

# Open System Energy Balance: (Control Volume analysis)

## □ Mass conservation

$$\dot{m}_{in} + \dot{m}_{gen} = \dot{m}_{out} + \dot{m}_{stored} \quad \text{--- (1)}$$

$$\rightarrow \dot{m} = \rho A V \xrightarrow{\text{velocity}} = A v / \rho$$

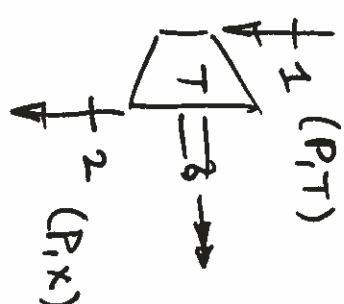
## □ Open System Energy conservation

$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out} + \dot{E}_{stored} \quad \text{--- (2)}$$

## + SF, SS, EE

$$(2) \Rightarrow \cancel{\frac{dE}{dt}} = \dot{Q} - \dot{W} + \dot{m}_1 \left( h_1 + \frac{V_1^2}{2} + g z_1 \right)$$

$$\begin{aligned} \dot{E}_{stored} &= 0 \\ \{ \text{Steady state} \} \end{aligned}$$



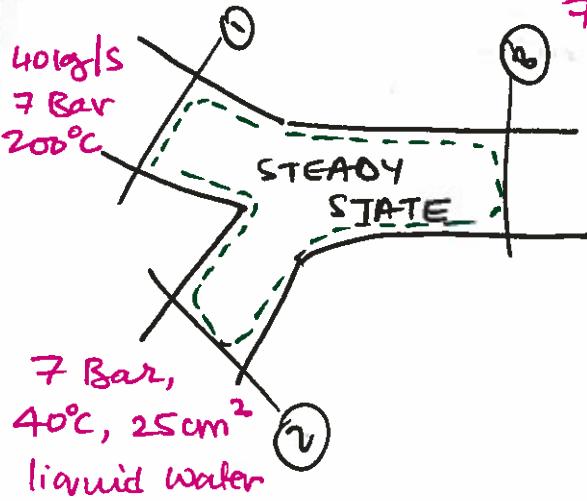
## → Closed System E-Balance: Special case of SFEE

$$\dot{m}_1 = 0 = \dot{m}_2, \quad dE/dt = 0$$

"Under what conditions  $\dot{Q} = \dot{W}$  can you ignore Bernoulli term from the SFEE"

[ms 4.1] } ms = MORAN + SHAPIRO

A feedwater heater operating at steady state has two inlets and one exit. At inlet 1, water vapor enters at  $p_1 = 7 \text{ bar}$ ,  $T_1 = 200^\circ\text{C}$  with a mass flow rate of  $40 \text{ kg/s}$ . At inlet 2, liquid water at  $p_2 = 7 \text{ bar}$ ,  $T_2 = 40^\circ\text{C}$  enters through an area  $A_2 = 25 \text{ cm}^2$ . Saturated liquid at 7 bar exits at 3 with a volumetric flow rate of  $0.06 \text{ m}^3/\text{s}$ . Determine the mass flow rates at inlet 2 and at the exit, in kg/s, and the velocity at inlet 2, in m/s.



7 Bar,  $0.06 \text{ m}^3/\text{s}$   
sat. liquid.

□ GIVEN: States of water @ 1, 2, 3,  
 $A_2 = 25 \text{ cm}^2$ ,  
 $\dot{V}_3 = 0.06 \text{ m}^3/\text{s}$ .

□ To Find:  
→  $m_2$ ,  $m_3$   
→  $v_2$  (velocity)

□ SOLUTION:

- Relevant properties @ 3 locations:

$$@1: P_1 = 7 \text{ Bar} = 7 \text{ atm.}, T_1 = 200^\circ\text{C} \\ = 0.7 \text{ MPa}$$

$$v_1 = \frac{1}{2} (0.35212 + 0.26088) \Rightarrow \\ (6 \text{ Bar}, 200^\circ) \quad (8 \text{ Bar}, 200^\circ) \\ [Table A-6]$$

$$v_1 = 0.306 \text{ m}^3/\text{kg}$$

$$@2: \text{Compressed liquid water} \quad \left. \begin{aligned} v_2 &\approx v_f (\text{T}_{\text{sat}} @ 7 \text{ Bar}) \\ v_2 &\approx 0.001108 \text{ m}^3/\text{kg} \end{aligned} \right\}$$

[Table A-5]

@3: Saturated liquid }  $V_3 = 0.001108 \text{ m}^3/\text{kg}$  (TAS)

