Ch. 14. Interphase Transport

Heat Transfer Coefficients, $q = h(\Delta T)$

- h can be measured or predicted

Prediction using solutions to Energy Transport Problems.

<u>Boundary Layer Flow</u>: Flat Plate @ T_0

$$\frac{T_0 - T}{T_0 - T_\infty} = 2\left(\frac{y}{\Delta\delta}\right) - 2\left(\frac{y}{\Delta\delta}\right)^3 + \left(\frac{y}{\Delta\delta}\right)^4$$

where $\Delta \equiv P_r^{-1/3}$

$$\delta = \sqrt{\frac{1260}{37}\left(\frac{\nu x}{v_\infty}\right)}$$

$$q_0 = h_{loc}\,(T_0 - T_\infty)$$

but $$q_0 = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = 2k(T_0 - T_\infty)\frac{1}{\Delta\delta}$$

$$= 2P_r^{1/3}k(T_0 - T_\infty)\sqrt{\frac{37}{1260}\underbrace{\left(\frac{v_\infty}{\nu x}\right)}_{\frac{Re}{x^2}}}$$

$$h_{loc} = \frac{q_0}{T_0 - T_\infty} = \frac{2kP_r^{1/3}\cancel{(T_0 - T_\infty)}\,Re^{1/2}}{\cancel{(T_0 - T_\infty)}\quad x}\sqrt{\frac{37}{1260}}$$

$$Nu_{loc} = \frac{h_{loc}\, x}{k} = \frac{2\cancel{k} Pr^{1/3} Re^{1/2} \sqrt{\frac{37}{1260}}\, \cancel{x}}{\cancel{k}\ \cancel{x}}$$

$$\boxed{= 2\sqrt{\frac{37}{1260}}\, Pr^{1/3} Re^{1/2}}$$

$$Pr = \frac{\alpha}{\nu}$$

$$Re = \frac{v_\infty x}{\nu}$$

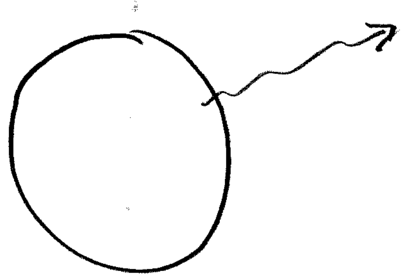
Ch 16. Energy Transport by Radiation

Radiation is a fundamentally different type of transport phenomena

- occurs in vacuum also (in absense of matl.)
- rate of transport is not proportional to a gradient in some property of the system.

  e.g. $q = -k \nabla T$ for conduction

Mechanism of Transport.



sphere at Temperature T.

$$\boxed{\lambda = \frac{c}{\nu}}$$

c: 2.998×10^{8} m/s

ν: frequency of electromagnetic (EM) wave

Photons: "particle" of EM wave

$\mathcal{E} = h\nu$

h → 6.626×10^{-34} J·s "Planck's" constant.

Atoms/Molecules are in an "excited" state due to the elevated temperature. Upon return to normal state, electromagnetic radiation is emitted.

Emission: emission of EM radiation when atoms/molecules transition from "excited" to "normal" states.

Absorption: atoms/molecules: "normal" to "excited".

Absorptivity $a_\nu \equiv \dfrac{q_\nu^{(a)}\,d\nu}{q_\nu^{(i)}\,d\nu}$ ← absorbed radiation $= f(\nu)$; ← incident "

thus $a_\nu = f(\nu)$ in general.

for $a_\nu =$ constant, $0 < a_\nu < 1$ "greybody"

$a_\nu =$ " , $a_\nu = 1$ "blackbody"

Emissivity $\varepsilon_\nu = \dfrac{q_\nu^{(e)}\,d\nu}{q_{\nu b}^{(e)}\,d\nu}$ ← emitted radiation $= f(\nu)$; → "blackbody" emission

$$\boxed{e_\nu = a_\nu}$$

Stefan-Boltzmann Law: Total Energy Flux.

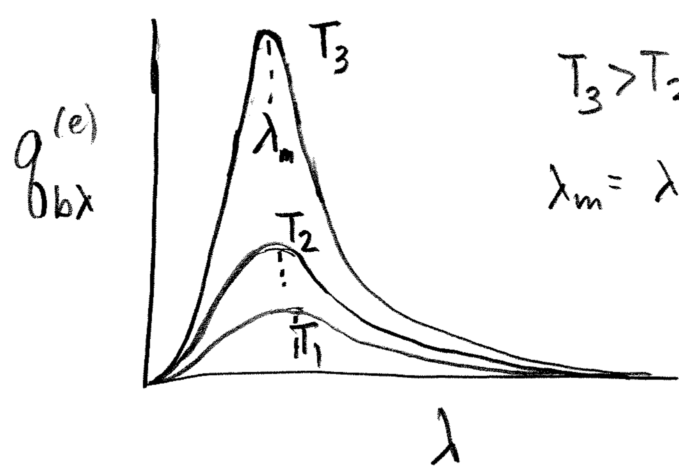
$$q_b^{(e)} = \sigma T^4 \qquad \text{"black body"}$$

σ = S-B constant = 1.355×10^{-12} cal/s·cm²·K

$$q^{(e)} = \varepsilon \sigma T^4 \qquad \text{"grey body"}$$

Planck Distribution Law:

$$q_{b\lambda}^{(e)} = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{ch/\lambda k T} - 1}$$



$T_3 > T_2 > T_1$

λ_m = λ where $q_{b\lambda}^{(e)}$ in maximum.

