Generalizations of Newton's Law of Viscosity, 1.2

in simple, steady-state shearing flow

$$\tau_{ij} = -\mu \frac{\partial v_j}{\partial x_i}$$

where j = one coord. direction.
i = another " ".

as shown in Section 1.1!

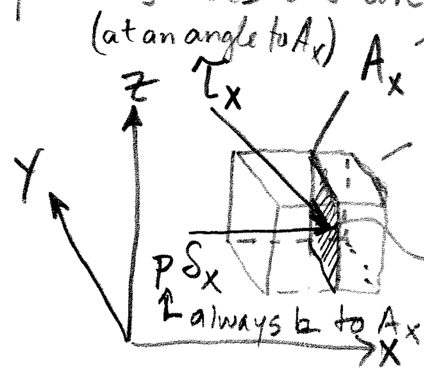
Most real flows are much more complicated where each component of velocity varies with time + space.

$$v_x(x,y,z,t) \qquad v_y(x,y,z,t) \qquad v_z(x,y,z,t)$$

Nine stress components will result from this general flow configuration.

For example - forces on an area perpendicular to x.

"remove $\frac{1}{2}$ of fluid in the cube! τ_x and $p\delta_x$ are the forces/area (A_x) to replace this fluid."



small volume element in flow field.

coordinate (x, y, z).

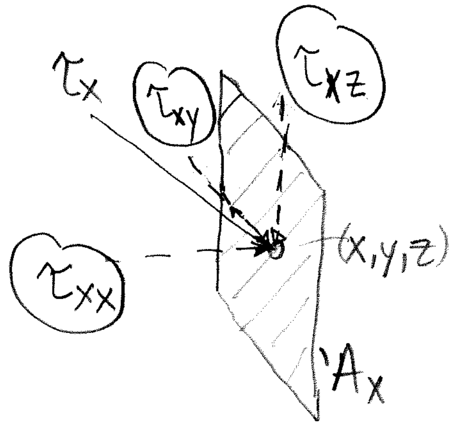
$p\delta_x$ is pressure stress acting normal to A_x

δ_x is a unit vector normal to A_x.

p is thermodynamic pressure of the fluid at (x,y,z).

τ_x is viscous force/unit area (stress) due to velocity gradients acting on A_x.

τ_x is a vector with components,



all which exert a force per unit area on A_x in the fluid due to velocity gradients in the fluid.

Thus, there are 9 viscous stress components, 3 each exerting a force on each area, A_x, A_y, and A_z.

Molecular Stress is the sum of pressure + viscous stress.

$$\pi_{ij} = p\,\delta_{ij} + \tau_{ij}$$

where i and j may be x, y, or z.

where

$$\delta_{ij} \equiv \text{Kronecker delta} \quad = 0 \text{ for } i \neq j$$
$$= 1 \text{ for } i = j$$

normal stresses, $\pi_{ii} = p + \tau_{ii}$

shear stresses, $\pi_{ij} = \tau_{ij} \qquad i \neq j$

Table 1.2-1 for a summary of Molecular Stress Tensor

How are τ_{ij} related to velocity gradients?

$$\tau_{ij} = -\mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + \left(\frac{2}{3}\mu - K\right)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\delta_{ij}$$

where

see Appendix B

K = dilatational viscosity
= 0 for ideal monoatomic gases.
assumed 0 for real gases.

note 2 properties, μ and K

In Tensor Notation.

$$\tau = -\mu\left(\nabla v + (\nabla v)^{+}\right) + \left(\frac{2}{3}\mu - K\right)(\nabla\cdot v)\,\delta$$

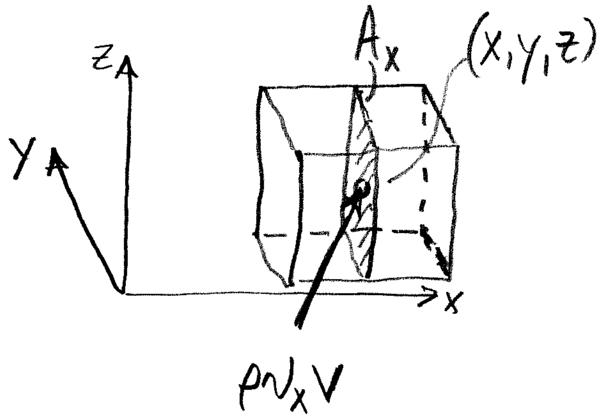
Appendix A

- δ = unit tensor
- ∇v = velocity gradient tensor
- $(\nabla v)^{+}$ = transpose of ∇v
- $\nabla\cdot v$ = "divergence" of velocity vector.

note: Ch 3 we show that incompressible fluids.
$(\nabla\cdot v) = 0$: so K term $\equiv 0$!

K is useful/important for
- sound absorption in polyatomic gases
- fluid flow containing gas bubbles.

Convective Momentum Transport, 1.7



v_x = x component of velocity vector (volumetric flow / unit area, A_x).

$\mathbf{v}$ = velocity vector

$\rho\mathbf{v}$ = momentum / unit volume – a vector quantity.

ρ = fluid density.

$\rho v_x \mathbf{v}$ = flux of momentum from regions of lesser x to regions of greater x by <u>bulk flow</u> (convection) through A_x.

3 components; $\rho v_x v_x$, $\rho v_x v_y$, $\rho v_x v_z$

For each of the other unit areas, A_y & A_z, there are 3 momentum flux components

Table 1.7-1 is a summary of the Convective Momentum Flux Components.

Combined Momentum Flux Tensor, ϕ

$$\phi = \pi + \rho v v = p\delta + \tau + \rho v v$$

molecular momentum flux tensor

Convective Momentum Flux Tensor

$$\phi_{xx} = \pi_{xx} + \rho v_x v_x = p + \tau_{xx} + \rho v_x v_x$$

$$\phi_{xy} = \pi_{xy} + \rho v_x v_y = \tau_{xy} + \rho v_x v_y$$

"combined flux of y-momentum through a surface perpendicular to x-direction, A_x."