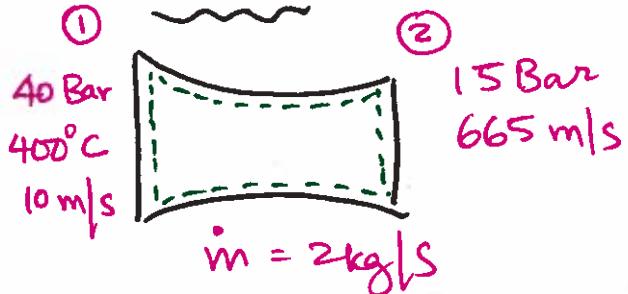
Mass conservation }  $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$   
 $\therefore \dot{m}_2 = \dot{m}_3 - \dot{m}_1$   
 $= \frac{\dot{V}_3}{\dot{V}_3} - \dot{m}_1 = \frac{0.06}{0.001108} - 40$   
 $\dot{m}_2 = \cancel{14} \cdot 15 \text{ kg/s}$  ← Please check

$$\therefore \dot{m}_2 = P_2 A_2 v_2 = \frac{A_2 v_2}{\dot{V}_3} \Rightarrow v_2 = \frac{\dot{m}_2 \dot{V}_3}{A_2}$$

$$v_2 = \frac{(14 \cdot 15)(0.001108)}{(25 \times 10^{-4})} \Rightarrow \boxed{v_2 = 6.27 \text{ m/s}}$$

[ms 4.3]

Steam enters a converging-diverging nozzle operating at steady state with  $p_1 = 40$  bar,  $T_1 = 400^\circ\text{C}$ , and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit,  $p_2 = 15$  bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in  $\text{m}^2$ .

GIVEN:To Find: Exit area of nozzle.SOLUTION:

SSEE:

(No boundary movement)

$$\cancel{\frac{dE}{dt}} = \cancel{\dot{Q} - \dot{W}} + \dot{m} \left( h_1 + \frac{v_1^2}{2} + g z_1 \right) \\ \text{Steady State} \quad \cancel{\dot{Q} - \dot{W}} - \dot{m} \left( h_2 + \frac{v_2^2}{2} + g z_2 \right)$$

negligible heat tran

$$\therefore h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} \quad \text{--- (I)}$$

$\frac{\text{kJ}}{\text{kg}}$        $\frac{\text{m}^2}{\text{s}^2}$   
 $\frac{\text{BTU}}{\text{kg}}$

$$@1: h_1 = h_1(40 \text{ Bar}, 400^\circ\text{C}) \quad \underbrace{\text{Sup'nt. steam}}_{\Rightarrow h_1 = 3214.5 \frac{\text{kJ}}{\text{kg}}}$$

$$(I) \Rightarrow h_2 = h_1 + \left( \frac{v_1^2 - v_2^2}{2 \times 1000} \right) t \text{ J} \rightarrow \text{kJ}$$

$$h_2 = 3214.5 + \left( \frac{10^2 - 665^2}{2000} \right)$$

$$h_2 = 2993.44 \text{ kJ/kg}$$

$$v_2 @ (P_2, h_2) = v_2(15 \text{ bar}, 2993.44 \text{ kJ/kg})$$

$$v_2 \approx 0.220 \text{ m}^3/\text{kg} \quad [\text{Supt steam}]$$

$$\dot{m}_2 = \dot{m}_1 = P_2 A_2 v_2 = \frac{A_2 v_2}{v_2}$$

$$\Rightarrow A_2 \approx \frac{\dot{m}_2 v_2}{v_2} \approx \frac{(0.22)(2)}{665}$$

$$A_2 \approx 661 \times 10^{-6} \text{ m}^2 \approx 6.61 \text{ cm}^2$$

[ms 4.4]

Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9 (90%), and the velocity is 30 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

Given: Steady state op. of steam turbine

$$\dot{m} = 4600 \text{ kg/h}, \dot{W}_T = 1000 \text{ kW}$$

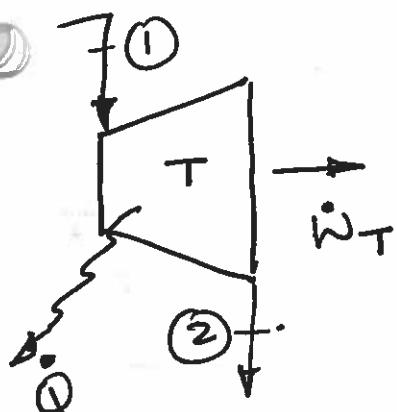
$$T_1 = 400^\circ\text{C}, P_1 = 60 \text{ Bar}, P_2 = 0.1 \text{ Bar}, x_2 = 90\%$$

$$v_1 = 10 \text{ m/s}, v_2 = 30 \text{ m/s}$$

To Find: Rate of H.T between turbine, surroundings.

