Ch9; Thermal Conductivity and Mechanisms of Energy Transport.

Mechanisms of Energy Transport

1. molecular energy transport – due to molecular motion
2. convective " " – due to bulk fluid motion
3. diffusive " " – due to interdiffusing mixtures (ch 19.3, 24.2).
4. radiation " " – ch 16.

Molecular Energy Transport – Conduction

→ conduction of heat is a familar concept to everyone

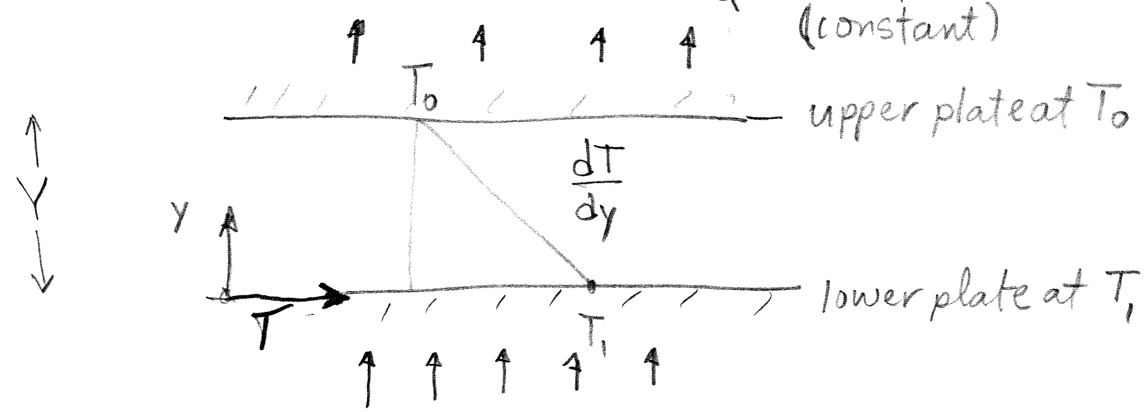
examples: a) holding a cold metal object vs a " wood "

b) "wet" cloths feel colder than dry cloths

c) cooking items like pots + pans are often fabricated with a layer of Cu or Al to distribute heat.

Fourier's Law of Conduction:

Q = rate of heat flow (constant)



observed $\frac{Q}{A} = k \frac{\Delta T}{Y}$ "Fourier's Law"

more accurately; $q_y = -k \frac{dT}{dy}$ "

where q_y = flux of heat in positive y direction.

$\frac{dT}{dy}$ = temperature gradient in the y direction

k = thermal conductivity [W/(m·K)] or [cal/(cm·s·°C)]

If temperature gradients exist in 3 coordinate directions

$$q_x = -k\frac{dT}{dx}, \quad q_y = -k\frac{dT}{dy}, \quad q_z = -k\frac{dT}{dz}$$

When the unit vector, δ_i, is (x) and 3 components Σ

$$\boxed{q = -k \nabla T}$$

q is a heat flux vector.
∇T is a gradient vector.

and k is the same in all 3 coordinate directions,
isotropic

For anisotropic materials (non cubic crystals, fibrous materials, laminates

$$q = -[K \cdot \nabla T]$$

where K (kappa) is the thermal conductivity tensor.

Thus, the heat flux vector does not point in the same direction as the temperature gradient.

Another important energy (heat) transport property is.

$$\alpha \equiv \frac{k}{\rho \hat{C}_p} \qquad [m^2/s] \text{ or } [cm^2/sec].$$ "thermal diffusivity"

where ρ = material density $[g/cm^3]$

$\hat{C}_p$ = heat capacity at constant pressure $[cal/(^\circ C \cdot g)]$

A dimensionless number often appearing in equation of change for energy transport is.

$$Pr \equiv \frac{\nu}{\alpha} = \frac{\hat{C}_p \mu}{k}$$ "Prandtl number"

$$Pe \equiv Re\, Pr$$ "Peclet number"

Summary of k, Pr values. Tables 9.1-2 – 9.1-5

gases: $0.01 \le k \le 0.10$ $[W/m \cdot {}^\circ K]$

liquids: $0.10 \le k \le 1.0$ "

liquid metals: $10 \le k \le 100$ "

solids: $0.01 \le k \le 1{,}000$ "

Temperature Dependence of k

gases: k ↑ with ↑T

liquids: k ↓ with ↑T

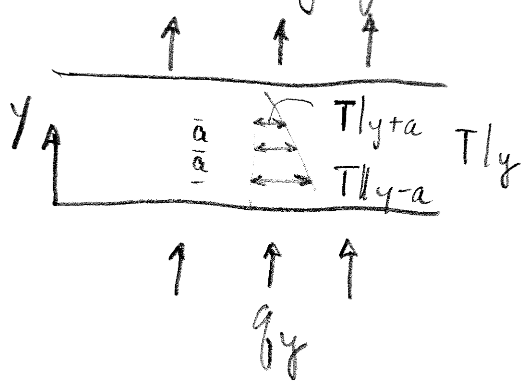
solids: k ↑ with ↑T

Pressure Dependence of k,

gases: $k \uparrow$ when $P \uparrow$

liquids: $k \uparrow$ as $P \uparrow$

Kinetic Theory of Gas: Prediction of k.



