|_0, \delta_0, I_{a0}, P_m$
 $P_m = \text{Re}\{E_0 I_{a0}^*\} = \text{constant}$
- 2) Perform system switching (Fault) etc
(Modify Y-Bus)
- 3) Use $I_{sc(0)} = \frac{E'_0}{jX'_d}$ and solve $V_{(0+)} = Y^{-1} I_{sc(0)}$ for all machines.
- 4) Calculate new machine currents based on $\frac{E'_0 - V}{jX'_d} = I_a(0+)$

- 5) Calculate machine electrical Power
 $P_e = \text{Re}\{E'_0 I_a^*(0+)\}$

$$\begin{aligned} 6) \quad \omega(t+\Delta t) &= \omega(t) + \left[\frac{d}{dt} (\omega(t)) \right] \Delta t \quad \text{from swing eqn.} \\ &= \omega(t) + \left[\frac{\pi f}{H} (P_m - P_e) \right] \Delta t \quad \text{est.} \end{aligned}$$

$$\begin{aligned} \delta(t+\Delta t) &= \delta(t) + \frac{d\delta(t)}{dt} \Delta t \\ &= \delta(t) + [\omega(t) - \omega_s] \Delta t \quad \text{est.} \end{aligned}$$

- 7) Calculate ~~new~~ est voltages behind $jX'_d \Rightarrow |E'| \angle \delta(t+\Delta t)$

- 8) Use $|E'| \angle \delta(t+\Delta t)$ (est) to determine $V(t+\Delta t) = Y^{-1} I_{sc}(t+\Delta t)$

- 9) Calculate $P_e = \text{Re}\{E'_{(t+\Delta t)} I_{a(t+\Delta t)}^*\}$

$$10) \quad \omega(t+\Delta t) = \omega(t) + \left[\frac{\frac{d\omega(t)}{dt} + \frac{d\omega(t+\Delta t)}{dt}}{2} \right] \Delta t$$

$$\delta(t+\Delta t) = \delta(t) + \left[\frac{\frac{d\delta(t)}{dt} + \frac{d\delta(t+\Delta t)}{dt}}{2} \right] \Delta t$$

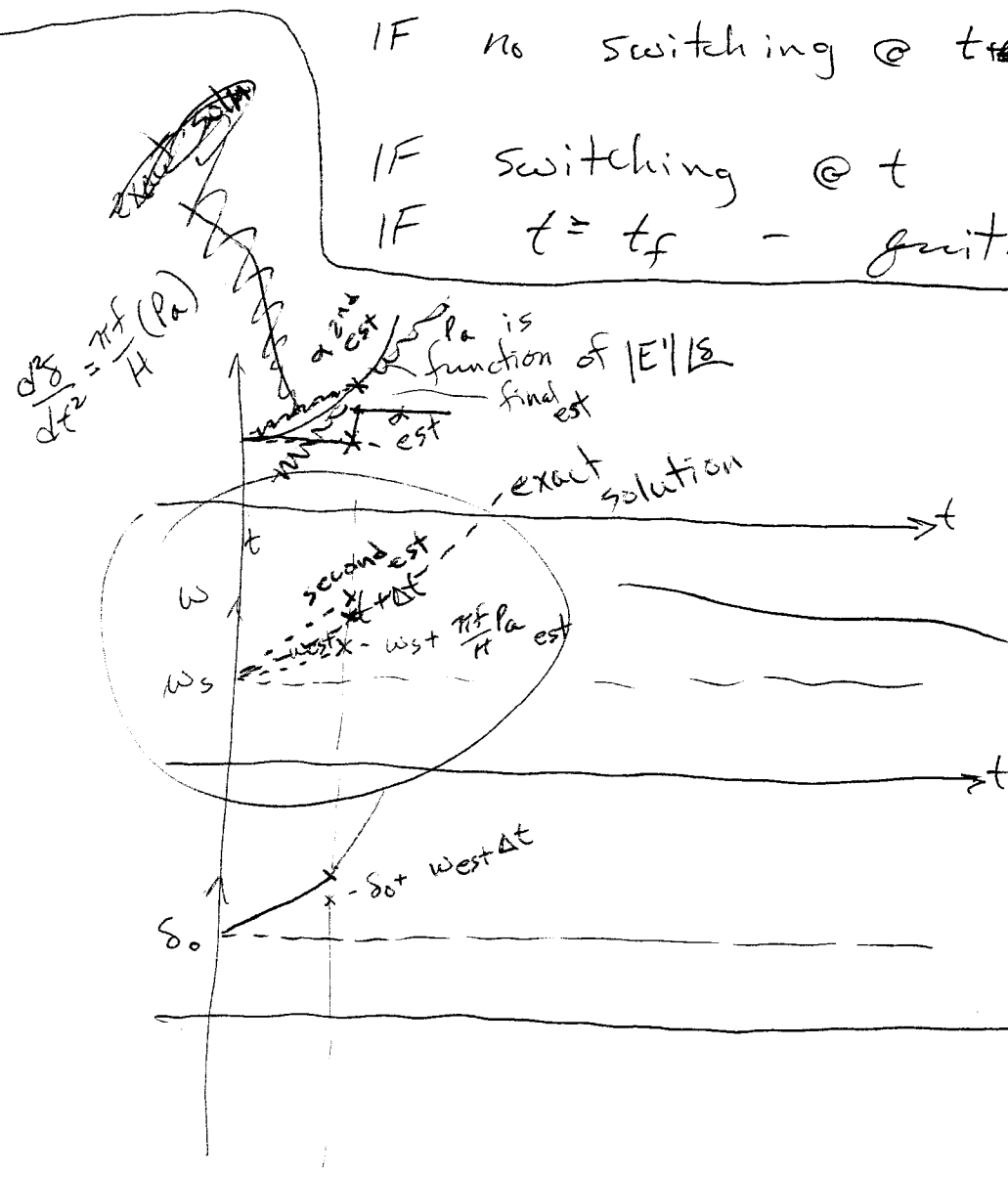
11) Calculate final est. of voltages behind $jXd \Rightarrow |E'|/L\delta(t+\Delta t)$

