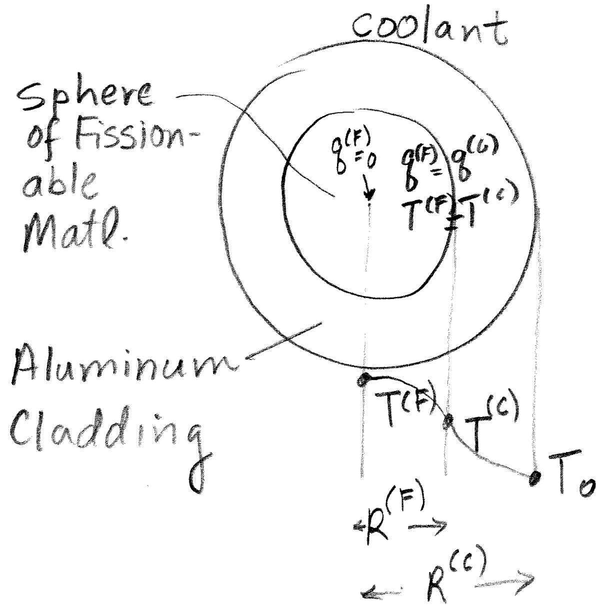
Heat Conduction With a Nuclear Heat Source: 10.3.



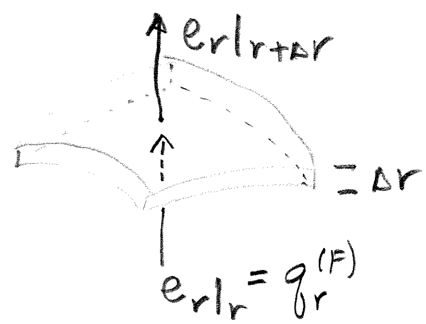
<u>Heat Source</u> - inner sphere.

$$S_n = S_{n0}\left[1+b\left(\frac{r}{R^{(F)}}\right)^2\right]$$

Assumptions,

$v_r = v_\theta = v_\phi = 0$

$T = f(r)$ only



Shell Balance: spherical shell

Fissionable matl:

rate of heat conduction in at r, $\quad q^{(F)}|_r \cdot 4\pi r^2$

" " " " out at $r+\Delta r$ $\quad (4\pi r^2 q_r^{(F)})|_{r+\Delta r}$

rate of heat generation by nuclear fission $\quad S_n \cdot 4\pi r^2 \Delta r$

$$4\pi\left((r^2 q_r^{(F)})|_r - (r^2 q_r^{(F)})|_{r+\Delta r}\right) + 4\pi r^2 \Delta r S_n = 0$$

÷ by $4\pi\Delta r$. $\lim \Delta r \to 0$

$$\lim_{\Delta r\to 0} \frac{(r^2 q_r^{(F)})|_{r+\Delta r} - (r^2 q_r^{(F)})|_r}{\Delta r} = S_n r^2$$

$$\frac{d}{dr}(r^2 q_r^{(F)}) = S_{n0}\left[1+b\left(\frac{r}{R^{(F)}}\right)^2\right] r^2 \qquad \text{(F), Fissionable Matl.}$$

$$\frac{d}{dr}\left(r^2 q_r^{(c)}\right) = 0 \qquad \text{(C), Cladding.}$$

Integrating:

(F): $r^2 q_r^{(F)} = S_{n_0}\left[\frac{r^3}{3} + \frac{b}{R^{(F)2}}\frac{r^5}{5}\right] + C_1^{(F)}$

$$\boxed{q_r^{(F)} = S_{n_0}\left[\frac{r}{3} + \frac{b}{R^{(F)2}}\frac{r^3}{5}\right]} + \frac{C_1^{(F)}}{r^2}$$

BC1 $\quad r=0,\ q_r^{(F)}$ is finite, $\therefore C_1^{(F)}=0$

(C): $r^2 q_r^{(c)} = C_1^{(c)} \longrightarrow q_r^{(c)} = \frac{C_1^{(c)}}{r^2}$

