$$(\Delta z W e_y|_y) - (\Delta z W e_y|_{y+\Delta y}) + (\Delta y W e_z|_z) - (\Delta y W e_z|_{z+\Delta z}) + (\Delta y W \Delta z)\rho g_z v_z = 0$$

$\div \Delta z W \Delta y$ let $\Delta z, \Delta y \to 0$

$$-\frac{\partial e_y}{\partial y} - \frac{\partial e_z}{\partial z} + \rho g_z v_z = 0$$

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$$e_y = \delta_y \cdot e = (\tfrac{1}{2}\rho v^2 + \rho \hat{H})\cancelto{0}{v_y} + [\tau \cdot v]_y + q_y$$

$$[\tau \cdot v]_y = \tau_{yx}\cancelto{0}{v_x} + \tau_{yy}\cancelto{0}{v_y} + \tau_{yz} v_z = -\mu\left(\frac{\partial v_z}{\partial y}\right) v_z$$

$$q_y = -k\frac{\partial T}{\partial y}$$

$$= -\mu\left(\frac{\partial v_z}{\partial y}\right) v_z - k\frac{\partial T}{\partial y}$$

$$-\frac{\partial e_y}{\partial y} = \mu\left(\frac{\partial v_z}{\partial y}\right)\frac{\partial v_z}{\partial y} + \mu v_z \frac{\partial^2 v_z}{\partial y^2} + k\frac{\partial^2 T}{\partial y^2}$$

$$e_z = \delta_z \cdot e = (\tfrac{1}{2}\rho v^2 + \rho\hat{H}) v_z + [\tau \cdot v]_z + q_z$$

$$v^2 = \cancelto{0}{v_x^2} + \cancelto{0}{v_y^2} + v_z^2 = v_z^2$$

$$\hat{H} = \hat{H}^\circ + \hat{C}_p(T - T^\circ) + \frac{1}{\rho}(p - p^\circ) \qquad (\text{pg } 286)$$

$$[\tau \cdot v]_z = \tau_{zx}\cancelto{0}{v_x} + \tau_{zy}\cancelto{0}{v_y} + \tau_{zz} v_z = -2\mu\left(\frac{\partial v_z}{\partial z}\right) v_z + \overset{(\nabla \cdot v)}{0}$$

$$q_z = -k\cancelto{0}{\frac{\partial T}{\partial z}} \qquad T = f(y) \text{ only}$$

$$= \tfrac{1}{2}\rho v_z^2 \cdot v_z + \rho\hat{H}^\circ v_z + \rho\hat{C}_p(T - T^\circ) v_z + (p - p^\circ) v_z - 2\mu\left(\frac{\partial v_z}{\partial z}\right) v_z$$

$$-\frac{\partial e_z}{\partial z} = 0 - 0 - 0 - 0 - v_z\frac{\partial p}{\partial z} - 0 - 0$$

note: $\frac{\partial v_z}{\partial z} = 0$ for this problem.

Combining terms: for $-\frac{\partial e_y}{\partial y} - \frac{\partial e_z}{\partial z} + \rho g_z v_z = 0$

$$\mu\left(\frac{\partial v_z}{\partial y}\right)^2 + \mu v_z \frac{\partial^2 v_z}{\partial y^2} + k\frac{\partial^2 T}{\partial y^2} - v_z\frac{\partial P}{\partial z} + \rho g_z v_z = 0$$

rearranging,

$$0 = k\frac{\partial^2 T}{\partial y^2} + \overbrace{\cancel{\mu\left(\frac{\partial v_z}{\partial y}\right)^2}}^{0 \text{ viscous heat generation}} + v_z\underbrace{\left(-\frac{\partial P}{\partial z} + \mu\frac{\partial^2 v_z}{\partial y^2} + \rho g_z\right)}_{\text{z-component of eqn. of motion: } = 0}$$

Table B.6, Eqn. B.6-3

or

$$k\frac{\partial^2 T}{\partial y^2} = 0$$

BC1 $y = -B$ $T = T_2$

BC2 $y = +B$ $T = T_1$

integrating: $T = C_1 y + C_2$

BC1 $\Rightarrow$ $T_2 = C_1(-B) + C_2 \longrightarrow C_2 = T_2 + BC_1$

BC2 $\Rightarrow$ $T_1 = C_1(+B) + C_2$

$T_1 = BC_1 + T_2 + BC_1 \longrightarrow \boxed{C_1 = -\frac{1}{2B}(T_2 - T_1) = -\frac{1}{2B}\Delta T}$

$C_2 = T_2 + BC_1 = T_2 - \frac{B}{2B}\Delta T = \boxed{T_2 - \frac{1}{2}\Delta T} = C_2$

$\therefore$ $T = C_1 y + C_2$

$$= -\frac{1}{2B}\Delta T y + T_2 - \frac{1}{2}\Delta T = \boxed{\bar{T} - \frac{1}{2}\Delta T\frac{y}{B}}$$

where $\bar{T} \equiv \frac{1}{2}(T_2 + T_1)$

Equation of Motion, y-component.

$$\mu \frac{d^2 v_z}{dy^2} = \frac{dp}{dz} + \rho g$$

$$= \frac{dp}{dz} + (\bar{\rho} - \bar{\rho}\bar{\beta}(T-\bar{T}) + \cdots) g$$

$$= \cancel{\left(\frac{dp}{dz} + \bar{\rho} g\right)}^{0} - \bar{\rho}\bar{\beta}(T-\bar{T}) g$$

but $T-\bar{T} = -\frac{1}{2}\Delta T \frac{y}{B}$

So

$$\mu \frac{d^2 v_z}{dy^2} = \frac{1}{2}\bar{\rho}\bar{\beta}\Delta T \frac{y}{B} g$$