properties of a monatomic gas.

$$\bar{u} = \sqrt{\frac{8\kappa T}{\pi m}}$$

$$Z = \frac{1}{4} n \bar{u}$$ collision frequency

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$ mean free path

$$\frac{1}{2} m \overline{u^2} = \frac{3}{2} \kappa T$$ mean kinetic energy/molec.

$$a = \frac{2}{3}\lambda$$ avg. distance between planes where last collision occurred

Energy Balance at y:

"heat flux across any plane at y" =

"rate of kinetic energy arriving from a plane at $y-a$" −

"rate of kinetic energy arriving from a plane at $y+a$"

$$q_y = Z \frac{1}{2} m \overline{u^2}\Big|_{y-a} - Z \frac{1}{2} m \overline{u^2}\Big|_{y+a}$$

$$= Z\left(\frac{3}{2}\kappa T|_{y-a} - \frac{3}{2}\kappa T|_{y+a}\right) = \frac{3}{2}\kappa Z\left(T|_{y-a} - T|_{y+a}\right)$$

$$T|_{y-a} = T|_y - a\frac{dT}{dy} = T|_y - \frac{2}{3}\lambda\frac{dT}{dy}$$

$$T|_{y+a} = T|_y + a\frac{dT}{dy} = T|_y + \frac{2}{3}\lambda\frac{dT}{dy}$$

$$q_y = \frac{3}{2}KZ\left(T|_y - \frac{2}{3}\lambda\frac{dT}{dy} - T|_y - \frac{2}{3}\lambda\frac{dT}{dy}\right).$$

$$= -2KZ\lambda\frac{dT}{dy}$$

$$= -2K\left(\frac{1}{4}\cancel{n}\bar{u}\right)\left(\frac{1}{\sqrt{2}\pi d^2\cancel{n}}\right)\frac{dT}{dy}$$

$$= -2K\left(\frac{1}{4}\sqrt{\frac{8KT}{\pi m}}\right)\left(\frac{1}{\sqrt{2}\pi d^2}\right)\frac{dT}{dy}$$

$$= -\frac{2}{4}\frac{\sqrt{8}}{\sqrt{2}}\left(\frac{\sqrt{KT/(\pi m)}}{\pi d^2}\right)\frac{dT}{dy}K$$

$$= -\left(\frac{\sqrt{KmT/\pi}}{\pi d^2}\right)\frac{K}{m}\frac{dT}{dy}$$

$$= -\frac{2}{3\pi}\frac{\sqrt{\pi mKT}}{\pi d^2}\hat{C}_v\frac{dT}{dy}.$$

where $\hat{C}_v = \frac{3}{2}\frac{K}{m}$

heat capacity at constant volume per mole gas.

From Fourier's Law,

$$q_y = -k\frac{dT}{dy}.$$

$$\therefore \boxed{k = \frac{2}{3\pi}\frac{\sqrt{\pi mKT}}{\pi d^2}\hat{C}_v}$$

Chapman - Enskog Formula:

$$k = \frac{25}{35} \frac{\sqrt{\pi m K T}}{\pi \sigma^2 \Omega_k} \hat{C}_v \quad \text{or} \quad k = 1.9891 \times 10^{-4} \frac{\sqrt{T/M}}{\sigma^2 \Omega_k}$$

[cal/(cm·s·°K)]

T [°K]

σ [Å]

$\Omega_k = \Omega_\mu$ in E.2.

$$k = \frac{15}{4} \frac{R}{M} \mu = \frac{5}{2} \hat{C}_v \mu$$

from Eqn. 9.3-13
and " 1.4-14.

Other Useful Formula, for Polyatomic gases.

$$k = \left(\hat{C}_p + \frac{5}{4} \frac{R}{M}\right) \mu$$

$\hat{C}_p = \frac{5}{2} \frac{R}{M}$ for monatomic gases.

$$P_r \equiv \frac{\hat{C}_p \mu}{k} = \frac{\hat{C}_p}{(\hat{C}_p + \frac{5}{4} R)}$$

Wed 2/22/06

Thermal Conductivity of Liquids, 9.4.

Bridgman's Theory; monatomic liquids.

Assumptions:

- atoms/molecules oscillate inside a "cage" having a volume roughly $= \tilde{V}/\tilde{N}$ volume/mole avagadro's no.
- heat is transfered by collisions with nearest neighbor molecules at the sonic velocity of the liquid, $v_s = \sqrt{\left(\frac{C_p}{C_v}\right)\left(\frac{\partial p}{\partial \rho}\right)_T}$ isothermal compressibility. ~1 for liquids

$$k = \frac{1}{3}\rho \hat{C}_v \bar{u} \lambda = \rho \hat{C}_v |u_y| a$$

molecular mass — $\frac{m\tilde{N}}{\tilde{V}}$ $\quad 3\frac{K}{m}$ $\quad v_s$ $\quad$ spacing between lattice planes in the liquid. $(\tilde{V}/\tilde{N})^{1/3}$.

Boltzman's Constant

$$k = 3\left(\tilde{N}/\tilde{V}\right)^{2/3} K v_s \longrightarrow \boxed{2.8\left(\tilde{N}/\tilde{V}\right)^{2/3} K v_s}$$