BC2 $\quad r = R^{(F)} \quad q_r^{(F)} = q_r^{(c)}$

$$S_{n_0}\left[\frac{R^{(F)}}{3} + \frac{b}{R^{(F)2}}\frac{R^{(F)3}}{5}\right] = \frac{C_1^{(c)}}{R^{(F)2}}$$

$$\therefore C_1^{(c)} = S_{n_0}\left[\frac{R^{(F)3}}{3} + \frac{b}{5}R^{(F)3}\right]$$

$$\boxed{q_r^{(c)} = S_{n_0}\frac{R^{(F)3}}{r^2}\left[\frac{1}{3} + \frac{b}{5}\right]}$$

Fourier's Law:

(F) $\qquad -k^{(F)}\frac{dT^{(F)}}{dr} = S_{n_0}\left[\frac{r}{3} + \frac{b}{5}\frac{r^3}{R^{(F)2}}\right]$

$$T^{(F)} = -\frac{S_{n_0}}{k^{(F)}}\left[\frac{r^2}{6} + \frac{b}{20}\frac{r^4}{R^{(F)2}}\right] + C_2^{(F)}$$

BC 3. $T^{(F)} = T^{(c)}$ at $r = R^{(F)}$

$$(C)\quad -k^{(c)}\frac{dT^{(c)}}{dr} = S_{n_0}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{r^2}$$

$$T^{(c)} = +\frac{S_{n_0}}{k^{(c)}}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{r} + C_2^{(c)}$$

BC4, $T^{(c)} = T_0$ at $r = R^{(c)}$

$$T_0 = \frac{S_{n_0}}{k^{(c)}}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{R^{(c)}} + C_2^{(c)}$$

$$C_2^{(c)} = T_0 - \frac{S_{n_0}}{k^{(c)}}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{R^{(c)}}$$

$$\therefore\ T^{(c)} = \frac{S_{n_0}}{k^{(c)}}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{r} + T_0 - \frac{S_{n_0}}{k^{(c)}}\left[\frac{1}{3} + \frac{b}{5}\right]\frac{R^{(F)^3}}{R^{(c)}}$$

$$\boxed{= T_0 + \frac{S_{n_0}R^{(F)^2}}{3k^{(c)}}\left(1 + \frac{3b}{5}\right)\left(\frac{R^{(F)}}{r} - \frac{R^{(F)}}{R^{(c)}}\right)}$$

Evaluating BC3. again.

$T^{(F)} = T^{(c)}$ at $r = R^{(F)}$.

