A power cycle produces 2.4×10^8 Btu and rejects 7.1×10^8 Btu of heat to the surroundings. Calculate the thermal efficiency of the cycle.

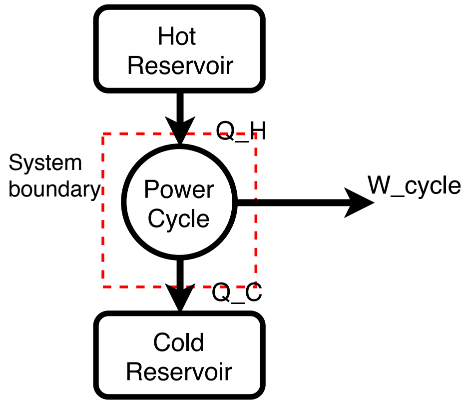

Figure 1: Schematic of power cycle

Solution:
The first law of thermodynamics is applied to the closed system that encloses the cycle. Heat and work exchanges occur but NO mass (of working fluid) influx or efflux occurs. For a power generating cycle such as described in figure 1, the *thermal efficiency*, η is simply given by:

$$\eta = \frac{W_{\text{output}}}{Q_{\text{input}}}$$

The first law of thermodynamics for a closed system:

$$\underbrace{\cancelto{U}{\Delta E}}_{\text{Cycle, no change}} = \oint \delta Q - \oint \delta W$$

$$\implies \underbrace{\cancelto{0}{\Delta U}}_{\text{Cycle, no change}} = \oint \delta Q - \oint \delta W$$

$$\begin{aligned}
\eta &= \frac{W_{\text{output}}}{Q_{\text{input}}} \\
&= \frac{W_{\text{cycle}}}{Q_H} \\
\therefore \eta &= \frac{2.4 \times 10^8}{W + Q_C} \\
&= \frac{2.4 \times 10^8}{(2.4 \times 10^8) + (7.1 \times 10^8)}
\end{aligned}$$

$$\eta = 25.26\%$$

An industrial refrigerator rejects heat at a rate of $24,750$ kJ/min to the surroundings. If the refrigeration cycle has a coefficient of performance (COP) of $b = 3.3$, determine Q_C and W_{cycle}, each in kJ/min.

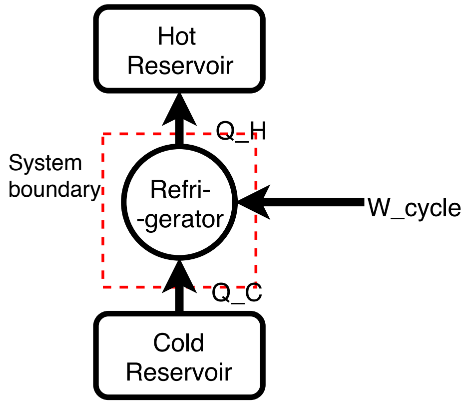

Figure 2: Schematic of refrigeration cycle

Solution:
The first law of thermodynamics is applied to the closed system that encloses the refrigerator cycle. Heat and work exchanges occur but NO mass influx or efflux (of refrigerant) occurs across the boundary of this system. For a power consuming refrigeration cycle such as described in figure 2, the *COP*, b is simply given by:

$$b = \frac{\text{desired (cooling) effect}}{\text{Work input}} = \frac{\dot{Q}_C}{\dot{W}_{\text{cycle}}}$$

The first law of thermodynamics for a closed system (in *rate form*):

$$\underbrace{\cancelto{U}{\Delta \dot{E}}}_{\text{Cycle, no change}} = \oint \delta\dot{Q} - \oint \delta\dot{W} \tag{1}$$

$$\implies \underbrace{\cancelto{0}{\Delta \dot{U}}}_{\text{Cycle, no change}} = \oint \delta\dot{Q} - \oint \delta\dot{W} \tag{2}$$

$$\oint \delta\dot{Q} = \oint \delta\dot{W} \tag{3}$$

$$\implies \dot{Q}_C + \dot{W}_{\text{cycle}} = \dot{Q}_H \tag{4}$$

$$b = \frac{\dot{Q}_C}{\dot{W}_{\text{cycle}}} \tag{5}$$

$$\therefore \dot{Q}_C = b \times \dot{W}_{\text{cycle}} \tag{6}$$

From equation 4:

$$\dot{Q}_H = \dot{W}_{\text{cycle}} + b\,\dot{W}_{\text{cycle}}$$
$$\therefore \dot{W}_{\text{cycle}} = \frac{\dot{Q}_H}{1+b}$$

$$\dot{W}_{\text{cycle}} = 5755.81\,\text{kJ/min}$$

From equation 6

$$\dot{Q}_C = b \times \dot{W}_{\text{cycle}}$$

$$\dot{Q}_C = 18994.2\,\text{kJ/min}$$

Some Notes:

1. Remember that for a **cycle**, the change in internal energy is zero. Internal energy is a function of state (or a state variable). In a cycle, the internal energy is returned to it's original or initial value.
2. The cyclic integral symbol, $\oint(\ldots)$, is used to describe heat and work exchanges in a **cycle**. The little circle in the integral sign depicts the "cyclic" nature of the process(es).
3. Just follow the direction arrows for the power cycle and the refrigerant to extract the closed system energy balance equation. An arrow coming into the system is treated as being positive while an arrow leaving the system is deducted, i.e.: for the power generating cycle: $Q_H - W_{\text{cycle}} - Q_C = 0 \implies Q_H = W_{\text{cycle}} + Q_C$ while for the refrigeration cycle (power absorbing): $\dot{Q}_C + \dot{W}_{\text{cycle}} - \dot{Q}_H = 0 \implies \dot{Q}_C + \dot{W}_{\text{cycle}} = \dot{Q}_H$.
4. For a power generating cycle, the useful heat input is consumed to (1) generate some power (2) the rest is wasted as "heat lost". For a power consuming cycle (like a refrigerator), useful power input is used to extract heat from a cold space (food compartments) and reject it to a warm space (the room).