BC1 $\quad y = -B \quad v_z = 0$

BC2 $\quad y = +B \quad v_z = 0$

$$v_z(y) = \frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu B} y^3 + C_1 y + C_2$$

BC1 $\quad 0 = \frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu B}(-B)^3 + C_1(-B) + C_2.$

$$C_2 = BC_1 + \frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu} B^2$$

BC2 $\quad 0 = \frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu B}(B)^3 + BC_1 + C_2.$

$$= \quad '' \quad + \quad '' \quad + BC_1 + \frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu} B^2$$

$$\boxed{C_1 = -\frac{\bar{\rho}\bar{\beta}\Delta T g}{12\mu} B}$$

$$\boxed{C_2 = 0}$$

$$v_z(y) = \frac{\bar{\rho}\bar{\beta}\Delta T g B^2}{12\mu}\left(\frac{y}{B}\right)^3 - \frac{\bar{\rho}\bar{\beta}\Delta T g B^2}{12\mu}\left(\frac{y}{B}\right).$$

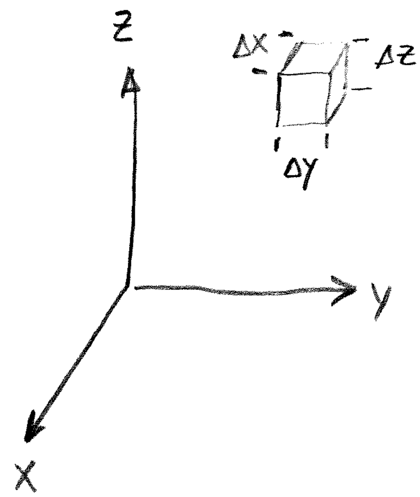
$$\boxed{= \frac{\bar{\rho}\bar{\beta}\Delta T g B^2}{12\mu}\left[\left(\frac{y}{B}\right)^3 - \left(\frac{y}{B}\right)\right]}$$

Grashof Number, $Gr \equiv \dfrac{\bar{\rho}^2 g \bar{\beta}\Delta T B^3}{\mu^2}$ ← bouyancy forces ← viscous forces

Dimensionless Velocity $\breve{v}_z \equiv \dfrac{B v_z \bar{\rho}}{\mu}$

$$\boxed{\breve{v}_z = \frac{1}{12} Gr\left(\left(\frac{y}{B}\right)^3 - \left(\frac{y}{B}\right)\right)}$$

Equations of Change For Non-isothermal Systems - Ch 11



Apply law of Conservation of Energy to a small Elemental Volume

"rate of increase of kinetic + internal energy" =

$$\Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho \hat{U}\right)$$

"net rate of kinetic + internal energy addn. by convection, conduction, and by work from stresses" +

$$\Delta y \Delta z \left(e_x|_x - e_x|_{x+\Delta x}\right) + \Delta x \Delta z \left(e_y|_y - e_y|_{y+\Delta y}\right) + \Delta x \Delta y \left(e_z|_z - e_z|_{z+\Delta z}\right)$$

"rate of work done on fluid by external forces"

$$\rho \Delta x \Delta y \Delta z \left(v_x g_x + v_y g_y + v_z g_z\right)$$

÷ by $\Delta x \Delta y \Delta z$, let $\Delta x, \Delta y, \Delta z \to 0$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho \hat{U}\right) = -(\nabla \cdot e) + \rho (v \cdot g)$$

Subst. for $e = (\frac{1}{2}\rho v^2 + \rho\hat{U})v + [\pi \cdot v] + q$

but $\pi = p\delta + \tau \longrightarrow [\pi \cdot v] = pv + [\tau \cdot v]$

and $\nabla \cdot e = \nabla \cdot (\frac{1}{2}\rho v^2 + \rho\hat{U})v) + \nabla \cdot pv + \nabla \cdot [\tau \cdot v]$

Eqn. of Energy – Kinetic + Internal Wed. 3/15/06

$$\boxed{\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2 + \rho\hat{U}) = -(\nabla \cdot (\tfrac{1}{2}\rho v^2 + \rho\hat{U})v) - (\nabla \cdot q) - (\nabla \cdot pv) - (\nabla \cdot [\tau \cdot v]) + \rho(v \cdot g)} \qquad (11.1\text{-}7)$$

does not include nuclear, radiative, electromagnetic or chemical sources of energy.

Including Potential Energy:

$$g = -\nabla\hat{\Phi} \qquad \hat{\Phi} = gh$$

$\rho(v \cdot g)$ term:

$$\begin{aligned}
\rho(v \cdot g) &= -(\rho v \cdot \nabla\hat{\Phi}) && \text{due to } \downarrow \\
&= -(\nabla \cdot \rho v\hat{\Phi}) + \hat{\Phi}(\nabla \cdot \rho v) && (A.4\text{-}19) \\
&= -(\nabla \cdot \rho v\hat{\Phi}) + \hat{\Phi}\frac{\partial \rho}{\partial t} && (3.1\text{-}4) \\
&= -(\nabla \cdot \rho v\hat{\Phi}) + \frac{\partial}{\partial t}(\rho\hat{\Phi}) && \hat{\Phi} \neq f(t)
\end{aligned}$$

$$\boxed{\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2 + \rho\hat{U} + \rho\hat{\Phi}) = -(\nabla \cdot (\tfrac{1}{2}\rho v^2 + \rho\hat{U} + \rho\hat{\Phi}) - (\nabla \cdot q) - (\nabla \cdot pv) - (\nabla \cdot [\tau \cdot v])}$$

Eqn. of Energy: Kinetic + Internal + Potential