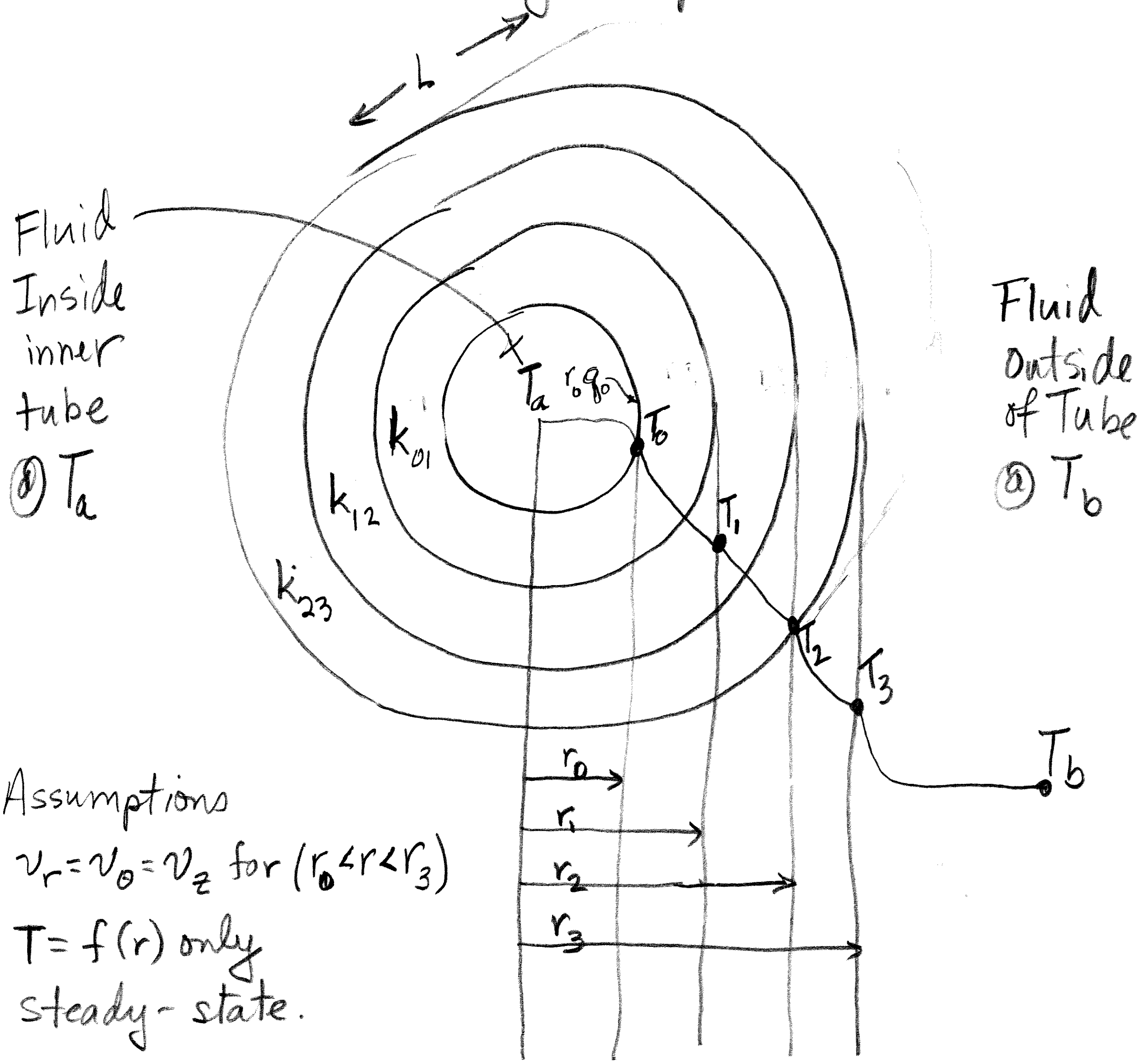
$$-\frac{S_{n_0}}{k^{(F)}}\left[\frac{R^{(F)^2}}{6} + \frac{b}{20}R^{(F)^2}\right] + C_2^{(F)} = T_0 + \frac{S_{n_0}R^{(F)^2}}{3k^{(c)}}\left(1 + \frac{3b}{5}\right)\left(1 - \frac{R^{(F)}}{R^{(c)}}\right)$$

$$C_2^{(F)} = T_0 + \frac{S_{n_0}R^{(F)^2}}{k^{(F)}}\left[\frac{1}{6} + \frac{b}{20}\right] + \frac{S_{n_0}R^{(F)^2}}{3k^{(c)}}\left(1 + \frac{3b}{5}\right)\left(1 - \frac{R^{(F)}}{R^{(c)}}\right)$$

$$T^{(F)} = -\frac{Sn_0}{k^{(F)}}\left[\frac{r^2}{6} + \frac{b}{20}\frac{r^4}{R^{(F)2}}\right] + T_0 + \frac{Sn_0 R^{(F)2}}{k^{(F)}}\left[\frac{1}{6} + \frac{b}{20}\right] + \frac{Sn_0 R^{(F)2}}{3k^{(c)}}\left(1 + \frac{3b}{5}\right)\left(1 - \frac{R^{(F)}}{R^{(c)}}\right)$$

$$\boxed{= T_0 + \frac{Sn_0 R^{(F)2}}{6k^{(F)}}\left\{\left(1 - \left(\frac{r}{R^{(F)}}\right)^2\right) + \frac{3b}{10}\left(1 - \left(\frac{r}{R^{(F)}}\right)^4\right)\right\} + \frac{Sn_0 R^{(F)2}}{3k^{(c)}}\left(1 + \frac{3b}{5}\right)\left(1 - \frac{R^{(F)}}{R^{(c)}}\right)}$$

Heat Conduction Through Composite Walls: 10.6



Assumptions

$v_r = v_\theta = v_z$ for $(r_0 < r < r_3)$

$T = f(r)$ only

steady-state.