$$q_b^{(e)} = 2\pi c^2 h \int_0^\infty \frac{\lambda^{-5}}{e^{ch/\lambda kT}-1}\, d\lambda$$

let $x = ch/\lambda kT$

$$dx = \frac{ch}{kT}\, d\lambda^{-1} = \frac{ch}{kT}(-\lambda^{-2})\, d\lambda$$

$$\boxed{d\lambda = -\frac{kT}{ch}\lambda^2\, dx}$$

also, $x = \frac{ch}{\lambda kT} \rightarrow x^5 = \left(\frac{ch}{kT}\right)^5 \lambda^{-5} \rightarrow \boxed{\lambda^{-5} = \left(\frac{kT}{ch}\right)^5 x^5}$

limits on $\int$

when $\lambda \to 0$, $x \to \infty$

$\lambda \to \infty$, $x \to 0$

$$q_b^{(e)} = 2\pi c^2 h \int_\infty^0 \frac{\left(\frac{kT}{ch}\right)^5 x^5}{e^x - 1}\left(-\frac{kT}{ch}\lambda^2\right) dx$$

$$= 2\pi c^2 h \int_0^\infty \frac{\left(\frac{kT}{ch}\right)^5 x^5}{e^x-1}\left(\frac{kT}{ch}\right)\left(\frac{ch}{xkT}\right)^2 dx$$

$$= 2\pi c^2 h \int_0^\infty \left(\frac{kT}{ch}\right)^4 \frac{x^3}{e^x-1}\, dx$$

$$= \frac{2\pi k^4 T^4}{c^2 h^3}\int_0^\infty \frac{x^3}{e^x-1}\, dx = \frac{2\pi k^4 T^4}{c^2 h^3}\left(\frac{\pi^4}{15}\right)$$

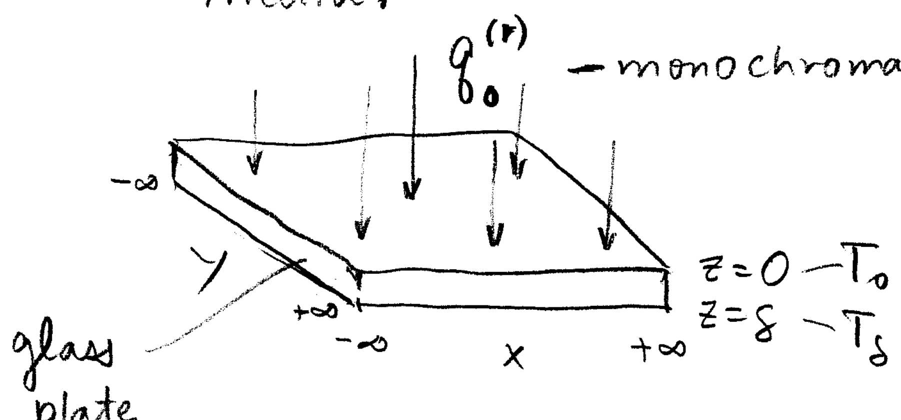
$$q_b^{(e)} = \int_0^\infty q_{b\lambda}^{(e)}\, d\lambda = \int_0^\infty \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{ch/\lambda k T} - 1}\, d\lambda$$

$$= \frac{2\pi k^4 T^4}{c^2 h^3}\left(\frac{\pi^4}{15}\right)$$

$$= \underbrace{\frac{2}{15}\,\frac{\pi^5 k^4}{c^2 h^3}}_{\textstyle\hookrightarrow\; 1.355 \times 10^{-12}\,\frac{cal}{s \cdot cm^2 \cdot K^4} = \sigma} T^4$$

see derivation next pg.

16B.4 Radiation + Conduction Through Absorbing Media:



assume: no reflection of q_0

no emission within glass
only absorption – if T low enough

· conduction in z-direction only.

Radiation Eqn

extinction coefficient

$$0 = -\frac{d}{dz} q_z^{(r)} - m_a q_z^{(r)} \qquad z=0 \quad q_z^{(r)} = q_0^{(r)}.$$

$$\frac{dq_z^{(r)}}{q_z^{(r)}} = -m_a dz \rightarrow \ln q_z^{(r)} = -m_a z + c$$

$$\ln q_0^{(r)} = c \longrightarrow \boxed{q_z^{(r)} = q_0^{(r)} \exp(-m_a z)}$$

Energy Eqn:

$$0 = k\frac{\partial^2 T}{\partial z^2} + m_a q_0^{(r)} \exp(-m_a z)$$

$$T(z) = -\frac{q_0^{(r)}}{m_a k} e^{-m_a z} + C_1 z + C_2.$$

$$T_0 = -\frac{q_0^{(r)}}{m_a k} e^0 + C_1(0) + C_2 \Rightarrow C_2 = T_0 + \frac{q_0^{(r)}}{m_a k}$$

$$T_\delta = -\frac{q_0^{(r)}}{m_a k} e^{-m_a \delta} + C_1 \delta + T_0 + \frac{q_0^{(r)}}{m_a k}$$

$$C_1 = \frac{T_\delta - T_0}{\delta} - \frac{q_0^{(r)}}{\delta m_a k}\left(1 - e^{-m_a \delta}\right)$$

$$T(z) = -\frac{q_0^{(r)}}{m_a k} e^{-m_a z} + \left(\left(T_\delta - T_0\right) - \frac{q_0^{(r)}}{m_a k}\left(1 - e^{-m_a \delta}\right)\right)\frac{z}{\delta}$$

$$+ q_0^{(r)}/m_a k + T_0$$

$$\boxed{T(z) - T_0 = \frac{q_0^{(r)}}{m_a k}\left(1 - e^{-m_a z}\right) + \left(\left(T_\delta - T_0\right) - \frac{q_0^{(r)}}{m_a k}\left(1 - e^{-m_a \delta}\right)\right)\frac{z}{\delta}}$$