Solution:

Steady state energy balance (SSEE):



$$\frac{dE}{dt} = \dot{Q} - \dot{W}_T + \dot{m}(h_1 + \frac{v_1^2}{2} + g z_1) - \dot{m}(h_2 + \frac{v_2^2}{2} + g z_2)$$

$$0 = \dot{Q} - \dot{W}_T + \dot{m}(h_1 + \frac{v_1^2}{2}) - \dot{m}(h_2 + \frac{v_2^2}{2})$$

$$h_1 = h_1(P_1, T_1)$$

$$= h_1(60 \text{ Bar}, 400^\circ\text{C}) = h_1(6 \text{ MPa}, 400^\circ\text{C})$$

$$\Rightarrow h_1 = 3178.3 \text{ kJ/kg}$$

Sup'nt.  
steam table

$h_2 = h_2(P_2, x_2) \equiv \text{Saturated L-v}$

$$= h_2(0.01 \text{ MPa}, 90^\circ\text{C})$$

OR  
10 kPa

$$h_2 = h_{f2} + x_2 h_{fg2} @ (10 \text{ kPa})$$

$$\boxed{h_2 = 2344.7 \text{ kJ/kg}}$$

$$0 = \dot{Q} - \dot{W}_T + \underbrace{\frac{4600}{3600}}_{+1000 \text{ kW}} \left[ 3178.3 \frac{\text{kJ/kg}}{} + \frac{10^2}{2000} \frac{\text{kJ/kg}}{} \right]$$

$$- \frac{4600}{3600} \left[ 2344.7 \frac{\text{kJ/kg}}{} + \frac{30^2}{2000} \frac{\text{kJ/kg}}{} \right]$$

$$\boxed{\dot{Q} = \dots}$$

[DIY]

[ms 4.5]

Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m<sup>2</sup>. At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

GIVEN: Air (ideal gas EoS),  $P_1 = 1 \text{ Bar}$ ,  $T_1 = 290 \text{ K}$ ,  $v_1 = 6 \text{ m/s}$

$$A_1 = 0.1 \text{ m}^2;$$

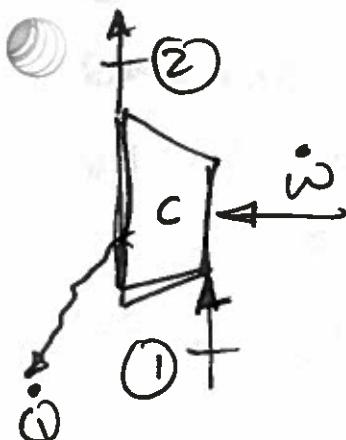
$$P_2 = 7 \text{ Bar}, T_2 = 450 \text{ K}, v_2 = 2 \text{ m/s}.$$

$$\dot{Q} = -180 \text{ kJ/min} = -3 \text{ kJ/s} = -3 \text{ kW}$$

To Find: Power input to compressor.

SOLUTION:

Steady state energy balance for an open system:



$$\frac{dE}{dt} = \dot{Q} - \dot{\omega} + \dot{m}(h_1 + \frac{v_1^2}{2} + g z_1) - \dot{m}(h_2 + \frac{v_2^2}{2} + g z_2)$$

$$\dot{\omega} = \dot{Q} + \dot{m}(h_1 + \frac{v_1^2}{2}) - \dot{m}(h_2 + \frac{v_2^2}{2})$$

$$\text{where: } \dot{m} = \frac{A_1 v_1}{\rho_1} = \frac{A_1 v_1}{R T_1 |P_1|} = \frac{(0.1)(6)}{\frac{287 \times 290}{100 \times 10^3}} =$$

$$[R = R_{\text{Sp,air}} = 287 \text{ J/kg-K}]$$

$$\boxed{\dot{m} = 0.721 \text{ kg/s}}$$

$$\left. \begin{array}{l} \text{use Table A-17} \\ \text{for air properties} \end{array} \right\} \quad \begin{array}{l} h_1 = h_1(290K) = 290.16 \text{ kJ/kg} \\ h_2 = h_2(450K) = 451.8 \text{ kJ/kg} \end{array}$$

$$\dot{w} = \dot{Q} + \dot{m} \left( h_1 + \frac{v_1^2}{2000} \right) - \dot{m} \left( h_2 + \frac{v_2^2}{2000} \right)$$

$$\boxed{\dot{w} = -119.4 \text{ kW}}$$