Energy Shell Balance:

region $r_0 < r < r_1$ (01)

rate of heat in at r — $2\pi r L \cdot q_r|_r$

" " " out at $r+\Delta r$ — $(2\pi r L\, q_r)|_{r+\Delta r}$

(no heat generation in region 01).

$$(2\pi r L q_r)\big|_r - (2\pi r L q_r)\big|_{r+\Delta r} = 0$$

$\div$ by $2\pi\, \Delta r\, L$, let $\Delta r \to 0$

$$\frac{d}{dr}(r q_r) = 0$$

integrating, $r q_r = r_0 q_0$ (q_0 = heat flux at $r = r_0$).
$\longrightarrow$ still unknown now!

Fourier's Law $\quad -k_{01} r \dfrac{dT}{dr} = r_0 q_0 \quad$ <u>region 01</u>

For other regions

$$-k_{12} r \frac{dT}{dr} = r_0 q_0 \qquad \text{region 12}$$

$$-k_{23} r \frac{dT}{dr} = r_0 q_0 \qquad \text{region 23}$$

Integrate each region eqn,

region 01 $\quad \displaystyle\int_{T_0}^{T_1} dT = -\frac{r_0 q_0}{k_{01}} \int_{r_0}^{r_1} \frac{dr}{r} = -\frac{r_0 q_0}{k_{01}} \ln \frac{r_1}{r_0}$

or $\boxed{T_0 - T_1 = \dfrac{r_0 q_0}{k_{01}} \ln \dfrac{r_1}{r_0}}$

region 12, $\quad T_1 - T_2 = \dfrac{r_0 q_0}{k_{12}} \ln \dfrac{r_2}{r_1}$

region 23, $\quad T_2 - T_3 = \dfrac{r_0 q_0}{k_{23}} \ln \dfrac{r_3}{r_2}$

Inside Tube Surface, $r_0(-)$ $\qquad T_a - T_0 = q_0/h_0$

Outside Tube Surface, $r_3\,(+)$ $\qquad T_3 - T_b = \dfrac{q_0\, r_0}{h_3\, r_3} = q_3$

$$T_a - T_b = (T_a - T_0) + (T_0 - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_b)$$

$$= q_0/h_0 + \frac{r_0 q_0}{k_{01}} \ln\left(\frac{r_1}{r_0}\right) + \frac{r_0 q_0}{k_{12}} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_0 q_0}{k_{23}} \ln\left(\frac{r_3}{r_2}\right) + \frac{q_0 r_0}{h_3 r_3}$$

$$= q_0 \left(\frac{1}{h_0} + \frac{r_0}{k_{01}} \ln\left(\frac{r_1}{r_0}\right) + \frac{r_0}{k_{12}} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_0}{k_{23}} \ln\left(\frac{r_3}{r_2}\right) + \frac{r_0}{h_3 r_3} \right)$$

or $q_0 = U_0 (T_a - T_b)$

where $U_0 = \left(\dfrac{1}{h_0} + \dfrac{r_0}{k_{01}} \ln\left(\dfrac{r_1}{r_0}\right) + \dfrac{r_0}{k_{12}} \ln\left(\dfrac{r_2}{r_1}\right) + \dfrac{r_0}{k_{23}} \ln\left(\dfrac{r_3}{r_2}\right) + \dfrac{r_0}{h_3 r_3} \right)^{-1}$

"overall heat transfer coefficient"

Wed 3/01/06