~~GOTO 4~~ $t = t + \Delta t$

IF no switching @ ~~t~~ ~~t~~ **GOTO 4**

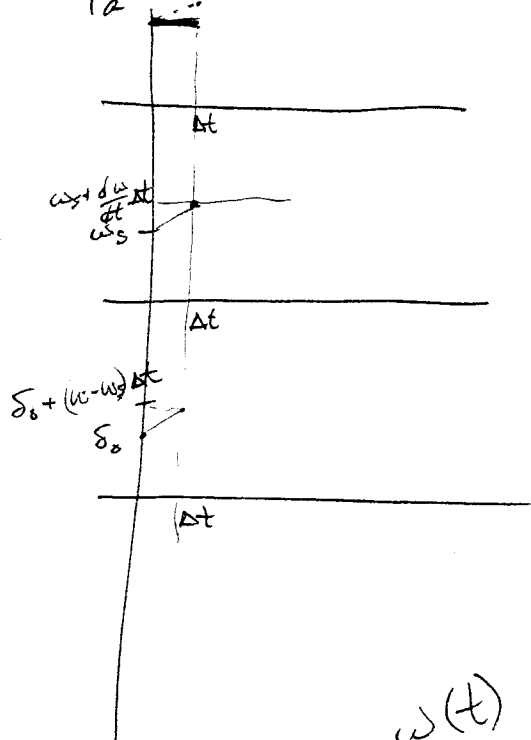
IF switching @ t **GOTO 2**

IF $t = t_f$ - quit.

