$Z = \frac{1}{4} n \bar{u}$ "frequency of molecular collision per unit area".

n = number density (molecules per volume).

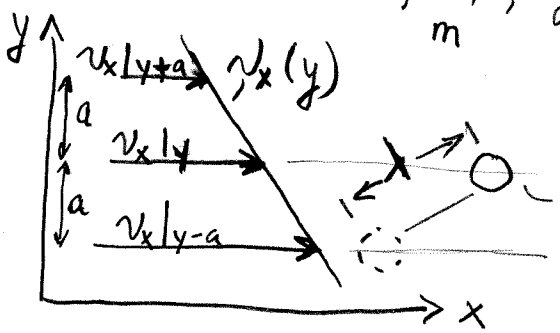
* Kinetic Theory is valid if $\frac{d}{\lambda} \ll 1$

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ "mean free path" avg. distance between collisions

Determine μ in terms of K, T, d, m { momentum is transported via collisions between atoms of the gas!

$a \sim \frac{2}{3}\lambda$

velocity gradient $\frac{dv_x}{dy}$



all molecular velocities are relative to the average velocity, $v_x(y)$.

Flux of x momentum in y-direction @ y.

$$\tau_{yx} = \sum m v_x|_{y-a} - \sum m v_x|_{y+a}$$

But for a linear velocity profile, dv_x/dy.

$$v_x|_{y\pm a} = v_x|_y \pm a\frac{dv_x}{dy} = v_x|_y \pm \frac{2}{3}\lambda\frac{dv_x}{dy}.$$

That is,

$$v_x|_{y-a} = v_x|_y - \frac{2}{3}\lambda \frac{dv_x}{dy} \quad \text{and}$$

$$v_x|_{y+a} = v_x|_y + \frac{2}{3}\lambda \frac{dv_x}{dy}$$

So τ_{yx} becomes.

$$\tau_{yx} = Zm\left(v_x|_y - \frac{2}{3}\lambda\frac{dv_x}{dy}\right) - Zm\left(v_x|_y + \frac{2}{3}\frac{dv_x}{dy}\right)$$

$$= -Zm\,\frac{4}{3}\lambda\frac{dv_x}{dy}$$

$$= -\frac{1}{4}n\bar{u}\,m\lambda\frac{dv_x}{dy}\frac{4}{3} = -\frac{1}{3}n\bar{u}m\lambda\frac{dv_x}{dy}.$$

$$= -\mu\frac{dv_x}{dy}.$$

So

$$\mu = \frac{1}{3}n\bar{u}m\lambda$$

$$= \frac{1}{3}n\left(\frac{8KT}{\pi m}\right)^{1/2} m \frac{1}{\sqrt{2}\,\pi d^2 n}$$

$$\boxed{= \frac{2}{3}\frac{(mKT/\pi)^{1/2}}{\pi d^2}}$$

eqn 1.4-9

$\mu \updownarrow$ as $\sqrt{T}$

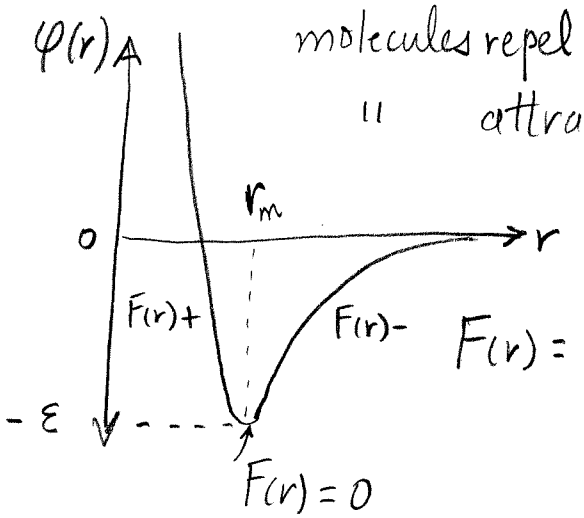
μ is independent of n (of pressure).

The prediction of eqn. 1.4-9 for μ independent of P agrees with experiments up to ~10 atm for $T > T_c$. The observed T dependence of μ is $T^{0.6}$ to $T^{1.0}$, $> T^{1/2}$.

To improve prediction of μ for gases, we need to account for molecular interactions.

Chapman-Enskog theory: Appendix D too.

Uses the Boltzmann eqn. and intermolecular potential energy function, $\varphi(r)$, to obtain molecular velocity distribution more accurately.



molecules repel for $r < r_m$

" attract for $r > r_m$.

$F(r) = -\frac{d\varphi}{dr}$ intermolecular force.

Lennard-Jones (6-12) potential.

$$\varphi(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

L-J Parameters:

- σ = characteristic (collision) diameter of molecules.
- ε = characteristic energy (maximum energy of attraction).

$\varepsilon/K = 0.77\,T_c$ $\quad \sigma = 0.841\,\tilde{V}_c^{1/3}$ or $= 2.44\,(T_c/P_c)^{1/3}$

$\varepsilon/K = 1.15\,T_b$ $\quad \sigma = 1.166\,\tilde{V}_{b,liq}^{1/3}$

$\varepsilon/K = 1.92\,T_m$ $\quad \sigma = 1.222\,\tilde{V}_{m,sol}^{1/3}$

ε/k and T in Kelvin, σ in Å (angstroms (10^{-10} m))

P in atm, $\tilde{V}$ in $cm^3/g\cdot mole$.

$$\mu = \frac{5}{16}\frac{\sqrt{\pi m k T}}{\pi \sigma^2 \Omega_\mu} \quad \text{or} \quad = 2.6693\times10^{-5}\frac{\sqrt{MT}}{\sigma^2\Omega_\mu}$$

[g/cm·s]

M is molecular wt. (g/g-mole)

T in [K]

Ω_μ = collision integral for viscosity

~1.0, a function of KT/ε (dimensionless T).

Table E.2

at low KT/ε (0.3-0.6), Ω_μ is 2.0-3.0 (low T or high ε).

at higher KT/ε (0.6-2.6), Ω_μ is 1.0-2.0 (higher T or low ε).

C-E Theory assumes monoatomic gas but has been shown to give good results for polyatomic gases too.

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