Method of Separation of Variables:

$$\phi_t = f(\eta)\, g(\tau)$$

$$\frac{\partial \phi_t}{\partial t} = \frac{\partial^2 \phi_t}{\partial \eta^2} \Rightarrow \frac{\partial}{\partial t}\left(f(\eta)\, g(\tau)\right) = \frac{\partial^2}{\partial \eta^2}\left(f(\eta)\, g(\tau)\right)$$

$$f\,\frac{\partial g}{\partial \tau} = g\,\frac{\partial^2 f}{\partial \eta^2} \Rightarrow \frac{1}{g}\frac{\partial g}{\partial \tau} = \frac{1}{f}\frac{\partial^2 f}{\partial \eta^2} = \underbrace{-c^2}_{\text{"a constant"}}$$

$$\frac{dg}{d\tau} = -c^2 g \quad , \quad \frac{d^2 f}{d\eta^2} + c^2 f = 0$$

$$g = A e^{-c^2 \tau} \qquad f = B \sin c\eta + C \cos c\eta$$

$$g_n = A_n \exp(-n^2\pi^2\tau)$$

$$\phi_t = gf = 0 \quad , \eta = 0 \qquad \underline{\underline{B.C.1}}$$

$$0 = B \sin 0 + C \cos 0 = \boxed{C = 0}$$

$$\phi_t = gf = 0 \quad , \eta = 1 \qquad \underline{\underline{B.C.2}}$$

either : $B = 0$ <u>or</u> $c = 0, \pi, 2\pi, \dots$

eigenvalues $\longrightarrow$ $c_n = n\pi \quad , n = 0, 1, 2, \dots$

$$f_n = B_n \sin n\pi\eta$$

$$\phi_t = \sum_{n=1}^{\infty} g_n f_n = \sum_{n=1}^{\infty} D_n \exp(-n^2\pi^2\tau) \sin n\pi\eta$$

$$\tau = 0 \quad , \phi_t = \phi_\infty = 1 - \eta$$

$$1 - \eta = \sum_{n=1}^{\infty} D_n \sin n\pi\eta$$

$$\int_0^1 (1-\eta)\sin m\pi\eta \, d\eta = \sum_{n=1}^{\infty} D_n \int_0^1 \sin(n\pi\eta)\sin(m\pi\eta)\, d\eta$$

$$\frac{1}{m\pi} = \frac{D_m}{2} \Longrightarrow D_m = \frac{2}{m\pi}$$

$$\therefore \phi_t = \sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right) \exp(-n^2\pi^2\tau)\sin(n\pi\eta)$$

$$\boxed{\phi = \phi_\infty - \phi_t = (1-\eta) - \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\right)\exp(-n^2\pi^2\tau)\sin(n\pi\eta)}$$

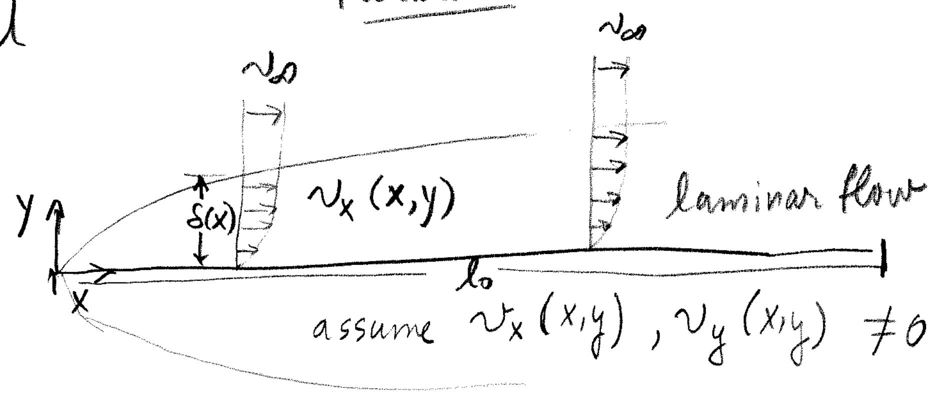
Boundary-Layer Theory 4.4

$v_\infty = v_e(x) = U$

$$v_e \frac{dv_e}{dx} = -\frac{1}{\rho}\frac{d\mathcal{P}}{dx}$$

for constant flow: $\frac{dv_e}{dx} = 0$

$\therefore \frac{d\mathcal{P}}{dx} = 0$



assume $v_x(x,y)$, $v_y(x,y) \neq 0$

$v_z = 0$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial \mathcal{P}}{\partial x} + \nu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)$$

$$v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho}\frac{\partial \mathcal{P}}{\partial y} + \nu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right)$$

Which term in these eqns are important?

Order of Magnitude Analysis!

$$\frac{\partial v_x}{\partial x} = O\left(\frac{v_\infty}{l_0}\right) = \frac{\partial v_y}{\partial y} \qquad \underline{\text{Eqn. of Continuity.}}$$

$$\frac{\partial v_y}{\partial y} = O\,\frac{v_\infty}{l_0} \Longrightarrow \int_0^{v_y} \partial v_y = \int_0^{\delta} \frac{v_\infty}{l_0}\,\partial y \Rightarrow v_y = O\, v_\infty \frac{\delta_0}{l_0}$$

$\therefore v_y << v_x$

x-component.

$$\left.\begin{aligned} v_x \frac{\partial v_x}{\partial x} &= O\left(v_\infty \frac{v_\infty}{l_0}\right) = O\left(\frac{v_\infty^2}{l_0}\right) \\ v_y \frac{\partial v_x}{\partial y} &= O\left(v_\infty \frac{\delta_0}{l_0} \frac{v_\infty}{\delta}\right) = O\left(\frac{v_\infty^2}{l_0}\right) \end{aligned}\right\} \quad lhs = O(rhs)$$

$$\left.\begin{aligned} \frac{\partial^2 v_x}{\partial x^2} &= O\left(\frac{v_\infty}{l_0^2}\right) \\ \frac{\partial^2 v_x}{\partial y^2} &= O\left(\frac{v_\infty}{\delta_0^2}\right) \end{aligned}\right\} \quad \frac{\partial^2 v_x}{\partial x^2} << \frac{\partial^2 v_x}{\partial y^2}$$

$$\nu \frac{\partial^2 v_x}{\partial y^2} = O\left(\nu \frac{v_\infty}{\delta^2}\right) = O\left(\frac{v_\infty^2}{l_0}\right)$$

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$$\frac{\delta_0}{l_0} = O\left(\sqrt{\frac{\nu}{v_\infty l_0}}\right) = O\left(\frac{1}{\sqrt{Re}}\right)$$

y-component

$$v_x \frac{\partial v_y}{\partial x} = O\left(v_\infty^2 \frac{\delta_0}{l_0^2}\right)$$

$$v_y \frac{\partial v_y}{\partial y} = O\left(v_\infty^2 \frac{\delta_0^2}{l_0^2} \cdot \frac{1}{\delta_0}\right) = O\left(v_\infty^2 \frac{\delta_0}{l_0^2}\right)$$

$$\frac{\partial^2 v_y}{\partial x^2} = O\left(v_\infty \frac{\delta_0}{l_0} \cdot \frac{1}{l_0^2}\right)$$