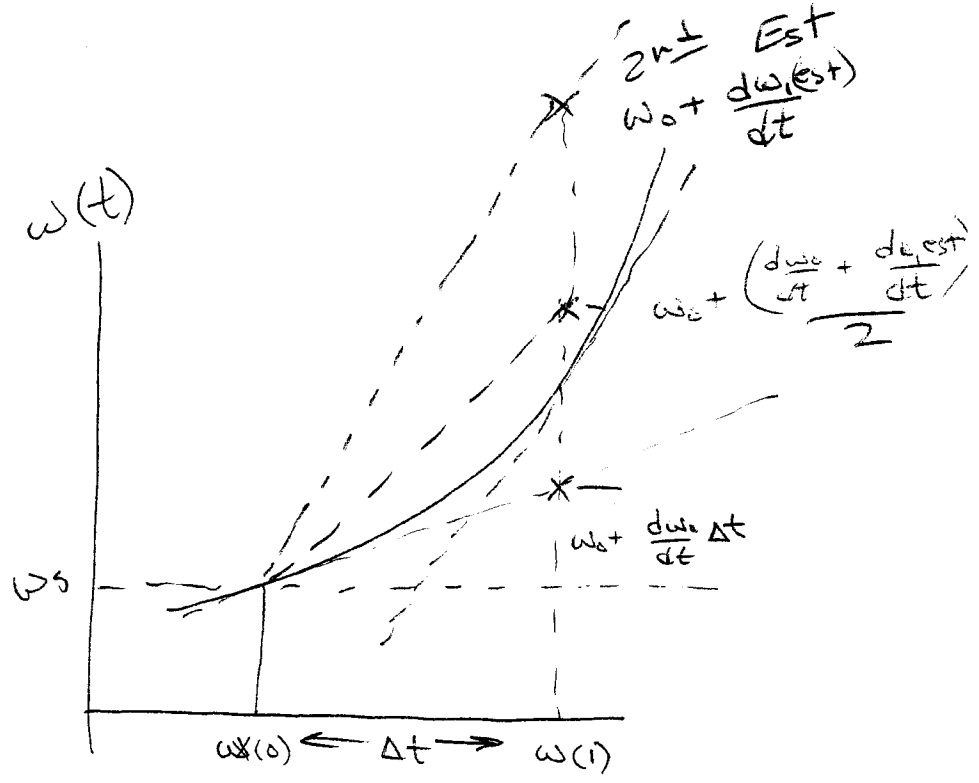


see next page

$$P_2 = \frac{H}{\pi f} \frac{d^2 \delta^0}{dt^2} = P_m - P_e$$

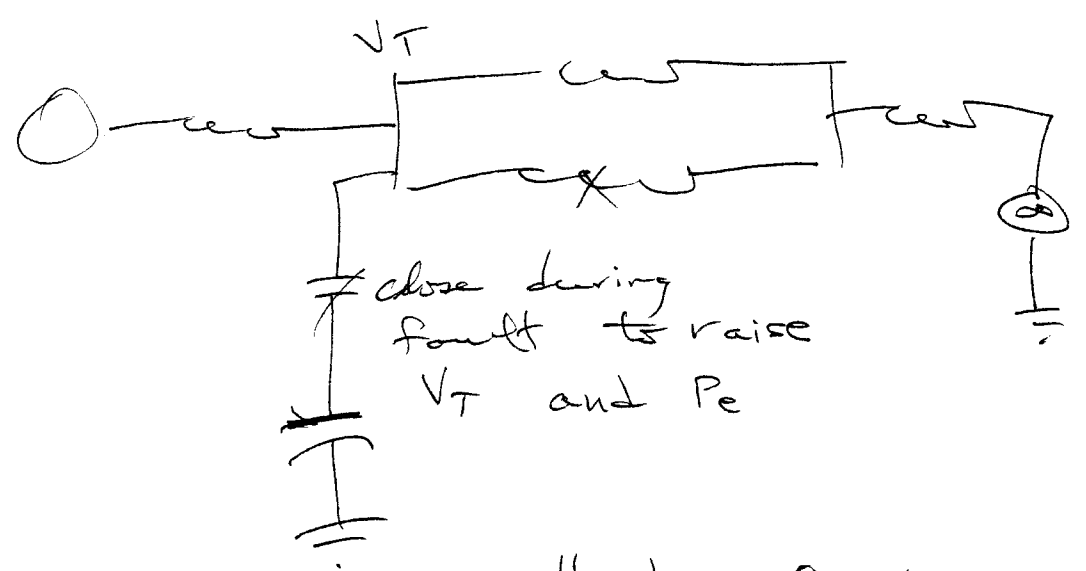


MODIFIED
EULER



Factors Affecting Stability

- Gov
- Exciter
- PSS (Power System Stabilizer)
works on Δf from f_s changes V_{int}
faster than Exciter ΔV controls
- STATIC VAR COMP



Hydro Quebec

Fast Valve

Braking Resist

EE486 - QUIZ I

A turbine/generator set manufactured by a European factory has a polar moment of inertia = 8800 ton-m². (This "ton" is a metric ton, equal to 2205 lbm.) Synchronous speed of the machine is 300 RPM, and its base MVA is 310 MVA.

Determine H on the machine's base.