Thermal Conductivity of Solids, 9.5

- mechanisms of energy transfer in solids is complex
- no general predicitive method is available.
- heat is conducted by molecular collisions and by free electrons in pure metals.

Pure Metals: heat conduction and electrical conduction occur by free electrons,

$$\frac{k}{k_e T} = L = \text{constant}$$

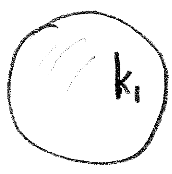
"Wiedemann-Franz-Lorenz" Eqn.

L = Lorenz No ⇒ $22 \rightarrow 29 \times 10^{-9}$ $\text{volt}^2/{}^\circ K^2$ @ 0°C.

Effective Thermal Conductivity of Composite Solids, 9.6

Spheres:

k_0 – continuous media



$$\frac{k_{eff}}{k_0} = 1 + \frac{3\phi}{\left(\frac{k_1 + 2k_0}{k_1 - k_0}\right) - \phi}$$

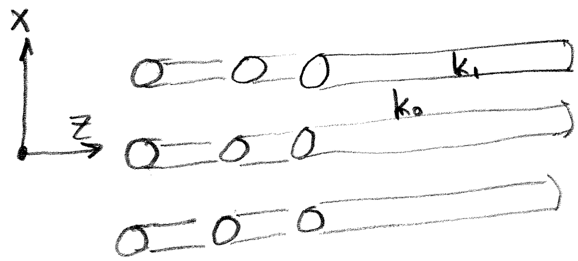
, ϕ = volume fraction solid in continuous phase (low values)

"by Maxwell"

Spheres: large ϕ, up to $\pi/6$ <u>Rayleigh</u>

$$\frac{k_{eff}}{k_o} = 1 + \frac{3\phi}{\left(\frac{k_1+2k_o}{k_1-k_o}\right) - \phi + 1.569\left(\frac{k_1-k_o}{3k_1-4k_o}\right)\phi^{10/3} + \cdots}$$

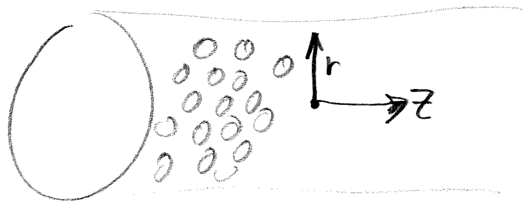
Cylinders / Non-spherical Inclusions.



$$\frac{k_{eff,zz}}{k_o} = 1 + \left(\frac{k_1 - k_o}{k_o}\right)\phi$$

$$\frac{k_{eff,xx}}{k_o} = 1 + \frac{2\phi}{\left(\frac{k_1+k_o}{k_1-k_o}\right) - \phi + \left(\frac{k_1-k_o}{k_1+k_o}\right)(.306\phi^4 + .0134\phi^8 + \cdots)}$$

Packed Beds of Porous Mat'l: (packed bed reactors).
fluid flows through bed.



$$K_{eff,rr} = \underbrace{\frac{1}{10}}_{\text{}}\ \rho\, \hat{C}_p\, v_0\, D_p$$ — particle diameter

ρ: fluid density

$\hat{C}_p$: fluid heat capacity

v_0: "superficial" velocity $= \dot{V}/A$

$\dot{V}$ — volumetric flowrate

A — empty bed cross-sectional area

$$\left.\begin{aligned} K_{eff,zz} &= \frac{1}{2}\rho \hat{C}_p v_0 D_p \\ K_{eff,rr} &= \frac{1}{10}\rho \hat{C}_p v_0 D_p \end{aligned}\right\} \quad \text{For } Re = D_p v_0 \rho/\mu > 200$$

Combined Energy Flux Vector, $\mathbf{e}$

1. convective flux vector
2. work flux vector
3. molecular heat flux vector

1. Convective Energy Flux Vector.
   - one important property of a fluid is it's energy content.

kinetic energy per unit volume.

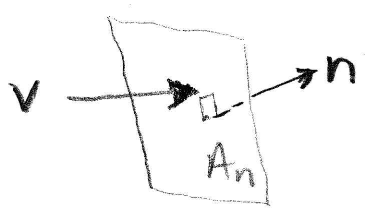
$$\frac{1}{2}\rho v^2 = \frac{1}{2}\rho\left(v_x^2 + v_y^2 + v_z^2\right)$$

internal energy per unit volume due to intra- and intermolecular potential energies

$$\rho \hat{U}$$

$$\boxed{\left(\frac{1}{2}\rho v^2 + \rho \hat{U}\right)\mathbf{v}} \rightarrow \text{convective energy flux vector}$$

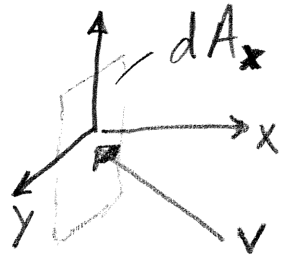
$$\left(\mathbf{n}\cdot\left(\frac{1}{2}\rho v^2 + \rho\hat{U}\right)\mathbf{v}\right) \rightarrow \text{convective energy flux through a surface normal to } \mathbf{n}$$



"a rate of flow of energy in a direction normal to A_n.

x-component,

$$\delta_x \cdot \left(\tfrac{1}{2}\rho v^2 + \rho \hat{U}\right) \mathbf{v} = \left(\tfrac{1}{2}\rho v^2 + \rho \hat{U}\right) v_x$$



similarly for y- and z- components

Molecular Work Flux Vector

- rate of work $\left(\frac{dW}{dt}\right)$ by a force (F), $\rightarrow$ $\mathbf{F}\cdot\mathbf{v}$

- in a fluid, the force per unit area that a fluid exerts on surrounding fluid is.,

$$\pi_x = p\,\delta_x + \tau_x \quad \text{– vector force acting on } A_x$$
$$\pi_y = p\,\delta_y + \tau_y \quad \text{– " " " " } A_y$$
$$\pi_z = p\,\delta_z + \tau_z \quad \text{– " " " " } A_z$$

- rate of work done on x, y, and z planes.

$$(\pi_x \cdot \mathbf{v}) = \pi_{xx} v_x + \pi_{xy} v_y + \pi_{xz} v_z \quad \text{-- } A_x$$
$$(\pi_y \cdot \mathbf{v}) = \pi_{yx} v_x + \pi_{yy} v_y + \pi_{yz} v_z \quad \text{-- } A_y$$
$$(\pi_z \cdot \mathbf{v}) = \pi_{zx} v_x + \pi_{zy} v_y + \pi_{zz} v_z \quad \text{-- } A_z$$

or combining.

$$[\pi \cdot \mathbf{v}] = \delta_x (\pi_x \cdot \mathbf{v}) + \delta_y (\pi_y \cdot \mathbf{v}) + \delta_z (\pi_z \cdot \mathbf{v})$$

Combined Energy Flux Vector, e

$$\boxed{e = \left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right)v + [\pi\cdot v] + q}$$

but $[\pi\cdot v] = pv + [\tau\cdot v]$

$$\boxed{e = \left(\tfrac{1}{2}\rho v^2 + \rho\hat{H}\right)v + [\tau\cdot v] + q}$$

: note $\rho\hat{U}v + pv \Rightarrow \rho(\hat{U} + p/\rho)v \Rightarrow$

$\rho(\hat{U} + p\hat{V})v \Rightarrow \rho\hat{H}v$

Fri 2/24/06